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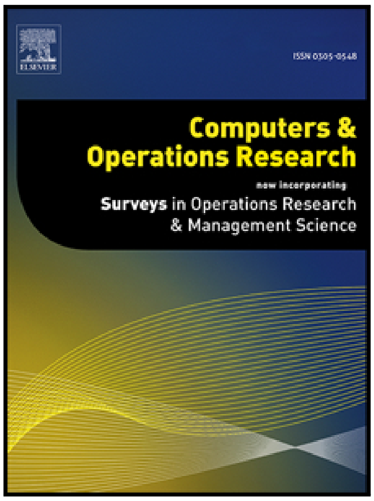

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Diego Gabriel Rossit, Daniele Vigo, Fernando Tohmé,
Mariano Frutos

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Highlights

- Relevance assessment of visual attractiveness for real-world routing plans.
- Literature review on visual attractiveness in routing and districting problems.
- Correlation analysis between the visual attractiveness KPIs.
- Suitability of diverse visual attractiveness KPIs in different contexts.

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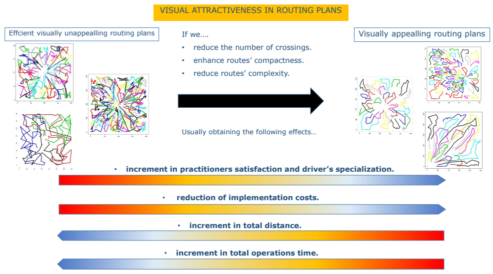
Graphical Abstract



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Visual Attractiveness in Routing Problems: a Review

Diego Gabriel Rossit[a,*], Daniele Vigo[b], Fernando Tohmé[c], Mariano Frutos[a]

[a]*IIESS, Department of Engineering, Universidad Nacional del Sur(UNS)-CONICET, Alem Avenue 1253, B8000CPB Bahía Blanca, Argentina*
[b]*Department of Electrical, Electronics, and Information Engineering, Università di Bologna, Viale del Risorgimento 2, 40136 Bologna, Italy*
[c]*INMABB, Department of Economics, Universidad Nacional del Sur(UNS)-CONICET, Alem Avenue 1253, B8000CPB Bahía Blanca, Argentina*

Abstract

Enhancing visual attractiveness in a routing plan has proven to be an effective way to facilitate practical implementation and positive collaboration among planning and operational levels in transportation. Several authors, driven by the requests of practitioners, have considered, either explicitly or implicitly, such aspect in the optimization process for different routing applications. However, due to its subjective nature, there is not a unique way of evaluating the visual attractiveness of a routing solution. The aim of this paper is to provide an overview of the literature on visual attractiveness. In particular, we analyze and experimentally compare the different metrics that were used to model the visual attractiveness of a routing plan and provide guidelines that planners and researchers can use to select the method that better suits their needs.

Keywords: visual attractiveness; vehicle routing problem
2010 MSC: 00-01, 99-00

1. Introduction

The Vehicle Routing Problem (VRP) is an important combinatorial optimization problem concerned with the optimal design of routes to be used by a fleet of vehicles to serve a set of customers (see, Toth and Vigo [74]). When

*Corresponding author
Email address: diego.rossit@uns.edu.ar (Diego Gabriel Rossit)

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solving a VRP many different objectives and constraints can be considered, depending on the application of interest. This leads to a large number of different variants: for example, the Capacitated VRP (CVRP), the Distance-Constrained VRP, the VRP with Time Windows (VRPTW), the VRP with Backhauls, the VRP with Pickup and Delivery (PDP) and the Periodic VRP (PVRP) (for a more complete overview, see Irnich et al. [24]). The conventional objectives pursued in these variants include minimizing the transportation costs, the travel distances or times, the required number of vehicles or the penalties for weak constraints violations, such as time windows violations.

Another objective that has been considered in the literature is the so-called "visual attractiveness" of the routes. Although a precise definition of visual attractiveness is not easy to state (see Constantino et al. [10]), many authors identify such subjective concept with a set of features that routes should exhibit:

- **compact** (see Bräysy and Hasle [7], Hollis and Green [23], Matis [49], Matis and Koháni [51], Poot et al. [59], Rossit et al. [64], and Tang and Miller-Hooks [73]).
- **not overlapping** (see Hollis and Green [23], Kim et al. [33], and Rossit et al. [64]) or **not crossing** each other (see Bräysy and Hasle [7], Lu and Dessouky [41], Matis [49], Poot et al. [59], Rossit et al. [64], and Tang and Miller-Hooks [73]),
- **not complex** (see Constantino et al. [10] and Gretton and Kilby [20]).

As will be extensively discussed later, visual attractiveness in routing assumes a high relevance in practical applications, where route compacteness and separation greatly enhances the acceptance by practitioners and eases the implementation of routing plans. Furthermore, in some cases compactness is even a design requirement, as in area-based distribution systems for parcel delivery (see, e.g., Schneider et al. [68]). The differences between the solutions that can be achieved by either pursuing the traditional objective of length (or cost) minimization or the maximization of visual attractiveness are quite evident. As an example, in Figures 1 and 2 we present the solutions obtained by optimizing a CVRP and a VRPTW instance, respectively, under these two different objectives. By examining the figures, it is evident that optimizing visual attractiveness provides, on the one hand, more compact and less overlapping routes which, on the other hand, are generally longer than those obtained by optimizing cost.

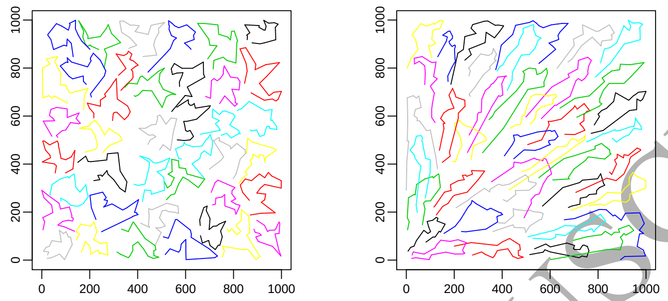

Figure 1: Different solutions to the CVRP instance X-n801-k40 from Uchoa et al.'s benchmark [77]. The left one maximizes the visual attractiveness and the right one minimizes the routing cost. Source: Rossit et al. [64].

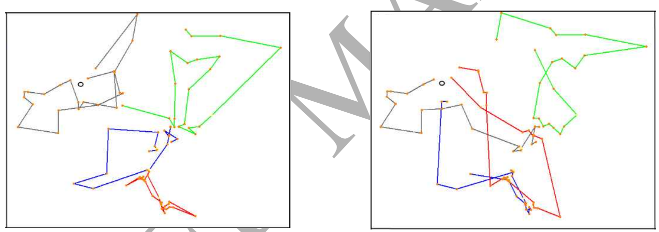

Figure 2: Different solutions of a real-life VRPTW instance, representing the daily distribution of Schweppes Australia Pty. Ltd. in a region of the city of Melbourne. The left one maximizes the visual attractiveness and the right one minimizes the routing cost. Source: Hollis and Green [23].

The aim of this paper is to systematize the research outputs in a field that has attracted a growing interest from both private companies and the academic community, generally motivated by the solution of real-life problems (see, e.g., Hollis and Green [23], Jang et al. [25], Kant et al. [28], and Kim et al. [33]). This paper is organized as follows: in Section 2 we justify the importance of visual attractiveness in the optimization of routing problems. In Section 3 we describe the research that has been already done in this field. In Section 4 we provide a description of the main measures that are available in the literature

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to model the visual attractiveness. In Section 5 we present a computational test to compare the visual attractiveness measures numerically. Finally, in Section 6 we present some conclusions.

2. Origin and Benefits of Visual Attractiveness in Practical Routing Applications

Despite the relative vagueness inherent in the definition of visual attractiveness, this concept has frequently been considered as central in the design of routing plans. To the best of our knowledge, the first use of this term was in Poot et al. [59] to express the requirements of their customers. They realized that some customers considered the results yield by the ORTEC[1] vehicle routing software were "poor". They highlighted that this was not only related to the traditional measures, such as cost, total number of vehicles used, or total distance traveled, but also to a set of non-standard indicators used by their customers to decide whether a plan is acceptable or not. Moreover, they pointed out that these non-standard measures were not adequately studied in the scientific literature. They stated that visually attractive plans seem to be more logical and closer to the traditional way of working, thus generating trust in the plan among both drivers and planners, "... which leads to fast acceptance of the system" (see Poot et al. [59]).

From then on, the literature has often highlighted the importance of taking into account visual attractiveness, and also expanded its foundations, based mainly on practical applications. According to Matis [49] if a set of routes overlaps, the drivers that should cover them will complain, thinking that the planning was quite inefficient. Moreover, "Practitioners tend to dislike routes that have been optimized for length and spread over quite different areas while crossing one another." (Mourgaya and Vanderbeck [53]). "Nice" solutions often require a much smaller effort for their practical implementation, reducing the

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time required to instruct the drivers about the routes and may have a more stable duration because they refer to more homogeneous areas in terms of traffic conditions (see Battarra et al. [1] and Schneider et al. [68]). This way, routes are subject to continuous refinement by exploiting the familiarity of drivers with the area and the clients served by the route (see Kant et al. [28] and Poot et al. [59]). Furthermore, if a customer cannot be served at the preferred time, as long as the vehicle stays in the same geographical area it is easier to return to serve the customer at a later time. Similarly, if a traffic jam or a road disruption occurs, it is easier to find alternative routes if the customers are distributed in a compact area (see Hollis and Green [23]).

More recently Battarra et al. [1] described some applications where route compactness is of major importance, such as the transportation of elderly people to recreation centers, where the users prefer to be picked up together with neighbors, or the case of "gated communities", which are residential or productive areas surrounded by walls for safety and protection reasons. Whenever more than one customer from a gated community requires service, these customers should be visited in sequence by the same vehicle. In fact, stops at the gates are time-consuming because the vehicle usually has to pass a checkpoint. Another practical example related to household newspaper delivery is mentioned in Bräysy and Hasle [7] and Hasle et al. [22]. In this case, it is not desirable to serve the same area with several carriers since neighboring subscribers may receive their newspaper at very different times.

Bosch [5] stated that practitioners tend to reject algorithm-made routes that, when vehicles have to drive to a distant region, also serve customers close to the path towards or back from that region. This is seen as an inefficient use of the vehicle capacity because planners consider that driving to a far region is the most expensive part of a route. Therefore, serving all the customers in the distant region by a single truck and send an additional truck to the region closer to the depot is seen as more efficient and preferable even with small cost inefficiency. Furthermore, planners often express two major requirements in interurban routing: each city should be visited with as few trucks as possible

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and all orders in a route should be close to each other.

From our point of view, the importance of visual attractiveness comes mainly from the fact that it has proven to be crucial in many practical applications. Investing large efforts and time in designing a (near)minimum-cost routing plan that turns out to be remarkably unattractive and, therefore, will be probably rejected or modified "on the fly" according to practitioners goals or preferences and beyond designers control, can result somewhat useless. Thus, enhancing customers satisfaction through the generation of visually attractive routing plans can effectively bound the implementation cost of the plan. Nevertheless, limiting this expense is not the only reason to include visual attractiveness in the optimization process. As it has been described in the specialized literature cited throughout this section, there are other benefits and special situations where "nice" routes are required. Moreover, in Section 2.2 we will present some discussion about the relation between cost and attractiveness metrics, which is not always unique.

2.1. Human perception in Traveling Salesman Problems

The reason why managers and practitioners tend to prefer (and consider "more efficient") compact and separated routes seems to be based on innate characteristics of humans. There are numerous papers that study the aspects that humans consider when seeking an optimal solution to the Traveling Salesman Problem (TSP), i.e., the special case of VRP with one uncapacitated vehicle. The importance of identifying these aspects relies on the clues obtained by discerning why people consider some solutions "nicer" (i.e., closer to the optimal) than others. For a thorough review of such works see MacGregor and Chu [47].

MacGregor and Ormerod [48] on the basis of some experimental work suggested that humans relate optimal solutions for the TSP to paths that follow the convex hull of the set of points, and called such property the "convex-hull hypothesis". However, the validity of these experiments have been criticized by other authors as Lee and Vickers [36], Van Rooij et al. [78], and Vickers et al. [79]. Furthermore, Van Rooij et al. [78] put forward the assumption that humans find non-crossing solutions to TSP as more optimal than those that have crossings between different paths (see also Vickers et al. [80]). This is supported by the fact that an optimal tour in the symmetric TSP, i.e., where the distance matrix satisfies the triangle inequality, does not intersect itself (Flood [14]). MacGregor et al. [46] compared both above-mentioned properties (i.e., the "convex-hull hypothesis" and the "crossing avoidance hypothesis") reaching the conclusion that they are not mutually exclusive. When asked to build optimal TSP tours, individuals tend to avoid crossings to reach interior points even when following the convex-hull boundary of a set of points as their main strategy.

The presence of clusters of nodes is also related to the quality of human-constructed solutions in TSP. Dry et al. [13] studied the relationship between human performance in solving the TSP and the spatial distribution of the nodes, concluding that humans find easier to solve (and usually obtain higher quality solutions) in instances where nodes are strongly clustered forming compact groups, as opposed to those in which nodes are uniformly distributed. Related to this, MacGregor [45] suggested that the human performance in solving the TSP is also influenced by the location of the customers, recognizing that instances in which they are located near the convex hull of the set of points are easier to solve for humans. Similarly, Vickers et al. [81] performed an experiment where people were asked to rank the aesthetic appeal of a series of drawings depicting TSP solutions without specifying their actual nature. In general, a positive correlation between aesthetic perception and compactness of a solution emerged from the experiment.

As a summary of these works, we can conclude that humans consider, when solving a TSP, characteristics very similar to those proper of the visual attractiveness concept introduced in Section 1, such as crossings and compactness. Therefore, solutions that include these characteristics are more attractive to humans and generally perceived as better than those without them.

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2.2. Visual attractiveness and cost in routing problems: always a negative correlation?

Vickers et al. [79] asked two different groups to construct TSP solutions under two different instructions: one group was asked to find the shortest solution while the other group was told to build the "most natural, attractive, or aesthetically pleasing" solution. The difference in length between the solutions obtained by the two groups was strikingly small. This is in line with the conclusions of the experiments of Ormerod and Chronicle [56] which indicate that humans tend to consider high-quality TSP solutions of simple instances (i.e., those having the majority of points to visit located close to the convex hull) as more attractive than suboptimal solutions. Similarly, Lu and Dessouky [41] observed that, when comparing two crossing-free solutions of the same instance of the TSP, the one closer to the convex hull is more visually attractive and has a higher probability of being shorter. However, Ormerod and Chronicle [56] pointed out that, when the instance is more complex (i.e., has a large number of points located far from the boundaries of the convex hull) this capacity of manually constructing good solutions deteriorates because following simple construction rules more frequently leads to suboptimal solutions. This means that the human innate ability to recognize the quality of a solution through visual inspection diminishes when the complexity of the problem increases.

In a broader view, complexity should not only be associated with the number of interior points but also with the inclusion of further restrictions, as happens in VRP with the introduction of time windows or vehicle capacity. Therefore, it is not necessarily true that visually attractive routes for the VRP are more efficient in terms of the traditional measures (see Bräysy and Hasle [7] and Poot et al. [59]). This negative correlation has been evinced in many different routing applications in the literature. For example, we find it in VRPTW (Hollis and Green [23] and Sahoo et al. [65]), CVRP (Dassisti et al. [11] and Rossit et al. [64]), Arc Routing Problem (ARP) (Constantino et al. [10] and Lum et al. [42]) and VRP with routing time limits (Tang and Miller-Hooks [73]). Jang et al. [25] allow modifications, suggested by the managers, to the solutions obtained by

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their algorithm in the context of a periodic TSP even if this implied a worsening of standard objectives. With these changes, the authors found it easier to implement the new plan because the managers were more willing to put in practice a set of routes they like. On the other hand, some specific exceptions are discussed in the literature. For example, in tests performed by Bosch [5], Lu and Dessouky [41], Poot et al. [59], and Zhou et al. [86] the addition of visual attractiveness constraints leads to both an enhancement of visual attractiveness and a cost reduction in the routing plan. For more details about the algorithms and benchmarks used by these authors see Section 3.1.

We can conclude that, although it is the most probable effect, we can not always assure that the inclusion of visual attractiveness diminishes the efficiency of the routing plan in terms of traditional objectives. However, because of the benefits of a "nice" routing plan pointed out at the beginning of this section, it is worth improving the visual attractiveness even when this comes at the expense of other standard objectives (Constantino et al. [10]).

3. Literature review

As previously mentioned visual attractiveness, as introduced by Poot et al. [59], is a relatively new concept in the routing literature. However, previous papers have already used similar concepts, mainly referring to route compactness. In addition, because many examples of the use of this concept can be found also in the districting optimization community, in Section 3.2 we incorporate a brief discussion of compactness within districting problems.

3.1. Attractiveness in Routing Problems

In Lu and Dessouky [41] and Zhou et al. [86] an insertion heuristic is presented to solve a multi-vehicle VRP with Pickup and Delivery with time windows (m-PDPTW) that considers a crossing-avoidance penalization to calculate the insertion costs (details on how to compute such penalization are given in Section 4). Both groups of authors stated that at the start of the construction

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of the routes, it is relatively easy to find feasible (and inexpensive) insertions without causing any crossings because each route contains few customers and many customers are available to extend the routes. However, when the construction progresses and routes reach their capacity saturation, the dispatching logic focus less on emphasizing visual attractiveness and more on optimizing the length and the use of the capacity. To adapt their algorithm to attractiveness preservation, the authors allow increasingly "uglier" insertions by reducing the crossing-avoidance penalty as the number of assigned customers increases. They tested their algorithm on instances derived from the Solomon's benchmark [69] and compared it with the performance of its Sequential Insertion Algorithm (SIA) and a parallel insertion heuristic obtaining generally better results in terms of visual attractiveness and standard objectives. Lu and Dessouky [41] also compared the number of vehicles and the travel time of their solutions with those of Li and Lim [38], achieving worse results in both objectives.

Tang and Miller-Hooks [73] presented an iterative heuristic that consists in a clustering-based algorithm in the context of a VRP with maximum travel time constraints. The heuristic works as follows: in the first stage, some seed customers are chosen and customers are assigned to the closest seed through a Semi-Assignment Problem (SAP). The schedule of visits and the routing time for each cluster is determined with a TSP heuristic algorithm. If some route travel time exceeds the maximum allowed duration, the SAP is solved using a different cost matrix in which distances from customers to overloaded cluster seeds are increased, while distances to feasible cluster seeds are reduced. If, after a certain number of iterations, SAP does not generate a feasible solution, the reassignment is done according to a different strategy. This is a Multi-objective Assignment Problem (MAP) that aims at minimizing both the number of customers closer to the center of another route and the sum of the travel times of all the routes, by explicitly limiting the travel time of each route. They tested their algorithm on real-life instances of the courier company FedEx and compared the results with the ones obtained with Tang and Hu's metaheuristic [72], experiencing the expected trade-off between visual attractiveness and standard objectives.

Kim et al. [33] developed a clustering-based algorithm to solve a VRPTW arising in waste collection with the aim of improving the visual attractiveness of the solutions. The heuristic consists of two stages: first, a capacitated clustering algorithm is used to estimate the number of clusters and then an extended insertion algorithm is used for routing the points within the clusters. The clustering starts with an estimation of the required number of clusters, according to the total workload, and a random selection of the seed-customers. Next, the remaining customers are assigned to the nearest seed-customer. After the first assignment, the algorithm calculates the cluster centroids and, subsequently, determines a centroid of the centroids (called "grand centroid"). Then, the customers are considered in decreasing order of their distance from the grand centroid and are assigned to the nearest feasible centroid. During the assignment, the feasibility of the clusters is maintained both in terms of capacity and maximum travel time, which is estimated through a simple TSP algorithm. The steps are repeated until there is no change in the clusters composition. In a second stage, the clusters are sorted in descending order of the number of included customers and for each cluster an insertion algorithm is applied to determine the routes. If at the end of the insertion step some customers remain unrouted, the number of clusters is increased and the process is repeated. Otherwise, routes are further improved by using a Simulated Annealing (SA) algorithm developed by Taillard et al. [71]. Sahoo et al. [65] presented a route-management application that improves the routing schedules of a waste collection company, taking into account route compactness. The algorithm they used has many similarities with that of Kim et al. [33] but used a slightly different clustering algorithm for generating an initial solution (namely, the K-means Variant Balanced Clustering Algorithm proposed in Kim et al. [32]) which does not use the concept of "grand centroid". They tested their algorithm on Solomon's VRPTW benchmark [69], achieving in general more visually appealing but less efficient solutions than the best known ones (BKS), while Kim et al. [33] used a set of real-world problems related to waste collection.

Another area in which companies show a clear interest in generating visually

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attractive routes is product distribution. Kant et al. [28] presented the results of a heuristic algorithm, implemented through the ORTEC vehicle routing software, that has produced substantial savings to the Coca-Cola Company. The algorithm is based on an insertion algorithm and subsequent local search to ensure that the solution is accepted by practitioners. Local search does not consider movements that would cause route overlaps and the algorithm incorporates a penalization mechanism to discourage non-compact routes. This penalty, called Clustering Penalty (CP), is proportional to the distance of all the customers in a route to the customer located in the intermediate position of that route. The CP value is adjusted during the algorithm use. The first times the dispatchers use the software, they set a high CP value to generate routes that are reasonably close to the routes they design in practice. Once the dispatchers and drivers become accustomed to the new routes, they can reduce the CP and calculate less expensive routes. This process, that was also suggested in Poot et al. [59], despite not producing an immediate cost reduction, allows a smoother and less problematic transition towards new and more efficient routes. Poot et al. [59] also worked with ORTEC clients. They adapted a savings algorithm (see Clarke and Wright [9]) to favor route compactness. In particular, they initially applied the algorithm to subsets of customers which are defined by taking into account the required type of vehicle, the time windows and the geographic location of the customers. To enhance compactness of the final solution, they also include a "region" factor in the calculation of the savings obtained by combining two routes. When applied to a set of problems of ORTEC clients, this algorithm performs better than an insertion algorithm, in particular with respect to the visual attractiveness of the routes. Additional studies on the modification of the ORTEC software to incorporate visual attractiveness issues are presented in Bosch [5]. Guided by the fact that experienced planners were able to improve the routing sequence by manually altering the solution, the author found that the inclusion of visual attractiveness constraints (based on Savelsbergh's circle covering method [67]) lead to a cost reduction for the distribution plan of the Zeeman chain store in the Netherlands.

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Another example can be found in Hollis and Green [23], where a complex heuristic algorithm that aims at finding visually attractive solutions is developed to support Schweppes Australia Pty. Ltd operations. The algorithm includes a set of operators organized in two different stages: the first one is a novel variation of the SIA of Solomon [69] and the second is a local search process based on the Guided Local Search Algorithm (GLSA) developed by Kilby et al. [30]. As other authors, Hollis and Green [23] recognized that constructing a routing plan uniquely based on insertion techniques can lead to elongated and non-compact routes. They found that when a route approaches its maximum allowed duration, the set of customers that can legally be inserted is restricted to the subset of those located in the zone that connects the depot with the bulk of the customers being served in the route. To avoid this, Hollis and Green [23] established an alternative insertion criterion: when a route approaches its maximum allowed duration only new customers which are progressively closer to an already visited customer can be inserted. Then, the objective function that guides the local search not only tries to reduce the standard routing costs, but also the overlapping of convex hulls associated with the routes. They tested their algorithm on both real-world instances from Schweppes Australia Pty. Ltd in the city of Melbourne and the Solomon's VRPTW benchmark [69], for which they obtained, as expected, larger total distances and number of routes than the BKS.

Gretton and Kilby [20] presented an application to enhance the visual attractiveness of the solutions obtained by the software application Indigo, designed by Kilby and Verden [31]. The algorithm is based on the Adaptive Large Neighborhood Search (ALNS) method, developed by Ropke and Pisinger [63], and repeatedly removes a large set of customers from a solution and reinserts them by using a simple heuristic algorithm. Gretton and Kilby [20] adopted an insertion algorithm that considers also visual attractiveness both in terms of distance of the customers to the route *median* (i.e., the customer which is closest to the geometric centroid of the route) and the sum of turn angles along the route, called *bending energy* (see Section 4.3 for more details). They reported a

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general summary of the tests performed on both benchmarks from the literature (Gehring and Homberger [17] and Solomon [69]) and some real-world instances.

In the context of arc routing problems, Constantino et al. [10] considered the Bounded overlapping MCARP (BCARP), which is a variant of the traditional Mixed Capacitated Arc Routing Problem (MCARP). To produce more spatially separated routes in the BCARP an upper bound on the number of nodes that are shared by more than one route is imposed. The authors solved small-size instances of BCARP through the integer programming solver CPLEX, and larger instances with a two-phases heuristic algorithm. First, they solved a SAP to create clusters of arcs starting from a set of seed arcs and then they determined the schedule inside the clusters by solving a simplified MCARP with no capacity constraints. The proposed algorithms were tested on Belenguer et al.'s MCARP benchmark [2]. Also in this case, the inclusion of visually attractiveness considerations led to worse solutions in terms of standard objectives.

Recently, Rossit et al. [64] presented a heuristic algorithm to optimize both visual attractiveness and standard cost in CVRP. An initial solution is found with the clustering-based algorithm used by Kim et al. [33] and improved with local search. The algorithm was tested on the CVRP instances proposed by Uchoa et al. [77] producing "nicer" solutions than the BKS but with larger total length.

3.2. Attractiveness in Districting Problems

Even though "visual attractiveness" as such is not mentioned in districting problems, many papers take into account some visual attributes during the optimization process. Muyldermans et al. [54] defined the districting as the partition of a large geographical region (or network) into smaller subareas (subnetworks) for organizational or administrative purposes, stating that a good partition should have the demand points within each district near to each other and near to the service center. In districting problems that are used to generate clusters of customers that later will be scheduled by using a VRP algorithm, there is a positive correlation between the compactness of the districts and the

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efficiency of subsequent routing plans (De Assis et al. [12], Kalcsics [27], and Mourgaya and Vanderbeck [53]). There are examples in context of winter gritting and road maintenance (Muyldermans et al. [54]), pickups and deliveries for a parcel company (González-Ramírez et al. [18]), waste collection (Mourão et al. [52]), and even stochastic problems that can arise in couriers companies (Lei et al. [37]). For example, Wong and Beasley [82] used visual inspection to evaluate the results of their heuristic in a districting problem. The authors recognized that "well clustered" districts as a characteristic feature of a "good solution". Although they do not explicitly define which characteristics should a district has to be considered "well clustered", the authors present some visual examples from which it can be inferred that they aim to obtain compact and minimum overlapping districts. We also find applications of districting not related to routing that also take into account compactness in other fields, such as politics (Bozkaya et al. [6], Niemi et al. [55], and Young [84]), commercial districting (López-Pérez and Ríos-Mercado [40], Ríos-Mercado and Escalante [61], and Ríos-Mercado and López-Pérez [62]), and energy management (Bergey et al. [3], De Assis et al. [12], and Yanık et al. [83]).

3.3. Solution approaches

Exact approaches are rare in routing or districting problems that incorporate some visual attractiveness aspect. One notable exception is Battarra et al. [1] who developed both a branch-and-cut and a branch-and-cut-and-price algorithm for the clustered VRP that were able to solve large-sized instances within reasonable computing time. Jarrah and Bard [26] have used a combination of column generation with heuristics methods. Mourgaya and Vanderbeck [53] used a column generation approach together with a rounding heuristic. The majority of the papers use heuristic approaches, and this is not surprising if we consider the challenging combination of the NP-hard nature of the routing problems and the fuzzy and multi-criteria definition of visual attractiveness. A list of the algorithms used in the main references from the literature is reported in Table 2 described in the next section.

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Another important aspect of visual attractiveness is the need of interaction with the planner. As visual attractiveness is a subjective concept, the opinion from experts is important for the success of the implementation. This is why some authors have considered the inclusion of the expert opinion during the optimization process (Matis [49] and Tang and Miller-Hooks [73]). For the same reasons, the visualization of the information and the solutions is also valuable. Therefore, some authors have incorporated Geographic Information System (GIS) technologies in their overall approaches (Matis [49] and Sahoo et al. [65]).

4. Measuring Visual Attractiveness

Although providing an unambiguous and rigorous definition of the visual attractiveness is difficult, building a single measurable parameter that can take into account all the subjective aspects of this concept is even harder.The majority of authors have used a set of indicators to deal with visual attractiveness (Constantino et al. [10], Gretton and Kilby [20], Hollis and Green [23], Matis [49], Poot et al. [59], Rossit et al. [64], and Tang and Miller-Hooks [73]). On the contrary, Lu and Dessouky [41] and Zhou et al. [86] used only the "crossing length percentage" indicator to measure visual attractiveness.

In this section, we describe the different ways of assessing visual attractiveness that were proposed in the VRP literature, which are also summarized in Table 2. We also present several of the formulas to compute visual attractiveness on routing plans that were proposed in the bibliography and will be used in Section 5 for a numerical analysis. With this purpose, we introduce some basic notation and definitions that will be used along this Section. Given a route I in a set of routes $\mathcal{K}$, let T_I be the set of customers assigned to route I and $T_{\mathcal{K}}$ be the set of customers of all routes in $\mathcal{K}$. Furthermore, let $dist(x, y)$ be the (Euclidean) distance between two points x and y. In several papers the authors used the location of the route for the computations of visual attractiveness measures. Although generally the location of the route is defined by

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its center, there is not a unique definition of the central position of route in the literature. On the one hand, some authors considered as the center of a route the geometric center (Hollis and Green [23], Matis [49], and Poot et al. [59]). On the other hand, other authors identified the center with one of the customers that are assigned to the route, as Kant et al. [28] who selected the customer located in the intermediate position of the route, as Kim et al. [32] or Rossit et al. [64] who chose the customer that has the minimum distance to the center of gravity, or as Gretton and Kilby [20] or Tang and Miller-Hooks [73] who selected the customer that minimizes the total distance from all the other customers assigned to the same route.

Some authors provided some insight about the situations in which a specific definition of the center of a route is more convenient than others. For example, Kim et al. [33] suggested that the center of the route should be the customer that has the minimum distance to the center of gravity when the distance measure is street distance. Instead, it should be the geometric center when distances are Euclidean or Manhattan.

Calculating the geometric center of gravity when the distance formula is difficult to compute, as it is in street distances, can be troublesome and, therefore, should be approximated by the nearest node in the graph. Furthermore, during the optimization process whenever the center of gravity changes, an additional effort is required because all the (Euclidean) distances from the customers to the new center have to be computed. This, however, does not apply to the case in which a customer is used as a center because the distances between the customers are an input data of the problem. Tang and Miller-Hooks [73] came to a similar conclusion choosing as the center of the route a point that coincides with a customer location, rather than the geometric center, since this location can be determined using the existing network of travel times. Finally, Gretton and Kilby [20] showed that for their algorithm the constant recalculation of the tour medians achieves a significant improvement in the visual attractiveness of the solutions and has only a limited impact on memory usage and computing time. In Table 1 we summarize the different definitions of the route center, providing

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for each of them an identifier and discussing the worst-case time complexity of its computation from scratch, where n is the total number of customers.

Table 1: Summary of the different definitions for the center of a route available in the literature.

Id.	Description	Time complexity	References
c_I^1	Geometric center calculated from the coordinates of the customers assigned to the route I.	$O(n)$	[23], [33], [49], and [59]
c_I^2	Customer located in the intermediate position of the route I.	$O(1)$	[28]
c_I^3	Customer with minimum distance to the center of gravity of route I.	$O(n)$	[32] and [64]
c_I^4	Customer that has the minimum total distance from all the other customer locations assigned to the same route I.	$O(n^2)$	[20] and [73]

4.1. Compactness

As mentioned in Section 3, compactness is one of the most widespread measures to represent the visual attractiveness of a solution. Despite from being an intuitive concept, compactness cannot be unequivocally defined (Kalcsics [27] and MacEachren [43]) and generally includes proximity measures between customers in the same route. Recently, Constantino et al. [10] classified the literature by distinguishing three types of compactness measures: i) similarity of the shape to standard geometric shapes (Jarrah and Bard [26]); ii) geographical/geometrical or visual compactness (Lei et al. [37] and Perrier et al. [58]); or iii) proximity between customers (Muyldermans et al. [54], Poot et al. [59], Salazar-Aguilar et al. [66], and Tang and Miller-Hooks [73]). In the same paper, a different classification was proposed by defining compactness measures based on: i) maximum travel times (González-Ramírez et al. [18] and Mourão et al. [52]) or Euclidean distances (De Assis et al. [12]); ii) the sum of Euclidean distances (Hollis and Green [23], Kant et al. [28], Kim et al. [33], Mourgaya and Vanderbeck [53], and Salazar-Aguilar et al. [66]); iii) the average and standard deviations of distances (or travel times) between customers and a reference point (Mourão et al. [52], Poot et al. [59], and Tang and Miller-Hooks [73]); or iv) and

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the perimeters of the zones (Lei et al. [37]) or perimeters and areas of the zones (Lin and Kao [39]).

Mourão et al. [52] considered two compactness measures: the average and the standard deviation of the distances between the points of the cluster and the seed of the cluster. Lei et al. [37] used the compactness measure developed in Bozkaya et al. [6], which is based on the quotient between the perimeter of the district and the total perimeter of the region. Similar measures were used in Lin and Kao [39] and Larsen [35]. Butsch et al. [8] considered two different compactness measures in the districting plan of an arc routing problem: local compactness on each single district, which is proportional to the sum of distances from each node to the median of the cluster, and global compactness on the entire districting plan, which is proportional to the overlap between the smallest axis-parallel rectangles enclosing the districts. In Matis [50] and Matis and Koháni [51] the authors evaluated compactness by considering the ratio between the area of the smallest non-convex polygon that includes all the nodes in a district and the area of the circle that has the same perimeter. However, in another paper Matis [49] measured the compactness as the ratio between the average distances of two intermediate customers in a route and the average length of the 20% longest segments in the route. Hollis and Green [23] and Kim et al. [33] measured route compactness by calculating the total sum of distances between each customer and the center of the route to which they are assigned. Tang and Miller-Hooks [73] used the average per customer of this distance whereas Rossit et al. [64] used the average per route of this value. Kant et al. [28] considered the sum of the distances between the customers scheduled in one route and the middle customer in that route (called the “center stop”). Poot et al. [59] adopted the average distance between any two customers in a route. Constantino et al. [10] used the average minimum traveling time between two demand units inside the service zones.

Closely related to the concept of compactness there is that of “route proximity” which is linked to the idea that customers should be assigned to the “nearest” route. In Hollis and Green [23], Matis [49], Rossit et al. [64], and

Tang and Miller-Hooks [73] the number of customers that are nearer to the center of another route than to one of the route to which they are assigned is used to evaluate visual attractiveness.

As to measuring compactness in routing problems, we present six metrics, identified as $COMP^a - COMP^f$, that consider the spatial and geographical compactness of the routes in a solution. In addition, we present three metrics, identified as $PROX^a - PROX^c$, that consider the route proximity linked with customers being assign to the nearest route. These measures are defined as follows:

- Compactness measure introduced by Matis [49]:

$$COMP_I^a = \frac{AvgDist_I}{AvgMaxDist_I} \tag{1}$$

where $AvgDist_I$ is the average distance of two consecutive customers in a route I and and $AvgMaxDist_I$ is the average distance of the 20% longest distances between two consecutive customers in the route I. The larger this value the more compact the solution is.

- Compactness measure introduced by Kim et al. [32]:

$$COMP^b = \sum_{I \in \mathcal{K}} \sum_{i \in T_I} dist(i, c_I^3) \tag{2}$$

where c_I^3 is the center of the route I defined by Kim et al. [32] (see Table 1). The smaller this value the more compact the solution is.

- Compactness measure based introduced by Kant et al. [28]:

$$COMP_I^c = \sum_{i \in T_I} dist(i, c_I^2) \tag{3}$$

where c_I^2 is the center of the route I defined by Kant et al. [28] (see Table 1). The smaller this value the more compact the solution is.

- Compactness measure introduced by Poot et al. [59]:

$$COMP_I^d = \frac{\sum_{i \in T_I} dist(i, c_I^1)}{|T_I|} \tag{4}$$

where c_I^1 is the center of the route I defined by Poot et al. [59] (see Table 1). The smaller this value the more compact the solution is.

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- Compactness measure introduced by Poot et al. [59]:

$$COMP_I^e = \frac{\sum_{i \in T_I} \sum_{\substack{h \in T_I \\ h \neq i}} dist(i,h)}{|T_I|} \tag{5}$$

  The smaller this value the more compact the solution is.

- Compactness measure introduced by Tang and Miller-Hooks [73]:

$$COMP^f = \frac{\sum_{I \in \mathcal{K}} \sum_{i \in T_I} dist(i, c_I^4)}{|T_{\mathcal{K}}|} \tag{6}$$

  where c_I^4 is the center of the route I defined by Tang and Miller-Hooks [73] (see Table 1). The smaller this value the more compact the solution is.

- Proximity measure introduced by Matis [49]:

$$PROX_I^a = 2 \cdot \left(1 - \frac{O_I'}{|T_I|}\right) - 1 \tag{7}$$

  where O_I' is the number of customers in route I that are nearer to the center of another route $J \in \mathcal{K}$, $J \neq I$. The center of the route considered is c_I^1 (see Table 1). The larger this value the better the solution is.

- Proximity measure introduced by Rossit et al. [64]:

$$PROX_I^b = \frac{O_I'}{|T_I|} \tag{8}$$

  where O_I' is the number of customers in route I that are nearer to the center of another route $J \in \mathcal{K}$, $J \neq I$. The center of the route used here is c_I^3 (see Table 1). The smaller this value the better the solution is.

- Proximity measure introduced by Tang and Miller-Hooks [73]:

$$PROX^c = \sum_{I \in \mathcal{K}} O_I' \tag{9}$$

  where O_I' is the number of customers in route I that are nearer to the center of another route $J \in \mathcal{K}$, $J \neq I$. The center of the route used here is c_I^4 (see Table 1). The smaller this value the better the solution is.

4.2. Routes overlap and crossings

The convex hull of a route is the smallest convex polygon that contains all its customers. The presence of customers that are included in more than one convex hull is often the most unattractive characteristic of a solution (Poot et al. [59]). In Figure 3 we can see the convex hulls of two different solutions to a VRPTW instance: one with a consistent overlap of convex hulls and the other one with reduced overlapping. In general, the depot is ignored in the construction of the convex hull. ome authors considered as attractiveness measure the number of customers that belong to more than one convex hull (Hollis and Green [23], Kim et al. [33], Poot et al. [59], and Rossit et al. [64]).

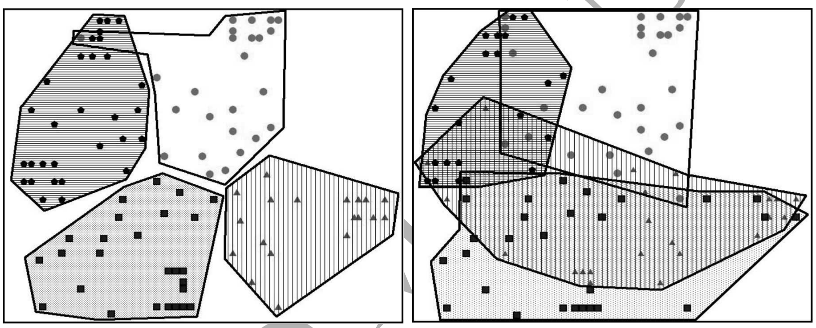

Figure 3: Convex hulls of routes for benchmark problem RC205 in Solomon [69] obtained with two different solution approaches. The left one considers visual attractiveness maximization and the right one cost minimization. Source: Sahoo et al. [65].

Kim et al. [33] and Rossit et al. [64] used Graham's algorithm [19] to determine the convex hull of the routes, whereas Hollis and Green [23] used the Boost C++ library (Gehrels et al. [15]). Both approaches have time complexity $O(n \log n)$. In the context of an arc routing problem Constantino et al. [10] proposed a "Route Overlapping Index" that compares the number of shared customers between the solutions. Also in the context of arc routing, Lum et al. [42] proposed a measure they called the "hull overlap" that is directly proportional to the intersection of the routes of each pair of convex hulls.

It is not hard to see that convex hull overlap is related to inter-route crossings, i.e., the intersection of arcs belonging to different routes, because these occur if and only if the convex hulls overlap (Hollis and Green [23]). Non-crossings

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among routes is a crucial aspect of visual attractiveness because routes without crossings are often seen as more efficient (Van Rooij et al. [78]). Poot et al. [59] considered both inter-route crossings and intra-route crossings, i.e., intersections between arcs of the same route. On the other hand, Matis [49] and Rossit et al. [64] considered only inter-route crossings. According to Tang and Miller-Hooks [73], it is more difficult to incorporate the computation of crossings in a computer algorithm than other visual attractiveness aspects. Moreover, computing crossings in real time during the optimization process is a very time-consuming issue since each pair of arcs has to be checked. A less computationally expensive alternative has been applied in Rossit et al. [64] where only the pair of arcs with middle points separated by a distance smaller than a given threshold value are checked. Finally, the crossings that may occur between arcs incident into the depot are not generally considered (Matis [49], Poot et al. [59], and Rossit et al. [64]) because many unavoidable intersections occur near the depot due to the high density of routes in this area.

In the category of routes overlap and crossings we implemented two different measures:

- Number of inter-route crossings ($Inter-C$) as in Matis [49]. Total number of crossings that occur between arcs belonging to two different routes. We do not consider the crossings that involve edges incident into the depot.
- Total number of customers that belong to more than one convex hull in a routing plan as in Kim et al. [33], Poot et al. [59], and Rossit et al. [64].

$$CH = \sum_{I \in K} CH_I \tag{10}$$

where CH_I in the number of customers of route I that are included in more than one convex hull. To determine the convex hull of a route we used the algorithm proposed by Graham [19].

4.3. Route complexity

We include here the characteristics related to each route individually and not to the overall routing plan. Among these aspects, Gretton and Kilby [20] proposed to measure visual attractiveness through the reduction of "bending energy", a concept taken from computer vision (Young et al. [85]). They defined the bending energy of a route as the sum of turn angles along the path. Therefore, plans with low bending energy have non-jagged transitions between customers. In support of this concept, we can mention the case presented in Bertero [4], where the routing schedule proposed for the waste collection system of an Argentinian city was modified because the municipality was reluctant to accept a plan with a large number of turns. Moreover, human solutions to TSP have a relatively low number of connections between boundary points (i.e., those near to the convex hull of the set) and points located inside the convex hull, which are also called "indentations" (MacGregor [44]). As can be seen in Figure 4, solutions with a high number of indentations have many jagged transitions between the visited points and, therefore, a high bending energy.

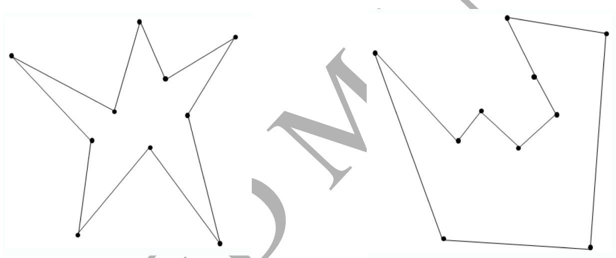

Figure 4: Different solutions for a TSP (left panel: solution with many jagged transitions; right panel: less jagged transitions). Source: MacGregor [44].

Intra-route crossings also affect visual attractiveness. Poot et al. [59] considered this measure in order to compare the solutions obtained by two different algorithms. In Lu and Dessouky [41] and Zhou et al. [86] intra-route crossings were the only visual attractiveness measure used. Moreover, they used a "crossing length percentage" (CLP) that expresses how much "entangled" the crossings are in the structure of the route. In fact, the number of crossings alone is not a proper quantity measure to evaluate the crossing level of a route, since the crossing level also depends on how deep the crossings are and whether the multiple crossings entangle each other. Dassisti et al. [11] also used CLP to compare the solutions obtained for the same CVRP instance by two different

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methods. These solutions are clearly suboptimal (Flood [14]). As the CLP is not a straightforward concept we present an example taken from Lu and Dessouky [41] later in this Section along with the formula.

Finally, Constantino et al. [10] considered the "Connectivity Index", an indicator proportional to the number of connected zones (or clusters). A zone is connected if it is possible to travel between any two points of the region without leaving it.

Related to route complexity (Section 4.3), i.e., characteristics that are linked to each route individually and not the overall routing plan, we consider three different measures.

- Number of intra-route crossings ($Intra - C_I$) as in Poot et al. [59]. Total number of crossings that occur between arcs belonging to the same route I. As for $Inter - C$, we do not consider crossings that occur in the first and last edge of a route.
- Crossing Length Percentage (CLP_I), introduced by Lu and Dessouky [41] and Zhou et al. [86]:

$$CLP_I = \frac{\sum_{e \in P_I} min(\beta_e, \lambda_I - \beta_e)}{\lambda_I} \tag{11}$$

where P_I is the set of inter-route crossing points of route I, β_e equals the route length of the portion within crossing point $e \in P_I$ and λ_I is the length of the route I. The smaller this value the better the solution is.

To better explain this measure let us consider the example depicted in Figure 5. The route in Figure 5(a) contains only one crossing. The CLP value for this route is $min\{4+4+2, 1+1+1\}/13 = 0.23$. The route in Figure 5(b) contains instead multiple crossings. The crossed lengths for the three crossings at points B, C and E yields $\overline{BC}+\overline{CD}+\overline{DE}+\overline{EF}+\overline{FB}$, $\overline{CD}+\overline{DE}+\overline{EF}+\overline{FB}+\overline{BG}+\overline{GC}$ and $\overline{EF}+\overline{FB}+\overline{BG}+\overline{GC}+\overline{CE}$ respectively. Hence, the CLP can be calculated as $CLP = (\overline{BC}+2*\overline{CD}+2*\overline{DE}+3*\overline{EF}+3*\overline{FB}+2*\overline{BG}+2*\overline{GC}+\overline{CE})/13 = 0.56$. This last example shows that segments $\overline{EF}$ and $\overline{FB}$ have been counted the

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largest number of times. Because of that, these segments (or set of customers) represent the most entangled portion of the route (Lu and Dessouky [41]) and, therefore, these are the ones that should be reassigned in order to make the route more visually attractive.

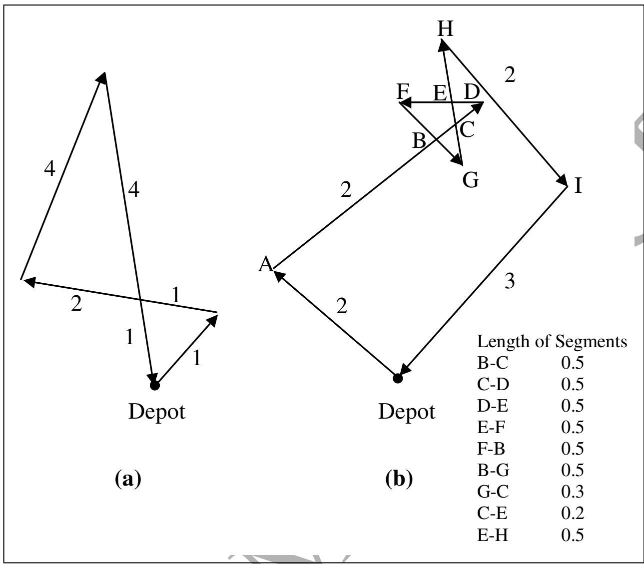

Figure 5: Data for the computation of Crossing Length Percentage (CLP). Source: Lu and Dessouky [41].

- Bending energy (BE) measure introduced by Gretton and Kilby [20]:

$$BE_I = \frac{\sum_{i=2}^{|T_I|}(alpha_{i-2,i-1,i})}{|T_I|} \tag{12}$$

where $alpha_{i-2,i-1,i}$ is the smallest angle, in radians, between the vectors formed by customers $i-2$ and $i-1$ and customers $i-1$ and i. The smaller this value the more visually attractive the solution is.

Table 2: Main reference on measuring visual attractiveness in Routing Problems

Authors/source	*Problem*	*Real-life case*	*Approach*	Measure considered		
				Compactness	Non-overlapping	Route complexity
Bosch [5]	VRPTW	Y	Clu/LS	Y	-	Y
Constantino et al. [10]	BCARP	-	Clu	Y	Y	Y
Gretton and Kilby [20]	VRPTW	Y	LNS	Y	-	Y
Hasle et al. [22] and Hasle et al. [21]	NEARP	Y	Clu/LS	Y	Y	-
Hollis and Green [23]	VRPTW	Y	LS/IA/Clu	Y	Y	-
Kant et al. [28]	SRP	Y	IA/LS/Clu	Y	-	-
Kim et al. [33]	MDVRPTW	Y	Clu/IA/LS	Y	Y	-
Lu and Dessouky [41] and Zhou et al. [86]	PDPTW	-	IA	-	-	Y
Matis [49]	SRP	-	SA/Mh/OH	Y	Y	-
Matis and Kohâni [51]	SRP	Y	Clu/TS	Y	-	-
Mourgaya and Vanderbeck [53]	PVRP	-	CG/OH	Y	-	-
Sahoo et al. [65]	VRPTW	Y	Clu/IA/LS	Y	-	-
Tang and Miller-Hooks [73]	VRPTL	Y	Clu	Y	Y	-
Poot et al. [59]	VRPTW	Y	SA	Y	Y	Y
Rossit et al. [64]	CVRP	-	Clu/LS	Y	Y	Y

Problem: BCARP: Bounded Capacitated Arc Routing Problem; CVRP: Capacitated VRP; MDVRPTW: multi-depot VRP with Time Windows; NEARP: Node-Edge Arc Routing Problem; PDPTW: Pick and Delivery with Time Windows; PVRP: Periodic VRP; SRP: Street Routing Problem; VRPTL: VRP with time limit; VRPTW: VRP with Time Windows.

Approach: CG: Column Generation; Clu: Clustering-based algorithm; IA: Insertion Algorithm; LNS: Large Neighbourhood Search; LS: Local Search; Mh: Metaheuristic; OH: other kind of Heuristic; SA : Saving Algorithm; TS: Tabu Search.

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5. Computational experimentation

In this Section, we present a test of the application of the 14 visual attractiveness measures from the literature that were outlined in the previous section to the well-known VRPTW benchmark proposed by Gehring and Homberger [17] (indicated, hereafter, as the GH99 instances). The use of VRPTW instances is motivated by the fact that the problem represents a good example of a constrained problem where the restrictions may have a strong influence on route compactness. For completeness, the results of the application to a large-scale CVRP benchmark are reported in Appendix B. Moreover, a comparison of the computational effort required to compute from scratch these visual attractiveness measures is reported in Table 3. The worst cases of this table consider the time to calculate, when needed, the center of the route (compactness measures) or the convex hull. This is why for $COMP^f$ and $PROX^c$ Tang and Miller-Hooks [73] reports a lower time complexity, since they did not consider the time to compute c_I^4.

Table 3: Worst-case Time Complexity of visual attractiveness measures.

Measure	Time complexity	Measure	Time complexity
$COMP^a$	$O(n)$	$PROX^b$	$O(n^2)$
$COMP^b$	$O(n)$	$PROX^c$	$O(n^2)$
$COMP^c$	$O(n)$	CH	$O(n^2)$
$COMP^d$	$O(n)$	$Inter-C$	$O(n \log n)$
$COMP^e$	$O(n \log n)$	$Intra-C$	$O(n \log n)$
$COMP^f$	$O(n^2)$	CLP	$O(n^2 \log n)$
$PROX^a$	$O(n^2)$	BE	$O(n)$

We computed the visual attractiveness for a subset of the GH99 instances for which the best known solution (BKS) data are available and the implementation was performed in C++. When the measure is defined for a specific route I, we defined as a global measure the average of the values computed for each route $I \in \mathcal{K}$. This happens for all measures above with the exception of $COMP^b$, $COMP^f$, $PROX^c$, CH, and $Inter-C$ that are already global measures. The

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GH99 instances data was taken from the Vehicle Routing Problem Repository [16] (VRP-REP) while the data of the BKS were obtained from the Transportation Optimization Portal of the Norwegian Foundation for Scientific and Industrial Research [75]. Out of the GH99 instances for which the BKS data is available in this website, we selected eighty-seven instances trying to obtain a balanced set in terms of the six Classes (C1, C2, R1, R2, RC1 and RC2) and the different instance sizes (200, 400, 600, 800 and 1000 customers). In particular, we selected three instances of each class and size, with the exception of the Class R2 with sizes 200, 400, 600 and 800 for which three BKS were not available. Following the classification of Solomon [69], C-type instances are those in which the customers are strongly clustered, while in R-type ones the customers are uniformly randomly distributed. The RC-type is an intermediate distribution between these two extremes. The numbers 1 and 2 in the class names are related to the scheduling horizon. In Class 1 the scheduling horizon is quite short, i.e., it allows only a few customers to be served by the same vehicle, whereas in Class 2 the horizon is longer.

In Table 4 we present the maximum, the minimum and the average value of the visual attractiveness measures for each Class of the GH99 (C, R and RC). The detailed information for each instance can be consulted in Appendix A. All the tables are based on the relative percentage deviation ($rel(x,t)$) for each measure x and for each instance t respect to the best measure of the whole group of instances (T) to which t belongs. T can be any of the classes of instances introduced in the aforementioned Solomon's classification for VRPTW [69], thus, it can be C, R or RC. Then, $rel(x,t)$ is defined as:

$$rel(x,t) = \frac{|Value(x,t) - Best_T(x)|}{|Worst_T(x) - Best_T(x)|} \cdot 100\%,\ T =\ C,\ R\ or\ RC \tag{13}$$

where $Best_T(x)$ and $Worst_T(x)$ are the best and worst values obtained for measure x in the type of instances T, respectively. For the majority of the measures the best result is the minimum value obtained, except for $COMP^a$ and $PROX^a$ for which the best result is the largest value obtained. $Value(x,t)$ is the value of the visual attractiveness measure x applied on instance t. Finally, we can observe that the larger the value of $rel(x,t)$, the worst the solution is in terms of visual attractiveness.

To analyze the relationship among the different measures we computed the bivariate correlation matrices for all pairs of measures, which are reported separately for Class C, R and RC in Figures 6 to 8, respectively. In the bottom-left half of each matrix we included the plot of each pair of visual attractiveness measures with a trend curve. In the top-right we included, from top to bottom, the correlation coefficients proposed by Pearson [57], Spearman [70], and Kendall [29], respectively. Pearson [57] coefficient measures the linear association between normal variables, instead Kendall [29] and Spearman [70] coefficients are nonparametric rank-order statistic measures (i.e., they do not assume any specific probabilistic distribution of the random variable) . Rank-order statistics are those for which the actual magnitude of any observation is used only in determining its relative position in the sample array. Therefore, the inclusion of the Kendall [29] and Spearman [70] coefficients allow for a more robust analysis. The interpretation of each coefficient is identical and their values vary between -1 and 1. A positive correlation coefficient reflects a positive relation: as one variable increases, the other variable increases too. Conversely, if the coefficient is negative, as one variable decreases, the other variable increases and vice-versa. The larger the absolute value of the coefficient the stronger the relation is.

In Figures 6 to 8 the negative correlation coefficient that $COMP^a$ and $PROX^a$ have with several other measures supports that the more visually attractive a solution, the smaller the value of $COMP^a$ and $PROX^a$ (see Section 5). $COMP^b$, $COMP^c$, $COMP^d$, $COMP^e$, $COMP^f$, and CH are strongly correlated in the three classes. Then, depending on the class of instances, some measures can be added to this group of highly correlated measures: $PROX^b$, $Intra-C$ and CLP in the case of Class C; $COMP^a$, $PROX^a$, $PROX^b$, $Inter-C$, $Intra-C$, and CLP in the case of Class RC; $Inter-C$, $Intra-C$, and CLP in the case of Class R. The correlation between $PROX^c$, CH and $Inter-C$ is also very strong in the three classes as is also between $Intra-C$ and CLP. Finally, BE does not seem to have any special relation with the

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Table 4: Summary values of the computation of visual attractiveness measures to BKS of GH99 benchmark.

Type of instance	Summary value	$COMP^a$	$COMP^b$	$COMP^c$	$COMP^d$	$COMP^e$	$COMP^f$	$PROX^a$	$PROX^b$	$PROX^c$	CH	$Inter-C$	$Intra-C$	CLP	BE
C-type	Maximum	0.59	89157.50	35.51	29.65	659.54	85.77	0.98	0.67	744.00	479.00	144.00	2.75	0.48	1.53
	Minimum	0.37	992.34	5.82	3.96	18.84	7.10	0.38	0.05	11.00	1.00	0.00	0.00	0.00	1.14
	Average	0.46	21169.85	19.78	15.91	253.41	32.94	0.65	0.40	238.07	153.93	47.47	0.79	0.14	1.32
R-type	Maximum	0.56	152009.00	126.37	102.44	3394.75	167.13	0.49	0.85	826.00	3125.00	1405.00	38.27	9.25	1.74
	Minimum	0.47	5332.35	21.06	15.43	90.68	26.87	-0.42	0.52	119.00	59.00	9.00	0.11	0.02	0.91
	Average	0.52	50580.14	56.95	43.80	1064.08	76.79	-0.04	0.71	448.19	876.11	350.30	8.11	1.78	1.35
RC-type	Maximum	0.49	160311.00	105.07	85.30	2916.48	167.81	0.43	0.87	793.00	2129.00	699.00	14.50	2.94	1.52
	Minimum	0.37	4633.71	16.15	12.64	66.47	22.28	-0.22	0.47	93.00	61.00	9.00	0.06	0.02	1.02
	Average	0.43	47081.01	47.49	38.19	929.06	68.53	0.10	0.68	395.67	640.73	182.70	5.54	1.11	1.26

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other measures, except for Class C were it is correlated to some compactness measures and with CLP. Another aspect about BE is that it has negative coefficients, something that was not expected from its definition. This shows, for the cases studied, that when visual attractiveness of the solutions increases in regard to BE it decreases in regard to the other measures.

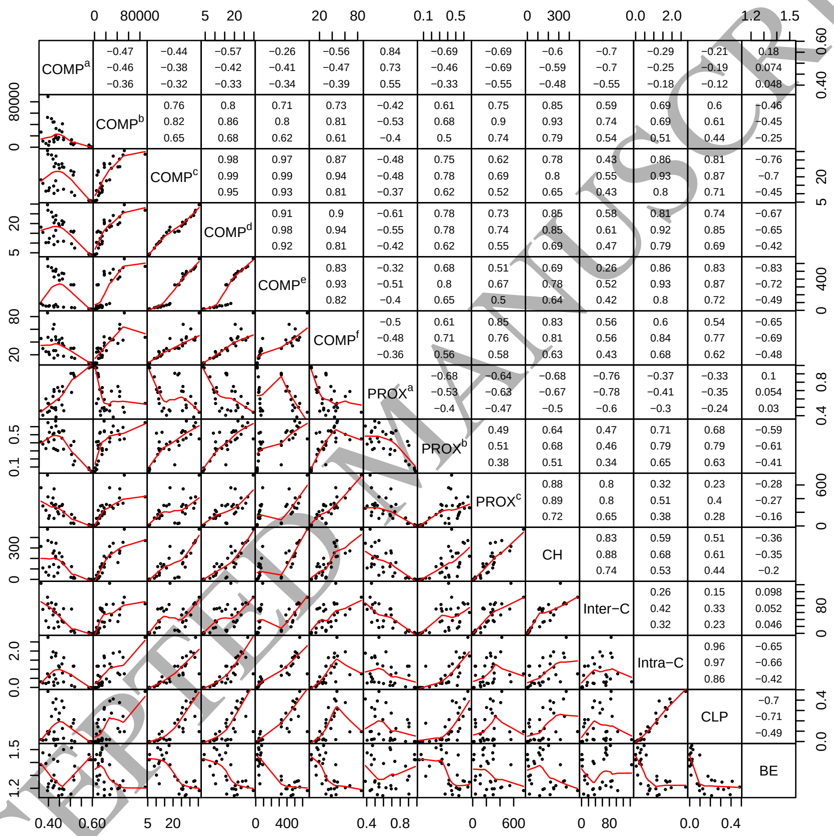

Figure 6: Correlation matrix for Class C of the GH99 VRPTW benchmark.

Furthermore, because visual attractiveness is quite a subjective concept, we consider that graphical information is also very important to carry out a thorough analysis. With this aim, we present some plots of the solution that we

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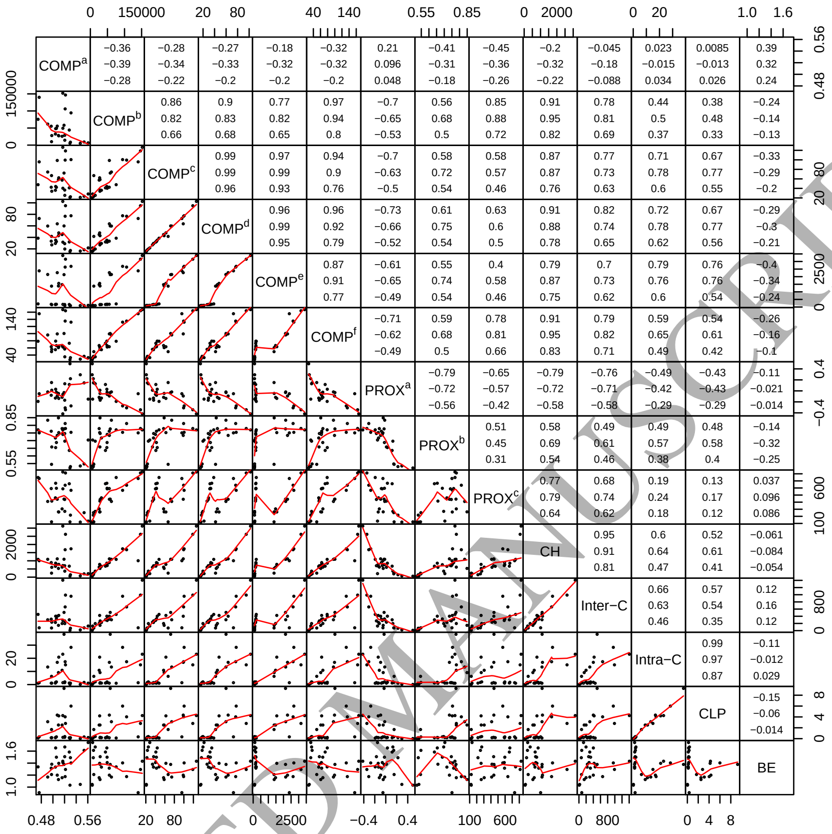

Figure 7: Correlation matrix for Class R of the GH99 VRPTW benchmark.

have used for our test in Appendix C[2].

5.1. Recommendations on the use of visual attractiveness measures

The correlation analysis performed on our experiment did not allow us to derive general relations between the various measures that permit to identify the most appropriate measures for each characteristic. However, based on the bibliographic analysis and the computational tests we can make some general recommendations:

[2]The rest of the plots are available upon request.

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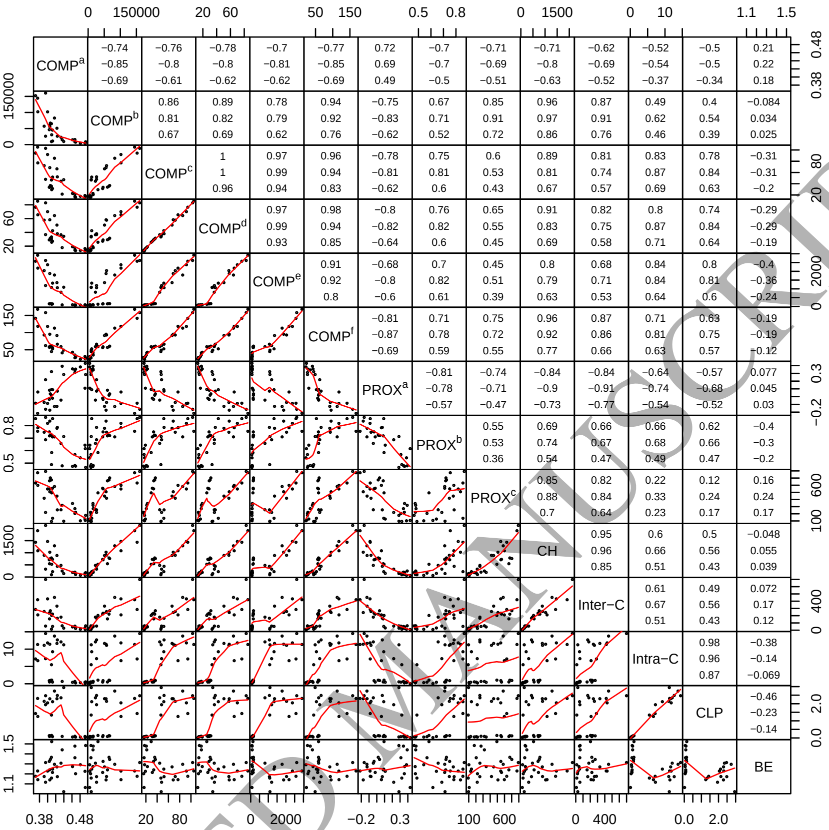

Figure 8: Correlation matrix for Class RC of the GH99 VRPTW benchmark.

- Bending energy is a concept that is particularly useful for urban route planning. In these applications, the routes should be repeated frequently in a short period of time. That is why routes with numerous jagged turns can have a negative impact on the tires and brakes of the vehicle. Moreover, many turns in the path of a vehicle can lead to larger routing times considering the extra time needed to turn left at crossroads with traffic lights (Lacomme et al. [34]). Conversely, in inter-city distribution plans where the nodes may represent a whole city and the details of actual paths are not specified, bending energy measures clearly makes no sense.

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- At least for the set of instances considered in this paper it does not seem worthwhile calculating the CLP measure because the number of average intra-route crossings ($Intra - C$) gives similar information and is faster and easier to compute (see Table 3).
- It seems appropriate to use just one measure out of either convex hull overlaps (CH) or inter-route crossings ($Inter - C$) because those measures are both very time-consuming to be computed and provide similar information. However, from Table 3 we can see that at least for our implementation, $Inter - C$ is faster to calculate. In addition, $PROX^c$ has a behavior similar to CH or $Inter - C$ and, therefore, could be used as a proxy.
- $COMP^b$, $COMP^c$ and $COMP^d$ produce similar results. Because the computation of $COMP^b$ and $COMP^d$ requires the distances from the center of gravity, we recommend the use of $COMP^c$.
- $PROX^a$ and $PROX^b$ produce very similar information. However, we recommend the use of $PROX^b$ because its computation does not require the calculation of the distances from the center of gravity.

6. Discussion and conclusion

The main objective of this paper was to organize the available literature so as to provide a reference point for future research on visual attractiveness aspects in routing problems. Even though in districting problems the objective of ensuring compactness has been effectively integrated into many of the implemented algorithms, within routing problems the literature is more scarce and is mainly driven by customer requests in real-life applications. Nevertheless, the bibliography analyzed here stresses the practical benefits of considering visual attractiveness measures in the optimizing process of routing problems. Routing patterns that are considered as "lab's plans", i.e., solutions that are very different from those generated by planners, are usually rejected by them. This can reduce the grade of collaboration and, therefore, increase the time and effort

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necessary for implementing the routing plan, becoming a huge obstacle to the organization. In addition, there are other practical benefits of visually attractive plans, as they may enhance drivers' specialization.

With the aim of producing visually attractive solutions, different authors have considered different measures in order to estimate the degree of visual attractiveness of a routing plan. We presented the main concepts and their formulas, applying them to the Best Known Solutions of a well-known VRPTW benchmark. Furthermore, based on the literature revision and the implementation tests, we outlined some preliminary recommendations about the usability of some measures in different contexts. We think that this will contribute to the work of other authors by allowing them to evaluate the measures that best suit their interests. Future work should start from such analysis to provide better general definitions of visual attractiveness. Another aspect where further research can be done is on the efficient integration of traditional objectives (e.g., the minimization of length, costs or number of vehicles) and visual attractiveness. To enhance visual attractiveness while not worsening excessively traditional objectives is a major challenge that has not been sufficiently addressed in the literature.

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Appendices

Appendix A. Detailed results of the tests on a VRPTW benchmark.

In this section we present the detailed results that led to Table 4, i.e., the value of the application of each visual attractiveness measure outlined in Section 4 to each of the eighty-seven GH99 benchmark instances. In Table A.1, A.2, and A.3 we present the results for Classes C1 and C2, Classes R1 and R2, and Classes RC1 and RC2, respectively.

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Table A.1: Computation of visual attractiveness measures to BKS of Class C of GH99 benchmark.

Instance	Length	$\|K\|$	$COMP^a$	$COMP^b$	$COMP^c$	$COMP^d$	$COMP^e$	$COMP^f$	$PROX^a$	$PROX^b$	$PROX^c$	CH	$Inter-C$	$Intra-C$	CLP	BE
C1_2_1	2679.09	20	4.43%	0.30%	0.39%	1.05%	0.02%	0.26%	1.86%	6.56%	0.14%	0.63%	2.08%	0.00%	0.00%	71.91%
C1_2_5	2678.7	20	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	71.58%
C1_2_9	2687.82	18	84.84%	2.82%	20.23%	24.78%	5.33%	21.60%	72.90%	58.41%	13.64%	8.16%	11.81%	10.10%	14.94%	23.65%
C1_4_2	7686.38	36	83.28%	10.28%	25.80%	32.81%	6.47%	16.91%	88.12%	77.56%	19.24%	24.90%	50.69%	21.21%	20.12%	81.03%
C1_4_4	6803.24	36	74.09%	9.38%	14.63%	20.37%	3.47%	9.82%	44.98%	75.73%	13.64%	5.86%	10.42%	4.04%	2.73%	40.92%
C1_4_8	7347.23	37	90.17%	6.41%	19.16%	24.32%	5.51%	22.75%	63.66%	45.09%	29.74%	14.23%	25.00%	7.86%	3.53%	64.12%
C1_6_1	14051.6	60	4.11%	3.25%	1.07%	2.10%	0.00%	0.06%	1.69%	3.34%	1.23%	0.00%	0.69%	0.00%	0.00%	82.03%
C1_6_4	13501.3	56	32.74%	6.81%	10.49%	13.51%	2.28%	9.12%	19.65%	20.14%	14.60%	2.09%	1.39%	1.30%	0.16%	58.76%
C1_6_7	15649.7	57	96.29%	11.02%	34.72%	44.86%	7.57%	28.88%	96.43%	53.96%	38.47%	22.59%	47.92%	8.93%	2.54%	85.48%
C1_8_3	24133.7	72	66.62%	14.44%	20.68%	26.52%	4.33%	15.44%	41.11%	41.21%	27.01%	12.76%	22.92%	8.59%	4.35%	68.34%
C1_8_6	26976.6	79	54.96%	13.87%	22.93%	27.13%	3.44%	13.51%	49.61%	43.82%	18.83%	31.17%	49.31%	11.05%	6.53%	91.92%
C1_8_7	26287.5	77	74.78%	14.30%	29.34%	37.36%	6.00%	24.77%	82.27%	45.24%	39.56%	29.50%	44.44%	10.86%	5.06%	72.60%
C1_10_2	41855.8	90	70.62%	22.69%	39.91%	48.86%	8.70%	24.68%	85.62%	43.12%	41.47%	42.05%	57.64%	16.97%	4.22%	100.00%
C1_10_6	43562	99	38.94%	13.88%	17.19%	22.05%	2.71%	11.28%	50.77%	33.25%	21.56%	25.31%	48.61%	4.04%	1.95%	95.59%
C1_10_8	42220.2	93	100.00%	29.09%	41.73%	55.26%	10.83%	48.92%	87.74%	58.30%	74.90%	65.69%	100.00%	11.34%	3.87%	61.69%
C2_2_1	1901.65	6	35.61%	3.86%	51.19%	45.26%	47.94%	29.05%	12.66%	12.67%	4.37%	2.30%	6.25%	42.42%	36.57%	38.68%
C2_2_2	1859.74	6	34.29%	6.43%	57.39%	47.30%	48.24%	29.61%	15.74%	59.27%	3.27%	4.39%	6.25%	42.42%	43.73%	31.00%
C2_2_7	1833.78	6	40.23%	5.77%	59.88%	46.97%	48.17%	29.27%	12.91%	51.43%	4.09%	3.97%	4.17%	24.24%	29.45%	3.08%
C2_4_3	4018.02	11	54.42%	18.61%	66.73%	65.24%	70.45%	34.16%	87.29%	80.35%	26.19%	30.75%	22.22%	59.50%	67.86%	12.20%
C2_4_4	3702.49	11	64.76%	18.49%	72.15%	62.15%	67.56%	40.08%	65.08%	78.85%	22.78%	24.27%	13.19%	42.98%	43.78%	0.00%
C2_4_8	4303.69	11	70.85%	18.06%	74.43%	69.54%	74.96%	52.36%	81.23%	100.00%	34.79%	46.03%	41.67%	62.81%	64.49%	26.99%
C2_6_10	7255.69	17	58.15%	29.54%	73.91%	67.92%	71.33%	49.40%	75.92%	65.94%	39.43%	45.19%	25.69%	47.06%	81.01%	1.90%
C2_6_2	8264.92	17	70.03%	36.27%	89.10%	78.54%	79.19%	46.44%	100.00%	90.01%	33.83%	71.97%	58.33%	68.45%	85.02%	28.54%
C2_6_7	7512.07	18	63.32%	32.76%	64.85%	57.29%	57.21%	45.99%	48.24%	77.32%	37.52%	52.51%	36.81%	40.40%	56.72%	13.58%
C2_8_2	12285	23	79.03%	55.88%	91.43%	85.19%	82.39%	45.51%	80.56%	87.30%	43.93%	73.85%	61.11%	60.08%	50.25%	29.63%
C2_8_7	11370.8	24	57.18%	45.64%	67.55%	60.54%	59.71%	39.13%	38.93%	80.74%	41.47%	33.47%	23.61%	28.79%	26.10%	13.66%
C2_8_9	11645.2	23	75.97%	48.95%	83.20%	75.77%	75.92%	68.05%	58.81%	73.45%	54.43%	52.09%	36.81%	71.15%	77.79%	15.77%
C2_10_3	16884.1	28	86.14%	100.00%	92.82%	89.02%	84.85%	51.25%	72.77%	96.84%	56.62%	77.41%	59.03%	100.00%	100.00%	17.10%
C2_10_5	16561.3	30	73.86%	49.38%	68.03%	63.97%	63.78%	77.11%	48.96%	62.69%	72.58%	56.69%	48.61%	25.45%	19.36%	19.99%
C2_10_8	16577.3	28	87.00%	58.42%	100.00%	100.00%	100.00%	100.00%	90.47%	64.39%	100.00%	100.00%	72.22%	27.27%	19.80%	10.01%
MAX			0.59	89157.50	35.51	29.65	659.54	85.77	0.98	0.67	744.00	479.00	144.00	2.75	0.48	1.53
MIN			0.37	992.34	5.82	3.96	18.84	7.10	0.38	0.05	11.00	1.00	0.00	0.00	0.00	1.14
AVERAGE			0.46	21169.85	19.78	15.91	253.41	32.94	0.65	0.40	238.07	153.93	47.47	0.79	0.14	1.32

Table A.2: Computation of visual attractiveness measures to BKS of Class R of GH99 benchmark.

Instance	Length	$\|K\|$	$COMP^a$	$COMP^b$	$COMP^c$	$COMP^d$	$COMP^e$	$COMP^f$	$PROX^a$	$PROX^b$	$PROX^c$	CH	$Inter-C$	$Intra-C$	CLP	BE
R1_2_1	4784.11	20	0.00%	0.89%	6.92%	7.20%	0.54%	4.21%	70.59%	55.80%	1.98%	5.09%	8.52%	2.46%	2.02%	100.00%
R1_2_4	3057.81	18	15.79%	0.00%	0.10%	0.98%	0.21%	1.25%	22.00%	3.96%	0.00%	0.62%	0.79%	0.15%	0.00%	16.42%
R1_2_8	2951.99	18	36.36%	0.14%	0.00%	0.00%	0.01%	0.00%	0.00%	0.00%	0.42%	0.00%	0.00%	0.00%	0.02%	14.84%
R1_4_1	10372.3	40	46.64%	6.11%	4.09%	5.04%	0.15%	5.83%	39.95%	53.45%	21.36%	10.01%	13.47%	5.08%	3.95%	91.14%
R1_4_5	9085.49	36	8.15%	2.63%	2.28%	3.44%	0.19%	6.87%	38.22%	8.03%	18.81%	4.66%	4.94%	4.00%	3.19%	84.35%
R1_4_9	8590.88	36	35.43%	2.21%	1.25%	2.08%	0.00%	4.13%	29.83%	2.73%	13.86%	2.87%	4.08%	2.55%	1.76%	69.71%
R1_6_2	18863.4	54	65.40%	18.45%	14.26%	16.13%	2.13%	20.06%	68.57%	59.85%	48.37%	19.63%	15.69%	2.62%	1.71%	50.97%
R1_6_5	19771.9	54	46.84%	15.93%	14.64%	15.67%	2.25%	21.38%	72.79%	68.74%	45.12%	20.87%	18.77%	3.54%	2.38%	71.78%
R1_6_9	18700.4	54	72.66%	15.42%	12.29%	13.22%	1.90%	20.74%	52.84%	41.96%	41.87%	17.71%	16.19%	2.47%	1.70%	56.70%
R1_8_2	32598.5	72	73.73%	34.07%	20.29%	20.65%	2.70%	26.01%	62.88%	73.32%	66.90%	27.14%	22.21%	2.47%	1.53%	48.80%
R1_8_5	33861.4	72	63.42%	29.71%	20.14%	19.84%	2.72%	28.16%	56.77%	50.79%	67.47%	22.96%	22.64%	4.99%	3.35%	70.73%
R1_8_9	32630	72	50.50%	29.96%	18.30%	18.67%	2.50%	25.40%	61.08%	66.87%	66.34%	21.23%	19.56%	3.49%	2.31%	60.04%
R1_10_2	49105.2	91	100.00%	47.86%	25.40%	26.58%	3.47%	36.90%	75.18%	76.09%	88.97%	33.56%	31.73%	2.33%	1.06%	51.43%
R1_10_5	51131.4	91	64.88%	29.26%	21.82%	22.62%	2.73%	37.18%	50.86%	40.72%	77.51%	25.93%	28.15%	4.26%	2.09%	89.18%
R1_10_9	49771.8	91	37.70%	26.46%	19.05%	19.08%	2.32%	32.89%	41.42%	34.94%	76.52%	19.99%	20.99%	2.73%	1.54%	81.00%
R2_2_9	3092.04	4	73.11%	3.20%	43.55%	30.16%	38.90%	16.11%	37.49%	7.24%	0.42%	6.07%	4.58%	16.09%	17.97%	36.16%
R2_4_1	9210.15	8	58.69%	14.89%	39.13%	37.17%	44.38%	25.95%	69.20%	70.61%	20.08%	23.55%	29.66%	54.41%	45.20%	58.50%
R2_4_5	7128.93	8	52.03%	13.93%	34.48%	33.87%	41.96%	28.38%	66.98%	78.61%	26.31%	18.10%	13.54%	28.21%	26.90%	29.83%
R2_4_9	6400.1	8	59.35%	15.62%	35.56%	31.56%	39.19%	24.14%	59.24%	100.00%	20.37%	15.46%	9.89%	23.62%	26.93%	26.13%
R2_6_1	18206.8	11	48.22%	36.01%	48.60%	51.06%	61.84%	52.40%	57.50%	78.00%	45.83%	33.86%	35.67%	100.00%	100.00%	56.30%
R2_6_5	15096.2	11	61.69%	34.28%	73.75%	55.32%	65.79%	51.58%	64.38%	64.92%	45.54%	34.61%	28.08%	44.02%	46.76%	34.89%
R2_6_9	13377.6	11	80.75%	38.36%	50.70%	50.87%	62.15%	49.48%	56.21%	95.31%	41.16%	33.69%	21.06%	34.73%	33.53%	31.10%
R2_8_1	28114.2	15	34.25%	60.70%	75.45%	71.37%	78.39%	75.01%	84.16%	71.11%	63.37%	54.66%	67.98%	73.95%	64.71%	59.29%
R2_8_5	24285.9	15	45.95%	68.68%	74.45%	70.39%	76.93%	77.19%	88.04%	88.71%	73.55%	52.80%	46.63%	45.30%	43.32%	31.98%
R2_10_1	42149.4	19	50.69%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	75.94%	100.00%	100.00%	100.00%	60.25%	45.80%	62.68%
R2_10_5	36186.6	19	45.29%	96.39%	92.79%	91.41%	92.68%	97.73%	91.82%	78.76%	98.02%	83.59%	70.27%	39.02%	30.09%	46.24%
R2_10_8	17484.1	19	97.76%	91.75%	70.95%	65.90%	69.42%	91.88%	59.79%	70.02%	86.99%	30.89%	5.01%	3.43%	3.33%	0.00%
MAX			0.56	152009.00	126.37	102.44	3394.75	167.13	0.49	0.85	826.00	3125.00	1405.00	38.27	9.25	1.74
MIN			0.47	5332.35	21.06	15.43	90.68	26.87	-0.42	0.52	119.00	59.00	9.00	0.11	0.02	0.91
AVERAGE			0.52	50580.14	56.95	43.80	1064.08	76.79	-0.04	0.71	448.19	876.11	350.30	8.11	1.78	1.35

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Table A.3: Computation of visual attractiveness measures to BKS of Class RC of GH99 benchmark.

Instance	Length	$\|K\|$	$COMP^a$	$COMP^b$	$COMP^c$	$COMP^d$	$COMP^e$	$COMP^f$	$PROX^a$	$PROX^b$	$PROX^c$	CH	$Inter-C$	$Intra-C$	CLP	BE
RC1_2_2	3249.05	18	1.25%	0.47%	3.38%	5.24%	0.90%	5.64%	49.68%	44.47%	4.86%	4.21%	3.19%	2.69%	2.27%	40.69%
RC1_2_4	2851.68	18	0.61%	0.00%	0.74%	0.00%	0.19%	0.89%	0.00%	37.38%	2.00%	0.00%	0.00%	0.00%	0.00%	8.32%
RC1_2_9	3081.13	18	10.46%	0.01%	1.74%	1.54%	0.37%	0.00%	7.27%	19.11%	1.29%	0.44%	0.58%	1.92%	1.15%	53.86%
RC1_4_1	8378.75	36	0.00%	2.66%	1.64%	3.15%	0.21%	3.05%	22.52%	0.00%	12.14%	5.66%	7.39%	3.46%	1.44%	92.24%
RC1_4_10	7517.93	36	20.04%	1.89%	0.00%	1.33%	0.07%	3.95%	11.58%	2.52%	18.43%	1.89%	2.17%	1.35%	1.21%	35.91%
RC1_4_5	8080.49	36	21.40%	2.61%	0.79%	2.10%	0.00%	3.02%	18.43%	25.65%	16.00%	4.84%	6.81%	2.88%	1.92%	67.86%
RC1_6_1	16581.3	55	21.42%	7.17%	5.23%	6.89%	0.56%	8.57%	13.36%	3.76%	25.57%	6.72%	6.09%	4.27%	2.79%	91.08%
RC1_6_5	16359.4	55	46.79%	6.73%	6.05%	7.16%	0.71%	12.84%	5.83%	3.10%	32.29%	8.99%	7.54%	4.27%	2.42%	83.60%
RC1_6_9	15668.4	55	26.16%	6.65%	7.43%	8.91%	1.00%	14.10%	11.17%	2.05%	35.86%	6.14%	5.22%	2.51%	1.32%	50.10%
RC1_8_1	30998	72	67.80%	16.62%	14.92%	17.91%	2.10%	25.11%	52.14%	29.59%	57.14%	22.44%	28.12%	6.92%	4.34%	100.00%
RC1_8_5	30277.1	72	72.51%	26.82%	19.71%	22.48%	2.87%	24.89%	72.05%	70.93%	65.14%	31.38%	32.61%	5.87%	4.01%	60.46%
RC1_8_9	28978.3	72	66.19%	26.01%	17.22%	20.02%	2.62%	23.33%	69.75%	60.94%	66.43%	32.25%	30.87%	2.98%	2.05%	38.38%
RC1_10_1	46272.1	90	70.42%	39.98%	23.27%	26.06%	3.22%	26.31%	61.07%	63.51%	87.00%	33.80%	37.54%	6.15%	3.54%	64.35%
RC1_10_5	45564.8	90	68.30%	38.77%	21.96%	24.59%	3.06%	25.04%	63.69%	64.53%	83.00%	35.06%	30.72%	6.85%	3.91%	68.67%
RC1_10_9	44280.8	90	72.10%	35.61%	20.00%	22.83%	2.84%	23.76%	53.73%	62.98%	81.14%	23.50%	21.30%	3.23%	2.27%	43.63%
RC2_2_3	2601.87	4	43.77%	3.26%	34.92%	29.83%	36.05%	18.30%	22.38%	61.68%	0.57%	3.82%	2.75%	77.50%	100.00%	0.00%
RC2_2_6	2873.12	4	66.00%	4.14%	40.10%	40.98%	44.22%	20.86%	52.76%	97.45%	6.43%	7.64%	6.23%	63.65%	71.82%	45.12%
RC2_2_8	2292.53	4	68.16%	3.09%	34.28%	28.45%	35.58%	15.39%	16.69%	5.54%	0.00%	1.50%	0.72%	37.69%	45.14%	27.17%
RC2_4_1	6682.38	11	49.21%	11.64%	31.88%	32.85%	26.07%	25.32%	73.53%	47.95%	23.29%	20.21%	24.78%	83.32%	79.63%	56.93%
RC2_4_2	6180.62	9	56.45%	13.81%	40.58%	34.97%	35.03%	30.20%	55.58%	79.91%	23.14%	18.33%	22.61%	87.31%	84.24%	30.01%
RC2_4_8	4793.06	8	80.01%	13.81%	35.68%	33.19%	39.42%	27.42%	32.83%	55.22%	18.00%	12.72%	6.96%	49.81%	65.79%	18.81%
RC2_6_1	13324.9	14	57.28%	29.74%	56.60%	52.19%	45.21%	47.54%	85.99%	78.19%	50.71%	39.94%	46.81%	80.22%	77.80%	49.55%
RC2_6_2	11555.5	12	71.56%	32.83%	58.98%	62.78%	63.40%	50.14%	93.62%	92.53%	59.29%	46.57%	33.91%	78.65%	77.65%	23.53%
RC2_6_6	11933.9	11	82.76%	33.79%	62.98%	62.03%	69.21%	48.56%	78.17%	70.30%	46.43%	40.38%	35.94%	76.40%	72.48%	30.25%
RC2_8_1	20981.1	18	62.21%	50.10%	77.78%	71.22%	60.39%	62.29%	86.28%	60.98%	69.86%	59.33%	64.20%	95.00%	93.09%	54.68%
RC2_8_5	19136	15	75.98%	62.50%	85.25%	79.00%	80.91%	65.67%	80.09%	88.03%	64.86%	67.99%	61.30%	81.31%	69.53%	41.17%
RC2_8_9	15360	15	95.48%	62.63%	68.46%	72.71%	74.20%	65.91%	57.35%	79.21%	72.14%	49.37%	29.28%	45.77%	42.25%	24.01%
RC2_10_1	30278.5	20	79.94%	100.00%	97.61%	96.62%	89.52%	93.25%	91.75%	100.00%	100.00%	100.00%	100.00%	100.00%	83.76%	58.22%
RC2_10_6	26645.6	18	95.85%	89.81%	100.00%	100.00%	100.00%	100.00%	100.00%	72.21%	97.29%	89.46%	59.57%	78.08%	78.28%	41.39%
RC2_10_9	22976.7	18	100.00%	94.82%	87.99%	87.90%	88.03%	81.98%	79.67%	97.36%	76.86%	66.44%	40.00%	49.23%	48.13%	30.73%
MAX			0.49	160311.00	105.07	85.30	2916.48	167.81	0.43	0.87	793.00	2129.00	699.00	14.50	2.94	1.52
MIN			0.37	4633.71	16.15	12.64	66.47	22.28	-0.22	0.47	93.00	61.00	9.00	0.06	0.02	1.02
AVERAGE			0.43	47081.01	47.49	38.19	929.06	68.53	0.10	0.68	395.67	640.73	182.70	5.54	1.11	1.26

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Appendix B. Tests on a CVRP benchmark.

In this section we present a similar analysis to the one presented for the GH99 benchmark but for the Uchoa et al.'s CVRP benchmark [77] instead. The datasets and the BKS were taken from the Capacitated Vehicle Routing Problem Library [76]. In Figure B.1 we present the correlations matrix and in Tables B.1 and B.2 we present the relative percentage deviation, i.e., $rel(x,t)$ from Eq. (13). For this CVRP benchmark we consider a unique group composed by all the instances of the Uchoa et al.'s CVRP benchmark [77] ($T = X$).

In Figure B.1 we can see that also for this CVRP benchmark there is a strong correlation between $COMP^c$, $COMP^d$, $COMP^e$, and $COMP^f$. However, compared to what happened with the VRPTW benchmark, $COMP^b$ is not in this group. $COMP^b$ is correlated with $PROX^b$, $PROX^c$, and CH. $COMP^a$ is correlated with $COMP^e$ and $PROX^b$. Once again, it is evident the strong relationship between $PROX^c$, CH and $Inter-C$, on the one hand, and between $Intra - C$ and CLP, on the other hand. BE is positively correlated with $COMP^a$ and negatively with $COMP^e$ and $PROX^b$.

Tables B.1 and B.2 show some peculiar result for a small set of instances which present intra-route crossings. It is known that in CVRP crossings within the same route is an evidence of sub-optimality (see Flood [14]). However, after applying a simple procedure to repair intra-route crossings based on the well-known two-opt operator, we found that the total length of the solutions generally remains unchanged, with the exception of instance X-n317-k53 in which it increases by one unit (see Table B.3). The reason for this unexpected behavior is that, as stated in Uchoa et al. [77], in this benchmark the TSPLIB convention of rounding distances between customers to the nearest integer applies (Reinelt [60]).

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Table B.1: Computation of visual attractiveness measures to BKS of Uchoa et al.'s benchmark [77] (Part A).

Instance	Length	$\|K\|$	$COMP^a$	$COMP^b$	$COMP^c$	$COMP^d$	$COMP^e$	$COMP^f$	$PROX^a$	$PROX^b$	$PROX^c$	CH	$Inter-C$	$Intra-C$	CLP	BE
X-n101-k25	27591	26	23.90%	1.06%	32.16%	34.11%	3.21%	18.42%	41.19%	32.64%	1.45%	2.00%	3.96%	0.00%	0.00%	69.94%
X-n106-k14	26362	14	52.49%	2.43%	24.96%	26.82%	7.23%	14.71%	25.52%	47.21%	2.37%	2.33%	1.49%	0.00%	0.00%	31.77%
X-n110-k13	14971	13	49.34%	5.24%	52.36%	53.42%	16.79%	26.49%	9.31%	23.90%	0.00%	0.67%	0.50%	0.00%	0.00%	21.74%
X-n115-k10	12747	10	67.73%	8.37%	60.30%	65.39%	40.16%	41.43%	36.28%	43.04%	3.02%	3.00%	0.50%	0.00%	0.00%	12.64%
X-n120-k6	13332	6	68.12%	15.01%	97.48%	93.77%	83.50%	60.01%	18.48%	61.23%	2.89%	2.17%	0.50%	0.00%	0.00%	4.37%
X-n125-k30	55539	30	18.21%	1.68%	30.14%	28.35%	2.73%	16.10%	75.77%	51.19%	3.68%	4.00%	7.92%	0.00%	0.00%	81.83%
X-n129-k18	28940	18	67.83%	5.62%	49.18%	45.01%	13.05%	30.69%	30.80%	45.54%	4.07%	3.67%	2.48%	0.00%	0.00%	14.83%
X-n134-k13	10916	13	73.21%	2.96%	26.21%	25.52%	15.84%	26.01%	33.54%	39.15%	3.68%	2.50%	1.49%	0.00%	0.00%	20.65%
X-n139-k10	13590	10	63.76%	11.44%	75.12%	74.71%	44.45%	42.17%	18.94%	55.39%	3.55%	1.33%	0.99%	0.00%	0.00%	5.66%
X-n143-k7	15700	7	74.96%	22.76%	100.00%	100.00%	100.00%	100.00%	30.15%	48.37%	5.78%	8.17%	3.47%	0.00%	0.00%	7.07%
X-n148-k46	43448	47	22.56%	1.65%	25.71%	24.94%	1.45%	14.31%	50.77%	27.22%	3.15%	1.83%	5.45%	0.00%	0.00%	87.16%
X-n153-k22	21220	23	50.72%	3.90%	29.83%	29.00%	8.71%	22.98%	79.63%	56.65%	9.33%	14.33%	9.41%	0.00%	0.00%	42.77%
X-n157-k13	16876	13	74.18%	5.26%	28.87%	28.31%	15.48%	19.32%	46.70%	57.79%	7.23%	4.17%	0.99%	0.00%	0.00%	3.89%
X-n162-k11	14138	11	69.62%	13.24%	61.33%	67.33%	42.92%	42.24%	38.02%	50.68%	6.83%	7.50%	1.98%	0.00%	0.00%	8.11%
X-n167-k10	20557	10	66.91%	14.15%	67.83%	68.63%	55.27%	60.95%	19.20%	30.70%	10.38%	3.67%	2.48%	0.00%	0.00%	12.06%
X-n172-k51	45607	53	0.00%	2.54%	24.75%	21.05%	1.57%	12.54%	40.06%	21.82%	4.86%	3.67%	4.46%	0.00%	0.00%	100.00%
X-n176-k26	47812	26	54.66%	12.10%	63.47%	66.79%	15.79%	45.88%	84.06%	70.79%	11.96%	11.00%	12.87%	0.00%	0.00%	29.15%
X-n181-k23	25569	23	55.76%	2.44%	19.37%	21.49%	7.08%	12.69%	21.75%	41.55%	8.28%	1.83%	0.50%	0.00%	0.00%	23.93%
X-n186-k15	24145	15	66.49%	19.39%	51.50%	58.06%	30.03%	39.68%	28.38%	64.05%	5.78%	5.50%	2.48%	0.00%	0.00%	10.13%
X-n190-k8	16980	8	100.00%	22.96%	39.84%	46.92%	54.88%	52.08%	64.12%	100.00%	14.72%	11.00%	2.48%	0.00%	0.00%	9.04%
X-n195-k51	44225	53	18.54%	3.11%	22.27%	21.12%	1.79%	11.31%	40.02%	30.84%	6.57%	4.33%	5.94%	0.00%	0.00%	71.36%
X-n200-k36	58578	36	59.84%	3.54%	13.69%	15.71%	2.78%	8.29%	39.90%	54.07%	7.10%	3.00%	5.45%	0.00%	0.00%	35.92%
X-n204-k19	19565	19	61.42%	9.38%	36.27%	41.57%	19.61%	24.75%	31.50%	51.76%	7.49%	6.50%	1.49%	0.00%	0.00%	17.10%
X-n209-k16	30656	16	72.48%	15.36%	53.91%	57.50%	32.66%	40.95%	21.01%	51.89%	11.56%	5.17%	4.46%	0.00%	0.00%	13.53%
X-n214-k11	10856	11	81.91%	15.91%	40.20%	47.07%	45.30%	31.39%	79.38%	86.68%	14.45%	26.33%	5.94%	0.00%	0.00%	0.38%
X-n219-k73	117595	73	13.14%	0.00%	10.43%	9.96%	0.00%	3.70%	0.00%	0.00%	2.50%	0.00%	0.00%	0.00%	0.00%	83.74%
X-n223-k34	40437	34	51.52%	8.73%	28.81%	31.60%	7.55%	18.56%	31.58%	46.31%	9.72%	7.83%	3.47%	0.00%	0.00%	31.62%
X-n228-k23	25742	23	72.67%	12.32%	32.12%	35.32%	15.58%	24.99%	60.27%	64.12%	13.93%	11.50%	8.42%	0.00%	0.00%	22.09%
X-n233-k16	19230	17	71.82%	16.91%	41.96%	44.51%	32.15%	35.28%	24.56%	44.98%	12.88%	4.50%	3.96%	0.00%	0.00%	17.86%
X-n237-k14	27042	14	65.81%	27.91%	60.18%	64.03%	50.95%	49.36%	39.83%	72.53%	17.08%	4.00%	1.49%	0.00%	0.00%	18.24%
X-n242-k48	82751	48	49.48%	7.72%	25.39%	27.59%	4.01%	15.61%	39.35%	49.75%	12.35%	6.33%	9.41%	0.00%	0.00%	42.59%
X-n247-k47	37274	51	52.15%	6.66%	18.10%	16.89%	3.72%	11.30%	74.96%	57.02%	13.40%	22.83%	20.30%	0.00%	0.00%	44.36%
X-n251-k28	38684	28	67.48%	11.69%	29.43%	33.25%	11.89%	19.26%	34.25%	59.04%	12.48%	5.50%	3.47%	0.00%	0.00%	16.71%
X-n256-k16	18880	17	75.29%	12.91%	32.58%	35.89%	26.43%	22.85%	35.93%	46.03%	11.04%	11.50%	1.49%	0.00%	0.00%	10.63%
X-n261-k13	26558	13	84.55%	44.75%	76.09%	86.01%	79.86%	75.40%	56.86%	92.89%	18.66%	26.33%	5.94%	0.00%	0.00%	1.06%
X-n266-k58	75478	58	43.06%	5.79%	17.24%	19.10%	2.15%	9.31%	30.97%	37.56%	12.35%	5.17%	5.45%	0.00%	0.00%	45.57%
X-n270-k35	35291	36	55.09%	8.01%	24.42%	26.75%	7.94%	17.65%	27.44%	51.47%	14.85%	4.50%	2.97%	0.00%	0.00%	19.77%
X-n275-k28	21245	28	66.27%	5.06%	15.91%	17.73%	9.05%	11.32%	24.12%	44.89%	15.77%	3.33%	0.50%	0.00%	0.00%	21.06%
X-n280-k17	33503	17	68.98%	41.75%	74.53%	78.06%	61.02%	62.04%	67.23%	79.69%	22.34%	28.00%	13.37%	0.00%	0.00%	2.44%
X-n284-k15	20226	15	90.55%	26.11%	30.28%	36.68%	37.70%	34.76%	50.44%	81.13%	21.55%	29.67%	7.43%	0.00%	0.00%	1.80%
X-n289-k60	95185	61	35.47%	11.79%	24.58%	26.24%	3.76%	13.48%	56.73%	49.04%	13.27%	9.00%	16.34%	0.00%	0.00%	55.33%
X-n294-k50	47167	51	56.48%	10.54%	26.17%	29.25%	5.84%	19.57%	47.27%	44.95%	14.72%	9.67%	11.39%	0.00%	0.00%	36.27%
X-n298-k31	34231	31	65.05%	18.55%	32.52%	36.74%	14.35%	21.76%	34.51%	63.67%	14.32%	8.83%	7.43%	0.00%	0.00%	16.20%
X-n303-k21	21744	21	78.53%	22.84%	35.37%	39.13%	27.44%	26.71%	55.79%	72.03%	20.50%	17.17%	4.95%	0.00%	0.00%	12.90%
X-n308-k13	25859	13	85.12%	47.18%	75.59%	85.34%	96.89%	73.16%	62.34%	91.33%	27.99%	32.83%	9.41%	0.00%	0.00%	0.00%
X-n313-k71	94044	72	33.95%	9.14%	22.38%	24.67%	3.58%	17.74%	73.81%	40.45%	16.16%	15.83%	18.81%	0.00%	0.00%	58.80%
X-n317-k53	78355	53	51.51%	2.08%	6.40%	6.71%	1.98%	3.43%	16.80%	31.15%	13.01%	1.00%	0.00%	56.62%	23.52%	33.14%
X-n322-k28	29866	28	64.90%	18.06%	32.90%	37.85%	18.82%	21.66%	21.15%	54.31%	16.29%	8.17%	4.46%	0.00%	0.00%	13.34%
X-n327-k20	27556	20	67.74%	28.70%	43.58%	48.77%	38.05%	29.25%	44.49%	65.36%	20.11%	17.83%	3.96%	0.00%	0.00%	8.66%
X-n331-k15	31103	15	75.01%	42.12%	64.56%	66.02%	68.38%	52.81%	36.61%	85.61%	25.76%	9.50%	3.47%	0.00%	0.00%	5.15%
X-n336-k84	139210	86	22.22%	13.75%	26.87%	28.93%	2.49%	14.88%	68.65%	44.88%	16.56%	7.50%	25.74%	0.00%	0.00%	79.05%
X-n344-k43	42099	43	53.36%	13.59%	22.56%	26.82%	8.33%	14.57%	35.69%	60.33%	19.05%	8.50%	8.42%	0.00%	0.00%	20.24%

Table B.2: Computation of visual attractiveness measures to BKS of Uchoa et al.'s benchmark [77] (Part B).

Instance	Length	$\|K\|$	$COMP^a$	$COMP^b$	$COMP^c$	$COMP^d$	$COMP^e$	$COMP^f$	$PROX^a$	$PROX^b$	$PROX^c$	CH	$Inter-C$	$Intra-C$	CLP	BE
X-n351-k40	25946	41	64.80%	9.22%	15.35%	16.83%	7.51%	11.98%	45.69%	67.11%	22.86%	25.50%	11.39%	0.00%	0.00%	15.48%
X-n359-k29	51509	29	70.04%	25.90%	33.91%	39.70%	21.61%	27.84%	38.12%	72.11%	26.41%	18.00%	8.42%	0.00%	0.00%	11.20%
X-n367-k17	22814	17	83.66%	34.72%	36.75%	39.25%	47.36%	32.43%	68.01%	79.13%	33.38%	46.50%	5.45%	0.00%	0.00%	3.80%
X-n376-k94	147713	94	32.92%	3.62%	9.43%	9.12%	0.68%	3.35%	3.68%	24.62%	9.33%	0.00%	0.00%	0.00%	0.00%	54.94%
X-n384-k52	66081	53	47.05%	13.47%	21.03%	23.07%	6.36%	12.44%	20.77%	58.20%	17.48%	5.17%	2.97%	0.00%	0.00%	23.72%
X-n393-k38	38269	38	68.62%	17.44%	25.14%	27.69%	12.63%	16.65%	28.36%	55.66%	21.94%	8.67%	5.45%	0.00%	0.00%	12.38%
X-n401-k29	66243	29	84.09%	29.07%	23.66%	30.02%	20.79%	24.49%	58.24%	78.33%	25.62%	29.67%	18.81%	0.00%	0.00%	9.05%
X-n411-k19	19718	19	80.71%	38.99%	33.86%	43.25%	48.27%	38.17%	77.99%	86.63%	32.33%	45.67%	8.91%	0.00%	0.00%	2.72%
X-n420-k130	107798	130	13.50%	5.73%	11.33%	10.09%	0.22%	3.97%	46.11%	33.43%	18.53%	5.67%	19.31%	0.00%	0.00%	83.86%
X-n429-k61	65501	62	49.53%	11.79%	17.86%	20.31%	5.66%	12.82%	21.60%	48.32%	27.07%	5.83%	7.43%	0.00%	0.00%	26.00%
X-n439-k37	36395	37	61.24%	20.07%	24.15%	27.62%	15.17%	16.58%	26.12%	56.75%	23.92%	7.33%	1.49%	0.00%	0.00%	12.83%
X-n449-k29	55358	29	82.07%	46.31%	44.14%	50.33%	34.82%	42.97%	62.95%	92.83%	37.06%	39.00%	20.30%	0.00%	0.00%	3.72%
X-n459-k26	24181	26	78.68%	22.70%	30.19%	28.33%	28.75%	20.49%	61.52%	76.36%	38.37%	29.17%	5.94%	0.00%	0.00%	4.72%
X-n469-k138	222070	140	19.65%	7.19%	12.80%	12.16%	0.43%	5.05%	49.85%	44.70%	22.08%	8.33%	25.74%	0.00%	0.00%	72.41%
X-n480-k70	89535	70	55.15%	11.34%	11.96%	13.60%	3.87%	8.24%	34.22%	56.33%	24.57%	11.33%	10.89%	42.87%	0.76%	24.93%
X-n491-k59	66633	60	69.00%	26.19%	19.73%	24.00%	8.13%	15.13%	41.39%	71.06%	35.61%	24.00%	20.30%	0.00%	0.00%	15.98%
X-n502-k39	69253	39	70.07%	14.02%	12.07%	14.70%	10.59%	10.91%	32.65%	66.33%	33.77%	10.83%	2.97%	0.00%	0.00%	9.47%
X-n513-k21	24201	21	77.17%	44.62%	46.58%	46.78%	59.59%	35.84%	38.62%	78.08%	39.03%	31.33%	8.42%	0.00%	0.00%	4.24%
X-n524-k137	154711	156	50.11%	25.16%	35.05%	30.83%	2.60%	18.10%	95.58%	43.30%	35.35%	27.17%	92.08%	0.00%	0.00%	51.18%
X-n536-k96	95122	97	55.67%	10.62%	7.03%	8.77%	2.68%	8.50%	48.84%	51.21%	34.95%	14.50%	16.34%	30.93%	1.91%	37.84%
X-n548-k50	86822	50	70.10%	23.98%	23.04%	25.16%	12.56%	15.45%	20.78%	69.97%	35.48%	6.00%	1.49%	0.00%	0.00%	12.56%
X-n561-k42	42756	42	74.54%	34.16%	28.04%	31.42%	19.85%	22.63%	37.11%	71.48%	39.82%	23.17%	13.37%	71.44%	2.49%	10.55%
X-n573-k30	50780	30	86.61%	40.37%	21.95%	23.43%	25.89%	24.82%	88.14%	97.60%	54.80%	68.67%	26.73%	100.00%	2.68%	0.89%
X-n586-k159	190543	159	29.91%	7.70%	8.78%	8.68%	0.48%	3.59%	43.59%	44.60%	29.30%	9.17%	19.80%	37.74%	11.47%	61.75%
X-n599-k92	108813	94	54.45%	17.12%	13.98%	16.09%	3.75%	8.43%	27.20%	51.93%	31.67%	12.67%	13.37%	0.00%	0.00%	27.66%
X-n613-k62	59778	62	70.68%	34.45%	23.23%	27.73%	11.91%	19.94%	46.28%	70.16%	44.68%	31.33%	21.29%	0.00%	0.00%	12.13%
X-n627-k43	62366	43	85.54%	33.34%	20.88%	23.59%	16.73%	17.71%	59.78%	93.84%	54.14%	40.17%	15.84%	0.00%	0.00%	3.83%
X-n641-k35	63839	35	76.78%	51.83%	34.83%	37.58%	33.06%	30.13%	52.11%	82.22%	57.03%	36.00%	11.39%	0.00%	0.00%	5.49%
X-n655-k131	106780	131	50.15%	2.87%	0.00%	0.00%	0.47%	0.00%	1.60%	21.09%	28.25%	0.00%	0.50%	45.81%	3.06%	38.55%
X-n670-k126	146705	134	47.02%	33.94%	30.53%	29.99%	5.20%	18.92%	100.00%	62.31%	53.22%	48.17%	76.73%	67.18%	100.00%	45.35%
X-n685-k75	68425	75	68.99%	35.53%	19.29%	21.14%	9.75%	17.69%	46.13%	69.42%	49.41%	39.00%	17.82%	0.00%	0.00%	22.67%
X-n701-k44	82292	44	86.25%	64.88%	31.55%	37.31%	26.53%	29.06%	61.33%	97.63%	60.84%	46.17%	25.74%	0.00%	0.00%	1.93%
X-n716-k35	43525	35	89.61%	51.87%	23.52%	26.01%	29.32%	34.75%	73.52%	92.90%	72.67%	96.00%	35.15%	0.00%	0.00%	3.85%
X-n733-k159	136366	160	52.51%	16.87%	10.32%	11.60%	1.45%	6.05%	43.60%	47.10%	42.05%	16.50%	20.79%	18.75%	2.87%	48.10%
X-n749-k98	77700	98	66.73%	20.92%	11.54%	13.14%	4.87%	10.44%	48.03%	64.43%	55.32%	38.17%	33.17%	30.60%	3.44%	19.91%
X-n766-k71	114683	71	66.15%	67.15%	36.46%	36.63%	18.07%	28.78%	92.42%	83.16%	72.40%	78.17%	50.50%	0.00%	0.00%	23.85%
X-n783-k48	72727	48	74.73%	63.59%	32.51%	36.06%	33.50%	35.36%	49.38%	81.37%	66.49%	53.17%	25.25%	0.00%	0.00%	7.48%
X-n801-k40	73587	40	80.05%	65.31%	33.08%	37.61%	36.31%	26.48%	47.17%	95.25%	67.28%	25.83%	10.89%	0.00%	0.00%	4.91%
X-n819-k171	158611	173	50.37%	9.92%	4.06%	4.36%	0.74%	1.86%	41.21%	47.13%	45.20%	13.83%	23.76%	17.34%	0.38%	42.34%
X-n837-k142	194266	142	60.41%	17.35%	9.05%	10.26%	2.45%	5.98%	35.00%	61.05%	51.12%	16.33%	21.78%	21.12%	4.40%	31.27%
X-n856-k95	89118	95	64.08%	22.08%	11.18%	12.54%	5.84%	7.58%	18.91%	54.75%	52.17%	5.50%	1.49%	0.00%	0.00%	16.54%
X-n876-k59	99715	59	90.31%	48.68%	19.16%	24.17%	18.32%	26.73%	77.52%	90.68%	83.57%	100.00%	65.84%	0.00%	0.00%	2.93%
X-n895-k37	54172	38	84.22%	77.72%	37.53%	40.61%	49.48%	37.18%	51.19%	89.68%	88.96%	57.83%	20.30%	0.00%	0.00%	1.54%
X-n916-k207	329836	208	49.65%	14.70%	6.90%	7.67%	0.88%	3.82%	44.90%	49.62%	51.38%	15.67%	26.73%	0.00%	0.00%	46.59%
X-n936-k151	133105	159	46.74%	42.19%	21.90%	21.56%	5.20%	13.98%	92.86%	66.97%	77.40%	67.17%	100.00%	37.74%	59.46%	43.56%
X-n957-k87	85672	87	69.96%	27.05%	13.66%	15.04%	9.08%	10.42%	27.32%	60.74%	71.22%	23.33%	7.43%	0.00%	0.00%	10.96%
X-n979-k58	119194	58	79.62%	62.02%	15.30%	19.05%	22.18%	28.14%	60.80%	83.83%	89.49%	75.33%	21.78%	51.73%	0.76%	7.61%
X-n1001-k43	72742	43	85.15%	100.00%	39.00%	43.57%	50.88%	44.53%	59.29%	91.09%	100.00%	92.33%	40.10%	0.00%	0.00%	0.76%
MAX(1)			0.82	214188.00	267.59	183.42	2355.24	506.68	0.84	0.80	796.00	600.00	202.00	0.03	0.01	2.43
MIN(1)			0.37	8675.00	22.02	16.23	16.71	23.70	-0.14	0.19	35.00	0.00	0.00	0.00	0.00	0.88
AVERAGE(1)			0.54	53927.15	98.66	72.14	506.56	141.17	0.40	0.56	245.42	119.29	26.99	0.00	0.00	1.26

Obs: (1) The maximum, minimum and average are calculated for the whole set of Uchoa et al.'s benchmark [77], i.e., Tables B.1 and B.2.

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Table B.3: Computation of visual attractiveness measures to crossing-free BKS of Uchoa et al.'s benchmark [77].

Instance	Length	$\|K\|$	$COMP^a$	$COMP^b$	$COMP^c$	$COMP^d$	$COMP^e$	$COMP^f$	$PROX^a$	$PROX^b$	$PROX^c$	CH	$Inter-C$	$Intra-C$	CLP	BE
X-n317-k53	78356	53	54.81%	2.08%	6.30%	6.71%	1.98%	3.43%	16.80%	31.15%	13.01%	1.00%	0.00%	0.00%	0.00%	33.17%
X-n480-k70	89535	70	56.77%	11.34%	11.96%	13.60%	3.87%	8.24%	34.22%	56.33%	24.57%	11.33%	10.89%	0.00%	0.00%	24.67%
X-n536-k96	95122	97	57.78%	10.62%	7.03%	8.77%	2.68%	8.50%	48.84%	51.21%	34.95%	14.50%	16.34%	0.00%	0.00%	37.68%
X-n561-k42	42756	42	75.98%	34.16%	28.04%	31.42%	19.85%	22.63%	37.11%	71.48%	39.82%	23.17%	13.37%	0.00%	0.00%	10.42%
X-n573-k30	50780	30	88.49%	40.37%	21.95%	23.43%	25.89%	24.82%	88.14%	97.60%	54.80%	68.67%	26.73%	0.00%	0.00%	0.66%
X-n586-k159	190543	159	31.96%	7.70%	8.81%	8.68%	0.48%	3.59%	43.59%	44.60%	29.30%	9.17%	19.80%	0.00%	0.00%	61.62%
X-n655-k131	106780	131	54.20%	2.63%	0.01%	0.00%	0.51%	0.19%	1.60%	21.09%	28.25%	0.00%	0.50%	0.00%	0.00%	38.30%
X-n670-k126	146705	134	47.28%	34.05%	30.50%	29.99%	5.19%	18.85%	100.00%	62.47%	53.22%	48.17%	76.73%	0.00%	0.00%	45.32%
X-n733-k159	136366	160	53.93%	16.87%	10.31%	11.60%	1.45%	6.05%	43.60%	47.10%	42.05%	16.50%	20.79%	0.00%	0.00%	48.04%
X-n749-k98	77700	98	69.04%	20.92%	11.54%	13.14%	4.87%	10.44%	48.03%	64.43%	55.32%	38.17%	33.17%	0.00%	0.00%	19.85%
X-n819-k171	158611	173	52.96%	9.92%	4.06%	4.36%	0.74%	1.86%	41.21%	47.13%	45.20%	13.83%	23.76%	0.00%	0.00%	42.28%
X-n837-k142	194266	142	62.07%	17.35%	9.05%	10.26%	2.45%	5.98%	35.00%	61.05%	51.12%	16.33%	21.78%	0.00%	0.00%	31.27%
X-n936-k151	133105	159	47.19%	42.21%	21.97%	21.56%	5.20%	13.97%	92.86%	66.97%	77.40%	67.17%	100.00%	0.00%	0.00%	43.54%
X-n979-k58	119194	58	82.86%	62.02%	15.30%	19.05%	22.18%	28.14%	60.80%	83.83%	89.49%	75.33%	21.78%	0.00%	0.00%	7.57%
MAX$^{(1)}$			0.82	214188.00	267.59	183.42	2355.24	506.68	0.84	0.80	796.00	600.00	202.00	0.03	0.01	2.43
MIN$^{(1)}$			0.38	8675.00	22.30	16.30	16.71	23.70	-0.14	0.19	35.00	0.00	0.00	0.00	0.00	0.88
AVERAGE$^{(1)}$			0.55	53927.15	98.61	72.13	506.56	141.17	0.40	0.56	246.09	119.15				

Obs: $^{(1)}$ The maximum, minimum and average are the ones calculated for Tables B.1 and B.2.

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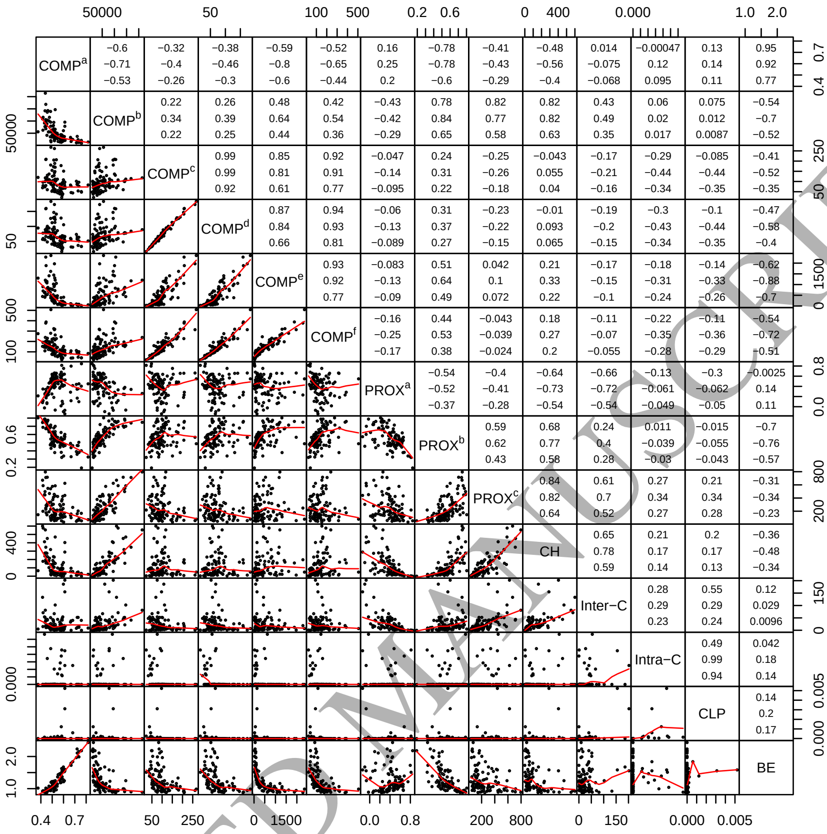

Figure B.1: Correlation matrix of Uchoa et al.'s CVRP benchmark [77].

Appendix C. Visual comparison.

In this section we aim at graphically illustrate the concept of visual attractiveness and the correlation with the measures described in Section 4. The routing plans are shown in groups of three images and there is one group for every measure. However, because there are strong correlations among some measures, a unique Figure is used to illustrate $COMP^c$, $COMP^d$, $COMP^e$ and $COMP^f$ (Figure C.3), $PROX^c$, CH and $Inter-C$ (Figure C.6), and $Intra-C$ and CLP (Figure C.7). The groups are homogeneous in terms of number of customers and the type of scheduling horizon (see Section 5). For

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example, in Figure C.1 the three instances have 200 clients and have a long scheduling horizon, i.e., it allows many customers to be served by the same vehicle.

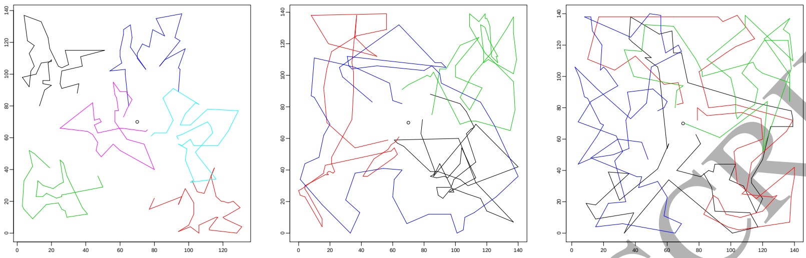

Figure C.1: From left to right instances C2_2_1, RC2_2_3 and R2_2_9 with a nice, intermediate and unattractive value of $COMP^a$. The three instances have 200 customers.

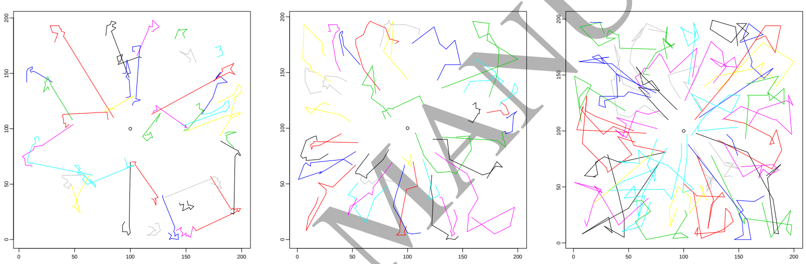

Figure C.2: From left to right instances C1_4_8, RC1_4_10 and R1_4_1 with a nice, intermediate and unattractive value of $COMP^b$. The three instances have 400 customers.

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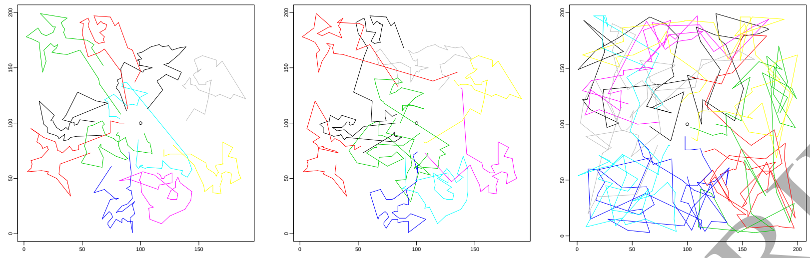

Figure C.3: From left to right instances C2_4_4, C2_4_8 and R2_4_1 with nice, intermediate and unattractive value of $COMP^c$ ($COMP^d$, $COMP^e$ or $COMP^f$). The three instances have 400 customers.

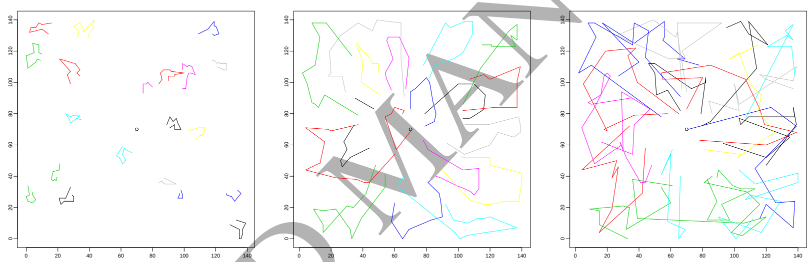

Figure C.4: From left to right instances C1_2_5, R1_2_8 and R1_2_1 with a nice, intermediate and unattractive value of $PROX^a$. The three instances have 200 customers.

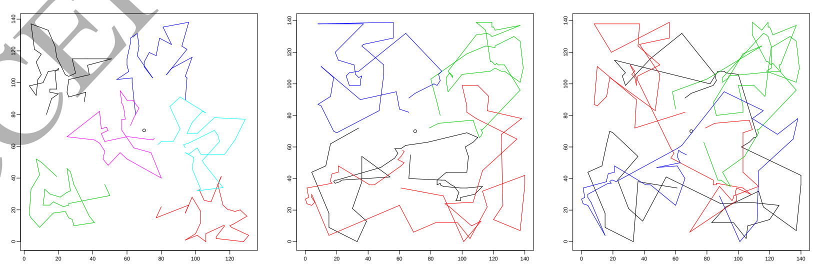

Figure C.5: From left to right instances C2_2_1, RC2_2_8 and RC2_2_6 with a nice, intermediate and unattractive value of $PROX^b$. The three instances have 200 customers.

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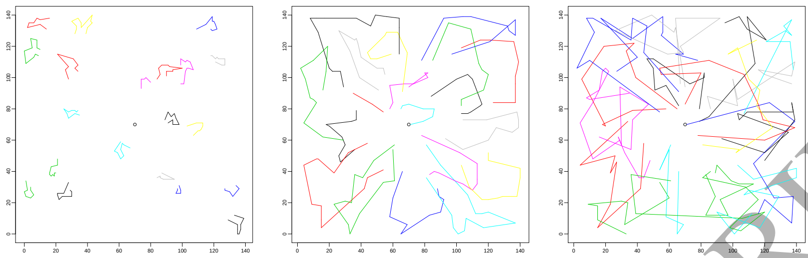

Figure C.6: From left to right instances C1_2_5, R1_2_4 and R1_2_1 with nice, intermediate and unattractive values of $PROX^c$ (CH or $Inter - C$). The three instances have 200 customers.

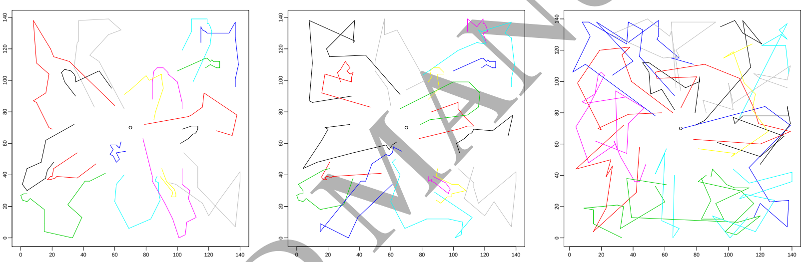

Figure C.7: From left to right instances RC1_2_4, RC1_2_9 and R1_2_1 with nice, intermediate and unattractive value of $Intra - C$ (or CLP). The three instances have 200 customers.

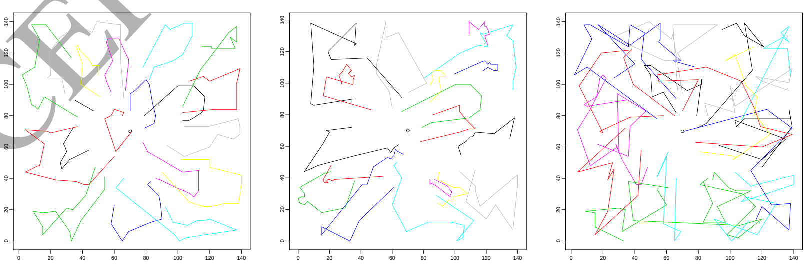

Figure C.8: From left to right instances R1_2_8, RC1_2_9 and R1_2_1 with a nice, intermediate and unattractive value of BE. The three instances have 200 customers.

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