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Simple methods to predict the minimum baking time of bread

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Abstract

Baking is a complex transformation process since many coupled physical phenomena take place within the product. For practical industrial purposes, it would be desirable to count on simple methods to predict accurately the process time. Unlike food preservation operations, two different process times can be defined: the critical or minimum time is determined by the complete dough/crumb transition and ensures the acceptability of the product; the quality time is given by a target value of a certain sensory attribute (e.g. surface colour), and it is associated with preference of consumers. Despite the existing physics-based models which aim to describe comprehensively the baking process, there is a gap between academic knowledge and the industrial practice and needs of design engineers. Therefore, in this work we explore three simple methods to predict the minimum baking time of bread, which are based on a previously developed and validated heat and mass transport model. All three simple methods (two heat transfer models and one regression equation) predict very well the critical time for a wide and common range of operating conditions; mean absolute relative error is 3.61%, 1.17% and 0.30%, respectively. The degree of difficulty regarding implementation of simple methods is also discussed. Finally, it is demonstrated that heat and mass transfer can be decoupled for certain calculations, by using appropriate simplifications based on knowledge of transport phenomena governing the process.

Keywords: Evaporation front, Moving boundary problem, Optimisation, Process design, Simulation

1. Introduction

One of the main interests of design engineers and equipment users is to count on simple and accurate prediction methods for the simulation of the process they are dealing with, and mainly for the calculation of process times as a function of material characteristics and operating conditions (Goi et al., 2008). Prediction of process times is important since they determine the residence times in equipment. However, it could be a difficult task to develop such simple methods in the case of complex processes like (bread) baking, where many coupled physical phenomena take place, i.e. multiphase heat and mass transport, water evaporation, volume expansion and formation of a porous structure, starch gelatinisation, crust development, browning reactions (Mondal & Datta, 2008; Purlis & Salvadori, 2009a; Nicolas et al., 2014). A similar situation can be found in other food operations with simultaneous heat and mass transfer and phase change, like freezing, thawing and drying.

Prediction methods can be divided into empirical-based and physics-based, or inductive and deductive methods (or models), respectively (Broyart & Trystram, 2003). The empirical or inductive approach aims

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to find a relationship between inputs (e.g. product characteristics and operating conditions) and outputs (e.g. process time), by using experimental data and a mathematical tool (i.e. black box model), e.g. response surface methodology, artificial neural networks, etc. The physics-based or deductive approach is based on transport phenomena models, occasionally coupled with kinetic models describing physicochemical changes in the product as a function of operating variables, e.g. browning development, degradation kinetics, etc. Both approaches are valid, with their advantages and limitations; the final decision will depend on the available resources and specific objectives. Nevertheless, when a complex process like baking is considered, implementing a physics-based method could be very difficult, since analytical solutions are not possible. Therefore, process time has to be calculated by using numerical methods, but the effort required to perform this task makes it impractical for the design engineer.

There are some cases in food process engineering where considerable research has been dedicated to develop analytical, semi-analytical or empirical, simple and accurate time process prediction methods that make use of simplifying assumptions and equations. In the case of freezing, the Planck's equation is the most widely known basic method; several simple models were developed afterwards by incorporating corrections to Planck's model, as well as numerical methods (Garca-Armenta et al., 2016). Similarly, simple methods for thawing time prediction are available, as the inverse operation to freezing (Goi et al., 2008). There also has been much research into prediction of chilling times by simple methods, e.g. Lin et al. (1996a,b). Another important operation is the drying of solids, where simple models of moisture transfer has been proposed for prediction of drying times, e.g. Sahin et al. (2002), Sahin & Dincer (2005). Equally essential for food industry, thermal processing operations have been subject of numerous studies to provide simple methods to predict pasteurisation and sterilisation times (and blanching times by analogy), having the Ball's formula and Stumbo's method as reference (Ghazala et al., 1991; Teixeira, 2006). Finally, "cooking charts" for shrimp were developed by using previously developed mathematical models to help the industry in the optimisation of thermal processing of shrimp and to enhance quality (Erdodu et al., 2003).

Unlike thermal processing, freezing and other preservation operations, baking is a transformation process with no microbiological risk a priori, and thus the definition of a process time is not straightforward. The end point of (bread) baking is generally established by assessing sensory attributes, in particular, the surface colour, which together with texture and flavour play a key role in the *preference* of the product by consumers (Purlis & Salvadori, 2007). However, when surface colour is used to determine the end point of baking, it is possible not to achieve a complete dough/crumb transition due to an incomplete starch gelatinisation (Purlis, 2011). That is, a complete starch gelatinisation ensures the sensory *acceptability* of the product because it determines the full transformation of dough into crumb, i.e. it ensures a minimum baking (Zanoni et al., 1995). Consequently, two different times have been identified in the baking process: a *critical time* (CT) and a *quality time* (QT) (Purlis, 2012). The CT is the minimum baking time, defined as the time necessary to achieve a complete transition of dough into crumb given by a complete starch gelatinisation; it has to be assessed at the coldest point of bread, where temperature has to reach 96 °C at least. The QT is defined as the time required to achieve the target value of a given quality attribute, relevant with regard to sensory preference of the product. For example, a target value of surface lightness representing the desired surface colour of bread, which can be established by sensory data obtained from preference of consumers. Overall, the CT is an objective parameter, while the QT is a subjective parameter, depending on various particular factors.

In the same direction as previous works regarding other food operations, the objective of this research is to develop simple methods to predict accurately the critical or minimum baking time of bread (CT), in order to help with design, optimisation and control of the process. Besides the efforts and advance in modelling comprehensively the product behaviour during baking, e.g. Zhang et al. (2005); Lucas et al. (2015); Nicolas et al. (2014, 2016, 2017), there exists a gap between such complex models and the actual industrial practice, especially at small and medium scale production. And to the best of the author's knowledge, no simple model (in the discussed terms) is available for baking time prediction in the open literature. For such aim, three methods are explored based on a previously developed and validated heat and mass transport model of bread baking. The critical time mainly depends on product properties and operating conditions, so it is expected that the proposed methods are of general application for baking and related processes.

2. Methodology

2.1. Case of study and general considerations

The case of study is conventional baking of French bread (without mould or tin, e.g. *baguette*) in a static or batch, indirect oven (e.g. electric baking oven). This is a typical case of traditional bread baking at small and medium scale production, still the major scale production of bread in countries with agricultural tradition, e.g. France, Argentina. In a conventional baking oven, the generated heat is transferred to the product by three modes: conduction, convection, and radiation. Heat conduction occurs from the hot solid surfaces in direct contact with the product. Such surfaces can be a baking support or any supporting device if no mould is used, e.g. sole, tray, grate, conveyor band. In order to obtain conclusions of general application, heat conduction from solid surfaces is not taken into account in this study; there exists a large diversity regarding this aspect of oven design and configuration. On the other hand, convection and radiation contributions can be studied more systematically. However, for sake of simplicity in the proposed prediction methods, radiation will be included into an "apparent" heat transfer coefficient, together with convection heating mode (Carson et al., 2006). Furthermore, steam injection is not considered in this study (for similar reasons as for conduction). An introduction to heat and mass transfer during baking can be found elsewhere (Purlis, 2016).

Bread is considered as an infinite cylinder of constant radius R (volume change is not considered), so the problem is reduced to a single dimension via the axial symmetry assumption. For initial conditions, uniform temperature (25 °C) and water content (0.65 kg/kg, dry basis; 39.4%, wet basis) are assumed.

The range of operating conditions is the following:

- Product radius (R): 0.025, 0.030, 0.035 m.

- Oven temperature (T_∞): 180, 200, 220, 240 °C.
- Apparent heat transfer coefficient (h): 10, 20, 30, 40 W/(m^2 K).
- Relative humidity in oven ambient is assumed to be negligible (RH = 0%; no steam injection).

The tested values of operating conditions are considered as representative and within the range of common practice for the proposed case of study (Carson et al., 2006), and also coincide with previous studies used as starting point of the present research (Purlis, 2011, 2012, 2014).

The critical time (CT) is calculated as the time necessary to reach 96 °C at bread core ($r = 0$).

2.2. Reference method

A previously developed and validated heat and mass transfer model (Purlis & Salvadori, 2009a,b) is taken as the reference method to obtain the *actual* critical times and to develop the simple methods. A similar procedure was used by Erdodu et al. (2003), and to design, optimise and obtain technological insights into the bread baking process (Purlis, 2011, 2012, 2014). It is worth to note that more complex models are available in the literature, e.g. Zhang et al. (2005); Lucas et al. (2015); Nicolas et al. (2014, 2016, 2017); however, the objective of this work is to develop simple methods to predict the minimum baking time by characterising the process based on knowledge about transport phenomena. In this way, the chosen model as reference has demonstrated to describe adequately the main features of bread baking for practical purposes.

The model includes the main distinguishing features of bread baking, i.e. the rapid heating of bread core and the development of a dry outer crust. Bread baking is considered as a moving boundary problem (MBP) where simultaneous heat and mass transfer with phase change occurs in a porous medium. Bread is modelled as a system containing three different regions: (i) *crumb*: wet inner zone, where temperature does not exceed 100 °C and dehydration does not occur; (ii) *crust*: dry outer zone, where temperature exceeds 100 °C and dehydration occurs; (iii) *evaporation front*: between the crumb and crust, where temperature is ca. 100 °C and water evaporates (liquid-vapour transition).

Mathematically, the MBP is formulated using a physical approach, where phase change is incorporated in the model by defining equivalent thermophysical properties. Major assumptions of the model are the following: (i) bread is homogeneous and continuous; the concept of porous medium is included through effective or apparent thermophysical properties; (ii) heat is transported by conduction inside bread according to Fourier's law, but an effective thermal conductivity is used to incorporate the evaporation-condensation mechanism in heat transfer; (iii) only liquid diffusion in the crumb and only vapour diffusion in the crust are assumed to occur; (iv) volume change is neglected.

Considering previous assumptions, governing equations are the following:

Heat balance equation:

$$\rho C_p \frac{\partial T}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left(rk \frac{\partial T}{\partial r} \right) \tag{1}$$

Mass balance equation:

$$\frac{\partial W}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(rD\frac{\partial W}{\partial r}\right) \tag{2}$$

Boundary condition at surface for heat balance states that heat arrives to bread by convection and radiation, through the apparent heat transfer coefficient, and is balanced by conduction inside the bread:

$$-k\frac{\partial T}{\partial r} = h\left(T - T_\infty\right) \tag{3}$$

For the mass balance, water migrating towards the bread surface is balanced by convective flux:

$$-D\rho_{\mathrm{s}}\frac{\partial W}{\partial r} = k_{\mathrm{g}}\left[a_{\mathrm{w}}P_{\mathrm{sat}}\left(T\right) - \left(\mathrm{RH}/100\right)P_{\mathrm{sat}}\left(T_\infty\right)\right] \tag{4}$$

At the centre of bread, i.e. $r = 0$:

$$\frac{\partial T}{\partial r} = 0 \tag{5}$$

$$\frac{\partial W}{\partial r} = 0 \tag{6}$$

For a more detailed description of the model, including thermophysical properties, the reader is referred to Purlis & Salvadori (2009a,b).

2.3. First method

The first simple method is a simplified version of the reference model, based on knowledge developed about transport phenomena occurring during baking; simplifications are the following:

- Mass transfer is neglected; moisture-dependent properties are evaluated using the initial water content value.
- Only crumb is considered, so thermophysical properties correspond to crumb zone only.
- Since the beginning of baking, while surface temperature is below 100 °C, convective flux condition at surface boundary is valid.
- When temperature at surface reaches 100 °C, a prescribed temperature condition is used until the end of the process, when the core temperature attains 96 °C.

Considering these simplifications, governing equations are the following:

Heat balance equation:

$$\left(\rho\,C_p\right)_{\mathrm{cb}}\frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(rk_{\mathrm{cb}}\frac{\partial T}{\partial r}\right) \tag{7}$$

Boundary condition at surface ($r = R$):

$$\text{if } T < 100\ ^\circ\text{C}: \quad -k_{\text{cb}} \frac{\partial T}{\partial r} = h\left(T - T_\infty\right) \tag{8}$$

$$\text{else}: \quad T = 100\ ^\circ\text{C} \tag{9}$$

At the centre of bread ($r = 0$):

$$\frac{\partial T}{\partial r} = 0 \tag{10}$$

Thermophysical properties for crumb (Purlis & Salvadori, 2009b):

$$k_{\text{cb}} = \frac{0.9}{1 + \exp\left[-0.1\left(T - 353.16\right)\right]} + 0.2 \tag{11}$$

$$\rho_{\text{cb}} = 180.61 \tag{12}$$

$$C_{p,\text{cb}} = 1000\, W\left(5.207 - 73.17 \times 10^{-4} T + 1.35 \times 10^{-5} T^2\right) + 5T + 25 \tag{13}$$

where $W = 0.65$ kg/kg (dry basis).

This first method attempts to simulate the moving boundary problem in a simpler manner than the original heat and mass transfer model, in order to make easier its implementation. The effects of crust formation are simplified by using a non-moving evaporation front at surface, and considering the product composed of dough/crumb only. These simplifying assumptions are based on the fact that crust is usually a thin layer (a few mm only) and that crumb maintains its initial water content during the process (Purlis & Salvadori, 2009a). That is, the main phenomena of the process are extracted and incorporated in a simpler way in a heat transfer model. This method is not able to predict changes related to mass transfer (e.g. weight loss), but it is worth to bear in mind that the specific objective of this work is to accurate predict the critical time in a simple way for industrial practical purposes.

2.4. Second method

The second simple method is in turn a simplified version of the first method: thermophysical properties are now considered to be constant. By exploring the variation of properties with temperature through Eqs. (11) and (13), it can be seen that thermal conductivity is the only property that presents a wide range of values within the temperature range of the simplified model (25-100 °C), due to the evaporation-condensation phenomenon (Purlis & Salvadori, 2009b). So, an optimisation routine is used to find an appropriate constant value for thermal conductivity in order to predict accurately the critical time. That is, we search the value of thermal conductivity that minimises the difference between the critical time predicted by the reference method and this second method, for each operating condition.

Next, all estimated 48 values of thermal conductivity, corresponding to the full range of operating conditions, are used to obtain a single average "optimum" value, k^*_{cb}. Finally, the critical time is recalculated using the second method with this k^*_{cb} value and the obtained results are compared against the reference method.

Besides the assumption of constant properties, all other simplifications proposed for the first method are maintained. For density, the same constant value is used (Eq. (12)); in the case of specific heat, an average value of 4.4843 kJ/(kg K) is used, calculated by using Eq. (13) in the temperature range 25-100 °C.

This second method also tries to simulate the moving boundary problem, but even in a simpler manner than the first method and the original heat and mass transfer model, to make even easier its implementation. It is expected then that results are more limited than the first method, beyond prediction of critical time.

2.5. Third method

The third simple method consists basically of a regression equation: results from the reference method (i.e. *actual* CT values) are used to obtain a simple prediction equation relating the material characteristics and operating conditions to the critical time. For this aim, we utilise basic concepts from dimensional analysis, where dimensionless groups or numbers are used to represent certain physical behaviour without necessarily depending on governing equations. In our case, we use the following (classical) groups: Fourier number or dimensionless time (Fo), dimensionless temperature (T^*), and Biot number (Bi). So, the following dimensionless relationship ψ can be established:

$$\mathrm{Fo} = \psi\left(T^*, \mathrm{Bi}\right) \tag{14}$$

This is the typical relationship of charts for solution of unsteady transport problems, e.g. Gurney-Lurie charts. Note that dimensionless position is not necessary for this problem since the centre ($r = 0$) is the only position of interest, i.e. $r^* = r/R = 0$. By analysing the (raw) results from the reference method, the following simple expression is proposed to be tested:

$$\mathrm{Fo} = a\left(T^*_{\mathrm{cr}}\right)^{-b} \tag{15}$$

where

$$b = c\left(\mathrm{Bi}\right)^{-d} \tag{16}$$

Dimensionless groups for our case are defined as follows:

$$\mathrm{Fo} = \frac{\alpha_0\,\mathrm{CT}}{R^2} \tag{17}$$

$$T^*_{\mathrm{cr}} = \frac{T_{\mathrm{cr}} - T_\infty}{T_0 - T_\infty} \tag{18}$$

$$\mathrm{Bi} = \frac{h\,R}{k_{\mathrm{cb},0}} \tag{19}$$

Subscript 0 indicates property evaluated at initial conditions; this is an arbitrary choice to simplify the calculation, as it is made in freezing or thawing (Garca-Armenta et al., 2016). T_{cr} is the core temperature value (96 °C) taken to calculate the critical time (CT).

A regression procedure is performed to found the value of constants a, c and d. Afterwards, a "baking chart" can be constructed for the evaluated range of operating conditions.

2.6. Simulations and numerical procedures

The heat and mass transport model (reference method) and its simplified versions (first and second simple methods) were solved using the finite element method; the numerical procedure was implemented in COMSOL Multiphysics 3.4 (COMSOL AB, Burlington, MA, USA) coupled with MATLAB R2007b (The MathWorks Inc., USA). In all cases, the 1D mesh consisted of 368 elements, where the maximum element size at the open boundary (surface) was set to 1×10^{-4} m (default values were used for the rest of parameters). This mesh ensured convergence and quality of results. In addition, time step was fixed to 0.01 min, for the same reasons.

The optimisation procedure for the second method was implemented by using an optimisation routine from MATLAB, i.e. *fminbnd* function (the algorithm is based on golden section search and parabolic interpolation). Similarly, regression procedure for the third method was performed by using the *lsqcurvefit* function of MATLAB (medium-scale optimisation using Levenberg-Marquardt method with line-search). In both cases, different initial search values were tested to ensure convergence.

3. Results and discussion

3.1. First method

Fig. 1 shows the comparison between critical times obtained by the first simple method and the reference method, while Table 1 presents the relative errors for prediction. Errors are calculated for each baking condition according to:

$$e\,(\%) = \frac{(t_{\mathrm{ref}} - t_{\mathrm{pred}})}{t_{\mathrm{ref}}} \times 100 \qquad (20)$$

Before analysing the prediction performance, a brief and general comment about the critical times shown in Fig. 1 and elsewhere: as it is expected from transport phenomena theory, CT diminishes for increasing intensity conditions (increasing h and T^*) and decreasing radius. Then, the mean relative error considering all 48 tested conditions is -3.61%. It can be seen from Fig. 1 and Table 1 that the simple method overestimates the critical time in all cases, i.e. the mean absolute relative error is 3.61%. This is mainly because of the non-moving evaporation front set at surface, a simplification for this first method. In the actual process, the evaporation front at 100 °C moves towards the core of the product during baking, so the thermal gradient is greater than in the stationary front situation, for the same temperature difference. Therefore, the centre achieves the critical temperature more rapidly in the reference model than in the simplified one. In addition, prediction errors are greater for more intensive heating conditions and increasing radius, due to a more rapid setting and advance of the evaporation front in the actual process, and the effect of increasing distance on the thermal gradient, respectively. Nevertheless, this systematic overestimation by the simple method could be considered as a safety factor in order to ensure the complete dough/crumb transition in all cases.

As an additional result, although it is not the objective of the present research, the first method predicts very well the temperature variation at bread centre. A representative example is shown in Fig. 2; this condition was chosen since it presents a similar prediction error (-3.99%) than the mean error for all conditions (-3.61%). It can be observed that profiles from both methods are almost identical. In the same way, the simple method is able to predict well the surface temperature variation until the prescribed temperature boundary condition is set. That is, it predicts well also the time required for the evaporation front to be established.

Regarding the implementation of the proposed method, it can be done by using relatively simple numerical methods, e.g. finite difference method. Also, it can be implemented in commercial software like COMSOL Multiphysics without any further complexity or programming skills. It is worth noting that, as with any other model, users have to take into account simplifications made and associated restrictions of the method to interpret results and extract conclusions.

3.2. Second method

The development of the second method consisted of two steps. Firstly, a constant value of thermal conductivity was calculated for each operating condition to match the corresponding reference critical time, according to the proposed optimisation procedure; obtained results are shown in Table 2. A clear trend is observed: thermal conductivity increases with heating intensity and product radius. The reason is associated with the previous discussion about increasing prediction errors for the first method: as the optimisation procedure attempts to match CT from both methods, the optimum values of thermal conductivity compensate the effects of moving evaporation front and increasing thermal gradient. That is, the simple method needs to increase heat transfer by conduction to equal the reference CT. In other words, the optimisation procedure is searching values of thermal conductivity that generate no prediction errors. So, to reduce values of Table 1 to zero, thermal conductivity values are higher with increasing intensity and product radius.

Secondly, the average "optimum" thermal conductivity k^*_{cb} was found to be 0.7826 W/(m K) (standard deviation = 0.0173); the minimum and maximum values were 0.7365 and 0.8299 W/(m K), respectively. This average k^*_{cb} was used to recalculate the critical times with a single thermal conductivity for all baking conditions, in order to have a simple prediction method. Comparison of results against the reference method are shown in Fig. 3, while prediction errors calculated by Eq. (20) are summarised in Table 3.

In this case, the simple method generates both positive and negative relative errors. Moreover, it can be observed the following trend by inspection of Tables 2 and 3: for operating conditions where the initial individual estimation (Table 2) is smaller than k^*_{cb}, the prediction error is positive, i.e. the reference CT is greater than the predicted one, according to Eq. (20). The reason is intrinsically related to previous discussion about compensation of the simplifications introduced in the simple method. Since the values of Table 2 simulate the reference CT, if a greater value is then used to recalculate the CT, it will result in a lesser CT due to a more rapid heat transfer, and thus, in a positive relative error. Similarly, negative prediction errors correspond to the operating conditions where initial thermal conductivity values are greater than k^*_{cb}. Finally, the mean absolute relative error of the second method is 1.17% (average of absolute values shown in Table 3).

Unlike the first method, this simplified model does not reproduce well the variation of core temperature during baking, as it can be seen in Fig. 4 (same operating condition as in Fig. 2). The typical sigmoid trend of core temperature is due to evaporation-condensation phenomenon, which is modelled through Eq. (11). A constant value of thermal conductivity generates instead a typical profile of pure conductive materials. However, the low prediction errors demonstrate the ability of the method to achieve the established objective.

In addition, this second method is more easy to implement than the first method, since all thermo-physical properties are assumed constant. In this way, numerical methods like finite difference method, or even the charts for solution of unsteady transport problems can be used to calculate the minimum baking time. Also important, analytical solutions of the heat transport equation can be used, e.g. Caro-Corrales & Cronin (2016).

3.3. Third method

Table 4 shows the regression results for modelling the full data set of reference CT values with Eqs. (15) and (16), while Fig. 5 presents the comparison of dimensionless critical times (Fo) between the reference method and regression equation, i.e. the goodness of the adjustment. Again, the simple method predicts accurately the critical times calculated by the reference method: the mean absolute relative error for Fo prediction is 0.30%, while the correlation coefficient is 0.9999. That is, the proposed relationship between the minimum baking time and operating conditions represents well the results provided by the reference method, taken as the *actual* CT in this work.

Obviously, this third method is not able to generate temperature profiles, as the previous ones. But, it is the simplest method of the ones explored in this research, since it can be easily implemented in a spreadsheet or even in a calculator. A similar procedure could be carried out for other critical values, i.e. different characteristic values for t and T defining the dimensionless time and temperature in Eqs. (17) and (18), respectively. This methodology has been used to construct the mentioned unsteady transport charts.

Finally, a "baking chart" can be generated for a certain range of operating conditions to have a graphical representation of the simple method, as it is shown in Fig. 6. As a reference, the range of adjusted values is 0.54-0.67 for T^* and 1.23-6.87 for Bi.

4. Conclusions

Three simple methods to predict the minimum (or critical) baking time of bread were proposed and tested by using a reference method, i.e. a previously developed and validated heat and mass transfer model for bread baking. The first and second methods are simplified versions of the reference transport model, so they can be catalogued as physics-based methods. On the other hand, the third method is a three-parameter regression equation, i.e. an empirical-based method. All three simple methods are able to predict accurately the critical time of baking. With regard to implementation, the second and third methods are the much easier to use considering industrial practice. Nevertheless, the first method can be also useful to predict the temperature variation at bread core without using a more complex model.

In addition, some important aspects regarding transport phenomena have been investigated through the simplifications proposed: we demonstrated that heat and mass transfer can be decoupled for certain calculations. That is, a relatively simple heat transfer problem can be proposed to simulate the process to accurately predict the temperature variation at bread core, considering practical processing times. By using appropriate (and still simple) boundary conditions, bread can be modelled as a single material (dough/crumb), where moisture content remains constant. An interesting challenge would be to decouple and deal only with the mass transfer aspects of the problem, so weight loss could be predicted in a simple manner also, for industrial purposes. This will be the focus of a future work.

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Broyart, B., & Trystram, G. (2003). Modelling of heat and mass transfer phenomena and quality changes during continuous biscuit baking using both deductive and inductive (neural network) modelling principles. *Food and Bioproducts Processing*, *81*, 316–326. doi:10.1205/096030803322756402.

Caro-Corrales, J., & Cronin, K. (2016). Nonsteady-state heat transfer. In *Advances in heat transfer unit operations: Baking and freezing in bread making* (pp. 79–112). CRC Press. doi:10.1201/9781315374543.

Carson, J. K., Willix, J., & North, M. F. (2006). Measurements of heat transfer coefficients within convection ovens. *Journal of Food Engineering*, *72*, 293–301. doi:10.1016/j.jfoodeng.2004.12.010.

Erdodu, F., Balaban, M. O., & Otwell, W. S. (2003). Construction of shrimp cooking charts using previously developed mathematical models for heat transfer and yield loss predictions. *Journal of Food Engineering*, *60*, 107–110. doi:10.1016/S0260-8774(03)00023-2.

Garca-Armenta, E., Carvajal, M. X. Q., & Alamilla-Beltrn, L. (2016). Freezing time calculations. In *Advances in heat transfer unit operations: Baking and freezing in bread making* (pp. 279–311). CRC Press. doi:10.1201/9781315374543.

Ghazala, S., Ramaswamy, H. S., Smith, J., & Simpson, B. (1991). Thermal process time calculations for thin profile packages: comparison of formula methods. *Journal of Food Process Engineering*, *13*, 269–282. doi:10.1111/j.1745-4530.1991.tb00073.x.

Goi, S. M., Oddone, S., Segura, J. A., Mascheroni, R. H., & Salvadori, V. O. (2008). Prediction of foods freezing and thawing times: Artificial neural networks and genetic algorithm approach. *Journal of Food Engineering*, *84*, 164–178. doi:10.1016/j.jfoodeng.2007.05.006.

Lin, Z., Cleland, A. C., Cleland, D. J., & Serrallach, G. F. (1996a). A simple method for prediction of chilling times: extension to three-dimensional irregular shapes. *International Journal of Refrigeration*, *19*, 107–114. doi:10.1016/0140-7007(95)00082-8.

Lin, Z., Cleland, A. C., Cleland, D. J., & Serrallach, G. F. (1996b). A simple method for prediction of chilling times for objects of two-dimensional irregular shape. *International Journal of Refrigeration*, *19*, 95–106. doi:10.1016/0140-7007(95)00081-X.

Lucas, T., Doursat, C., Grenier, D., Wagner, M., Trystram, G., & Flick, D. (2015). Modeling of bread baking with a new, multi-scale formulation of evaporation-condensation-diffusion and evidence of compression in the outskirts of the crumb. *Journal of Food Engineering*, *149*, 24–37. doi:10.1016/j.jfoodeng.2014.07.020.

Mondal, A., & Datta, A. K. (2008). Bread baking - A review. *Journal of Food Engineering*, *86*, 465–474. doi:10.1016/j.jfoodeng.2007.11.014.

Nicolas, V., Glouannec, P., Ploteau, J.-P., Salagnac, P., & Jury, V. (2017). Experiment and multiphysic simulation of dough baking by convection, infrared radiation and direct conduction. *International Journal of Thermal Sciences*, *115*, 65–78. doi:10.1016/j.ijthermalsci.2017.01.018.

Nicolas, V., Salagnac, P., Glouannec, P., Ploteau, J.-P., Jury, V., & Boillereaux, L. (2014). Modelling heat and mass transfer in deformable porous media: Application to bread baking. *Journal of Food Engineering*, *130*, 23–35. doi:10.1016/j.jfoodeng.2014.01.014.

Nicolas, V., Vanin, F., Grenier, D., Lucas, T., Doursat, C., & Flick, D. (2016). Modeling bread baking with focus on overall deformation and local porosity evolution. *AIChE Journal*, *62*, 3847–3863. doi:10.1002/aic.15301.

Purlis, E. (2011). Bread baking: Technological considerations based on process modelling and simulation. *Journal of Food Engineering*, *103*, 92–102. doi:10.1016/j.jfoodeng.2010.10.003.

Purlis, E. (2012). Baking process design based on modelling and simulation: Towards optimization of bread baking. *Food Control*, *27*, 45–52. doi:10.1016/j.foodcont.2012.02.034.

Purlis, E. (2014). Optimal design of bread baking: Numerical investigation on combined convective and infrared heating. *Journal of Food Engineering*, *137*, 39–50. doi:10.1016/j.jfoodeng.2014.03.033.

Purlis, E. (2016). Heat and mass transfer during baking. In *Advances in heat transfer unit operations: Baking and freezing in bread making* (pp. 173–188). CRC Press. doi:10.1201/9781315374543.

Purlis, E., & Salvadori, V. O. (2007). Bread browning kinetics during baking. *Journal of Food Engineering*, *80*, 11071115. doi:10.1016/j.jfoodeng.2006.09.007.

Purlis, E., & Salvadori, V. O. (2009a). Bread baking as a moving boundary problem. Part 1: Mathematical modelling. *Journal of Food Engineering*, *91*, 428–433. doi:10.1016/j.jfoodeng.2008.09.037.

Purlis, E., & Salvadori, V. O. (2009b). Bread baking as a moving boundary problem. Part 2: Model validation and numerical simulation. *Journal of Food Engineering*, *91*, 434–442. doi:10.1016/j.jfoodeng.2008.09.038.

Sahin, A. Z., & Dincer, I. (2005). Prediction of drying times for irregular shaped multi-dimensional moist solids. *Journal of Food Engineering*, *71*, 119–126. doi:10.1016/j.jfoodeng.2004.10.024.

Sahin, A. Z., Dincer, I., Yilbas, B. S., & Hussain, M. M. (2002). Determination of drying times for regular multi-dimensional objects. *International Journal of Heat and Mass Transfer*, *45*, 1757–1766. doi:10.1016/S0017-9310(01)00273-3.

Teixeira, A. (2006). Thermal processing of canned foods. In *Handbook of Food Engineering, 2nd Edition* (pp. 745–797). CRC Press. doi:10.1201/9781420014372.

Zanoni, B., Peri, C., & Bruno, D. (1995). Modelling of starch gelatinization kinetics of bread crumb during baking. *LWT - Food Science and Technology*, *28*, 314–318. doi:10.1016/S0023-6438(95)94458-3.

Zhang, J., Datta, A. K., & Mukherjee, S. (2005). Transport processes and large deformation during baking of bread. *AIChE Journal*, *51*, 2569–2580. doi:10.1002/aic.10518.

Nomenclature

a, c, d	Parameters in Eqs. (15) and (16)
a_{w}	Water activity
Bi	Biot number
C_p	Specific heat (J/(kg K))
D	Water (liquid or vapour) diffusion coefficient ($\mathrm{m^2/s}$)
e	Relative error (%)
Fo	Fourier number
h	Heat transfer coefficient (W/($\mathrm{m^2}$ K))
k	Thermal conductivity (W/(m K))
k_{g}	Mass transfer coefficient (kg/(Pa $\mathrm{m^2}$ s))
P_{sat}	Saturation vapour pressure (Pa)
R, r	Radius, radial coordinate (m)
RH	Relative humidity (%)
T	Temperature (K)
t	Time (s)
T^*	Dimensionless temperature
W	Water content, dry basis (kg/kg dm)

Greek symbols

α	Thermal diffusivity ($\mathrm{m^2/s}$)
ρ	Density ($\mathrm{kg/m^3}$)

Subscripts

0	(Evaluated at) Initial condition
∞	Ambient (oven)
cb	Crumb
cr	Critical value
pred	Predicted value
ref	Reference value
s	Solid

Table 1. Relative errors (%) calculated through Eq. (20) for prediction of critical times by first simple method, for all operating conditions.

		h (W/(m^2 K))			
R (m)	T_∞ (°C)	10	20	30	40
0.025	180	−0.23	−0.80	−1.97	−2.54
	200	−0.26	−1.35	−3.03	−4.69
	220	−0.38	−1.94	−4.15	−6.18
	240	−0.51	−2.55	−5.30	−7.74
0.030	180	−0.18	−1.19	−2.67	−4.36
	200	−0.40	−1.90	−3.99	−6.09
	220	−0.58	−2.64	−5.14	−7.88
	240	−0.86	−3.61	−6.65	−9.50
0.035	180	−0.34	−1.71	−3.48	−5.35
	200	−0.54	−2.63	−5.05	−7.35
	220	−0.87	−3.60	−6.60	−9.25
	240	−1.16	−4.67	−8.20	−11.12

Table 2. Thermal conductivity (W/(m K)) values estimated by the optimisation procedure for the second simple method, for all operating conditions.

		h (W/(m^2 K))			
R (m)	T_∞ (°C)	10	20	30	40
0.025	180	0.7365	0.7652	0.7662	0.7759
	200	0.7656	0.7707	0.7759	0.7843
	220	0.7830	0.7751	0.7828	0.7941
	240	0.7886	0.7782	0.7852	0.8074
0.030	180	0.7497	0.7650	0.7697	0.7804
	200	0.7735	0.7712	0.7799	0.7932
	220	0.7833	0.7770	0.7892	0.8074
	240	0.7854	0.7817	0.7999	0.8180
0.035	180	0.7597	0.7667	0.7759	0.7871
	200	0.7743	0.7737	0.7864	0.8020
	220	0.7797	0.7812	0.7986	0.8163
	240	0.7793	0.7879	0.8090	0.8299

Table 3. Relative errors (%) calculated through Eq. (20) for prediction of critical times by second simple method, for all operating conditions.

		h (W/(m^2 K))			
R (m)	T_∞ (°C)	10	20	30	40
0.025	180	1.74	1.25	1.44	0.85
	200	0.77	0.86	0.83	−0.15
	220	0.00	0.65	0.00	−1.36
	240	−0.41	0.40	−0.88	−2.79
0.030	180	1.49	1.44	1.24	0.30
	200	0.47	0.99	0.20	−1.15
	220	0.00	0.66	−0.72	−2.81
	240	−0.23	0.10	−1.90	−4.31
0.035	180	1.21	1.45	0.87	−0.46
	200	0.48	0.90	−0.45	−2.29
	220	0.17	0.22	−1.86	−3.97
	240	0.12	−0.52	−3.18	−5.73

Table 4. Regression results for the third simple method, i.e. Eqs. (15) and (16). Confidence intervals for parameters correspond to 95% confidence.

Parameter	Value	Confidence interval	Residual sum of squares (RSS)
a	0.1389	$[0.1381, 0.1397]$	
c	1.7385	$[1.7288, 1.7481]$	2.4201×10^{-5}
d	0.8932	$[0.8790, 0.9073]$	

Figure captions

Fig. 1. Critical times (min) obtained by reference method and first simple method. Symbols indicate different values of h (W/(m^2 K)): ○, 10; ×, 20; △, 30; +, 40. Colours indicate different values of R (m): blue, 0.025; black, 0.030; red, 0.035. For the same symbol (h) and colour (R), oven temperature increases from right to left (180, 200, 220, 220 °C). Solid line represents perfect correlation.

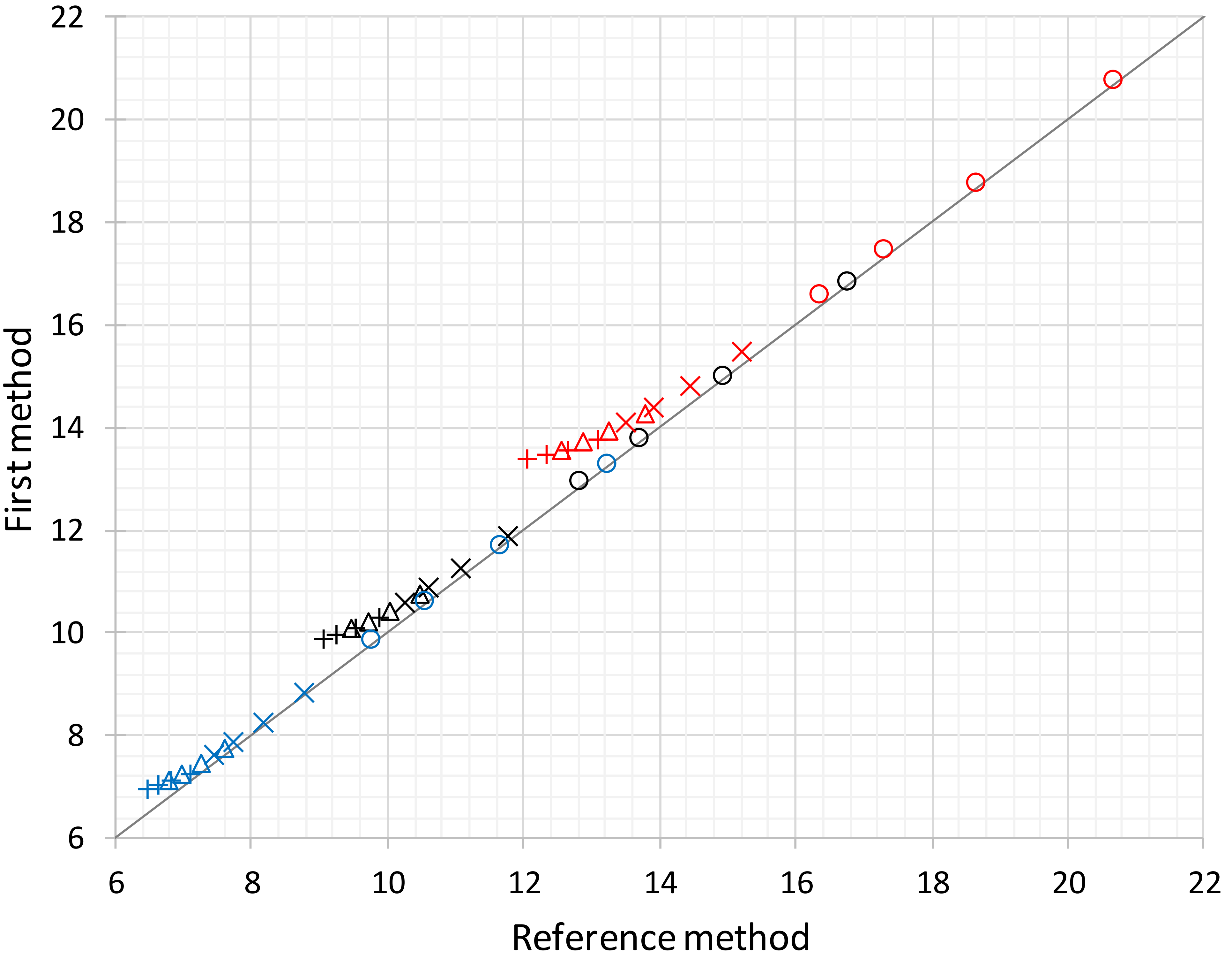
Fig. 2. Temperature variation at centre and surface of bread with $R = 0.030$ m, for baking at 200 °C, with $h = 30$ W/(m^2 K). Solid lines represent reference method and symbols, first simple method.

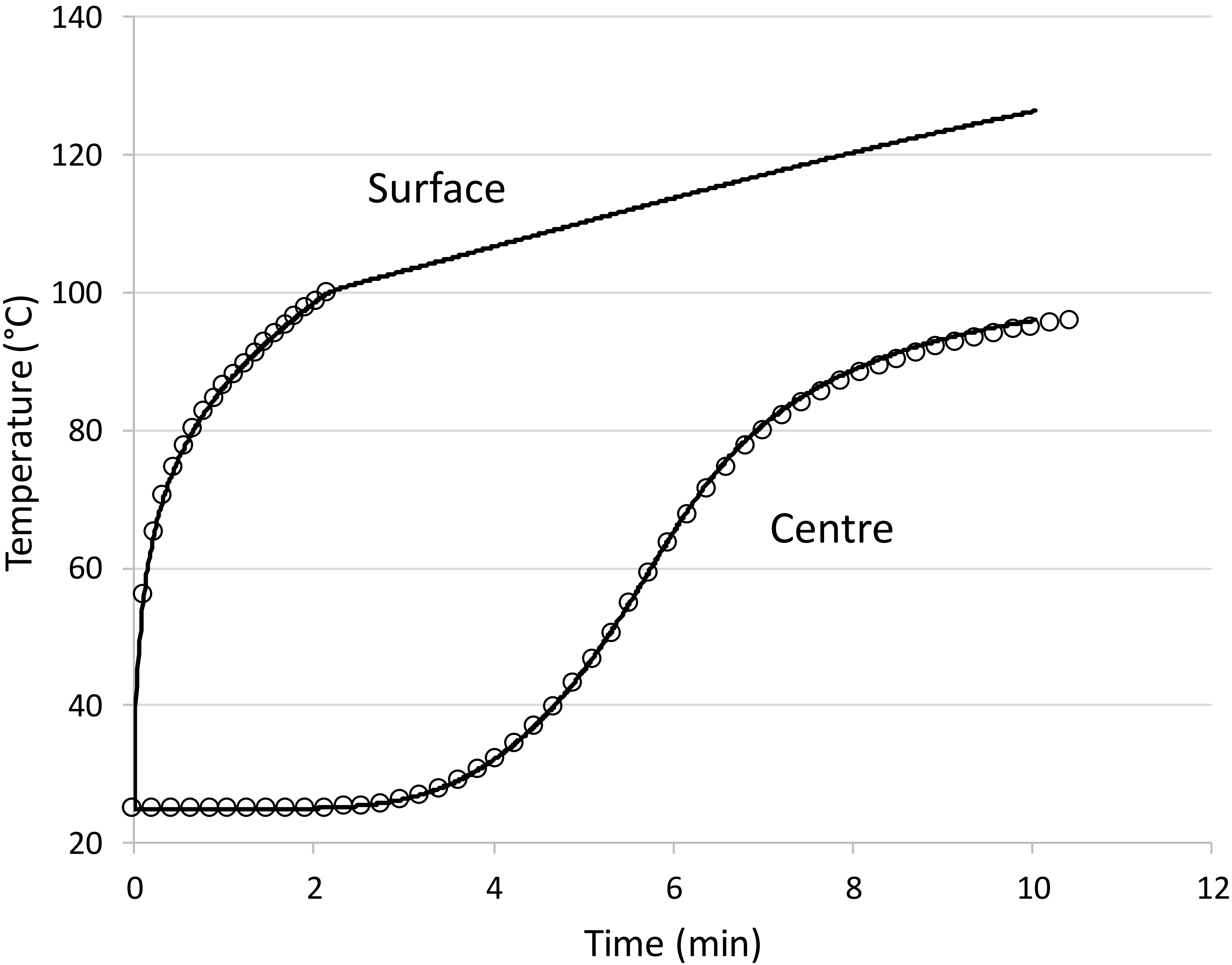
Fig. 3. Critical times (min) obtained by reference method and second simple method. Symbols indicate different values of h (W/(m^2 K)): ○, 10; ×, 20; △, 30; +, 40. Colours indicate different values of R (m): blue, 0.025; black, 0.030; red, 0.035. For the same symbol (h) and colour (R), oven temperature increases from right to left (180, 200, 220, 220 °C). Solid line represents perfect correlation.

Fig. 4. Temperature variation at centre of bread with $R = 0.030$ m, for baking at 200 °C, with $h = 30$ W/(m^2 K), obtained by the reference and second methods.

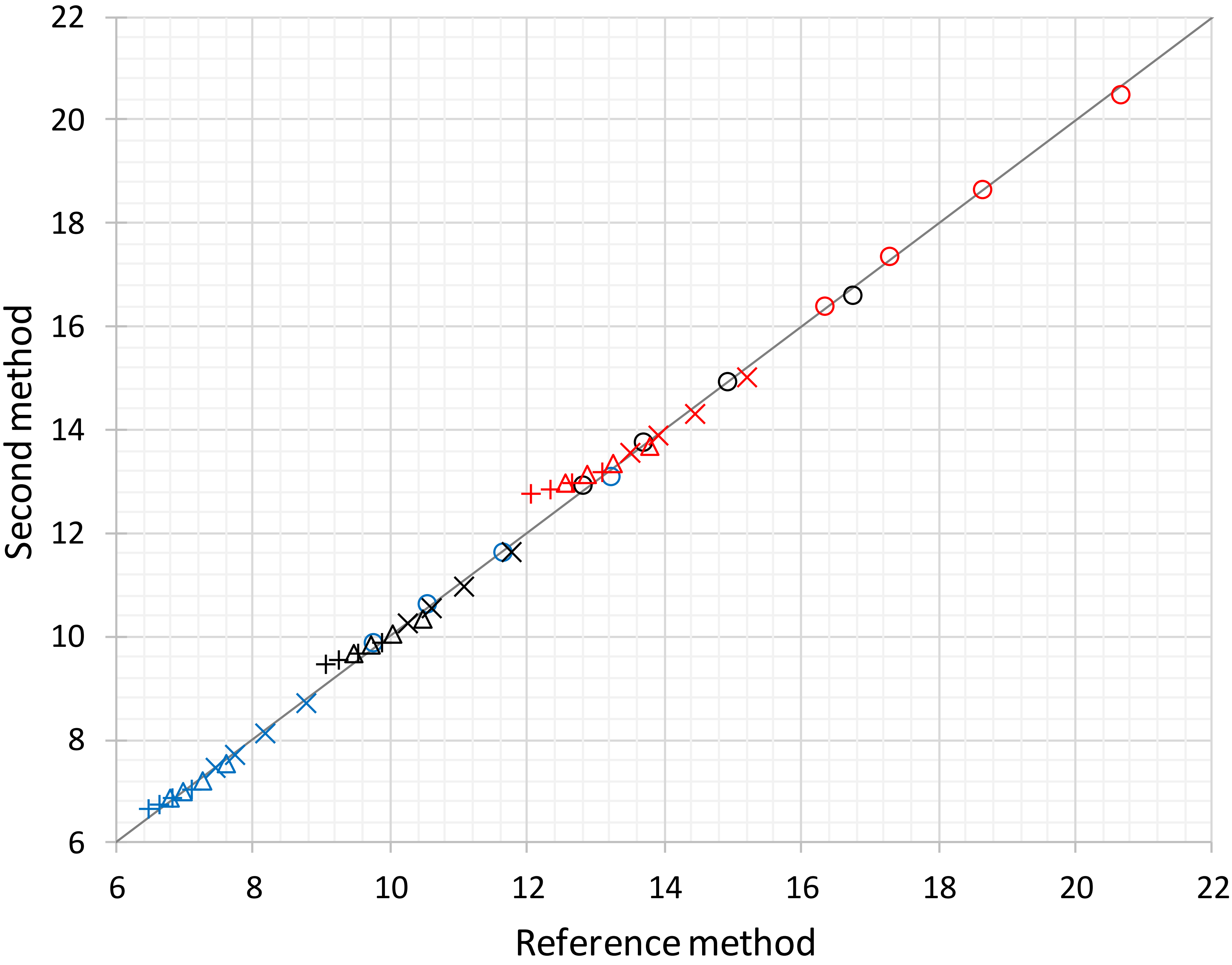
Fig. 5. Dimensionless critical times (Fo, symbols) obtained by reference method and third simple method. Solid line represents perfect correlation.

Fig. 6. Baking chart obtained from the third simple method, by using Eqs. (15) and (16). Fo, T^* and Bi are defined in Eqs. (17)-(19), respectively. Dashed lines indicate extrapolated values. Solid lines account for values within the tested value of operating conditions.

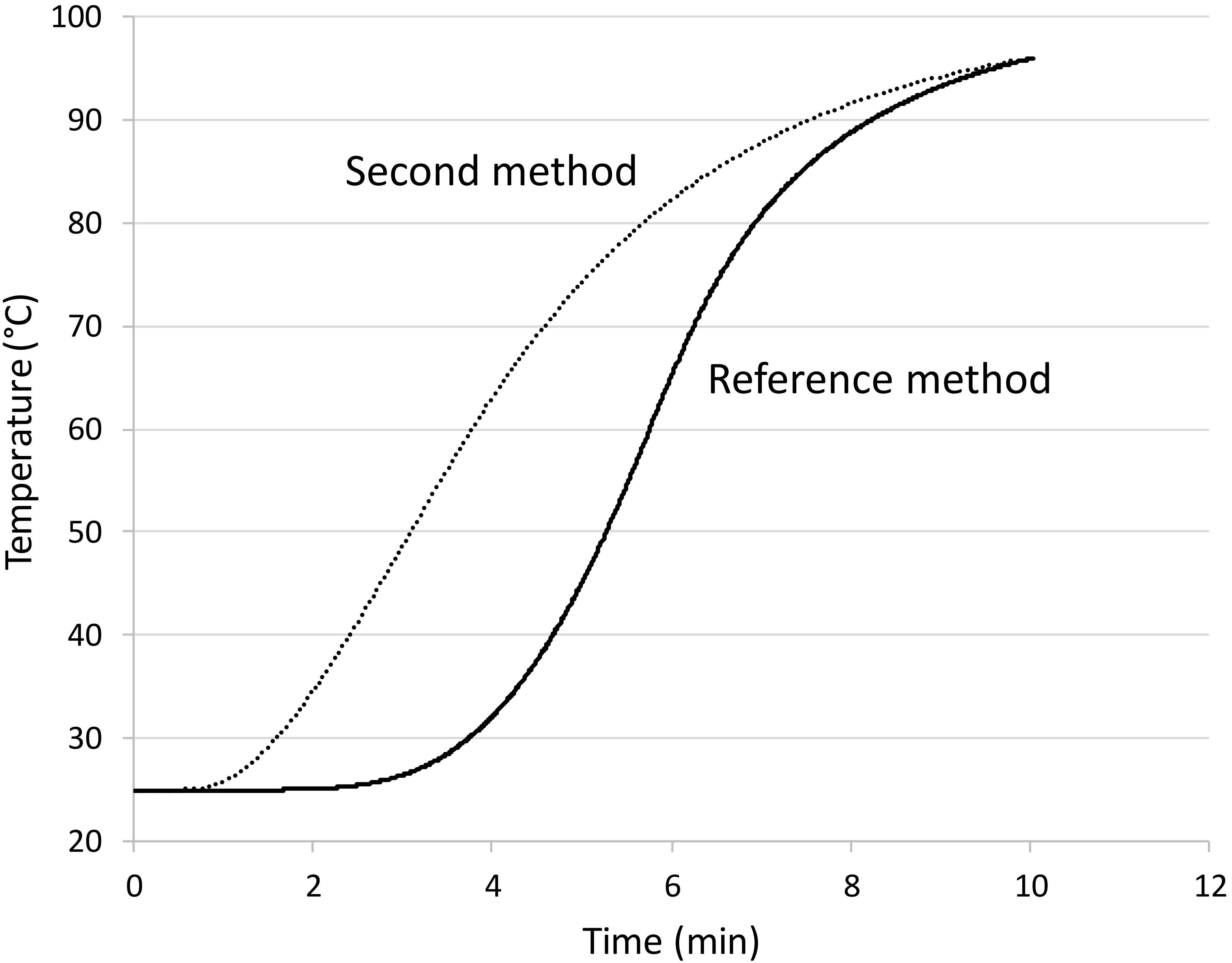

22
20
18
16
14
12
10
8
6
First method
6
8
10
12
14
16
18
20
22
Reference method



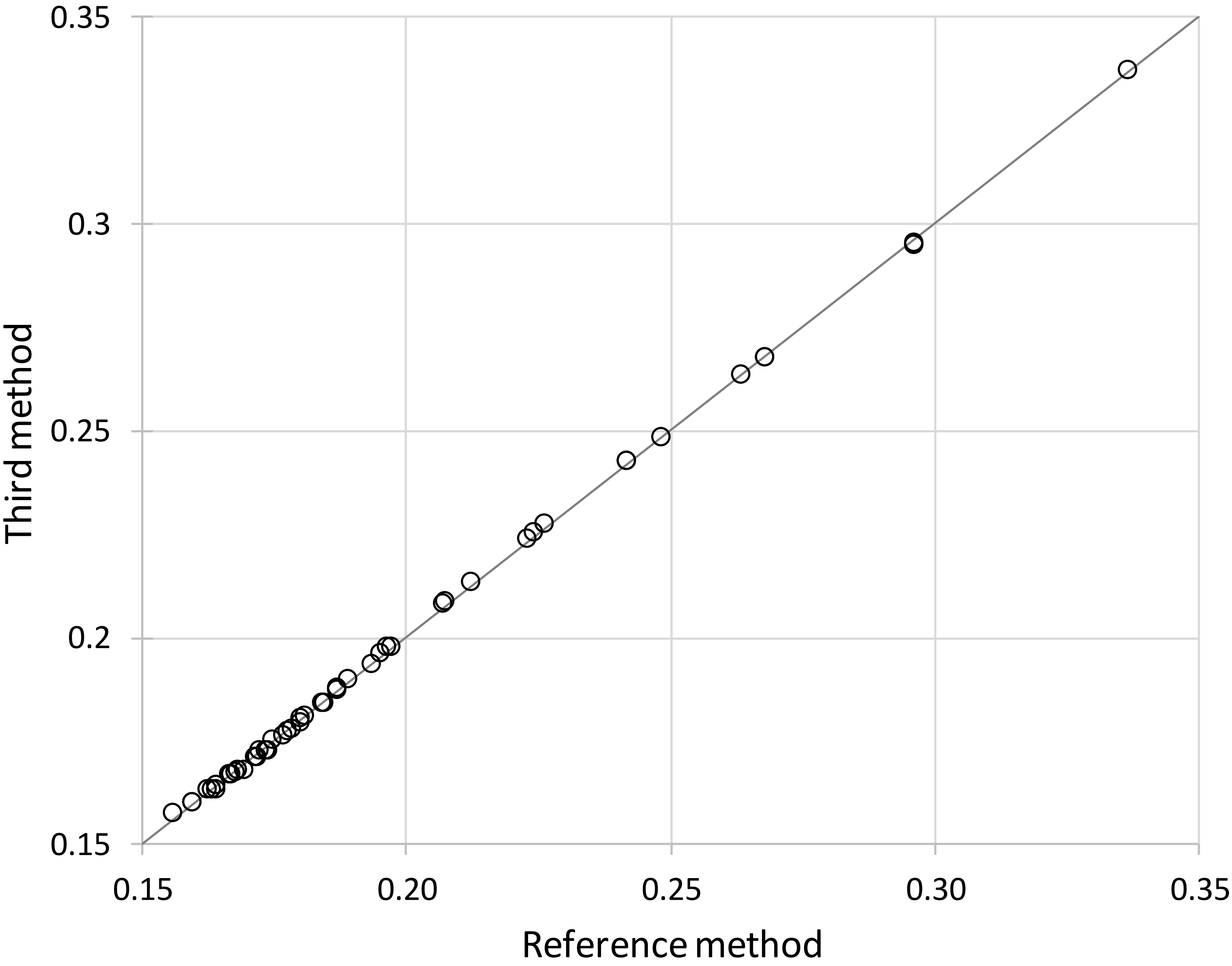
140
120
100
80
60
40
20
0
2
4
6
8
10
12
Temperature (°C)
Time (min)
Surface
Centre



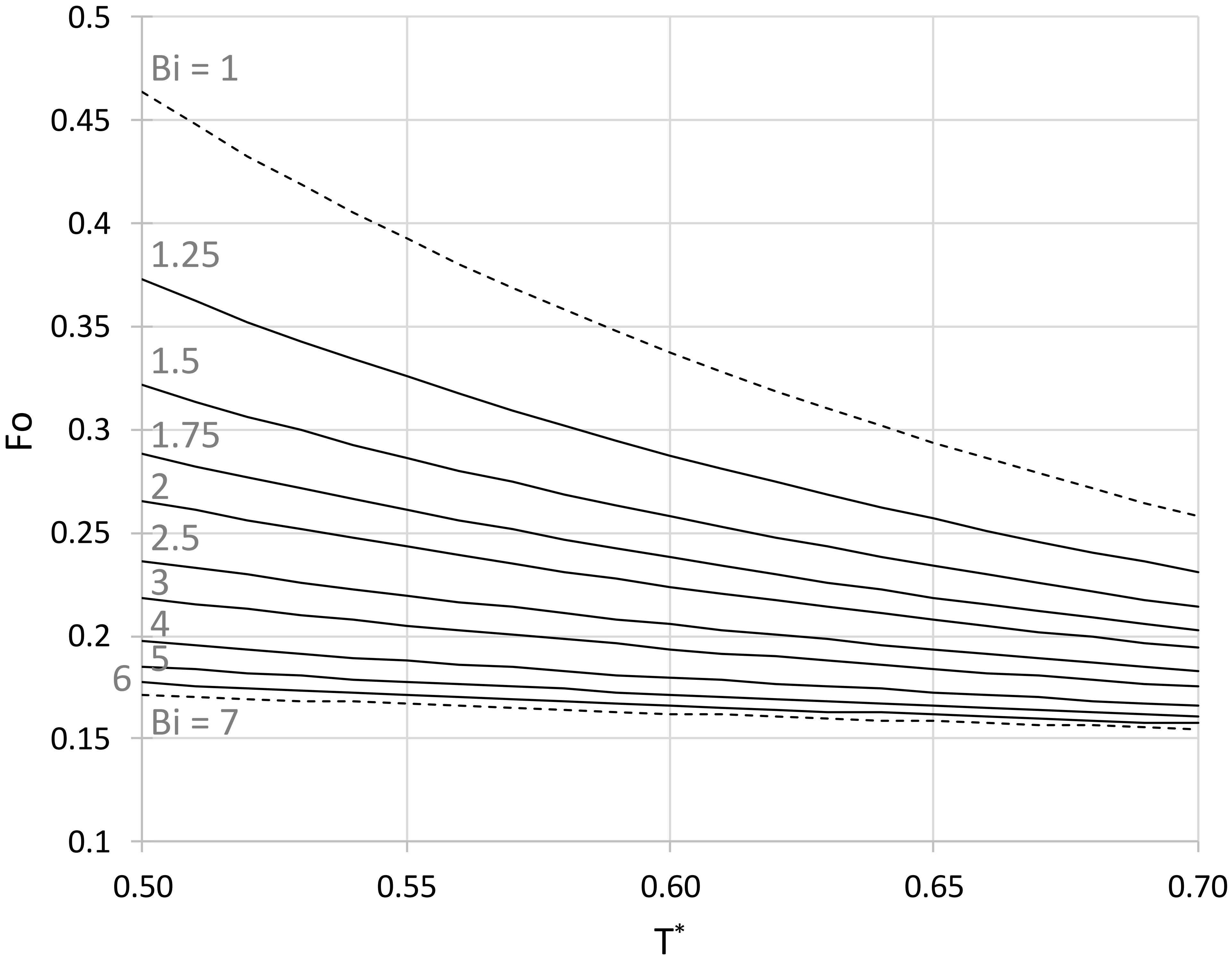
22
20
18
16
14
12
10
8
6
Second method
6
8
10
12
14
16
18
20
22
Reference method



100
90
80
70
60
50
40
30
20
Temperature (°C)
Second method
Reference method
0
2
4
6
8
10
12
Time (min)



0.35
0.3
0.25
0.2
0.15
Third method
0.15
0.20
0.25
0.30
0.35
Reference method



Bi = 1
1.25
1.5
1.75
2
2.5
3
4
5
6
Bi = 7
Fo
T*
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.50
0.55
0.60
0.65
0.70

Highlights

- Baking is a complex transformation process where many coupled physical phenomena take place.
- There exists a gap between complex models and actual industrial practice, mainly for process times prediction.
- Three simple methods are presented to predict accurately the minimum baking time of bread.
- Explored methods are based on a previously developed and validated heat and mass transfer model.
- All simple methods are able to predict accurately the minimum baking time, with different degree of difficulty regarding implementation.