Longitudinal Magnification Drawing Mistake

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Lateral magnification in image formation by positive lenses, mirrors, and dioptrics is usually appropriately developed in most optics textbooks.[1–9] However, the image of a three-dimensional object occupies a three-dimensional region of space. The optical system affects both the transverse and the longitudinal dimensions of the object and, in general, does it in different ways. The magnification in the direction of the optical axis (the longitudinal magnification) is seldom treated. In several texts, the concept of longitudinal magnification is not even considered. Symmetrical objects (such as arrows) are used and their images appear laterally inverted. It is not shown how a longitudinally nonsymmetric object is imaged. One of the few books where this subject is well treated is in the textbook by Hecht.[10] We have repeatedly verified in our classes that there is some confusion related to this subject. Students tend to believe that the image is longitudinally symmetric with respect to the lens optic center. Some prestigious texts commit the same mistake. In addition, a very nice optics book,[11] a catalogue of optical hardware,[12] a worldwide scientific magazine,[13] a paper in an optics journal,[14] and a Spanish encyclopedia,[15] for example, have also been found to contain this error in drawing the image of a three-dimensional object formed by a positive lens. In this paper we suggest that the teaching of longitudinal magnification should be done with some care and we include a figure showing a properly drawn image.

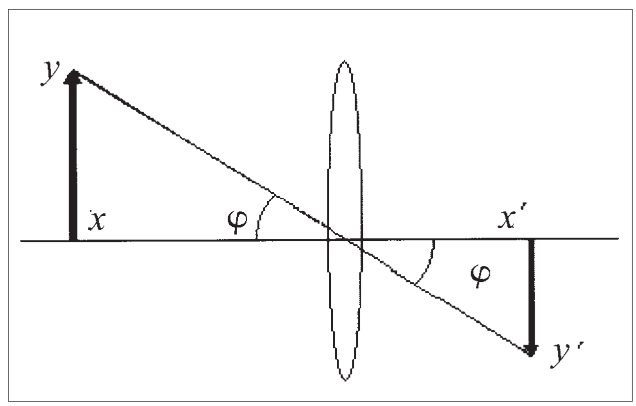

Fig. 1. Calculation of the lateral magnification. The ray through the center of the lens is essentially undeviated.

The well-known thin-lens equation is

$$\frac{1}{x} + \frac{1}{x'} = \frac{1}{f}, \qquad (1)$$

where x is the object distance, x' is the image distance, and f is the focal length of the lens.

Lateral magnification $m \equiv \frac{y'}{y}$ is usually obtained by tracing the ray going through the center of the lens (see Fig. 1). This ray does not deviate significantly, so that, by using the equality of the tangents of the angles φ, we obtain

$$m \equiv \frac{y'}{y} = \frac{-x'}{x} = 1 + \frac{-x'}{f}, \qquad (2)$$

and using

 DOI: 10.1119/1.1639966

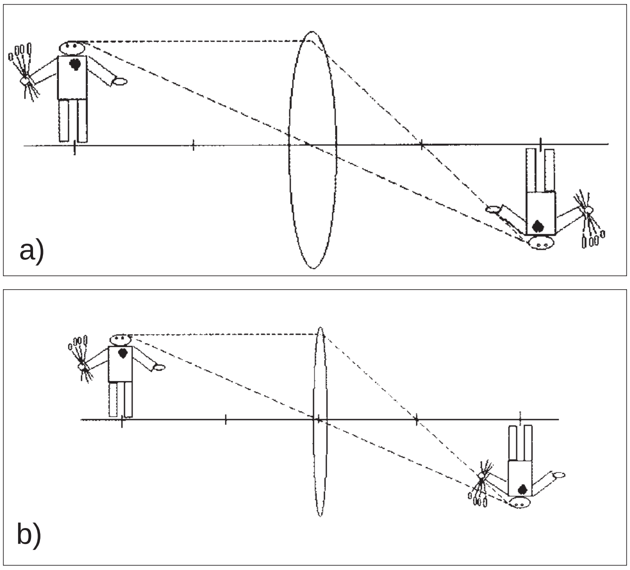

Fig. 2. (a) Incorrect drawing: The orientation of the image in the longitudinal direction is reversed. (b) The correct orientation of the image (the small variation in the value of the longitudinal magnification is not shown).

$$x' = \frac{xf}{x-f} \tag{3}$$

results in

$$m = 1 + \frac{x}{x-f} = \frac{f}{x-f}. \tag{4}$$

The calculation of the longitudinal magnification $L \equiv dx'/dx$ required a little exercise in calculus. By taking the derivative in Eq. (3), we obtain

$$L = \frac{dx'}{dx} = \frac{f(x-f) - xf}{[x-f]^2} = \frac{-f^2}{[x-f]^2} = -m^2, \tag{5}$$

where the result in Eq. (4) has been used.

The longitudinal magnification L is related to the lateral magnification m by $L = -m^2$. The negative sign means that if an arrow is pointing toward the lens, its real image will point away from the lens.

This may be verified by elementary ray tracing and, of course, experiment. The result follows: The ordering of the points in the image space in the direction of the optical axis is *the same* as in the object. To make it more evident it is convenient to find the image of more than one object point.

It seems that image inversion in the transverse direction leads students (and sometimes teachers) to also conclude that the same inversion occurs in the longitudinal direction. Usually they draw the image of an asymmetric object as in Fig. 2(a) while the correct way is as shown in Fig. 2(b). Longitudinal inversion indeed occurs in images produced by spherical and plane mirrors and in some cases in holography. Some particular situations showing that longitudinal inversion does not occur in image formation by lenses can be found in Ref. 10.

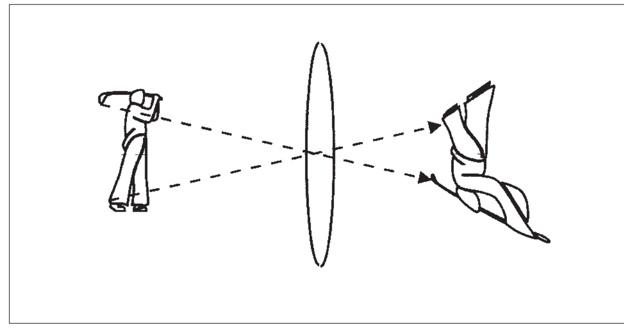

Fig. 3. Approximate distortions in the image of the 3-D object due to the longitudinal magnification.

Most of the time, the longitudinal magnification seriously distorts the image of the 3-D object (see Fig. 3). This distortion is minimal in the neighborhood of the regions where $m = \pm 1$ (when the object position is near $x = 0$ or $x = 2f$) and maximal near the object position surrounding the object focus. In this last region, some parts of the image suffer a dramatic change as small variations in the object position are introduced.

The (real) image of a point travels to infinity as the point approaches to the object focus of the positive lens. It reappears (as virtual, on the other side) when the focus position is surpassed. So, a 3-D object located with regions behind and in front of the object focus results in an image with two separated parts. One of them is inverted, real, strongly magnified, and very far. The other part is upright, virtual, but also very far and strongly magnified.

In conclusion, we suggest that the topic of the longitudinal magnification should be more carefully treated, particularly in the case of the images of three-dimensional objects. Students should work with images of asymmetrical objects and trace rays coming from points at different distances from the lens. The results are simple to understand and provide a good example of how nature sometimes does the unexpected.

Acknowledgments

This work was supported by grants of the University of La Plata, CONICET and CIC, Argentina. Helpful comments by G. Molesini and R. Arizaga are gratefully acknowledged.

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PACS code: 01.40Gb, 42.10D, 42.78

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etcetera...

The Chandler Wobble

etcetera... Editor
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"Although the 18th century Swiss mathematician Leonhard Euler predicted that the Earth should wobble on its axis at a pace of around once a year, it wasn't until 1891 that American businessman and amateur scientist Seth Carlo Chandler Jr. detected this wobble through analysis of stellar observations. Once every 14 months, Chandler found, Earth's spin axis wanders near the geographic pole within a rough circle anywhere from 3 to 6 meters across. If the off-kilter motion resulted from a single nudge to the tilted spinning top that is Earth, calculations showed it would have faded away in a few decades. Something must keep pumping energy into the wobble..."

The rest of the news story describes qualitatively the interaction of the atmosphere and the oceans that is thought to maintain this wobble.[1]

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