Economics Letters 171 (2018) 115–118

Contents lists available at ScienceDirect

Economics Letters

journal homepage: www.elsevier.com/locate/ecolet

Procuring substitutes with (fine-tuned) first-price auctions[☆]

Leandro Arozamena[a,*], Federico Weinschelbaum[a], Elmar G. Wolfstetter[b,c]

[a] Universidad Torcuato Di Tella and CONICET, Buenos Aires, Argentina
[b] Humboldt-University at Berlin, Germany
[c] Korea University, Seoul, Republic of Korea

HIGHLIGHTS

- A firm buys imperfectly substitutable inputs from two contractors.
- The buyer announces demand functions and contractors simultaneously bid unit prices.
- We show that the firm has an incentive to announce demands that over-state input substitutability and understate its willingness to pay with respect to their true values.

ARTICLE INFO

Article history:
Received 16 January 2018
Received in revised form 5 April 2018
Accepted 11 July 2018
Available online xxxx

JEL classification:
C72
D44

Keywords:
Procurement
Auctions
Mechanism design

ABSTRACT

Suppose a firm uses inputs that are substitutes. Each input is supplied by a single contractor. The firm would like to make suppliers compete. However, since inputs are imperfect substitutes, resorting to winner-take-all competition may not be an attractive option. We allow the firm to use a modified first-price auction. It announces demand functions for each input and contractors simultaneously bid unit prices and sell according to announced input demands. We show that the firm has an incentive to announce demands that overstate input substitutability and understate its willingness to pay. In the extreme inputs are treated as perfect substitutes even if goods are independent.

1. Introduction

Suppose a firm uses two inputs that are substitutes to generate output according to a smooth production function. Each input is supplied by a single contractor subject to linear cost functions. Contractors' unit costs are their private information. The firm would like to make both suppliers compete. However, because their products are imperfect substitutes, it would rather not resort to any form of winner-take-all competition.

We examine a context where the firm, to which we refer as "buyer", procures those inputs using a modified first-price (reverse) auction. There, the buyer announces demand functions for each input which depend on both unit prices; given these announced demand functions, contractors simultaneously bid unit prices that determine how many units each of them sells.

The degree of competition induced by the auction, and therefore its profitability for the buyer, depend upon the degree of substitutability between the two inputs and the buyer's maximum willingness to pay, as reflected in the announced demand functions. If the degree of substitutability is low, or the maximum willingness to pay is high, contractors will choose high bids that translate into a low profit for the buyer. However, the buyer may strategically engineer or fine-tune the demand functions by announcing demands that exhibit a higher than true degree of substitutability and/or a lower than true maximum willingness to pay. Strategically fine-tuned demand functions can contribute to increase the buyer's expected profit by inducing contractors to compete more fiercely; however, they may also distort the input mix. Therefore, the optimal degree of strategic fine-tuning must trade off its benefits and costs.

In the present note we allow the buyer to announce demand functions with parameters that may differ from true demands, so

☆ We are grateful to Joaquín Coleff, Guido Martirena, and an anonymous referee for their comments on an earlier version of this note. Arozamena and Weinschelbaum acknowledge financial support from *FONCyT*, *PICT* 2014-3367.

* Corresponding author.
E-mail addresses: larozamena@utdt.edu (L. Arozamena), fweinschelbaum@utdt.edu (F. Weinschelbaum), elmar.wolfstetter@rz.hu-berlin.de (E.G. Wolfstetter).

https://doi.org/10.1016/j.econlet.2018.07.021
0165-1765/

as to induce more aggressive bidding. If costs are perfectly correlated the buyer can extract the maximum surplus by extremely overstating substitutability. Whereas if costs are independent private values, full surplus extraction is not feasible, although it is still optimal to overstate substitutability and understate the buyer's maximum willingness to pay.

Although the use of auctions in procuring inputs that are substitutes has, to the best of our knowledge, not been tackled in the literature, there are a number of related contributions. Anton and Yao (1989, 1992) analyze "split award auctions", where a buyer procures multiple units of a given good from several suppliers. There, the intention of multiple sourcing is to minimize the impact of decreasing returns to scale or to maintain future competition among suppliers,[1] whereas in the present study the different suppliers produce different goods.

In another connected line of work Klemperer (2010) studies product-mix auctions with multiple goods. There, suppliers submit multiple bids, specifying prices for different goods, and the auction determines which bids are accepted, at most one from each buyer, whereas in our setting each supplier sells at most one good. Saban and Weintraub (2015) analyze a government procurement agency that runs an auction to construct a menu of differentiated goods with unit prices, which will then be demanded by heterogeneous buyers.[2] There, the trade-off is between the variety of goods included in the menu and the intensity of competition among suppliers.

2. The model

A firm (buyer) purchases two inputs from two independent contractors. Let q_i be the quantity of input i bought from contractor $i \in \{1, 2\}$. Inputs are imperfect substitutes. To ensure tractability, the buyer's revenue function, R, is assumed quadratic,

$$R(q_i, q_j) = \alpha(q_i + q_j) - \frac{\beta}{2(1+\sigma)}(q_i^2 + q_j^2 + 2\sigma q_i q_j). \tag{1}$$

Thus, profit maximization yields the linear "true" input demand functions, $Q_i^t(p_i, p_j)$:

$$Q_i^t(p_i, p_j) = \frac{\alpha}{\beta} - \frac{1}{\beta(1-\sigma)}\left(p_i - \sigma p_j\right). \tag{2}$$

There, α represents the buyer's maximum willingness to pay for each input, and $1/\beta$ the size of the market. The parameter σ reflects input substitutability, and ranges from $\sigma = 0$, when inputs are independent, to σ approaching 1, when they become perfect substitutes.

The assumed revenue and the associated *true* demand functions are borrowed from Shubik and Levitan (1980) who introduced them in the context of Bertrand market games with differentiated products to remedy an undesirable feature of the often used Bowley specification (Bowley, 1924; Singh and Vives, 1984), in which an increase in the measure of substitutability reduces the size of the market and makes it shrink away completely as products become perfect substitutes.[3]

Contractors are subject to linear cost functions. Their unit costs x_i are their private information. We will examine below two different cases, in which X_1 and X_2 are either perfectly correlated or *i.i.d.* random variables, drawn from the continuous *c.d.f.* F, with support $[c, d]$, where $0 \le c < d < \alpha$. As a rule we denote random variables by capital and realizations by lowercase letters.

Inputs are procured by a fine-tuned first-price reverse auction. There, the buyer announces demand functions, $Q_i(p_i, p_j)$,

$$Q_i(p_i, p_j) = \frac{a}{b} - \frac{1}{b(1-s)}\left(p_i - sp_j\right), \quad a > d,\ b > 0,\ s \in [0, 1), \tag{3}$$

defined for $a > d$, $b > 0$, $s \in [0, 1)$, where the parameters (a, b, s) may differ from the true demand parameters, (α, β, σ). Based on these announced demand functions the buyer asks contractors to simultaneously bid unit prices, p_1, p_2. After bids are submitted, they are revealed, and then contractors must deliver the quantities, $Q_i(p_i, p_j)$ in exchange for payments $p_i Q_i(p_i, p_j)$.

The buyer chooses the parameters of the announced demand functions, (a, b, s), in such a way that it maximizes its expected profit.

A special case comes up if the buyer wants to announce demand functions in which inputs are treated as perfect substitutes ($s = 1$), in which case the functions (3) are not defined. This is relevant if costs are perfectly correlated.

3. Results

As a benchmark case, first suppose unit costs are perfectly correlated and this fact is common knowledge among contractors. Then, each supplier knows not only his unit cost but also that of his competitor, while unit costs are unknown to the buyer.

We will show that, by announcing demand functions that treat inputs as perfect substitutes, the buyer can extract the maximum surplus. Because the functions (3) are not defined for $s = 1$, the announced demand functions require a different format in that case.

For simplicity, suppose $x_2 = x_1 = x$.[4] Compute the surplus maximizing input levels:

$$q^*(x) = \arg\max_q R(q, q) - 2xq = \frac{\alpha - x}{\beta}, \tag{4}$$

and consider the announced demand functions:

$$Q^c(p_i, p_j) = \begin{cases} \frac{2}{b}(a - p_i) & \text{if } p_i < p_j \\ \frac{1}{b}(a - p_i) & \text{if } p_i = p_j \\ 0 & \text{if } p_i > p_j. \end{cases} \tag{5}$$

Proposition 1. *(Benchmark) If unit costs are perfectly correlated the buyer can extract the maximum surplus by announcing the demand functions (5) with the parameter values $(a, b, s) = (\alpha, \beta, 1)$.*

Proof. If the buyer announces these demand functions, $p_1 = p_2 = x$ is evidently an equilibrium, and there is no other equilibrium. In that unique equilibrium, the buyer procures the input quantities $Q_i^c(x, x) = (\alpha - x)/\beta = q^*(x)$ from each of the two contractors. These quantities maximize the surplus and fully extract it, at each level of x. □

This simple mechanism extracts the maximal surplus, regardless of the size of the true substitution parameter σ (even if goods are independent). Given the well-known results in Crémer and McLean (1988), it is not surprising that one can find mechanisms that achieve full surplus extraction if costs are correlated. However, it may be surprising that simple belief-free mechanisms can achieve that goal, though it relies on correlation being perfect.[5]

[1] See also Klotz and Chaterjee (1995), Inderst (2008) and Chaturvedi et al. (2014), and the critical assessment of second-sourcing in Riordan and Sappington (1989).

[2] Albano and Sparro (2008) study a similar setting.

[3] We express these demand functions in the slightly more convenient yet equivalent form, introduced by Collie and Le (2015).

[4] If unit costs are perfectly correlated but not the same, one can still achieve full surplus extraction by "handicapping" the contractor with the lower unit cost.

[5] Crémer and McLean (1988) implies that full surplus extraction is feasible under the weaker requirement that there is some stochastic dependency between unit

Now suppose that unit costs are *i.i.d.* random variables drawn from the continuous *c.d.f.* F, with support $[c, d]$, where $0 \leq c < d < \alpha$.

In this case, an auction cannot possibly achieve full surplus extraction because prices cannot be driven down to unit cost levels. In addition, setting $s = 1$ and stipulating the demand functions as in (5) is not desirable, because in that case only one good will be procured, which forgoes the benefit of using both inputs. However, as we will show, the buyer will strategically engineer demand functions and announce $Q_i(p_i, p_j)$, with $s > \sigma$ and $a < \alpha$. By overstating substitutability and understating its maximum willingness to pay for inputs the buyer benefits from inducing bidders to bid more aggressively.

Because unit costs are contractors' independent private information, contractors now face uncertainty concerning their rival's cost. Therefore, the equilibrium bid strategies, $(p_1(x_1), p_2(x_2))$, must be a Bayesian Nash equilibrium and solve:

$$p_i(x_i) = \arg\max_p (p - x_i) \int_c^d Q_i(p, p_j(x_j)) dF(x_i) \quad i, j = 1, 2, i \neq j. \tag{6}$$

Solving the best response problem of contractor i gives:

$$p_i(x_i) = \frac{1}{2}\big(a(1-s) + sE(p_j(X_j)) + x_i\big). \tag{7}$$

Invoking the (symmetric) equilibrium requirement $p_i(x) = p_j(x) = p(x)$ yields $E(p(X)) = \frac{a(1-s)+\bar{x}}{2-s}$, and one obtains the symmetric Bayesian Nash equilibrium strategy:

$$p(x) = \frac{2a(1-s) + (2-s)x + s\bar{x}}{2(2-s)}, \quad \text{where } \bar{x} := E(X). \tag{8}$$

Evidently, the announced market size parameter, b, does not affect bidding. Therefore, the buyer has no incentive to set $b \neq \beta$, and for convenience we normalize $b = \beta = 1$.

Given the symmetric equilibrium strategy, the associated equilibrium quantities supplied are[6]:

$$Q_i^e(\mathbf{x}; a, s) = Q_i(p(x_i), p(x_j)) = \frac{2a - \bar{x}}{2(2-s)} - \frac{x_i - sx_j}{2(1-s)}, \tag{9}$$

where $\mathbf{x} := (x_1, x_2)$. The buyer's profit, as a function of suppliers' unit costs and announced parameters, is

$$\begin{aligned}\pi_0(\mathbf{x}; a, s) =& R(Q_1^e(\mathbf{x}; a, s), Q_2^e(\mathbf{x}; a, s)) - p(x_1)Q_1^e(\mathbf{x}; a, s) \\ & - p(x_2)Q_2^e(\mathbf{x}; a, s) \\ =& \alpha \sum_{i=1}^{2} Q_i^e(\mathbf{x}; a, s) - \frac{1}{2(1+\sigma)} \\ & \left(\sum_i Q_i^e(\mathbf{x}; a, s)^2 + 2\sigma Q_1^e(\mathbf{x}; a, s) Q_2^e(\mathbf{x}; a, s)\right) \\ & - \sum_{i=1}^{2} p(x_i) Q_i^e(\mathbf{x}; a, s).\end{aligned} \tag{10}$$

The buyer chooses (a, s) to maximize his expected profit:

$$\max_{a,s} \Pi(a, s) := \int_c^d \int_c^d \pi_0(\mathbf{x}; a, s) dF(x_1) dF(x_2). \tag{11}$$

We can now show that the buyer has an incentive to strategically set parameter values (a, s) that systematically differ from the true demand parameters (α, σ).

Proposition 2. *The profit maximizing buyer announces demand functions that exhibit a higher than true substitution parameter, $s > \sigma$, and a lower than true willingness to pay parameter, $a < \alpha$.*

Proof. The first-order conditions of the buyer's maximization problem are[7]:

$$\partial_a \Pi(a, s) = \frac{2\alpha(2-s) - 2a(3-2s) + \bar{x}(1-s)}{(2-s)^2} = 0 \tag{12}$$

$$\begin{aligned}&\partial_s \Pi(a, s) \\ &= \frac{1}{2}\Big(\frac{-4a^2(1-s) + 2s\bar{x}^2 + a(2-s)(4\alpha - 2\bar{x}) + 2\bar{x}(2-3s)}{(2-s)^3} \\ &\quad - \frac{2(s-\sigma)Var(X)}{(1+\sigma)(1-s)^3} + \frac{(2\bar{x} - 4\alpha)\bar{x}}{(2-s)^2}\Big) = 0.\end{aligned} \tag{13}$$

From (12), it follows that

$$a = \frac{\alpha(2-s) + \bar{x}(1-s)}{3-2s}. \tag{14}$$

Consequently,

$$a - \alpha = \frac{\alpha(2-s) + \bar{x}(1-s)}{3-2s} - \alpha = -\frac{(1-s)(\alpha - \bar{x})}{3-2s} < 0.$$

Hence, the buyer understates his willingness-to-pay for inputs. To show that he overstates the measure of substitutability, substitute the solution of a, by (14), in the first-order conditions (13), and one has:

$$\frac{2(\alpha - \bar{x})^2}{(3-2s)^2} - \frac{(s-\sigma)Var(X)}{(1+\sigma)(1-s)^3} = 0. \tag{15}$$

It is straightforward to check that the *LHS* in (15) is positive if $s \leq \sigma$. Furthermore, as $\sigma < 1$, that *LHS* goes to $-\infty$ when $s \to 1$. It then follows that (15) will be satisfied for some $s > \sigma$. □

The buyer induces stronger competition between suppliers by announcing a demand system that reflects more substitutability, and less overall willingness to pay, than would follow from his actual production technology.

The results described so far have been derived under the implicit assumption that equilibrium input quantities are positive for all profiles of unit costs. However, as is clear by examining the expression for equilibrium quantities in (9), that need not be the case. At least for some values of a and s, if a supplier is significantly less efficient than its rival, the *LHS* of (9) may become negative, which is of course not permitted.

One would like to generalize our results by constraining demands so that input quantities are nonnegative for any parameter configuration. This, however, would complicate the analysis substantially. Moreover, examining the case where interior solutions hold is informative, as the buyer's and contractors' incentives are all that drives the results that follow. Anyway, for any cost distribution, there is a set of parameter values such that input quantities are indeed positive. We now characterize that set.

Clearly, we have

$$\begin{aligned}Q_i^e(x_i, x_j) &= \frac{2a - \bar{x}}{2(2-s)} - \frac{x_i - sx_j}{2(1-s)} \geq \frac{2a - \bar{x}}{2(2-s)} - \frac{d - sc}{2(1-s)} \\ &= Q_i^e(d, c)\end{aligned}$$

costs. However, the mechanisms that achieve this when unit costs are not perfectly correlated are highly sensitive with respect to the assumed common beliefs, which limits their application.

[6] Note that if x_i is significantly larger than x_j, it may be the case, for some values of a, s, that $Q_i^e(\mathbf{x}; a, s)$ becomes negative. Here, we proceed as if equilibrium quantities were necessarily positive. We will later confirm that there is a sizeable parameter region where quantities are indeed positive for all cost profiles.

[7] Note that $Y := X_1 - X_2$ gives $E(Y^2) = E(Y)^2 + Var(Y) = 2Var(X)$, using the fact that $X := X_1 = X_2$ are *i.i.d.* random variables.

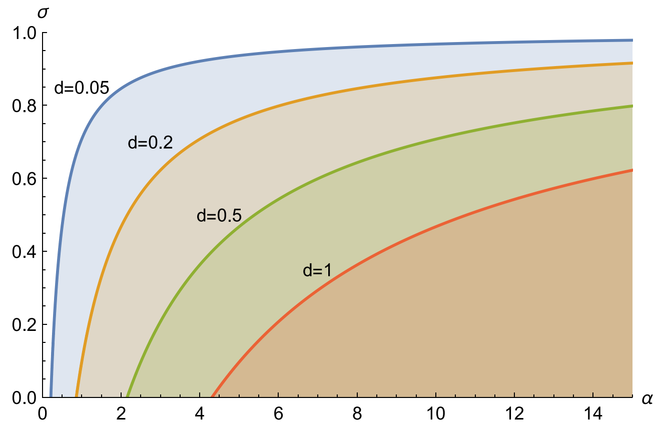

Fig. 1. Feasible sets of (α, σ) for the uniform distribution with support $[0, d]$.

Therefore, input quantities are positive for all cost profiles if $Q_i^e(d, c) \geq 0$.

Whether this inequality holds or not depends on the parameters (α, σ), and the probability distribution, particularly the difference between the boundaries of its support. For any cost distribution, though, quantities will be positive if α is large and σ is small enough. When α grows and σ falls, the buyer selects a higher value of a and a lower value of s, thus making any non-negativity constraint for input quantities less likely to bind. Fig. 1 depicts the region of parameter values (α, σ) where input quantities are always positive when costs are distributed uniformly on $[0, d]$. This exercise can easily be replicated for other probability distributions, because all that matters are its expected value and variance.[8]

Naturally, the non-negativity issue is less likely to arise the smaller the distance between the highest and the lowest possible contractor costs.

We close with an example.

Example 1. Consider the parameter specification $(c, d, \alpha, \sigma) = (0.15, 0.3, 1.5, 0.2)$ and assume uniformly distributed unit costs, which implies $\bar{x} = 0.225, Var(X) = 0.001875$. Then, the optimal parameters of the announced demand functions are $(a, s) = (1.4146, 0.9226)$. Equilibrium input quantities are non-negative for all draws of (x_1, x_2) because, using the assumed parameters and optimal values of (a, s): $Q_i^e(x_i, x_j) = \frac{2a-\bar{x}}{2(2-s)} - \frac{x_i - sx_j}{2(1-s)} \geq \frac{2a-\bar{x}}{2(2-s)} - \frac{d-sc}{2(1-s)} = Q_i^e(d, c) = 0.1642 > 0, \ \forall \left(x_i, x_j\right)$.

Evidently, in this example it is optimal to treat inputs as highly substitutable, $s = 0.9226$, even though the true substitution parameter is as low as $\sigma = 0.2$.

Appendix A. Supplementary material

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.econlet.2018.07.021.

References

Albano, G., Sparro, M., 2008. A simple model of framework agreements: Competition and efficiency. J. Publ. Procuremen 8, 356–378.

Anton, J., Yao, D., 1989. Split awards, procurement, and innovation. Rand J. Econ. 20, 538–552.

Anton, J., Yao, D., 1992. Coordination in split award auctions. Q. J. Econ. 105, 681–701.

Bowley, A.L., 1924. The Mathematical Groundwork of Economics, Oxford University Press, Oxford.

Chaturvedi, A., Beil, D., Martínez-de Albéniz, V., 2014. Split-award auctions for supplier retention. Manage. Sci. 60, 1719–1737.

Collie, R., Le, V., 2015. Product differentiation, the volume of trade and profits under Cournot and Bertrand duopoly. Int. J. Econ. Bus. 22, 73–86.

Crémer, J., McLean, R.P., 1988. Full extraction of surplus in Bayesian and dominant strategy auctions. Econometrica 56, 1247–1258.

Inderst, R., 2008. Single sourcing versus multiple sourcing. Rand J. Econ. 39, 1–21.

Klemperer, P., 2010. The product-mix auction: A new auction design for differentiated goods. J. Eur. Econom. Assoc. 8, 526–536.

Klotz, D., Chaterjee, K., 1995. Dual sourcing in repeated procurement competitions. Manage. Sci. 20, 1317–1327.

Riordan, M.H., Sappington, D.E., 1989. Second sourcing. Rand J. Econ. 2006, 41–58.

Saban, D., Weintraub, G., 2015. Procurement Mechanisms for Differentiated Products. Working Paper, Stanford University.

Shubik, M., Levitan, R., 1980. Market Structure and Behavior, Harvard University Press.

Singh, N., Vives, X., 1984. Price and quantity competition in a differentiated duopoly. Rand J. Econ. 15, 546–554.

[8] See Appendix A for a *Mathematica* file where the feasible sets are computed.