Reduced Complexity MLSD Receivers for Nonlinear Optical Channels

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***Abstract*— We present a novel maximum likelihood sequence detection (MLSD) receiver structure for nonlinear channels. This scheme is derived by treating the NLC as a multiple input/multiple output system. Then, orthogonal signal components are computed using a special form of space-time whitened matched filter (ST-WMF) obtained by a modified Gram–Schmidt orthogonalization of the Volterra kernels of the NLC. The MLSD receiver consists of the ST-WMF followed by a Viterbi detector (VD) with multidimensional branch metrics. The space orthogonalization and noise whitening achieved by the ST-WMF provide an efficient way to reduce the receiver complexity in the presence of highly dispersive NLC. Complexity reduction is crucial in practical applications such as intensity modulation/direct detection (IM/DD) optical channels. As an example, the number of states of the VD in ST-WMF-MLSD required on a 10 Gb/s, 700 km, IM/DD fiber-optic link is reduced eight times compared with an oversampled MLSD.**

***Index Terms*— MLSD, nonlinear channel, non-Gaussian noise, optical-fiber communications, whitened matched filter.**

I. Introduction

OPTIMUM receivers for nonlinear channels affected by additive white Gaussian noise (AWGN) have been extensively studied in the literature. Much of the early work in this area has been focused on the compensation of nonlinearities in satellite communications [2]. A traditional architecture of the optimal receiver in nonlinear channels consists of a matched-filter bank (MFB) followed by a maximum likelihood sequence detector (MLSD) [2]. The use of oversampling and MLSD (OS-MLSD) has also been proposed to implement the optimal receiver in the presence of nonlinearities (see [3] and references therein).

Multigigabit fiber-optic communication systems are one of the challenging applications of nonlinear receivers of great current interest [4]. In this application the receiver must compensate the linear fiber dispersion as well as nonlinearities caused by lasers, optical modulators, the fiber Kerr effect, photo-detectors, and other components of the link. In particular, chromatic dispersion (CD) and polarization-mode dispersion (PMD), in combination with the quadratic response of the photo-detector, become major factors that limit the reach of intensity modulation/direct detection (IM/DD) optical-transmission systems at data rates $\geq$10 Gb/s [4], [5]. MLSD-based receivers for transmissions over IM/DD fiber channels have been investigated in the literature [4], [6], [7]. It has been found that a $\sim$3 dB-penalty at a bit-error-rate (BER) of 10^{-3} can be achieved by OS-MLSD in the range of $250 - 1000$ km [6], [8]. Unfortunately, when the fiber length approaches or exceeds 500 km, optimal OS-MLSD receivers become difficult to implement as a result of the large channel memory (e.g., an 8192-state MLSD is required to compensate 700 km of fiber at 10 Gb/s with 3 dB penalty [6]). Complexity grows quadratically with the symbol rate and quickly becomes intractable at $\geq$10 Gb/s. Several techniques to reduce MLSD complexity in IM/DD systems have been proposed (e.g., [9]–[11]). Most of them are based on the bandwidth optimization of electrical and optical filters. However this may be difficult to implement in installed optical networks. Recently, *time* channel shortening (CS) proposed in [12] has been used in nonlinear satellite channels with Gaussian noise [13]. However, both *space and time* channel shortening should be adopted to further reduction of complexity in transmissions over highly dispersive nonlinear channels such as long IM/DD fiber-optic links. Furthermore, CS as proposed in [13] must be modified to consider the non-Gaussian and signal-dependent nature of the noise in IM/DD optical channels. Therefore complexity reduction of MLSD receivers for nonlinear channels continues to be an active research area of great interest in applications such as IM/DD fiber optic links [5].

In this letter we present a new MLSD receiver architecture for nonlinear channels. Our work builds upon structures such as that proposed by Stern [14], which consist of a whitened matched filter bank followed by a Viterbi detector (VD). The structure proposed here is derived by treating the nonlinear channel as a multiple input / multiple output (MIMO) system (see Fig. 1 and associated text in Section II). Unlike previous works, we use a novel representation of the received signal obtained by a Gram-Schmidt-like orthogonalization of the kernels of a Volterra series expansion of the channel. This procedure yields a special form of space-time whitened matched filter (ST-WMF) [1] whose baud-rate-sampled outputs are sufficient statistics with independent noise components in both space and time. Combined with the minimum phase property [1] of the response of each ST-WMF branch, the former provides an efficient way to reduce the complexity of MLSD in nonlinear channels. As discussed in connection with equation (14) in Section II, the technique proposed here offers a smooth

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tradeoff between performance and complexity. As complexity is progressively reduced, performance degrades in a graceful manner. As an example of application, we evaluate the performance of the ST-WMF-MLSD receiver in IM/DD fiber-optic systems. Since the input noise is signal-dependent and non-Gaussian [4], we use a memoryless nonlinear transformation of the received signal before the ST-WMF in order to Gaussianize the noise [6], [7]. Our results show that the number of states of the VD in IM/DD optical links can be drastically reduced with the proposed ST-WMF-MLSD (e.g., $\sim$ 8 times lower than in OS-MLSD).

II. Nonlinear Channel Model

The noisy received signal is given by

$$r(t) = s(t) + z(t), \tag{1}$$

where $s(t)$ is the noise-free signal and $z(t)$ is the noise component, which is assumed to be a white Gaussian process with power spectral density N_0. Component $s(t)$ can be expressed by using its Volterra-series expansion. For example, in optical IM/DD systems we get[1]

$$s(t) = \sum_k a_k \left[h_0(t-kT) + \sum_{m=1}^{N-1} a_{k-m} f_m(t-kT) \right], \tag{2}$$

where $h_0(t)$ is the linear kernel, $f_m(t)$ is the m-th second-order kernel [15], a_k is the k-th symbol at the input of the nonlinear channel, $1/T$ is the symbol rate, and N is the total number of kernels.

Next we derive an alternative representation of the nonlinear signal $s(t)$. Without loss of generality, we consider here the Volterra-series expansion given by (2) with dominant linear kernel, $h_0(t)$. Let $\mathcal{H}_0$ be the *signal space* spanned by the set $\{h_0(t-kT)\}$ [1]. In this letter, we assume that signal spaces are Hilbert spaces with inner product defined as $\int_{-\infty}^{\infty} x(t)y^*(t)dt$, where superscript * denotes complex conjugate. From the *projection theorem*, the nonlinear kernels $f_m(t)$ can be uniquely expressed as

$$f_m(t) = \sum_n \lambda_n^{(0,m)} h_0(t-nT) + g_m^{(0)}(t), \tag{3}$$

where $g_m^{(0)}(t)$ is *orthogonal* to the signal space $\mathcal{H}_0$, i.e.,

$$\int_{-\infty}^{\infty} g_m^{(0)}(t) h_0^*(t-jT)dt = 0, \quad m = 1, ..., N-1, \forall j, \tag{4}$$

while $\int_{-\infty}^{\infty} |g_m^{(0)}(t)|^2 dt$ is minimum [1]. We highlight that the first summation in eq. (3) is the *projection* of $f_m(t)$ onto $\mathcal{H}_0$. Define $\mathcal{G}_m^0$ as the signal space spanned by $\{g_m^{(0)}(t-kT)\}$. For $x(t) \in \mathcal{H}_0$ and $y(t) \in \mathcal{G}_m^0$, from (4) note that $\int_{-\infty}^{\infty} x(t)y^*(t)dt = 0$, therefore $x(t)$ and $y(t)$ are orthogonal signals [1]. Replacing (3) in (2) and operating, we can obtain

$$s(t) = s_0(t) + \overline{s}_0(t), \tag{5}$$

[1]The DC term of the series expansion is omitted.

where

$$s_0(t) = \sum_k \left[a_k + \sum_{m=1}^{N-1} a_k a_{k-m} \otimes \lambda_k^{(0,m)} \right] h_0(t-kT), \tag{6}$$

$$\overline{s}_0(t) = \sum_k \sum_{m=1}^{N-1} a_k a_{k-m} g_m^{(0)}(t-kT), \tag{7}$$

with operator $\otimes$ denoting convolution. Notice that $s_0(t) \in \mathcal{H}_0$ and $\overline{s}_0(t) \in \mathcal{G}_m^0$, therefore signals $s_0(t)$ and $\overline{s}_0(t)$ are orthogonal (see (4)). Next we focus on $\overline{s}_0(t)$. Denoting $h_1(t) = g_1^{(0)}(t)$, eq. (7) can be rewritten as

$$\overline{s}_0(t) = \sum_k a_k \left[a_{k-1} h_1(t-kT) + \sum_{m=2}^{N-1} a_{k-m} g_m^{(0)}(t-kT) \right].$$

Similarly to (5), $\overline{s}_0(t)$ can be expressed as

$$\overline{s}_0(t) = s_1(t) + \overline{s}_1(t), \tag{8}$$

where

$$s_1(t) = \sum_k \left[a_k a_{k-1} + \sum_{m=2}^{N-1} a_k a_{k-m} \otimes \lambda_k^{(1,m)} \right] h_1(t-kT),$$

$$\overline{s}_1(t) = \sum_k \sum_{m=2}^{N-1} a_k a_{k-m} g_m^{(1)}(t-kT), \tag{9}$$

with $\lambda_n^{(1,m)}$ chosen to satisfy $\int_{-\infty}^{\infty} g_m^{(1)}(t) h_1^*(t-jT)dt = 0$, $m = 2, ..., N-1, \forall j$. Thus, note that $\overline{s}_1(t)$ is orthogonal to the signal spaces spanned by both $\{h_0(t-kT)\}$ and $\{h_1(t-kT)\}$. Repeating the processing on (9) and generalizing, we can get

$$s(t) = \sum_{n=0}^{N-1} s_n(t) = \sum_{n=0}^{N-1} \sum_k b_k^{(n)} h_n(t-kT), \tag{10}$$

where $h_n(t)$ is the response of the n-th channel *path*,

$$b_k^{(0)} = a_k + \sum_{m=1}^{N-1} a_k a_{k-m} \otimes \lambda_k^{(0,m)}, \tag{11a}$$

$$b_k^{(n)} = a_k a_{k-n} + \sum_{m=n+1}^{N-1} a_k a_{k-m} \otimes \lambda_k^{(n,m)}, \quad 0 < n < N-1, \tag{11b}$$

$$b_k^{(N-1)} = a_k a_{k-N+1}, \tag{11c}$$

with

$$\int_{-\infty}^{\infty} h_m(t) h_n^*(t-jT)dt = 0 \qquad m \neq n, \forall j. \tag{12}$$

From (10) and (12) note that

$$\int_{-\infty}^{\infty} s_m(t) s_n^*(t)dt = 0, \qquad m \neq n. \tag{13}$$

Fig. 1 shows a MIMO representation of the nonlinear channel with a traditional MFB [2]. For an efficient design of the receiver, the described processing should be done in order to satisfy

$$\mathcal{E}^{(0)} \geq \mathcal{E}^{(1)} \geq \cdots \geq \mathcal{E}^{(N-1)}, \tag{14}$$

with $\mathcal{E}^{(n)} = E\{|b_k^{(n)}|^2\} \int_{-\infty}^{\infty} |h_n(t)|^2 dt$ and $E\{.\}$ denoting expectation. As we shall show later, condition (14) gives rise to *spatial* channel compression in IM/DD applications, which can be exploited to reduce complexity.

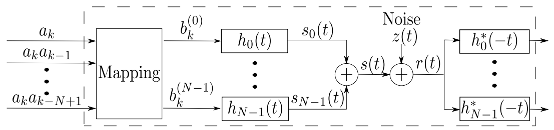

Fig. 1. MIMO model of the nonlinear channel with MFB.

III. ST-WMF-MLSD FOR NONLINEAR CHANNELS

The MLSD receiver chooses the sequence $\{a_k\}$ that minimizes the metric

$$J = \int_{-\infty}^{\infty} |r(t) - s(t)|^2 \, dt. \tag{15}$$

Let $\phi_n(t)$ be the impulse response of the filter with Fourier transform (FT) given by

$$\Phi_n(\omega) = \frac{H_n(\omega)}{\gamma^{(n)} M_n(e^{j\omega T})}, \quad n = 0, ..., N-1, \tag{16}$$

where $H_n(\omega)$ is the FT of $h_n(t)$, $\gamma^{(n)}$ and $M_n(z)$ are defined by the folded spectrum factorization $S_n(z) = (\gamma^{(n)})^2 M_n(z) M_n^*(1/z^*)$ with $S_n(z)$ being the Z-transform of the sequence $\rho_k^{(n)} = \int_{-\infty}^{\infty} h_n(t) h_n^*(t - kT) dt$ (see eq. (5.80) in [1] for more details). The set $\{\phi_n(t - kT)\}$ forms an orthonormal basis for the signal space spanned by $\{h_n(t-kT)\}$. Furthermore, we choose $M_n(z)$ to be minimum phase. Define $\tilde{r}_n(t)$ as the projection of $r(t)$ onto de signal space spanned by $\{h_n(t - kT)\}$, that is,

$$\tilde{r}_n(t) = \sum_k \tilde{r}_k^{(n)} \phi_n(t - kT), \tag{17}$$

where $\tilde{r}_k^{(n)} = \int_{-\infty}^{\infty} r(t) \phi_n^*(t-kT) dt$. From (12), and following the procedure used in [1], Section 10.2.4, metric (15) can be expressed as

$$J = \int_{-\infty}^{\infty} \left\| \tilde{\mathbf{r}}(t) - \sum_k \mathbf{H}(t - kT) \mathbf{b}_k \right\|^2 dt, \tag{18}$$

where $\mathbf{H}(t)$ is an $N \times N$ diagonal matrix with

$$diag\{\mathbf{H}(t)\} = \begin{bmatrix} h_0(t) & h_1(t) & \cdots & h_{N-1}(t) \end{bmatrix}, \tag{19}$$

while $\mathbf{b}_k = [b_k^{(0)} b_k^{(1)} \cdots b_k^{(N-1)}]^T$, and

$$\tilde{\mathbf{r}}(t) = \begin{bmatrix} \tilde{r}_0(t) & \tilde{r}_1(t) & \cdots & \tilde{r}_{N-1}(t) \end{bmatrix}^T, \tag{20}$$

with $b_k^{(n)}$ and $\tilde{r}_n(t)$ given by (11) and (17), respectively.

On the other hand, from (16) it is possible to show that

$$\sum_k \mathbf{H}(t - kT) \mathbf{b}_k = \sum_k \Phi(t - kT) \left[\mathbf{M}_k \otimes \mathbf{b}_k \right], \tag{21}$$

where $\Phi(t)$ and $\mathbf{M}_k$ are $N \times N$ diagonal matrices:

$$diag\{\Phi(t)\} = \begin{bmatrix} \phi_0(t) & \phi_1(t) & \cdots & \phi_{N-1}(t) \end{bmatrix}, \tag{22a}$$

$$diag\{\mathbf{M}_k\} = \begin{bmatrix} \gamma^{(0)} m_k^{(0)} & \cdots & \gamma^{(N-1)} m_k^{(N-1)} \end{bmatrix}, \tag{22b}$$

with $m_k^{(n)}$ being the inverse FT of $M_n(e^{j\omega T})$. From (17) notice that (20) can be expressed as

$$\tilde{\mathbf{r}}(t) = \sum_k \Phi(t - kT) \tilde{\mathbf{r}}_k, \tag{23}$$

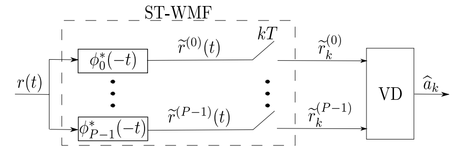

Fig. 2. Block diagram of the P-dimensional ST-WMF-MLSD receiver.

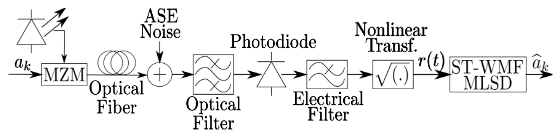

Fig. 3. IM/DD fiber-optic system with ST-WMF-MLSD receiver.

where $\tilde{\mathbf{r}}_k = [\tilde{r}_k^{(0)} ... \tilde{r}_k^{(N-1)}]^T$ (see (17)). From (18), (21), and (23) it can be shown that MLSD reduces to minimize

$$J = \| \tilde{\mathbf{r}}_k - \mathbf{M}_k \otimes \mathbf{b}_k \|^2 . \tag{24}$$

Let $\mathbf{w}_k$ be the N-dimensional vector with the noise components of the baud rate samples $\tilde{\mathbf{r}}_k$. From (12) and (16), the power spectral density of $\mathbf{w}_k$ results $\mathbf{S}_w = N_0 \mathbf{I}$, where $\mathbf{I}$ is the identity matrix. Therefore, the minimization of (24) can be easily implemented by using a VD with *multidimensional* Euclidean branch metrics. Fig. 2 shows a block diagram of the ST-WMF-MLSD receiver with *dimension* P (i.e., P is the number of filters in the bank). If $P = N$, all the paths of the nonlinear channel are used by the receiver.

IV. ST-WMF-MLSD IN IM/DD OPTICAL SYSTEMS

Next we analyze the proposed ST-WMF-MLSD receiver in transmissions over IM/DD fiber-optic systems with on-off keying (OOK) modulation. We focus on two key aspects of ST-WMF-MLSD: its performance (in comparison with current solutions based on OS-MLSD), and its ability to reduce complexity (e.g., number of states of VD). Complexity reduction is possible owing to (*i*) the minimum-phase property of the equivalent channel response provided by ST-WMF, and (*ii*) condition (14). The latter gives rise to *space compression*, which reduces the ST-WMF dimension, P. This is achieved by using the *most important* P paths of the nonlinear channel.

Fig. 3 depicts the optical system under consideration. The transmitter modulates the intensity of the transmitted signal using NRZ OOK modulation. The standard single mode fiber (SMF) introduces chromatic and polarization mode dispersion, as well as attenuation. Optical amplifiers are deployed periodically along the fiber to compensate the attenuation, also introducing amplified spontaneous emission (ASE) noise in the signal. ASE noise is modeled as AWGN in the optical domain. The received optical signal is filtered by an optical filter, and then converted to a current with a PIN diode or avalanche photodetector. The resulting photocurrent is filtered by an electrical filter. The noise component after the electrical filtering is non-Gaussian and signal-dependent [4]. Therefore, the electrical signal is first processed by a memoryless nonlinear transformation. It has been found that after a square root transformation, the noise can be assumed Gaussian and signal-independent [6], [7]. Futhermore, channel nonlinearities can also be reduced by using the square root transformation [16], which improves the space compression used to reduce

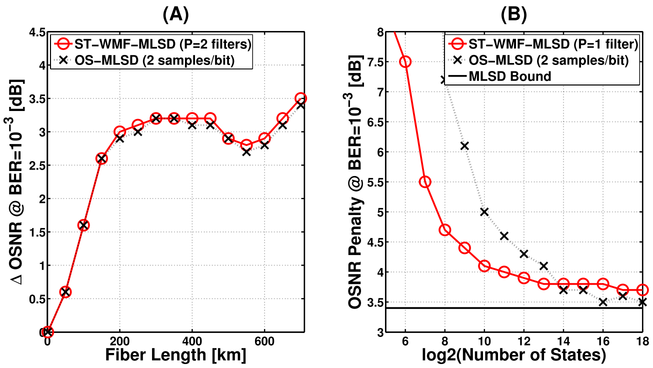

Fig. 4. (A): OSNR penalty at $BER = 10^{-3}$ versus fiber length with unconstrained complexity receivers. (B): OSNR penalty at $BER = 10^{-3}$ versus number of states of the VD with $L = 700$ km.

the receiver dimension (i.e., most of the channel energy is concentrated on the linear kernel). The split-step Fourier method is used to compute the propagation of optical signals through the fiber. Oversampled linear and nonlinear kernels are extracted from the electrical signal after the square root transformation. The oversampled factor is $T/T_s = 16$. Then, we compute $h_n(kT_s)$ and $\lambda_k^{(n,m)}$ according to (14). For the channels analyzed in this letter, $\mathcal{E}^{(0)} + \mathcal{E}^{(1)}$ is $\sim 99\%$ of the total signal energy. The baud rate channel response matrix $\mathbf{M}_k$ (22b) can be easily obtained from (16). Since the noise after the square root transformation is approximately Gaussian and signal-independent [7], the theory proposed in [17] is used to evaluate the bit error probability. All the kernels of the nonlinear channel are used to compute the error probability, independently of the receiver dimension, P.

Data rate is $1/T = 10$ Gb/s and the transmitted pulse shape has an unchirped Gaussian envelope $e^{-t^2/2T_0^2}$ with $T_0 = 36$ ps. We use a Lorentzian optical filter and a fourth-pole Butterworth electrical filter with bandwidths of 15 and 10 GHz, respectively. The fiber dispersion is $D = 17$ ps/(nm-km).

Fig. 4-A shows the penalty of the optical signal-to-noise ratio (OSNR) at BER $= 10^{-3}$, as a function of the fiber length, L. We present results for two *unconstrained* complexity receivers (i.e., without reduction of the number of states of the VD): ST-WMF-MLSD with $P = 2$ and OS-MLSD with 2 samples/bit (note that the branch metric *dimensions* of both VD's are the same). From Fig. 4-A we observe that both receivers have essentially the same performance. We also highlight that the OSNR penalty is ~ 3 dB for $L > 250$ km, which agrees very well with that reported in [6], [11].

Fig. 4-B depicts the OSNR penalty at BER$= 10^{-3}$ versus the number of states of the VD for $L = 700$ km. In this channel, $\mathcal{E}^{(0)}$ represents 97.5% of the total energy, therefore ST-WMF-MLSD with $P = 1$ should capture most signal information, as verified from Fig. 4-B. We also present results for OS-MLSD with 2 samples/bit, where the reduction of states is achieved by truncation and optimization of the sampling phase in order to minimize BER (8 uniformly distributed phases in the interval $T/2$ were tested). From Fig. 4-B, we verify that the number of states of the VD at a penalty of 4.6 dB can be reduced from 2048 to 256 with ST-WMF-MLSD and $P = 1$!. Notice that this performance is achieved by using a VD with one sample per bit [8]. We emphasize that these benefits widely outperform the extra complexity required by the linear filter and the channel estimator Moreover, numerical results not included here due to space limitations show that imperfect channel knowledge sligthly degrades the performance of OS and ST-WMF MLSD ($\sim$ 0.2 dB). These facts make the ST-WMF-MLSD an atractive technique to increase the distance of IM/DD fiber-optic links.

V. Conclusion

We have proposed a new ST-WMF-MLSD receiver structure for nonlinear channels. The new technique offers a smooth tradeoff between performance and complexity. In transmissions over IM/DD fiber-optic links, the number of states of the VD can be significantly reduced with low performance degradation. Finally, the ST-WMF-MLSD can also be used with different modulation formats such as DQPSK [5], [9].

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