

# Simultaneous correlation of complementary set of sequences using a transpose generation approach

Marcos A. Funes\*, Patricio G. Donato, Matías N. Hadad, Daniel O. Carrica, Mario Benedetti

Universidad Nacional de Mar del Plata – CONICET, J.B. Justo 4302, Mar del Plata, Argentina

## ARTICLE INFO

### Article history:

Available online xxxx

### Keywords:

Complementary set of sequences  
Correlation  
Efficient architectures

## ABSTRACT

Complementary sets of sequences are currently being applied to signal coding, radar, and multi-user systems, among others. Their particular mathematical properties make them adequate for multi-emission and noisy environments. Nowadays sustained efforts are being devoted to reduce the calculations involved in the generation and/or correlation of these signals by means of recursive algorithms. Some authors have proposed efficient algorithms that are based on modular architectures made up of adders, multipliers and delays. This work introduces a new approach to correlation algorithms of complementary sets of sequences, which is based on a transposition of the generation process. This approach allows to notoriously reduce calculations, and enables the simultaneous correlation of  $M$  sequences, without adopting time multiplexing schemes or complex parallel implementations. The correlation algorithm is theoretically demonstrated and its calculation performance is evaluated in a hardware reconfigurable platform. A comparison with other algorithms is included, considering the amount of calculations as a function of the length of the sequences.

© 2012 Elsevier Inc. All rights reserved.

## 1. Introduction

Complementary sequences were defined by Golay [1] as a pair of finite sequences of two kinds of elements with the property that the number of pairs of like elements with any given separation in one sequence is equal to the number of pairs of unlike elements with the same separation in the other sequence. This concept was extended to complementary sets of sequences (CSS) by Tseng and Liu [2], who generalized their mathematical properties. The distinctive feature of these sequences is the possibility of signal detection under high noise level conditions thanks to their correlation properties. Additionally, their orthogonality property renders them suitable for applications where many digital signals are transmitted over the same physical channel. Golay sequences and complementary sets of  $M$  sequences (M-CSS) are currently being applied to signal coding [3], radar [4,5], multi-emission systems [6], channel estimation [7] and sensor systems [8].

The application of an M-CSS in any of the previously mentioned areas involves the use of logical architectures for generation and correlation. Regarding Golay sequences, Budisin [9] and Popovic [10] proposed efficient architectures able to perform both operations. Taking into account that Golay sequences can be considered a particular case of M-CSS, it is logical to think that a

similar solution can be developed for the latter. Some authors, [11–13], proposed efficient architectures to generate and correlate an M-CSS based on recursive methods that reduce computational load and hardware complexity as compared to the *straightforward* implementation [14]. Despite the fact that these modular architectures herald significant progress in the implementation of an M-CSS-based systems, the correlation approach by these authors is based on the recursive method proposed by Popovic [10] for pairs of complementary sequences. Popovic's correlation algorithm executes the correlation of a single input signal with the two sequences of the complementary pair. Therefore, two options for processing a pair of sequences exist: the first one is time multiplexing and the other one is architecture duplication. In the former, the architecture is the same, though the processing time doubles. In the latter, processing is much faster, though the architecture duplicates (with the consequent hardware requirement). This problem accentuates with M-CSS, in which the possibilities are either time multiplexing  $M$  sequences composing a macro-sequence, or using  $M$  architectures.

Previous works [15] applying Golay sequences provide a solution to the simultaneous correlation algorithm based on a correlation as a transposition of the generation process. According to said approach, two sequences are simultaneously correlated with neither time multiplexing nor correlation architecture duplication. Considering that these pairs are a particular case of complementary sets, this paper proposes the generalization of the correlation algorithm for M-CSS. This approach allows for a regular and modular architecture which executes the correlation of  $M$  inputs on a

\* Corresponding author.

E-mail addresses: mfunes@fi.mdp.edu.ar (M.A. Funes), donatopg@fi.mdp.edu.ar (P.G. Donato), mhadad@fi.mdp.edu.ar (M.N. Hadad), carrica@fi.mdp.edu.ar (D.O. Carrica), mbenedet@fi.mdp.edu.ar (M. Benedetti).

simultaneous basis, with the consequent calculation reduction. The correlation algorithm is mathematically demonstrated, and is verified with simulations. The proposed correlator is implemented in a hardware platform in order to evaluate its resources consumption and calculation efficiency.

## 2. Complementary sequences

Complementary pairs of sequences (also known as Golay sequences [1]) are defined as a pair of sequences,  $\{S_{1,N}[k]; S_{2,N}[k]\}$  of length  $L = 2^N$  binary elements ( $\pm 1$ ), where  $N \in \mathbb{N} - \{0\}$ . The number of pairs of like elements with any given separation in one sequence is equal to the number of pairs of unlike elements with the same separation in the other sequence. The particular property of these sequences is that the addition of their periodic autocorrelation functions is a Kronecker delta of amplitude  $2 \cdot L$ . This can be expressed as follows:

$$Y[k] = C_1[k] + C_2[k] = 2 \cdot L \cdot \delta[k] \quad (1)$$

where  $C_1[k]$  and  $C_2[k]$  are the autocorrelations of  $S_{1,N}[k]$  and  $S_{2,N}[k]$ , respectively.

Golay defines synthesis rules, based on concatenation or interleaving, to generate complementary pairs of different length from shorter ones. These rules can be recursively applied to generate diverse pairs, as proposed by Budisin [9]:

$$\begin{aligned} S_{1,0}[k] &= \delta[k] \\ S_{2,0}[k] &= \delta[k] \\ S_{1,n}[k] &= S_{1,n-1}[k] + w_{1,n} \cdot S_{2,n-1}[k - D_n] \\ S_{2,n}[k] &= S_{1,n-1}[k] - w_{1,n} \cdot S_{2,n-1}[k - D_n] \end{aligned} \quad (2)$$

where  $S_{1,n}[k]$ ,  $S_{2,n}[k]$  are the pairs of sequences at the  $n$ -th iteration,  $D_n = 2^{n-1}$  is a delay and  $w_{1,n}$  is a seed coefficient with value  $\pm 1$ .

Eq. (2) can be expressed by matrices and  $Z$  transform.

$$\begin{bmatrix} S_{1,n}[Z] \\ S_{2,n}[Z] \end{bmatrix} = \begin{bmatrix} +1 & +1 \\ +1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & w_{1,n} \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & z^{-2^{n-1}} \end{bmatrix} \cdot \begin{bmatrix} S_{1,n-1}[Z] \\ S_{2,n-1}[Z] \end{bmatrix} \quad (3)$$

or

$$\mathbf{S}_n[Z] = \mathbf{H}_2 \cdot \mathbf{W}_{2,n} \cdot \mathbf{D}_{2,n} \cdot \mathbf{S}_{n-1}[Z] \quad (4)$$

where

- $\mathbf{H}_2$  is a Hadamard matrix of order 2.
- $\mathbf{W}_{2,n}$  is a seed matrix of order 2 of the  $n$ -th iteration.
- $\mathbf{D}_{2,n}$  is a delay matrix of order 2 of the  $n$ -th iteration.

Golay sequences correlation can also be performed by recursive algorithms, as first proposed by Popovic [10] and later optimized by Donato et al. [15]. The correlator of Popovic is a digital matched filter which simultaneously performs the correlation of one input signal with both sequences of the Golay pair. The correlator proposed by Donato et al. is an architecture that realizes an inverse generation process, and thereby performs the simultaneous correlation of two inputs. This recursive algorithm is summed up in these two equations:

$$\begin{aligned} C_{1,n-1}[k] &= C_{1,n}[k - D_n] + C_{2,n}[k - D_n] \\ C_{2,n-1}[k] &= w_{1,n} \cdot C_{1,n}[k] - w_{1,n} \cdot C_{2,n}[k] \end{aligned} \quad (5)$$

where  $C_{1,n}[k]$ ,  $C_{2,n}[k]$  are a pair of sequences at the input of the  $n$ -th stage of the correlator and  $C_{1,n-1}[k]$ ,  $C_{2,n-1}[k]$  are the outputs

of the same stage. By rewriting this algorithm with matrices and  $Z$  transform,

$$\mathbf{C}_{n-1}[Z] = \mathbf{D}'_{2,n} \cdot \mathbf{W}_{2,n} \cdot \mathbf{H}_2 \cdot \mathbf{C}_n[Z] \quad (6)$$

where

$$\mathbf{D}'_{2,n} = \begin{bmatrix} z^{-2^{n-1}} & 0 \\ 0 & 1 \end{bmatrix} \quad (7)$$

and the correlator output is:

$$Y[k] = C_{1,0}[k] + C_{2,0}[k] = 2 \cdot L \cdot \delta[k - L] \quad (8)$$

where  $C_{1,0}[k]$ ,  $C_{2,0}[k]$  are the correlation outputs at the last stage and the output of  $Y[k]$  is a Kronecker delta of amplitude  $2 \cdot L$  delayed  $L$  samples due to the iterative process.

Tseng and Liu [2] generalized the mathematical properties of Golay sequences giving rise to Complementary Sets of Sequences (CSS). An M-CSS is defined as a set of  $M$  sequences (where  $M = 2^m$ , and  $m \in \mathbb{N} - \{0\}$ )<sup>1</sup> of length  $L = M^N$  elements ( $N \in \mathbb{N} - \{0\}$ ), where each one is a binary element ( $+1$  or  $-1$ ). A set is characterized by the sum of their autocorrelations:

$$Y[k] = \sum_{i=1}^M C_i[k] = M \cdot L \cdot \delta[k] \quad (9)$$

where  $C_i$  is the autocorrelation of the sequence  $S_{i,N}[k]$ :

$$C_i[k] = \sum_{j=1}^L S_{i,N}[j] \cdot S_{i,N}[j + k] \quad (10)$$

The autocorrelation addition of the  $M$  sequences generates a Kronecker delta of amplitude  $M \cdot L$ , with null sidelobes, just like with Golay sequences.

Recursive algorithms were developed for CSS. Álvarez et al. [11, 16] proposed an efficient architecture to generate and correlate a set of four complementary sequences. Afterwards, De Marziani et al. [12] generalized the generation and correlation architectures for M-CSS. Subsequently, this proposal was improved by Pérez et al. [13], who obtained a more efficient architecture, especially for interleaved sequences schemes. Even though these correlation algorithms imply significant progress, the correlation approach used by these authors is based on the recursive method proposed by Popovic [10] for Golay sequences, where the correlation is performed between a single input signal and all the sequences of the complementary set. Then, for all cases, it is necessary to multiply the number of correlators by  $M$ , or else, use an interleaved transmission scheme to process the signal (with the consequent reduction of the transmission rate).

## 3. Proposed correlation approach

In order to reduce the amount of calculations required to perform the correlation, or the number of correlators, without decreasing the processing speed, it is necessary to change the correlation scheme. Instead of designing the correlator as a matched filter with increasing delays, the current proposal applies a different concept. Just like with Golay sequences, the correlation of an M-CSS can be performed as an inverse generation process based on the transposition of the generation procedure. This section develops and demonstrates this concept for any M-CSS, irrespective of its length.

<sup>1</sup> In this sense, the particular case of  $m = 1$  are Golay sequences.

The first step to develop the new correlation algorithm is to express the generation equation for  $M$  sequences in matrix notation. This generation approach is based on the generation rules presented in [2] with the particularity that the generation matrix is split into a Hadamard Matrix and a seed matrix.

$$\begin{bmatrix} S_{1,n}[z] \\ \vdots \\ S_{M,n}[z] \end{bmatrix} = \begin{bmatrix} H_{\frac{M}{2}} & H_{\frac{M}{2}} \\ H_{\frac{M}{2}} & -H_{\frac{M}{2}} \end{bmatrix} \cdot \begin{bmatrix} W_{\frac{M}{2},n-1} & 0 \\ 0 & w_{r,n} \cdot W_{\frac{M}{2},n-1} \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & z^{-M^{n-1}} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & z^{-(M-1)M^{n-1}} \end{bmatrix} \cdot \begin{bmatrix} S_{1,n-1}[z] \\ \vdots \\ S_{M,n-1}[z] \end{bmatrix} \quad (11)$$

or

$$\mathbf{S}_n[z] = \mathbf{H}_M \cdot \mathbf{W}_{M,n} \cdot \mathbf{D}_{M,n} \cdot \mathbf{S}_{n-1}[z] \quad (12)$$

where

- $\mathbf{H}_{\frac{M}{2}}$  is a Hadamard matrix of order  $\frac{M}{2}$ , with  $\mathbf{H}_1 = 1$ .
- $\mathbf{W}_{\frac{M}{2},n-1}$  is a seed matrix of order  $\frac{M}{2}$ , with  $\mathbf{W}_{2,0} = \begin{bmatrix} 1 & 0 \\ 0 & w_{1,0} \end{bmatrix}$ .
- $w_{r,n}$  is a seed coefficient, with  $r = \log_2 M$ .
- $z^{-(M-i)M^{n-1}}$  are delays with  $i = 1, \dots, (M-1)$ ,  $i \in \mathbb{N}$ .

This equation allows the generation of a set of  $M$  sequences of length  $M \cdot L$  from a set of  $M$  sequences of length  $L$ . By repeating this process  $N$  times ( $N \in \mathbb{N} - \{0\}$ ), an M-CSS of length  $M^N$  can be generated. Eq. (13) represents this iterative process:

$$\mathbf{S}_N = \prod_{n=1}^N (\mathbf{H}_M \cdot \mathbf{W}_{M,n} \cdot \mathbf{D}_{M,n}) \mathbf{S}_0 \quad (13)$$

The next step in the algorithm development consists in performing the correlation as a transposition of the generation process. The underlying idea of the proposed correlator is to iteratively decompose an M-CSS (generated with (13)) into a shorter one, until it is reduced to a set of single Kronecker deltas.

In order to better understand this process, let's consider a vector  $\mathbf{S}_N$  as an M-CSS, generated with (13) using the  $\mathbf{H}_M \cdot \mathbf{W}_M$  matrices, at the input of the correlator. If the product of  $\mathbf{W}_M \cdot \mathbf{H}_M$  (the transposition of  $\mathbf{H}_M \cdot \mathbf{W}_M$ ) with  $\mathbf{S}_N$  is performed, a scaled version of the sequences of  $\mathbf{S}_{N-1}$  are obtained. Eqs. (14) and (15) show this process using  $M = 4$  for simplicity reasons:

$$\begin{aligned} \mathbf{S}_n &= \begin{bmatrix} S_{1,n} \\ S_{2,n} \\ S_{3,n} \\ S_{4,n} \end{bmatrix} \\ &= \begin{bmatrix} S_{1,n-1} + S_{2,n-1} \cdot z^{-4^{n-1}} + S_{3,n-1} \cdot z^{-2 \cdot 4^{n-1}} + S_{4,n-1} \cdot z^{-3 \cdot 4^{n-1}} \\ S_{1,n-1} - S_{2,n-1} \cdot z^{-4^{n-1}} + S_{3,n-1} \cdot z^{-2 \cdot 4^{n-1}} - S_{4,n-1} \cdot z^{-3 \cdot 4^{n-1}} \\ S_{1,n-1} + S_{2,n-1} \cdot z^{-4^{n-1}} - S_{3,n-1} \cdot z^{-2 \cdot 4^{n-1}} - S_{4,n-1} \cdot z^{-3 \cdot 4^{n-1}} \\ S_{1,n-1} - S_{2,n-1} \cdot z^{-4^{n-1}} - S_{3,n-1} \cdot z^{-2 \cdot 4^{n-1}} + S_{4,n-1} \cdot z^{-3 \cdot 4^{n-1}} \end{bmatrix} \end{aligned} \quad (14)$$

The product  $\mathbf{W}_{4,n} \cdot \mathbf{H}_4 \cdot \mathbf{S}_n$  (considering  $\mathbf{W}_{4,n} = \mathbb{I}_4$ ) is:

$$\begin{bmatrix} +1 & +1 & +1 & +1 \\ +1 & -1 & +1 & -1 \\ +1 & +1 & -1 & -1 \\ +1 & -1 & -1 & +1 \end{bmatrix} \cdot \begin{bmatrix} S_{1,n} \\ S_{2,n} \\ S_{3,n} \\ S_{4,n} \end{bmatrix} = 4 \cdot \begin{bmatrix} S_{1,n-1} \\ S_{2,n-1} \cdot z^{-4^{n-1}} \\ S_{3,n-1} \cdot z^{-2 \cdot 4^{n-1}} \\ S_{4,n-1} \cdot z^{-3 \cdot 4^{n-1}} \end{bmatrix} \quad (15)$$

As a result, four sequences of length  $\frac{L}{4}$  are obtained, each one delayed  $4^{n-1} = \frac{L}{4}$  with respect to the previous one (being  $L$  the length of the sequences of  $\mathbf{S}_N$ ). These sequences must be synchronized by a proper delay in order to obtain a new M-CSS ( $\mathbf{C}_{n-1} = M \cdot \mathbf{S}_{n-1} \cdot z^{-3 \cdot 4^{n-1}}$ ). Then, the proposed correlation algorithm for the  $n$ -th iteration is:

$$\mathbf{C}_{n-1} = \mathbf{D}'_{M,n} \cdot \mathbf{W}_{M,n} \cdot \mathbf{H}_M \cdot \mathbf{C}_n \quad (16)$$

where  $\mathbf{D}'_{M,n}$  is the delay matrix:

$$\mathbf{D}'_{M,n} = \begin{bmatrix} z^{-(M-1)M^{n-1}} & 0 & \cdots & 0 \\ 0 & z^{-(M-2)M^{n-1}} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 1 \end{bmatrix} \quad (17)$$

Note that the delays are inversely ordered with respect to (11).

Generalizing, the formulation for  $N$  stages is:

$$\mathbf{C}_0 = \prod_{n=N}^1 (\mathbf{D}'_{M,n} \cdot \mathbf{W}_{M,n} \cdot \mathbf{H}_M) \cdot \mathbf{C}_N \quad (18)$$

and the addition of the  $M$  correlation outputs is:

$$Y[k] = \sum_{i=1}^M C_{i,0}[k] \quad (19)$$

Fig. 1 depicts the block diagram of the proposed correlator for  $M = 4$ . As shown in this figure, delays are placed at the output of each stage unlike in the generator (according to Eq. (16)). The gains and adders represent the product  $\mathbf{W}_{M,n} \cdot \mathbf{H}_M$ .

Despite the fact that other generation approaches are used in some applications, each of them allows to obtain different families of complementary sequences. The proposed correlator can be applied in these cases by means of a suitable transformation. Appendix A at the end of this work summarizes other generation approaches and the required transformation matrices.

### 3.1. Example

Given an M-CSS generated with (13) and assuming that the input of the correlator is the generator output ( $\mathbf{C}_N = \mathbf{S}_N$ ), then the correlator output is:

$$\mathbf{C}_0 = \prod_{n=N}^1 (\mathbf{D}'_{M,n} \cdot \mathbf{W}_{M,n} \cdot \mathbf{H}_M) \cdot \prod_{n=1}^N (\mathbf{H}_M \cdot \mathbf{W}_{M,n} \cdot \mathbf{D}_{M,n}) \mathbf{S}_0 \quad (20)$$

Assuming an M-CSS generated with  $N = 2$ , the output of the first correlation stage is:

$$\begin{aligned} \mathbf{C}_1 &= (\mathbf{D}'_{M,2} \mathbf{W}_{M,2} \mathbf{H}_M) \cdot \mathbf{S}_2 \\ \mathbf{C}_1 &= (\mathbf{D}'_{M,2} \mathbf{W}_{M,2} \mathbf{H}_M) (\mathbf{H}_M \mathbf{W}_{M,2} \mathbf{D}_{M,2}) \cdot \mathbf{S}_1 \\ &= \mathbf{D}'_{M,2} \mathbf{W}_{M,2} \mathbf{H}_M \mathbf{H}_M \mathbf{W}_{M,2} \mathbf{D}_{M,2} \mathbf{S}_1 \end{aligned} \quad (21)$$

Computing some associations,

$$\begin{aligned} \mathbf{C}_1 &= \mathbf{D}'_{M,2} \mathbf{W}_{M,2} (\mathbf{H}_M \mathbf{H}_M) \mathbf{W}_{M,2} \mathbf{D}_{M,2} \mathbf{S}_1 \\ &= \mathbf{D}'_{M,2} \mathbf{W}_{M,2} (M \cdot \mathbb{I}_M) \mathbf{W}_{M,2} \mathbf{D}_{M,2} \mathbf{S}_1 \\ &= M \cdot \mathbf{D}'_{M,2} \mathbf{W}_{M,2} \mathbf{W}_{M,2} \mathbf{D}_{M,2} \mathbf{S}_1 \end{aligned} \quad (22)$$

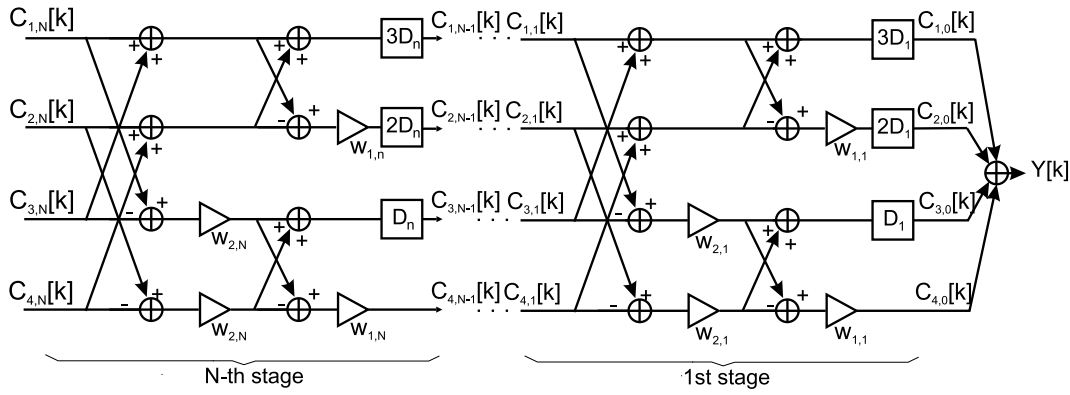


Fig. 1. Proposed M-CSS correlator of  $N$  stages for  $M = 4$ .

where  $M$  is a scalar and  $\mathbb{I}_M$  is the identity matrix. In the same way,  $\mathbf{W}_{M,2} \cdot \mathbf{W}_{M,2} = \mathbb{I}_M$  when the seeds are  $\pm 1$ . After such simplification, the following is obtained:

$$\begin{aligned} \mathbf{C}_1 &= M(\mathbf{D}'_{M,2} \mathbf{D}_{M,2}) \mathbf{S}_1 \\ &= M \cdot (z^{-(M-1)M} \cdot \mathbb{I}_M) \mathbf{S}_1 \end{aligned} \quad (23)$$

Then, if  $\mathbf{S}_1 = (\mathbf{H}_M \mathbf{W}_{M,1} \mathbf{D}_{M,1}) \cdot \mathbf{S}_0$ ,

$$\begin{aligned} \mathbf{C}_0 &= (\mathbf{D}'_{M,1} \mathbf{W}_{M,1} \mathbf{H}_M) \cdot M(z^{-(M-1)M} \cdot \mathbb{I}_M) \mathbf{S}_1 \\ &= M \cdot z^{-(M-1)M} (\mathbf{D}'_{M,1} \mathbf{W}_{M,1} \mathbf{H}_M) \\ &\quad \cdot (\mathbf{H}_M \mathbf{W}_{M,1} \mathbf{D}_{M,1}) \mathbf{S}_0 \end{aligned} \quad (24)$$

and after iteratively processing (24), the result is:

$$\mathbf{C}_0 = M^2 \cdot z^{-(M^2-1)} \cdot \mathbb{I}_M \cdot \mathbf{S}_0 \quad (25)$$

The output of the correlator ( $Y$ ) is the addition of the  $M$  correlation outputs. Being  $\mathbf{C}_0 = [C_{1,0} \ C_{2,0} \ \dots \ C_{M,0}]^T$ ,

$$Y = \sum_{i=1}^M C_{i,0} = M \cdot M^2 \cdot z^{-(M^2-1)} \quad (26)$$

### 3.2. Simulations

In order to illustrate the operation of the correlator described in this section, a simulation is conducted. Fig. 2 displays the waveforms of the proposed correlator in which a 4-CSS of length  $L = 64$  ( $N = 3$ ) generated with a  $\mathbf{W} = \mathbb{I}_4$  is present at the input.

The signals at the input of the correlator are considered to be those corresponding to the output of the generator of Eq. (11). As explained above, in each correlation stage, the length of the sequences is divided by  $M$  and the amplitude is multiplied by the same factor.

$$\mathbf{C}_2 = (\mathbf{D}'_{4,3} \mathbf{W}_{4,3} \mathbf{H}_4) \cdot \mathbf{C}_3 = (\mathbf{D}'_{4,3} \mathbf{W}_{4,3} \mathbf{H}_4) \cdot \mathbf{S}_3 \quad (27)$$

from (21), (22) and (23):

$$\mathbf{C}_2 = 4 \cdot z^{-48} \cdot \mathbb{I}_4 \cdot \mathbf{S}_2 \quad (28)$$

Fig. 2 demonstrates that the output of the first correlation stage is a 4-CSS with a 48-sample delay, scaled by 4. By repeating this process with the next stage:

$$\mathbf{C}_1 = 16 \cdot z^{-60} \cdot \mathbb{I}_4 \cdot \mathbf{S}_1 \quad (29)$$

A new 4-CSS of amplitude 16 and length 4 is obtained, as verified in the simulation (Fig. 2). By repeating this process once again and adding the outputs, the Kronecker delta of amplitude  $M \cdot L = 256$  is obtained, as expected.

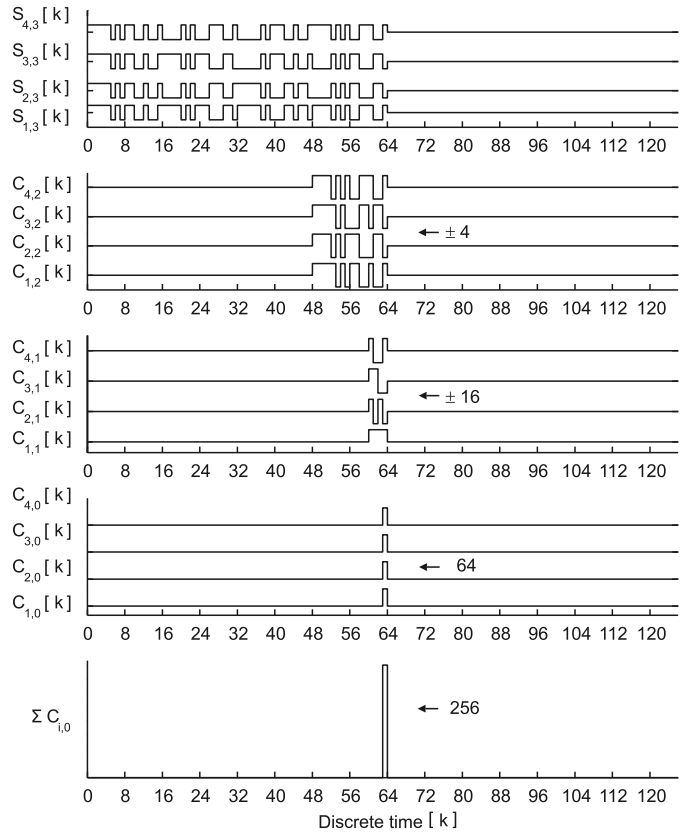


Fig. 2. 4-CSS correlation using the transposed generation approach.

## 4. Hardware implementation of the proposed algorithm

The proposed correlator implementation is synthesized in hardware (Field Programmable Gate Array – FPGA) by using adders (or subtractors) and delays, as shown in Fig. 1. The parameters needed to define these hardware resources are  $M = 2^m$ ,  $N$ ,  $W$  and the ADC (Analog to Digital Converter) quantization  $q$ . This last parameter increases stage by stage as a result of the arithmetic operations with the sequences. Considering that a typical FPGA is composed of 4 inputs LUTs (Look Up Tables) and Flip Flops (FFs), the correlator resources consumption can be expressed by these two variables.

The structure of the correlator, as presented in the previous section, is composed of  $N$  stages and a final addition. Each stage includes  $M \cdot m$  additions and  $M \frac{M-1}{2}$  shift registers of length  $\sum_{n=1}^N M^{N-n}$ . Eqs. (30) and (31) express the required logic re-

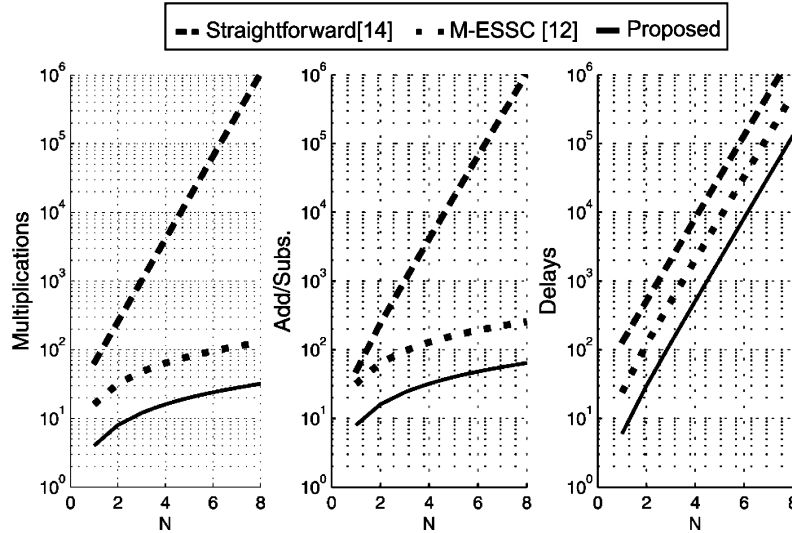


Fig. 3. Amounts of operations needed to perform 4-CSS correlations.

Table 1

Implementation results for 4-CSS and 8-CSS.

$M$	4					8		
$N$	1	2	3	4	5	1	2	3
FGs	110	208	322	452	598	328	661	1066
FFs	92	344	1364	5456	21 836	372	2920	23 388
Clock [MHz]	149.1	83.3	57.8	44.2	35.8	105.4	57.3	39.4

sources in terms of LUTs and Flip Flops by considering the wordlength increment of every arithmetic operation:

$$LUTs = M \cdot \left( m \cdot N \cdot q + m \cdot N \frac{1 + m \cdot N}{2} \right) + \sum_{n=1}^m 2^{m-n} \cdot (q + m \cdot N + n) \quad (30)$$

$$FFs = M \frac{M-1}{2} \cdot \left( \sum_{n=1}^N M^{N-n} \cdot (q + n \cdot m) \right) \quad (31)$$

In order to evaluate the proposed architecture and the resources consumption estimation, the correlator was implemented in a real platform with a Xilinx Spartan 3 development kit (3S1500FG320-5) and Xilinx ISE software. The FPGA applied contains 13312 slices, each with the following elements in common: 2 logic function generators (FG), 2 storage elements (FF), wide-function multiplexers, carry logic and arithmetic gates. Table 1 summarizes the implementation results for  $M = 4$  and  $M = 8$ , respectively. The parameters evaluated were the set size  $M$  and the number of stages  $N$ , with a quantization of the input sequences  $q = 8$ . The same kind of seed matrix was used for all the implementations.

Table 1 shows the implementation results. Note that the number of calculations impacts directly on the hardware regarding resources utilization. Such utilization increases as the number of stages does, as well with the set size. Hence, processing speed reduces inversely with these parameters.

## 5. Efficiency of the proposed correlator

The main contribution of the proposed correlator is the reduction of calculation resources with regard to other architectures, with no loss of transmission rate. Considering the simultaneous transmission of all the sequences of an M-CSS, the correlator proposed by De Marziani et al. (M-ESSC [12]) executes  $(M-1)$  unnec-

Table 2

Number of calculations for an M-CSS correlation.

Operations	Straightforward [14]	M-ESSC [12]	Proposed
Multiplications	$M^{N+2}$	$\frac{M^2}{2} \cdot N \cdot \log_2(M)$	$\frac{M}{2} \cdot N \cdot \log_2(M)$
Add/Subs.	$M^2(M^N - 1)$	$M^2 \cdot N \cdot \log_2(M)$	$M \cdot N \cdot \log_2(M)$
Delays	$2 \cdot M^{N+2}$	$\frac{M^2}{2} \cdot (M^N - 1)$	$\frac{M}{2} \cdot (M^N - 1)$

essary correlations, and hence  $M$  correlators are needed to obtain the sum of autocorrelations, just like in the *straightforward* correlation [14].

Another advantage of the proposed correlator is the interconnection of the outputs of one stage with the inputs of the following one. Due to the generation approach used in this proposal, the outputs of each stage are ordered in the same way as the inputs. This does not occur in other generation approaches, as those presented in [12,11], in which the interconnection between stages is governed by an algorithm. Even though this drawback was later solved with the re-arrangement proposed in [17], the outputs order still differs from that of the inputs.

Table 2 summarizes the number of calculations necessary to perform  $Y[k]$  for an M-CSS of  $N$  stages; and Fig. 3 exemplifies the number of operations for a 4-CSS with  $N = 1, \dots, 8$ . The reduction of any of the amounts of operations considered is important, especially for larger sequences. Note that the number of calculations of the M-ESSC and that of the proposed correlator are related by a scale factor  $M$  (set size).

### 5.1. Hardware implementation comparison

Since it was demonstrated in [12] that the M-ESSC is more efficient than the *straightforward* correlator, the implementation of the proposed correlator will be compared with the M-ESSC implementation presented in [17]. Table 3 shows the resources consumption rated in slices, Flip-Flops and LUTs for both architectures. In order to correctly compare both correlators, the proposed correlator was



**Table 3**  
Proposals comparison.

$M = 4, L = 64$	Perez et al. [17]	Proposed
Slices	349	289
LUTS	578	446
FFs	(not reported)	140
Frequency	33.8 MHz	57.11 MHz

implemented in the same FPGA reported by Perez et al. with the option of SRL (Luts based shift registers) synthesis enabled, as it was reported in the paper. This option allows Flip-Flops consumption reduction by synthesizing the shift registers as state machines. As shown in this table, the resources consumption is about 20% lesser and the transmission rate about 68% higher with respect to M-ESSC. Also bear in mind that the M-ESSC requires  $M$  correlators to perform the simultaneous correlation of all the sequences of a complementary set.

## 6. Conclusions

This paper presents an improved correlation algorithm for M-CSS. It is based on a transposition of the generation process, similar to the one employed for Golay sequences [15]. The approach herein simplifies the correlation algorithm, allowing a recursive regular and modular calculation scheme that simultaneously performs  $M$  correlations of  $M$  inputs. It also allows a more efficient implementation requiring lower hardware complexity with respect to the straightforward and other previously proposed correlation schemes. The achieved reduction is almost  $M$  times for all cases. This approach can also be applied to systems where the M-CSS has been generated with other algorithms, by using an adequate linear transform (see Appendix A). The calculation efficiency encouraged by this contribution represents another essential step forward in the practical application of these sequences.

## Acknowledgments

This work was supported by the Universidad Nacional de Mar del Plata, the CONICET and the ANPCyT. The authors work with the Department of Electronics at the Universidad Nacional de Mar del Plata, Argentina.

## Appendix A. Other generation approaches

Some authors have introduced other approaches for the recursive generation of complementary sets of sequences. In [11], Álvarez et al. proposed an efficient architecture to generate and correlate a set of four complementary sequences (4-CSS). It is a simplification of the Tseng and Liu [2] algorithm based on the use of seed coefficients and different permutation delays. This generation approach can be resumed with the generation matrix  $\Gamma$ , which comprises the sign of the elements and the generation seed for the 4-CSS:

$$\mathbf{S}_n[z] = \Gamma_{4,n} \cdot \mathbf{D}_{4,n} \cdot \mathbf{S}_{n-1}[z] \quad (32)$$

where

$$\Gamma_{4,n} = \begin{bmatrix} 1 & -1 & w_{1,n} & w_{1,n} \\ 1 & 1 & w_{1,n} & -w_{1,n} \\ w_{2,n} & -w_{2,n} & -w_{1,n} \cdot w_{2,n} & -w_{1,n} \cdot w_{2,n} \\ w_{2,n} & w_{2,n} & -w_{1,n} \cdot w_{2,n} & -w_{1,n} \cdot w_{2,n} \end{bmatrix} \quad (33)$$

Later on, De Marziani et al. [12] generalized the generation and correlation architectures to M-CSS. This formulation is based on a generation matrix  $\Lambda_{M,n}$  summed up in (34). This matrix is generated in a recursive way, producing interleaves between matrices of

lower order,  $\Lambda_{\frac{M}{2},n}$  and the  $n$ -th generation seed. The choice of a different generation seed,  $W$ , enables the generation of different sets of complementary sequences.

$$\Lambda_{M,n} = \begin{bmatrix} \Lambda_{\frac{M}{2},n} \oplus (w_{r,n} \cdot (-\Lambda_{\frac{M}{2},n})) \\ \Lambda_{\frac{M}{2},n} \oplus (w_{r,n} \cdot \Lambda_{\frac{M}{2},n}) \end{bmatrix} \quad (34)$$

where  $\oplus$  denotes interleaving between elements of both matrices, and  $\Lambda_{(n)}^{(1)}$  is

$$\Lambda_{2,n} = \begin{bmatrix} 1 & w_{1,n} \\ 1 & -w_{1,n} \end{bmatrix} \quad (35)$$

This approach allows to obtain sets of  $M$  sequences of length  $M^n$ . It is worth noticing that, with this approach, the implementation of an M-CSS does not result in an ordered architecture. Unlike the approach presented in this paper, the outputs of a stage do not correspond to the inputs of the next stage.

Regardless of the approach adopted, a relationship can be distinguished, which allows to transform a complementary set into another one. Both, Álvarez and De Marziani generation matrices can be transformed to match the product  $\mathbf{H} \cdot \mathbf{W}$ .

For example, for a 4-CSS set proposed by Álvarez, with  $w_{1,n} = 1$  and  $w_{2,n} = -1$ , the transformation matrix  $\mathbf{T}$  can be obtained with:

$$\mathbf{T}_{4,n} = \Gamma_{4,n} \cdot \mathbf{W}_{4,n}^{-1} \cdot \mathbf{H}_{4,n}^{-1} \quad (36)$$

where

$$\mathbf{T}_{4,n} = 0.5 \cdot \begin{bmatrix} -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ -1 & -1 & 1 & -1 \\ -1 & -1 & -1 & 1 \end{bmatrix} \quad (37)$$

This means that a set of complementary sequences generated with the previously proposed approach is transformed into a set by Álvarez with the transformation presented in (36). The same procedure can be conducted to transform an M-CSS into a set by De Marziani. An example of this transformation for a 4-CSS is presented in (38) and it is valid for any seed.

$$\mathbf{T}_{4,n} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad (38)$$

This means that the approach by De Marziani differs from that proposed herein only in the sequences ordering for generation. In this case, it is possible to find the  $\mathbf{T}$  matrix for any M-CSS, regardless of the seed coefficients.

## References

- [1] M. Golay, Complementary series, IRE Trans. Inf. Theory 7 (2) (1961) 82–87, <http://dx.doi.org/10.1109/TVT.1961.1057620>.
- [2] C.-C. Tseng, C. Liu, Complementary sets of sequences, IEEE Trans. Inform. Theory 18 (5) (1972) 644–652, <http://dx.doi.org/10.1109/TVT.1972.1054860>.
- [3] H. Wang, X. Gao, B. Jiang, X. You, W. Hong, Efficient MIMO channel estimation using complementary sequences, IET Commun. 1 (5) (2007) 962–969, <http://dx.doi.org/10.1049/iet-com:20060581>.
- [4] A. Vazquez Alejos, D. Muhammad, H. Ur Rahman Mohammed, Ground penetration radar using Golay sequences, in: Proceedings of the 2007 IEEE, Region 5 Technical Conference, pp. 318–321, <http://dx.doi.org/10.1109/TPSD.2007.4380327>.
- [5] T.R. Qureshi, M.D. Zoltowski, R. Calderbank, A. Pezeshki, Unitary design of radar waveform diversity sets, Digit. Signal Process. 21 (5) (2011) 552–567, <http://dx.doi.org/10.1016/j.dsp.2010.09.006>.
- [6] C. De Marziani, J. Ureña, A. Hernandez, M. Mazo, F. Alvarez, J. Garcia, J. Villadangos, A. Jimenez, Simultaneous measurement of times-of-flight and communications in acoustic sensor networks, in: Proc. IEEE International Workshop on Intelligent Signal Processing, 2005, pp. 122–127, <http://dx.doi.org/10.1109/WISP.2005.1531644>.

- [7] P. Donato, C.D. Marziani, D. Carrica, M. Funes, J. Ureña, Algorithm for complementary-derived orthogonal sequences applied to measurements in noisy channels, *IET Signal Process.* 4 (6) (2010) 610–617, <http://dx.doi.org/10.1049/iet-spr.2009.0117>.
- [8] P.G. Donato, J. Ureña, M. Mazo, C.D. Marziani, A. Ochoa, Design and signal processing of a magnetic sensor array for train wheel detection, *Sens. Actuators A Phys.* 132 (2) (2006) 516–525, <http://dx.doi.org/10.1016/j.sna.2006.02.043>.
- [9] S. Budisin, Efficient pulse compressor for Golay complementary sequences, *Electron. Lett.* 27 (3) (1991) 219–220, <http://dx.doi.org/10.1049/el:19910142>.
- [10] B. Popovic, Efficient Golay correlator, *Electron. Lett.* 35 (17) (1999) 1427–1428, <http://dx.doi.org/10.1049/el:19991019>.
- [11] F. Alvarez, J. Ureña, M. Mazo, A. Hernandez, J. Garcia, J. Jimenez, Efficient generator and pulse compressor for complementary sets of four sequences, *Electron. Lett.* 40 (11) (2004) 703–704, <http://dx.doi.org/10.1049/el:20040438>.
- [12] C. De Marziani, J. Ureña, A. Hernandez, M. Mazo, F. Alvarez, J. Garcia, P. Donato, Modular architecture for efficient generation and correlation of complementary set of sequences, *IEEE Trans. Signal Process.* 55 (5) (2007) 2323–2337, <http://dx.doi.org/10.1109/TSP.2006.890931>.
- [13] M. del Carmen Pérez, J. Ureña, Á. Hernández, C.D. Marziani, A. Jiménez, W. Marnane, Hardware implementation of an efficient correlator for interleaved complementary sets of sequences, *J. Univers. Comput. Sci.* 13 (3) (2007) 388–406.
- [14] M. Hahn, E. Friedman, E. Titlebaum, A comparison of analog and digital circuit implementations of low power matched filters for use in portable wireless communication terminals, *IEEE Trans. Circuits Syst., 2 Analog Digit. Signal Process.* 44 (6) (1997) 498–506, <http://dx.doi.org/10.1109/82.592584>.
- [15] P. Donato, M. Funes, M. Hadad, D. Carrica, Optimised Golay correlator, *Electron. Lett.* 45 (7) (2009) 380–381, <http://dx.doi.org/10.1049/el.2009.2923>.
- [16] F.J. Álvarez, Á. Hernández, J. Ureña, M. Mazo, J.J. García, J.A. Jiménez, A. Jiménez, Real-time implementation of an efficient correlator for complementary sets of four sequences applied to ultrasonic pulse compression systems, *Microprocess. Microsyst.* 30 (1) (2006) 43–51, <http://dx.doi.org/10.1016/j.micpro.2005.04.003>.
- [17] M.C. Pérez, J. Ureña, A. Hernandez, C. De Marziani, A. Ochoa, W.P. Marnane, FPGA implementation of an efficient correlator for complementary sets of sequences, in: *Proc. Int. Conf. Field Programmable Logic and Applications, FPL'06*, 2006, pp. 1–4, <http://dx.doi.org/10.1109/FPL.2006.311293>.



**Marcos A. Funes** was born in Argentina in 1974. He was awarded his B.S. degree in Electronics Engineering by the Universidad Nacional de Mar del Plata (UNMDP), Mar del Plata, Argentina, in 1999 and his Ph.D. in Electronics by Universidad Nacional de Mar del Plata (UNMDP), Mar del Plata, Argentina, in 2007. In 1999, he joined the Department of Electronics, Universidad Nacional de Mar del Plata (UNMDP), as Assistant Professor and Research Assistant. Since 2009 he joined the Consejo Nacional de Investigaciones Científicas y Técnicas (CONICET) as research assistant. His current research interests include high density programmable logic devices and digital signal processing.



**Patricio G. Donato** was born in Argentina in 1975. He was awarded his B.S. degree in Electronics Engineering by the Universidad Nacional de la Patagonia “San Juan Bosco”, Patagonia, Argentina, in 2000, and his Ph.D. degree in Electronics by the Universidad de Alcalá, Alcalá de Henares, Spain,

in 2005. Presently he works for the Universidad Nacional de Mar del Plata (UNMDP), Mar del Plata, Argentina. His main research interests include digital signal processing, local positioning systems (LPS), power line communications and synchronism methods. Dr. Donato is a member of the Consejo Nacional de Investigaciones Científicas y Técnicas (CONICET), Argentina.



**Matías N. Hadad** was born in Argentina in 1985. He was awarded his B.S. degree in Electronics Engineering by the Universidad Nacional de Mar del Plata (UNMDP), Mar del Plata, Argentina, in 2010, where he is currently working toward the Ph.D. in Electronics. His research interests include signal processing, power line communications and programmable logic devices.



**Daniel O. Carrica** was born in Dolores, Argentina, in 1958. He was awarded his B.S. degree in Engineering from the Universidad Nacional de Mar del Plata, Argentina, in 1984 and the M.Sc. degree in Electronics from the Universidad Politécnica de Madrid, Spain, in 1992. In 1984, he joined the Department of Electronics, Universidad Nacional de Mar del Plata, as a Research Assistant. From 1990 to 1991, he was an Associate Scientist at the European Organization for Nuclear Research (CERN), Geneva, Switzerland. From 1994 to 1996, he was Head of the Department of Electronics, UNMDP where he is currently an Associate Professor. His current research interests include motion control and power electronics.



**Mario Benedetti** was born in Italy in 1945. He received the Ing. degree in Telecommunications Engineering from the Universidad Nacional de La Plata, La Plata, Argentina, in 1968. From 1968 to 1983, he was with the Laboratorio de Electrónica Industrial, Control e Instrumentación, Universidad Nacional de La Plata, working on developing electronic instruments. From 1970 to 1983, he was also an Associate Professor with the Department of Electrical Engineering at the same university. He spent two years as a Fellow at the Conseil Européen pour la Recherche Nucleaire, Geneva, Switzerland. Since 1985, he has been a Full Professor with the Department of Electrical Engineering, Universidad Nacional de Mar del Plata, Mar del Plata, Argentina. He is currently the Head of the Laboratorio de Instrumentación y Control, Departamento de Electrónica, Universidad Nacional de Mar del Plata, and a member of the National Scientific and Technical Research Council (CONICET), Buenos Aires, Argentina. He has served as a Lecturer for numerous short courses presented to the industry and other universities. His current research interests include power electronics and EMC.

Switzerland. Since 1985, he has been a Full Professor with the Department of Electrical Engineering, Universidad Nacional de Mar del Plata, Mar del Plata, Argentina. He is currently the Head of the Laboratorio de Instrumentación y Control, Departamento de Electrónica, Universidad Nacional de Mar del Plata, and a member of the National Scientific and Technical Research Council (CONICET), Buenos Aires, Argentina. He has served as a Lecturer for numerous short courses presented to the industry and other universities. His current research interests include power electronics and EMC.