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ABSTRACT
Rational Expectations (RE) is typically interpreted as: (i) an equivalence between the probability distribution of future outcomes informing agents´ decisions and the objective distributions; or: (ii) a correspondence between the expectations of agents and those generated by professionally validated models. Both definitions differ, unless absolute validity is counterfactually attributed fallible models built by economists. Another ambiguity arises with the model-consistency notion, since what is considered relevant theory has varied over time and across researchers, especially in Macroeconomics. These issues affect the logic and significance of analytical procedures for treating expectations, and seem particularly pertinent when studying crises.

RESUMEN

La noción de expectativas racionales (RE) se interpreta típicamente como (i) una equivalencia entre la distribución de probabilidades de eventos futuros que guía las decisiones de los agentes y la distribución que de hecho caracteriza a esa evolución futura; o, (ii) una correspondencia entre las expectativas de los agentes y las distribuciones que serían generadas por la teoría relevante, profesionalmente validada. Ambas definiciones son distintas a menos que, contrafácticamente, se atribuya validez plena a los falibles y mutables modelos elaborados por los economistas. Una ambigüedad adicional surge con la noción de modelo- consistencia, porque lo que se considera teoría relevante ha variado con el tiempo y suele diferir entre analistas, particularmente en Macroeconomía. Estas cuestiones afectan a la lógica y el significado de los procedimientos usados en la representación de las expectativas, y serían especialmente centrales en el estudio de fenómenos de crisis.

1 Introduction

For several decades already, the Rational Expectations (RE) notion has been at the center of macroeconomic analysis. In its concrete applications, RE has been generally identified with the coincidence between the anticipations that a model attributes to the agents and those that would be generated by the same current model. The literature has certainly explored alternatives, such as expectations based on various statistical learning schemes, behavior incorporating model uncertainty, or simple heuristics such as extrapolative expectations (De Long, Shleifer, Summers & Waldmann, 1990; Sargent, 1993; Hansen & Sargent, 2000; Choi & Mertens, 2006; Lansing, 2006; Evans & Honkapohja, 2001; Barberis, Greenwood, Jin & Shleifer, 2016). However, RE in its typical form has retained its prominence as a presumed sign of rigorous reasoning (almost to the point of embodying a methodological precept), and as the default option in macro modeling, against which other possible representations of agents' forecasts are charged with a heavy burden of proof.

In this article, we shall not be concerned particularly with the practical pertinence of RE for the study of macro phenomena (for a discussion focused on crises see, for example, Heymann and Leijonhufvud (2014)), or with its usefulness as a practical heuristic device. Rather, we concentrate on the logic of RE as an analytical construct. We will indicate (Section 2) that, as commonly understood in Macroeconomics, RE is an ambiguous, ill-defined concept, which, when given a specific definition, varies between (i) the statement that the probability distributions of the outcomes anticipated by agents equal the objective distributions characterizing in fact those outcomes[1] and (ii) the idea that the expectations of agents incorporate at each moment the knowledge derived from economic analysis. We shall then propose (Sections 3-6) that the usual practice of RE modeling implements neither one of those definitions and, consequently, can hardly be given a precise interpretation.

The argument leading to that conclusion is quite unsophisticated, and follows directly from the straightforward recognition that economic analysis is an ongoing activity, so that its practitioners, who are continuously and consciously engaged in refining, extending, or revising their understanding of the economy, should naturally recognize that (i) their constructs do not possess the status of "true models" of the system of interest, and that (ii) those provisional analytical objects have been evolving and will continue to evolve. Failure to incorporate these obvious facts has implied that the presumed implementation of the notion that expectations are correct in probabilities has confused the model that the analyst tentatively posits at present with the generator of objectively correct expectations. When invoking the meaning of RE as consistency with the relevant theory, the usual practice has brought about an unwarranted assimilation of the current model, which corresponds to the analyst's state of knowledge at this date, with the engine for making forecasts that the agents have used in the past and will use in the future.

We will suggest that these problems are not simply a matter of analytical esthetics, but they have concrete consequences for the assessment of the significance and consistency of the respective models. By the self-referential nature of the object of study, the economist tries to understand and to represent social outcomes generated by the behavior of agents who are also in the business of making practical sense of the same system. Thus, the job of the researcher has intrinsically an aspect of "applied epistemology". This calls for a reflection on the feature, the dynamics and the possible real- world implications of the analytical work itself. If the time varying and fallible nature of the representations built by economists is explicitly taken into account, "model consistent" expectations could then be viewed as those based on analytical schemes prevalent or influential when the anticipations are made. From an applied point of

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[1] Also expressed as $PLM = ALM$: the perceived law of motion is identical with the actual law of motion governing the performance of the system.

view, the argument may be especially relevant for the study of debt crises, characterized by a widespread sense of discomfort with previous representations of the economy and an active search for lessons from the experience among both agents and analysts.

2 Rational Expectations: An Ambiguous Notion

In the celebrated work that introduced the RE framework, Muth (1961) expresses the concept by the statement:

> "... expectations of firms (or more generally, the subjective probability distribution of outcomes) tend to be distributed, for the same information set, about the prediction of the theory (or the 'objective' probability distribution of outcomes)".

That explanation has substantive content, much more than the vague pronouncements often encountered, such as "agents make no systematic mistakes". However, an ambiguity is present in the parenthetical observation: the objective distribution cannot be equated automatically with the prediction of the theory. The distinction between the two meanings would be particularly relevant in Macroeconomics, a field where the search for appropriate representations seems far from having converged. One way of trying to avoid the duality of meaning is by conflating the three perspectives of the analyst, the agent, and the real world evolution of the system:

> "A rational expectations equilibrium is a fixed point of this mapping (from perceived law of motion to actual law of motion). From a practical perspective, an important property of a rational expectations model is that it imposes a communism of models and expectations. If we define a model as a probability distribution over a sequence of outcomes, possibly indexed by a parameter vector, a rational expectations equilibrium asserts that the same model is shared by (1) all of the agents within the model, (2) the econometrician estimating the model, and (3) nature, also known as the data generating mechanism. Different agents might have different information, but they form forecasts by computing conditional expectations with respect to a common joint density, that is, a common model." (Sargent T., 2008, p. 878)

However, the identity between the forecast produced by the economist's model and that which would result from the actual law of motion can only be sustained if the model is assumed to generate strictly optimal predictions (in probability), not subject to any improvement. But this contradicts the obvious fact that economic theories and models concerning a given set of phenomena vary substantially over time (as they may also do across researchers). Therefore, the two potential concepts ("prediction of theory" and "objective distribution") must be distinguished to accommodate the fact that theories do change over time.[2]

In what follows we shall stress the difference between the (current) model built by the economist and the model that would produce outcomes matching the actual data generating processes. Accordingly, we shall deal with two separate concepts of Rational Expectations: RE1, representing the equivalence between the stochastic law of motion of the system of interest as perceived by agents and the actual law of motion, and RE2, denoting the correspondence of the forecasts generated by agents with those that the economist would derive from her model.

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[2] As Daniel Aromi has remarked to us, one could argue that what may seem a "change in knowledge" which implies a revision of previous beliefs and modes of analysis does not introduce in fact an actual novelty, but results in fact from an existing meta scheme, which dictates a re specification of the existing model as new information arrives. However, what an analyst as such does know is embodied in explicit arguments (the agent, by contrast, may rely on tacit perspicacity). If the presumptive model choosing device does not have an open accessible expression, there is effectively no such grand analytical tool.

So as to permit a discussion of policy issues like the Lucas critique, we will distinguish between the expectations of the government and those formed by the private sector (taken as a whole: in order to focus the argument, we will abstract away matters regarding second and higher order expectations within those sets of agents). Thus, RE1.1 will denote the case where both policy makers and private individuals base their expectations on the ALM (actual law of motion of the system), while RE1.2 indicates that the private sector acts as if it was aware of the ALM; while the government projects future conditions using the (fallible) model of the analyst; similarly with RE2.1 (private sector: model consistent expectations, government: $PLM = ALM$) and RE2.2 (both model consistent).[3]

3 General Setup

The actual evolution of the system is specified as a function of the realized history and the looking ahead behavior of agents (private and public). This behavior is determined by decision rules predicated on the system's history and on the probability distribution of the future values of the variables of interest, as perceived by the agents when the expectations are formed. The economy is also potentially hit by random shocks: these have the nature of intrinsically stochastic exogenous impulses. That is:

$$Y^{t'} = F\left(Y_{(t-1)'}, H^P\left(Y_{(t-1)'}, \phi_{(t-1)}^{eP}\left(Y^{t'}\right)\right), H^G\left(Y_{(t-1)'}, \phi_{(t-1)}^{eG}\left(Y^{t'}\right)\right), Z^{t'}\right) \qquad (3.1)$$

Where:

- Y is the vector of relevant variables; Y_t its value at t.

- $Y^{t'}$ indicates the system's current and future states: $Y^{t'} = [Y_t, ..., Y_{t+j}, ...]$.

- $Y_{(t-1)'}$ denotes the history of the system up to $t-1$.

- F is the function that specifies the current state and the random future path of the system as a consequence of the past history, the behavior of agents and the sequence of present and future shocks.

- H summarizes the decision rules of agents (P and G), which determine behavior as function of the observed history of the system and the anticipated distribution of the state[4]. For the sake of the argument, H will be interpreted in the usual way, as the result of an overall optimization on the part of agents.

- ϕ denotes a probability distribution; ϕ_t^{ei} is a perceived distribution that embodies the expectations of agent i (P, G) at t.

- $Z^{t'}$ is the (random) sequence of present and future actually exogenous external shocks (in contrast with "error terms" representing "measures of ignorance" for agents).

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[3] Since the discussion concentrates on the specification of RE notions, we exclude from the analysis the general case where the working schemes of reasoning that guide the anticipations of the agents are imperfect, and also differ from the forecasts produced by the economist's model.

[4] These actual behavior- inducing functions are given, and fixed over time, although the history- dependence embodied in H can lead to changes of the reduced expression of the decision rule as a "reaction function".

The probabilistic actual law of motion will then be given by:

$$\phi\left(Y^{t'}\right) = \phi\left(F\left(Y_{(t-1)'}, H^P\left(Y_{(t-1)'}, \phi^{eP}_{(t-1)}\left(Y^{t'}\right)\right), H^G\left(Y_{(t-1)'}, \phi^{eG}_{(t-1)}\left(Y^{t'}\right)\right), Z^{t'}\right)\right) \quad (3.2)$$

In this setting, agents, private and public, will be assumed to behave as if they followed expectations given by:

$$\phi^{ei}_{(t-1)}\left(Y^{t'}\right) = \phi\left(F_{i,(t-1)}\left(\begin{array}{c} Y_{(t-1)'}, H^P_{i,(t-1)}\left(Y_{(t-1)'}, \left(\phi^{eP}_{(t-1)}\left(Y^{t'}\right)\right)_{i,(t-1)}\right), \\ H^G_{i,(t-1)}\left(Y_{(t-1)'}, \left(\phi^{eG}_{(t-1)}\left(Y^{t'}\right)\right)_{i,(t-1)}\right), Z^{t'}_{i,(t-1)} \end{array}\right)\right) \quad (3.3)$$

Where:

- ϕ indicates a probability distribution; $\phi^{ei}_{(t-1)}\left(Y^{t'}\right)$ is the probability of future events perceived by agent $i(i = P, G)$ at time $t - 1$.

- $F_{i,t}$ is the representation used by agent i at t of the function that defines the evolution of the system given past history, the behavior rules of agents and their expectations, and random shocks.

- $H^P_{i,(t-1)}$ denotes the decision function of sector P as perceived by i at $t - 1$. The private sector is assumed to be homogenous, in actuality and perception, so that for all t $H^P_{P,t} = H^P$: the private agent understands his own behavior and the aggregate implications of that of his peers (likewise with $G : H^G_{G,t} = H^G$).

- $\left(\phi^{ej}_{(t-1)}\left(Y^{t'}\right)\right)_{i,(t-1)}$ is the perception of agent i at $t-1$ of the expectations formed at $t-1$ by the agents belonging to group j.

- $Z^{t'}_{i,(t-1)}$ is the representation that i makes at $t-1$ of the process generating the sequence of random (from the viewpoint of the agent) impulses on the system; these perceived shocks may be different from the truly exogenous disturbances which effectively influence the actual motion of the system.

Similarly, the economist, as an outside observer, would form anticipations on the basis of a dated model, $M, (t - 1)$:

$$\phi_{M,(t-1)}\left(Y^{t'}\right) = \phi\left(F_{M,(t-1)}\left(\begin{array}{c} Y_{(t-1)'}, H^P_{M,(t-1)}\left(Y_{(t-1)'}, \left(\phi^{eP}_{(t-1)}\left(Y^{t'}\right)\right)_{M,(t-1)}\right), \\ H^G_{M,(t-1)}\left(Y_{(t-1)'}, \left(\phi^{eG}_{(t-1)}\left(Y^{t'}\right)\right)_{M,(t-1)}\right), Z^{t'}_{M,(t-1)} \end{array}\right)\right)$$
$$(3.4)$$

The previous formulations raise questions about high order beliefs, including those that refer to what an individual (agent or analyst) thinks about the future evolution of her own conceptual and practical expectation building apparatus. We simplify our discussion by the assumption that the expectations of sector P are common knowledge for its members, and that G can be treated as a single agent. The time varying nature of the schemes that people use to form anticipations in a learning environment will have to be addressed when dealing with model consistent expectations.

4 Fully knowledgeable economic agents: RE1.1

Under this assumption, agents behave as if they knew the actual law of motion of the relevant variables. We will work here with the case where the condition is presumed to hold for the whole set of economic actors, private and public (RE1.1). This means that, in practice, they have internalized in full the systematic elements of the system's dynamics; from their perspective, the randomness of the future path of the economy is limited to objectively stochastic shocks[5], irreducible by any potential accumulation of experiences or learning efforts. The working knowledge of private agents includes, in particular, the history contingent decision rules of the government; the government, in turn knows as much as can be known about the present and future behavior patterns of the private sector.

In this case, as noted in the literature, the system's performance can be described as a fixed point equilibrium, where the law of motion perceived by the agents coincides with the actual law of motion generated by their behavior given those expectations:

$$\phi^* \left(Y^{t'} \right) = \phi \left(F \left(Y_{(t-1)'}, H^P \left(Y_{(t-1)'}, \phi^* \left(Y^{t'} \right) \right), H^G \left(Y_{(t-1)'}, \phi^* \left(Y^{t'} \right) \right), Z^{t'} \right) \right) \qquad (4.1)$$

Where $\phi^* \left(Y^{t'} \right)$ indicates the actual and perceived (by agents) probability distribution of future events.

Some direct implications of RE1.1 as a maintained hypothesis are:

- Irrelevance of multiple equilibria:

  The fixed point equation (4.1) can admit as solutions multiple distributions for given "fundamentals" (functions F, H, and truly exogenous shocks Z). This opens the possibility of economy wide coordination effects which may condition the laws of motion of the system, and make its evolution dependent on random variables ("sunspots") that signal the way in which those coordination effects will operate. But this does not imply any indeterminacy, or allows unexplained (for the agents) jumps in the behavior of the economy. Under RE1.1, the assumption that agents behave as if they knew the true law of motion entails that, for practical purposes, the coordination game has been played at the beginning of time, including the choice of sunspots, and the agreed upon response to their realizations. Operationally, sunspots are as much a part of the set of external impulses as the fundamental shocks, and their generating processes and impacts on the economy are perfectly known for both sets of variables. Therefore, in fact, there is a single, pre-determined contingent path that the agents both know and generate by their behavior.

- No regrets, no possible learning for agents:

  If the behavior functions H generate truly optimal contingent decisions, and agents have, and know they have, fully correct probabilistic expectations, no event can cause concrete regrets to economic actors, in the sense of retrospective desires of having acted differently in the past given the circumstances and the information available when the behavior was chosen. With RE1.1, agents may subjectively feel sorry for bad luck in the realization of external shocks (sunspots included), but would certainly stick to their original history-dependent plans and expectations, and would not engage in the useless task of trying to improve their already as if perfect knowledge of the economy. There can be no search for lessons by economic actors after what looks as an economic crash: under RE1.1 this cannot

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[5]For the sake of the discussion, one may as well suppose that agents are informed with infinite precision of the past histories of the relevant variables and make no "rounding errors" in their as if computations, so that they need not be concerned with chaotic deterministic dynamics.

have been other than the perfectly understood, unavoidable consequence of a well identified outside shock whose likelihood the agents have always taken into account correctly.

- Practical immateriality of economic analysis:

  The perfect knowledge of economic actors with RE1.1 operates under the as if umbrella: it refers to the determination of actual behavior, and not necessarily to the coherent expression of arguments and well formulated beliefs. Economic analysts could then perform a culturally worthy intellectual activity by building conceptual structures that try to provide explicit, coherent approximate representations of what the agents do and know. But, in the RE1.1 world, economists are working for no practical purpose: what they may have to say can have no bearing on the determination of actual expectations or behavior.

- Efficient markets, debt sustainability by assumption:

  Under RE1.1, every trader decides as if aware of the asset price generating process. Information may be costly to obtain, and agents can (optimally) decide which data to gather and which to leave unobserved (as in Grossman and Stiglitz (1976)). But, given those constraints, the practical knowledge that drives the agents' information processing activity is presumed to be as perfect as possible. Therefore, no individual can improve the outcome of her market participation. In addition, debt sustainability holds without question: formal default may well occur, but assets are valued on the basis of correct state contingent expectations of repayment and the corresponding probabilities of occurrence, so that outcomes at each state realize the implicit understanding of the parties when defining the debt contract (Guzman & Heymann, 2015).

- Irrelevance of the Lucas Critique:

  In practice, private agents are apt to change their behavioral responses to observables according to the state of expectations, including particularly anticipations about the course of government policies. If some news do modify existing perceptions about how policies will be determined in the future, this can be expected to feed into the current decision patterns of agents. That effect only requires that agents be aware of information about a switch in the government's behavior patterns, without restricting expectations to a particular form or degree of accuracy. However, the usual discussions on the issue are based on a particular scenario where, before a certain date, agents confidently expect the constancy of the prevailing reaction functions that determine the sequence of policy instruments and have adapted well their behavior to that belief; these expectations are suddenly disrupted by the rare event of a zero ex ante probability regime change, after which agents redefine their decision rules presuming that the new policy scheme will be permanent.

  That argument conflicts with the RE1.1 assumption. Under RE1.1, the expectations of agents incorporate the full dynamics of the behavior of others. Here, the very notion of a regime shift becomes questionable. If the government changes an elementary reaction function, it is either the result of a current fundamentally random shock (which can only happen as a realization of a variable with a known probability distribution belonging to the set of variables Z that agents have always taken into consideration), or it is motivated as part of the overall government's history dependent decision rule (implementing a "meta regime", say); in both cases, it cannot be taken as an arbitrary act of fiat, and the future evolution of policies will be perceived by private agents in accordance to their correct knowledge of the process that generates the government's behavior. In turn, a RE1.1 government understands all that, and has determined its actions in the past, and will continue to do so in the future, with a precise recognition of the response that they will elicit: a superficial change in some "shallow" parameter will not disrupt those plans, based as they are on encompassing knowledge.

In any case, the RE1.1 assumption itself is not implemented in usual practice. An economist who wishes to work strictly within the RE1.1 framework must represent the behavior of agents whose practical knowledge about the economy is superior to that embodied in the models that the analyst has built in the past and, realistically, will produce in the future. Therefore, the use of model consistent schemes to generate expectations contradicts the RE1.1 hypothesis.[6] The analyst should not attribute to the agents expectations based on the probability distributions generated by the model. The uncertainty perceived by the model builder as such, in possession of his analytical constructs, fallible and provisional as they are, and restricted to a limited range of validity, will certainly be different from that of the RE1.1 agents.[7]

In summary, the analyst who maintains the RE1.1 criterion looks for a model of the type:

$$Y^{t'} = F_{M,(t-1)}\left(Y_{(t-1)'}, H^P_{M,(t-1)}\left(Y_{(t-1)'}, \phi^*\left(Y^{t'}\right)\right), H^G_{M,(t-1)}\left(Y_{(t-1)'}, \phi^*\left(Y^{t'}\right)\right), Z^{t'}_{M,(t-1)}\right)$$
(4.2)

Where, as before, ϕ^* indicates the true probability distribution of future events, which by assumption bases the expectations that guide the behavior of agents. As a consequence, under the RE1.1 assumption, the model would produce the distribution:

$$\phi_{M,(t-1),\phi^*}\left(Y^{t'}\right) = \phi\left(F_{M,(t-1)}\left(\begin{array}{c} Y_{(t-1)'}, H^P_{M,(t-1)}\left(Y_{(t-1)'}, \phi^*\left(Y^{t'}\right)\right), \\ H^G_{M,(t-1)}\left(Y_{(t-1)'}, \phi^*\left(Y^{t'}\right)\right), Z^{t'}_{M,(t-1)} \end{array}\right)\right)$$
(4.3)

The notation, as above, indicates that $F_{M,(t-1)}$, $H^P_{M,(t-1)}$, $H^G_{M,(t-1)}$ and $Z^{t'}_{M,(t-1)}$ are functions and shocks pertaining to the model M considered relevant by the analyst at time $t-1$. As part of its specification, the model includes the provision that the expectations of private agents and the government are generated by the true probability distribution $\phi^*\left(Y^{t'}\right)$. Thus, to be consistent with its RE1.1 assumption, the model should incorporate a conjecture about how agents who know more than the analyst may evaluate the future dynamics of the system. Instead, the usual practice would look for a current model consistent fixed point given by:

$$\phi_{M,(t-1)}\left(Y^{t'}\right) = \phi\left(F_{M,(t-1)}\left(\begin{array}{c} Y_{(t-1)'}, H^P_{M,(t-1)}\left(Y_{(t-1)'}, \phi_{M,(t-1)}\left(Y^{t'}\right)\right), \\ H^G_{M,(t-1)}\left(Y_{(t-1)'}, \phi_{M,(t-1)}\left(Y^{t'}\right)\right), Z^{t'}_{M,(t-1)} \end{array}\right)\right)$$
(4.4)

The difference between both expressions, (4.3) and (4.4), shows that using the model to represent expectations misses the distinguishing characteristic of RE1.1 as the limit case where agents have already obtained implicitly the knowledge that the analyst aspires to attain, and make explicit, but not in the foreseeable future. Moreover, if expectations are somehow observable, a firm belief in RE1 should make the economist adapt her theory to make it compatible with the perceptions of agents, rather than trying to represent those perceptions using her a priori preferred model.

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[6] If the analyst as an economic agent is part of the $PLM = ALM$ collective, his tacit knowledge as a decision maker should generate better anticipations than his explicitly formulated model. Consequently, the economist who believes in RE1.1 should not rely on the model to guide his economic behavior.

[7] It has been argued (Sargent, 2010) that macroeconomic models of the RBC or NK tradition built before the crisis of the years 2000 were designed to work under normal economic conditions, not during financial disruptions or economic breakdowns. But if it is understood that those constructs leave relevant phenomena out of consideration, it cannot be said that the model generated expectations are rational, because agents would be ignoring the relevant fact of the incomplete validity of the model. RE1.1 agents would contemplate the chances of crises and breakdowns with their true probabilities, so that the representation of the normal times itself would have to be different from that produced under the assumption of model-based expectations.

A simple example may serve to illustrate these points. Consider an economist who studies a certain asset market on the basis of a Lucas (1978) tree model. Dividends on the asset are actually generated by a process: $d_t = \bar{d} + f(x_{t-1}) + \varepsilon_t$. The function $f(x_{t-1})$ is fundamentally deterministic, and its argument x_{t-1} is publicly observable at $t-1$. The term ε_t denotes a truly stochastic shock, which represents "rare", low-probability tail events (Barro, 2005; Chen, Dou & Kogan, 2015). Assume that the analyst understands that $f(x_{t-1})$ is non-random, but has not "cracked the code" of the generating function; and that, from her viewpoint (and that of the agents who potentially follow her perspective) that term is readily described as a stochastic impulse with a well defined distribution. Moreover, the data set available to the analyst does not include any realization of the "big shocks" ε, or evidence that they may sometime occur. For the economist, then, the error term in the value of the dividend is defined simply by the pseudo random variable $f(x_{t-1})$.

Suppose, for the sake of the argument, that the economist can approximate well the observed yields of the asset with a satisfactory theory using the observed distribution of the pseudo random variable. These results may seem to validate the model, with no further questions to pose. However, the conclusion would disregard an important piece of relevant information from the perspective of the analyst, if the economist firmly believes that, for the RE1 agents, the realizations of x_{t-1} are perfectly foreseeable, and do not constitute a risk factor. Therefore, in order to rationalize market behavior, the researcher should reject her apparently plausible result, and look for a "hidden" effect (hidden for her, not the agents), which she has not observed, and does not emerge from her contemporaneous theory.

5   Model consistency: RE2.2

The second notion of rational expectations (model consistency) would downgrade the quality of agents' expectations from the high planes of RE1 to the level of professional competence. But professional standards and analytical constructs are not immutable. What does model consistency of expectations then mean when applied to past or prospective behavior?

5.1   Universal (private and public) model-consistency: RE2.2

Now, agents are assumed to know what the analyst knows about the system, not more, not less. However, this is not a strictly defined notion, since the preferred model of the analyst, as an active worker, has changed over time, and can be expected to continue evolving. Typically, macroeconomic models built under the rational expectations cover take the present model and use it to generate expectations for all times. That is, for every τ, the practice is to assume that:

$$\phi_{M,(t-1)}\left(Y^{\tau'}\right) = \phi\left(F_{M,(t-1)}\left(\begin{array}{l} Y_{(\tau-1)'}, H^P_{M,(t-1)}\left(Y_{(\tau-1)'}, \phi_{M,(t-1)}\left(Y^{\tau'}\right)\right), \\ H^G_{M,(t-1)}\left(Y_{(\tau-1)'}, \phi_{M,(t-1)}\left(Y^{\tau'}\right)\right), Z^{\tau'}_{M,(t-1)} \end{array}\right)\right) \quad (5.1)$$

But, if $\tau < t$, at that time the analyst was holding, and applying to represent the expectations of the agents populating the economy, a vintage τ model:

$$\phi_{M,(\tau-1)}\left(Y^{\tau'}\right) = \phi\left(F_{M,(\tau-1)}\left(\begin{array}{l} Y_{(\tau-1)'}, H^P_{M,(\tau-1)}\left(Y_{(\tau-1)'}, \phi_{M,(\tau-1)}\left(Y^{\tau'}\right)\right), \\ H^G_{M,(\tau-1)}\left(Y_{(\tau-1)'}, \phi_{M,(\tau-1)}\left(Y^{\tau'}\right)\right), Z^{\tau'}_{M,(\tau-1)} \end{array}\right)\right) \quad (5.2)$$

Therefore, if both conditions hold:

- When viewed from t, the expectations derived from the model built at τ do not satisfy rational expectations, contrary to the claim made at the time the older vintage model was current.

- Since the model produced at t is presumably considered an improvement relative to past models $M\tau$, agents in the current model are assumed to have had at τ superior knowledge compared to the analyst: they are implicitly represented as benefiting in advance from the learning of the economist in the $t - \tau$ interval.

- Consequently, the model$-t$ consistent expectations for all times implement neither the $PLM = ALM$ criterion (as seen in the previous section), nor the correspondence of the forecasts of agents with those resulting from the models in use at the time (τ) when the expectations were formed.

A conceivable criterion for implementing the RE.2 assumption on historical data could be to combine the specification of the model currently proposed by the economist with expectations drawn from the model "in use" at the time when the anticipations were made. Thus, the economy at time t would be represented by an up to date scheme, but the forecasts imputed to agents would correspond to the analytical schemes prevalent at τ, when those anticipations were worked out:

$$
\phi_{M,t,(\tau-1)}\left(Y^{\tau'}\right) = \phi\left(F_{M,t}\left(\begin{array}{c} Y_{(\tau-1)'}, H_{M,t}^{P}\left(Y_{(\tau-1)'}, \phi_{M,(\tau-1)}\left(Y^{\tau'}\right)\right), \\ H_{M,t}^{G}\left(Y_{(\tau-1)'}, \phi_{M,(\tau-1)}\left(Y^{\tau'}\right)\right), Z_{M,t}^{\tau'} \end{array}\right)\right) \tag{5.3}
$$

There, $\phi_{M,t,\tau}\left(Y^{\tau'}\right)$ indicates the probability distribution that the economist would generate for the forward-looking sequence starting at a previous date $\tau < t$, assuming that the representation of the functions (F, H^P, H^G) corresponds to the latest vintage model M, t, but the predictions of the agents made at $\tau-1$ were based on the current model at that time, $M, (\tau-1)$.

This formulation has special features. In particular, it presumes that expectations are automatically formed using the model held by the analyst at each date, as if the agents had coordinated unanimously on that scheme, and relied on its applicability at all future periods. This assumption may be questionable, since all actors concerned are, and should know they are, in a learning environment (the model has replaced previous vintage constructs, and will be superseded in the future), so that future expectations will be formed differently than today. The treatment of this point raises nontrivial questions. In any case, the use of dated descriptions of beliefs about future economic conditions seems hardly avoidable in macroeconomic contexts, where both theoretical constructs and popular frames of mind have shown substantial variability.

This form of model-consistency represents agents as people with provisional and fallible expectations-forming instruments, far from the limit of RE1. Here, one need not rule out costly "knowledge failures" like those that emerge in crises, where agents discover that their information-processing tools have been inappropriate, and are impelled to engage in a search for substitutes, with unclear outcomes.

The criterion "places agents and analysts on a similar footing"; however, it may be compatible with an asymmetric role of "expert opinion" influencing the expectations and behavior of agents, if the choice of the incumbent model M, t can be supposed to be driven mainly by the evolution of some type of professional consensus. Such a scenario leaves room for real-world effects of economic analysis, particularly in the form of policy advice, although there

would be no presumption that this advice is based on an accurate approximation of the actual data-generating model.

In the RE2 setting, by assumption, the designated expert knows that the government shares (or follows) her representation of the economy and is ready to apply its conclusions.[8] But the analyst recognizes that her incumbent model, which she is prepared to use for the evaluation of policy alternatives, is also incorporated into the expectations of private agents. Does this contemplate the (as if) capacity of those agents to second- guess the economist and anticipate her recommendations? How will the private sector assess the potential durability of the newly determined policies given that the reformulation is part of a recurrent process as the economist, and the agents themselves, go about revising their working models? The implementation of the general notion of model- consistent expectations would then lead to nontrivial questions regarding the features of economic analysis for the purpose of macro policy design, and the meaning and formulation of propositions akin to the Lucas critique.

6 Heterogeneous expectations types: RE1.2 and RE2.1

For completeness, we discuss briefly the cases where one sector (P or G) is assumed to be endowed with knowledge of the actual law of motion, while the other follows model-consistent expectations.

6.1 RE1.2: Fully knowledgeable private sector, model-consistent government

In these conditions, private expectations would be given by the true distribution, taking into account the correctly anticipated (in distribution) sequence of the future decision rules the government would choose, given the history of the system observed at the time and the expectations generated by the model that the policymaker will be using. In order to implement type 1 rational expectations, the agent must, and does, form correct anticipations of the evolution of the decision-making schemes employed by the government and the expectations that shall be fed into them. In this sense, the private sector knows more about the thought- process of the policymaker and the economist (who produces the model-consistent forecasts used by G) than those players themselves. The analyst faces a problem analogous to that which appeared under RE1.1, of modeling the beliefs and the behavior of agents with an implicit understanding of the system superior to that of the observer; as a policy advisor, she must also factor into her exercises that changes in the "policy regime" derived from her work cannot be automatically represented as unanticipated shocks when studying the potential response of the economy.

6.2 RE2.1: Fully knowledgeable government, model-consistent private sector

Now the analyst can accompany, or influence, the perceptions and behavior of the private sector, but would have nothing to contribute to policymaking. Whatever the model at hand tells about the economy, and about the consequences of alternative policy choices, the government would possess superior implicit knowledge, including the recognition that the economist can at best help in making explicit, imperfectly, part of the practical wisdom that G has attained. From an analytical perspective, as discussed when dealing with RE2.2, the economist working within the RE2.1 framework must specify the meaning to be assigned to the model-consistency notion, taking into account as an observed fact that incumbent models do change over time.

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[8] The situation brings memories of the discussion of Tinbergen (1952) where the policymaker determines the preferences that will decide choices, and the economist supplies to the best of her knowledge the model that maps tradeoffs between objectives. The scenario differs from that in Sargent (1999), where the government has been conducting its learning on the basis of representations misspecified from the contemporaneous point of view of the analyst.

7 Concluding remarks

The model currently proposed by an economist is neither the ultimate, as-good-as -can-possibly-be instrument to generate expectations, nor a timeless object which has been in use already in the past and will continue in use in the future. Economic theories form part of an ongoing, individual and collective historical learning process with its ups and downs, comings and goings. In a self- referential discipline, where the agents who populate the system under study actively search for ways to understand their environment and predict its evolution, the historicity of analytical frameworks constitutes one of the observables that the theory should recognize, and contemplate explicitly when relevant.

We have argued that the usual procedure employed in macroeconomic rational expectations models implements no clearly defined meaning of the concept. On the one hand, that practice fails to incorporate the implications of the inferiority of the expectations generated by an economist's model compared with those formed by hypothetical agents with practical command of the actual laws of motion of the variables. On the other hand, if referring strictly to the model-consistency notion, the assimilation of the expectations of past, present and future agents with those based on the presently proposed model places the economist in the position of endowing the agents whose historical behavior she studies with an understanding of the economy above that of the relevant economic theory of the times when they were acting, and also above that of the agents which she once qualified as rational because they followed the now obsolete older-vintage model.

These problems have practical consequences, especially for the analysis of large-scale fluctuations. Macroeconomic crises frustrate plans and expectations on a massive scale. The behaviors thus invalidated in various historical episodes do not spring merely from psychological exuberance, but in many instances reflect sober, considered choices, possibly influenced, or rationalized, by established professional pronouncements. As a matter of fact, crises are typically perceived as a challenge to once prevailing modes of analysis and beliefs about the working on the economy, and they motivate widespread reconsiderations of theories and opinions. In a way, those extreme events reflect the fallibility of the popularly predominant theories themselves. It would thus matter that macroeconomic analysis recognizes the evolution of the mental models used by agents to make anticipations in connection with the process of change of its own representations.

The rational expectations approach meant to take expectations out of the set of variables which merit a specific analysis, since they would be represented by the economist's currently preferred model. The possible gain in terms of parsimony does not seem to compensate the losses from ignoring a crucial element in the actual behavior of agents and from leaving out dynamics of behaviorally relevant beliefs which can have strong macroeconomic impacts.

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