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Gravito-magnetic currents in the inflationary universe from WIMT

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Abstract Using the Weitzenböck representation of a Riemann-flat 5D spacetime, we study the possible existence of primordial gravito-magnetic currents from Gravito-electromagnetic Inflation (GEMI). We found that these currents decrease exponentially in the Weitzenböck representation, but they are null in a Levi-Civita representation because we are dealing with a 5D Riemann-flat spacetime without structure or torsion.

1 Introduction and motivation

It is well known that magnetic monopoles have been elusive as regards detection, despite the efforts. Such monopoles arise as a theoretical possibility from the dual formulation of an electrodynamic theory [1]. The fate of primordial monopoles is very closely linked to the history of the very early universe. Preskill [2] realized that this possible monopole production could create a crisis for cosmology, implying far more monopoles than observational limits allow. Because the expected energy scale of grand unification is quite high, the geometrical size of a monopole core must be quite small. Linde [3] and Vilenkin [4] independently pointed out that such monopoles could expand exponentially in the context of inflationary cosmology.

However, there is an even more interesting possibility, which arises from a theory that extends and unifies conceptually the electrodynamics with a theory of gravity: gravito-electrodynamics. This theory was first outlined in 2006 [5,6] in a cosmological context and later studied in greater detail [7–9], but the dual formalization has not yet been addressed. In this paper we shall study the dual formalism from a 5D vacuum and we will try and formalize their dual streams, which in a gravito-electrodynamic context are related gravito-magnetic currents. This formalism is inspired by the Induced Matter Theory (IMT), which is based on the assumption that ordinary matter and physical fields that we can observe in our 4D universe can be geometrically induced from a 5D Ricci-flat metric with a space-like noncompact extra dimension on which we define a physical vacuum [10–12]. The Campbell–Magaard theorem [13–17] serves as a ladder to move between manifolds whose dimensionality differs by one. This theorem, which is valid in any number of dimensions, implies that every solution of the 4D Einstein equations with arbitrary energy–momentum tensor can be embedded, at least locally, in a solution of the 5D Einstein field equations in vacuum. Because of this, the stress–energy may be a 4D manifestation of the embedding geometry. An extension of the IMT was realized recently using the Weitzenböck Induced Matter Theory (WIMT) [18]. This approach makes possible a geometrical representation of a 5D vacuum (with a zero curvature in the Weitzenböck representation), on a nonzero curvature tensor (in the sense of the Levi-Civita representation).

2 Weitzenböck Induced Matter Theory (WIMT)

We consider the basic elements for the extension of the IMT to a geometrical description with the Weitzenböck connections. The connections are constructed from certain 5D vielbeins related to the transformation defined by

$$\vec{e}_{a} = e^{A}_{\ a}\vec{E}_{A}, \tag{1}$$

where $\vec{E}_A$ is an element of a base $\{\vec{E}_A\}$ that we shall call "initial base" (IB). In our case we shall work with a 5D

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Minkowski space. Furthermore $\overrightarrow{e}_a$ is an element of the "final base" (FB), which is obtained through the transformation (1). In general, $\{\overrightarrow{E}_A\}$ cannot be coordinated.[1]

We can write the components of any tensor through the 5D vielbein $e^A_{\ a}$ and their inverses $\bar{e}^a_{\ A}$, which comply with $e^A_{\ a}\bar{e}^b_{\ A} = \delta^b_a$ and with $e^A_{\ a}\bar{e}^a_{\ B} = \delta^A_B$. In particular for the metric tensor, we have

$$g_{ab} = e^A_{\ a} e^B_{\ b} g_{AB}. \tag{2}$$

If we use the Weitzenböck connections ${}^{(W)}\Gamma^c_{ab} = \bar{e}^c_N \overrightarrow{e}_b(e^N_a)$, it can be seen that

$$\begin{aligned} {}^{(W)}\nabla_{\overrightarrow{e}_b}(\overrightarrow{E}_A) &= {}^{(W)}\nabla_{\overrightarrow{e}_b}(e^a_A \overrightarrow{e}_a) \\ &= \underbrace{e^a_A\{{}^{(W)}\Gamma^c_{ab} - e^c_N \overrightarrow{e}_b(e^N_a)\}}_{} \overrightarrow{e}_c = 0. \\ &\quad [{}^{(W)}\nabla_{\overrightarrow{e}_b}(\overrightarrow{E}_A)]^c = [\overrightarrow{E}_A]^c{}_{;\,b} = \bar{e}^c_{A;\,b}. \end{aligned} \tag{3}$$

It can be seen that the expressions $[{}^{(W)}\nabla_{\overrightarrow{e}_b}(\overrightarrow{E}_A)]^c = [\overrightarrow{E}_A]^c{}_{;\,b}$ represent the c-component of the application of the derivative operator characterized by the Weitzenböck connections with respect to the b-component of the base of the FB over the A-component of the vector of the ST. In general, for any vector field $\overrightarrow{A} = A^c \overrightarrow{e}_c$,[2] we have $[\overrightarrow{A}]^c{}_{;\,b} = A^c{}_{;\,b}$.

When the field $\overrightarrow{A}$ is the A-component of the IB, $\overrightarrow{A} = \overrightarrow{E}_A$, the vector field is given by $\overrightarrow{E}_A = \bar{e}^a_A \overrightarrow{e}_a$ and it is clear that $[\overrightarrow{E}_A]^c = \bar{e}^c_A$. In this sense the connections of Weitzenböck imply $\bar{e}^c_{A;\,b} = 0$, and the vielbeins are seen as coefficients of the development of one base in another base. The expression (3) is fulfilled if we are working with the Weitzenböck connections.

Now we consider a 5D spacetime described by the metric g_{AB} in the IS and g_{ab} describing the FS. It is obvious that the latter space is Weitzenböck-flat in the sense that the Riemann tensor constructed through this kind of connection is null: ${}^{(W)}R^a_{bcd} = 0$. However, it cannot be Riemann-flat with respect to the Levi-Civita connections: ${}^{(lc)}R^a_{bcd} \neq 0$. The Riemann tensor written with the Weitzenböck representation for the spacetime characterized by the metric g_{ab} is given by

$$\begin{aligned} {}^{(W)}R^a_{bcd} &= \overrightarrow{e}_b({}^{(W)}\Gamma^a_{dc}) - \overrightarrow{e}_c({}^{(W)}\Gamma^a_{db}) \\ &+ {}^{(W)}\Gamma^n_{dc}\,{}^{(W)}\Gamma^a_{nb} - {}^{(W)}\Gamma^n_{db}\,{}^{(W)}\Gamma^a_{nc} - C^n_{cb}\,{}^{(W)}\Gamma^a_{dn} = 0, \end{aligned} \tag{4}$$

where ${}^{(W)}\Gamma^a_{bc}$ are the Weitzenböck connections and C^a_{bc} are the coefficients of the structure of the FB, which can be expressed through $C^a_{bc} = \bar{e}^a_N \vec{e_c}(e^N_b) - \bar{e}^a_N \vec{e_b}(e^N_c) = {}^{(W)}\Gamma^a_{bc} - {}^{(W)}\Gamma^a_{cb}$, when the absence of structure of the IB makes the Weitzenböck torsion null. Both representations are related by the expression

$${}^{(W)}\Gamma^a_{bc} = {}^{(lc)}\Gamma^a_{bc} - {}^{(W)}K^a_{bc}, \tag{5}$$

where the Weitzenböck contortion ${}^{(W)}K^a_{bc}$ is related to the Weitzenböck torsion ${}^{(W)}T^a_{bc}$,

$$\begin{aligned} {}^{(W)}K^a_{bc} &= \frac{g^{ma}}{2}\{g_{bm;c} + g_{mc;b} - g_{bc;m}\} \\ &+ \frac{g^{ma}}{2}\{{}^{(W)}T^n_{cm} g_{bn} + {}^{(W)}T^n_{bm} g_{nc} - {}^{(W)}T^n_{cb} g_{nm}\}. \end{aligned} \tag{6}$$

Here, we have considered the nonzero non-metricity $g_{ab;\,c} = {}^{(W)}Q_{abc}$. When $g_{ab;\,c} = 0$, the tensor ${}^{(W)}K^a_{bc}$ reduces to the well-known contortion tensor in the Weitzenböck representation ${}^{(W)}K^a_{bc} = \frac{g^{ma}}{2}\{{}^{(W)}T^n_{cm} g_{bn} + {}^{(W)}T^n_{bm} g_{nc} - {}^{(W)}T^n_{cb} g_{nm}\}$.[3]

On the other hand, if the Weitzenböck torsion becomes zero (this holds when the IB has no structure), we have ${}^{(W)}K^a_{bc} = \frac{g^{ma}}{2}\{g_{bm;c} + g_{mc;b} - g_{bc;m}\}$.

By contracting the null-tensor ${}^{(W)}R^a_{bcd}$ we obtain the following tensors: ${}^{(W)}S_{bc} = {}^{(W)}R^a_{bca}$ (which is antisymmetric) and ${}^{(W)}R_{cd} = {}^{(W)}R^a_{acd}$ (which is symmetric), that is[4]

$$\begin{aligned} {}^{(W)}S_{bc} = {}^{(W)}R^a_{bca} &= \overrightarrow{e}_b({}^{(W)}\Gamma^a_{ac}) - \overrightarrow{e}_c({}^{(W)}\Gamma^a_{ab}) \\ &+ {}^{(W)}\Gamma^n_{ac}\,{}^{(W)}\Gamma^a_{nb} \\ &- {}^{(W)}\Gamma^n_{ab}\,{}^{(W)}\Gamma^a_{nc} - C^n_{cb}\,{}^{(W)}\Gamma^a_{an} = 0, \end{aligned} \tag{7}$$

$$\begin{aligned} {}^{(W)}R_{cd} = {}^{(W)}R^a_{a(cd)} &= \overrightarrow{e}_a({}^{(W)}\Gamma^a_{(dc)}) - \overrightarrow{e}_{(c}({}^{(W)}\Gamma^a_{d)a}) \\ &+ {}^{(W)}\Gamma^n_{(dc)}\,{}^{(W)}\Gamma^a_{na} - {}^{(W)}\Gamma^n_{(d|a}\,{}^{(W)}\Gamma^a_{n|c)} - C^n_{(c|a}\,{}^{(W)}\Gamma^a_{|d)n} = 0. \end{aligned} \tag{8}$$

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[1]
- Capital Latin letters $A, B, C, ..., H = 0, 1, 2, 3, 4$ run on the 5D "initial space" (IS).
- Lowercase Latin letters $a, b, c, ..., h = 0, 1, 2, 3, 4$ run on the 5D "final space" (FS).
- Greek letters $\alpha, \beta, ... = 0, 1, 2, 3$ run on the 4D hypersurface embedded in the FS.
- Indices $i, j, k, ... = 1, 2, 3$ and $I, J, K, ... = 1, 2, 3$ run on the 3D pure space of the FS and IS, respectively.

[2] We denote by ";" the covariant derivative

$$\begin{aligned} \overrightarrow{A} &= A^b \overrightarrow{e}_b = A^0 \overrightarrow{e}_0 + A^i \overrightarrow{e}_i \Rightarrow \\ \nabla_{\overrightarrow{e}_b}(\overrightarrow{A}) &= \nabla_{\overrightarrow{e}_b}(A^0 \overrightarrow{e}_0) + \nabla_{\overrightarrow{e}_b}(A^i \overrightarrow{e}_i) \\ &= \overrightarrow{e}_b(A^0)\overrightarrow{e}_0 + A^0 \underbrace{\nabla_{\overrightarrow{e}_b}(\overrightarrow{e}_0)}_{\Gamma^c_{0b}\overrightarrow{e}_c} + \overrightarrow{e}_b(A^i)\overrightarrow{e}_i + A^i \Gamma^c_{ib} \overrightarrow{e}_c \\ &= (\overrightarrow{e}_b(A^0) + A^0 \Gamma^0_{0b} + A^i \Gamma^0_{ib})\overrightarrow{e}_0 \\ &\quad + (\overrightarrow{e}_b(A^i) + A^0 \Gamma^i_{0b} + A^j \Gamma^i_{jb})\overrightarrow{e}_i. \end{aligned}$$

Therefore $[\nabla_{\overrightarrow{e}_b}(\overrightarrow{A})]^0 = A^0{}_{;\,b}$. This is not only valid for the superscript 0, but also for the other indices, so that $[\nabla_{\overrightarrow{e}_b}(\overrightarrow{A})]^c = A^c{}_{;\,b}$.

[3] In this work we shall use this definition with zero nonmetricity.

[4] The expressions ${}^{(W)}\Gamma^n_{(d|a}\,{}^{(W)}\Gamma^a_{n|c)}$ indicate the symmetrization of the indices d and c, inside the parentheses, but excepting the indices a and n inside the vertical bars "|".

From (4), (5), (7), and (8) we obtain the expressions for the corresponding curvature tensors with the Weitzenböck representation by means of those of the Levi-Civita representation (and vice versa). Hence, we have

$$\begin{aligned}{}^{(W)}R^a_{bcd} = {}& {}^{(lc)}R^a_{bcd} - \vec{e}_b({}^{(W)}K^a_{dc}) + \vec{e}_c({}^{(W)}K^a_{db})\\ &- {}^{(W)}K^n_{dc}\,{}^{(W)}K^a_{nb} - {}^{(lc)}\Gamma^n_{dc}\,{}^{(W)}K^a_{nb} - {}^{(W)}K^n_{dc}\,{}^{(lc)}\Gamma^a_{nb}\\ &+ {}^{(W)}K^n_{db}\,{}^{(W)}K^a_{nc} + {}^{(lc)}\Gamma^n_{db}\,{}^{(W)}K^a_{nc} + {}^{(W)}K^n_{db}\,{}^{(lc)}\Gamma^a_{nc}\\ &+ C^n_{cb}\,{}^{(W)}K^a_{dn}.\end{aligned} \tag{9}$$

Using (7) and (8) in the last expression we found the analogous equations for ${}^{(W)}S_{bc}$ and ${}^{(W)}R_{cd}$, or for ${}^{(lc)}S_{bc}$ and ${}^{(lc)}R_{cd}$. Usually it is simple to determinate the latter, because they comply with the transformation ${}^{(lc)}S_{bc} = e^B_b e^C_c\,{}^{(lc)}S_{BC} = 0$ and ${}^{(lc)}R_{cd} = e^D_d e^C_c\,{}^{(lc)}R_{CD}$, where, if the IB has no structure, the tensors with capital indices are calculated in a coordinate base with the Levi-Civita connections.

Now we shall consider the Einstein equations with the Weitzenböck representation. We shall try to obtain the effective 4D equations after making a constant foliation from a 5D Weitzenböck vacuum. Taking (8), we obtain

$$\begin{aligned}\left.\overbrace{{}^{(W)}R_{\zeta\delta}}^{5D}\right|_{l=l_0} = {}& \left.\overbrace{\vec{e}_\alpha({}^{(W)}\Gamma^\alpha_{(\zeta\delta)}) - \vec{e}_{(\zeta}({}^{(W)}\Gamma^\alpha_{\delta)\alpha}) + {}^{(W)}\Gamma^\nu_{(\zeta\delta)}\,{}^{(W)}\Gamma^\alpha_{\nu\alpha} - {}^{(W)}\Gamma^\epsilon_{(\delta|\alpha}\,{}^{(W)}\Gamma^\alpha_{\nu|\zeta)} - C^\nu_{(\zeta|\alpha}\,{}^{(W)}\Gamma^\alpha_{|\delta)\nu}}^{5D}\right|_{l=l_0}\\ &+ [(\vec{e}_4({}^{(W)}\Gamma^4_{(\zeta\delta)}) - \vec{e}_{(\zeta}({}^{(W)}\Gamma^4_{\delta)4}))\\ &+ ({}^{(W)}\Gamma^4_{(\zeta\delta)}\,{}^{(W)}\Gamma^\alpha_{4\alpha} + {}^{(W)}\Gamma^\nu_{(\zeta\delta)}\,{}^{(W)}\Gamma^4_{\nu4}\,{}^{(W)}\Gamma^4_{(\zeta\delta)}\,{}^{(W)}\Gamma^4_{44}\\ &- {}^{(W)}\Gamma^4_{(\delta|\alpha}\,{}^{(W)}\Gamma^\alpha_{4|\zeta)} - {}^{(W)}\Gamma^\nu_{(\delta|4}\,{}^{(W)}\Gamma^4_{\nu|\zeta)}\,{}^{(W)}\Gamma^4_{(\delta|4}\,{}^{(W)}\Gamma^4_{4|\zeta)})\\ &- (C^4_{(\zeta|\alpha}\,{}^{(W)}\Gamma^\alpha_{|\delta)4} + C^4_{(\zeta|4}\,{}^{(W)}\Gamma^4_{|\delta)\nu} + C^4_{(\zeta|4}\,{}^{(W)}\Gamma^4_{|\delta)4})]|_{l=l_0} = 0.\end{aligned} \tag{10}$$

In this work we shall deal with canonical metrics [19–21]. An interesting example is [22]

$$\mathrm{d}S^2 = \left(\frac{l}{l_0}\right)^2 h_{\alpha\beta}(y^\gamma)\,\mathrm{d}y^\alpha \mathrm{d}y^\beta - \mathrm{d}l^2, \tag{11}$$

where l is related to the noncompact extra dimension and l_0 is a constant. After making the constant foliation, we obtain $\overbrace{{}^{(W)}\Gamma^\epsilon_{\beta\alpha}}^{5D}|_{l=l_0} = \overbrace{{}^{(W)}\Gamma^\epsilon_{\beta\alpha}}^{4D}$, where $\overbrace{{}^{(W)}\Gamma^\epsilon_{\beta\alpha}}^{4D}$ is a Weitzenböck connection defined on the embedded 4D hypersurface obtained through the foliation: $l = l_0$. It makes it possible to obtain the effective 4D Ricci–Weitzenböck tensor,

$$\begin{aligned}\overbrace{{}^{(W)}R_{\zeta\delta}}^{4D} = {}& -[(\vec{e}_4({}^{(W)}\Gamma^4_{(\zeta\delta)}) - \vec{e}_{(\zeta}({}^{(W)}\Gamma^4_{\delta)4}))\\ &+ ({}^{(W)}\Gamma^4_{(\zeta\delta)}\,{}^{(W)}\Gamma^\alpha_{4\alpha} + {}^{(W)}\Gamma^\nu_{(\zeta\delta)}\,{}^{(W)}\Gamma^4_{\nu4} + {}^{(W)}\Gamma^4_{(\zeta\delta)}\,{}^{(W)}\Gamma^4_{44}\\ &- {}^{(W)}\Gamma^4_{(\delta|\alpha}\,{}^{(W)}\Gamma^\alpha_{4|\zeta)} - {}^{(W)}\Gamma^\nu_{(\delta|4}\,{}^{(W)}\Gamma^4_{\nu|\zeta)} - {}^{(W)}\Gamma^4_{(\delta|4}\,{}^{(W)}\Gamma^4_{4|\zeta)})\\ &- (C^4_{(\zeta|\alpha}\,{}^{(W)}\Gamma^\alpha_{|\delta)4} + C^4_{(\zeta|4}\,{}^{(W)}\Gamma^4_{|\delta)\nu} + C^4_{(\zeta|4}\,{}^{(W)}\Gamma^4_{|\delta)4})]|_{l=l_0},\end{aligned} \tag{12}$$

which is symmetric with respect to the indices ζ, δ. The antisymmetric tensor is obtained as $\overbrace{{}^{(W)}S_{\beta\zeta}}^{5D}|_{l=l_0} = \overbrace{{}^{(W)}R^a_{\beta\zeta a}}^{5D}|_{l=l_0} = 0$, so that we obtain

$$\begin{aligned}\overbrace{{}^{(W)}S_{\beta\zeta}}^{4D} = {}& -[(\vec{e}_\beta({}^{(W)}\Gamma^4_{4\zeta}) - \vec{e}_\zeta({}^{(W)}\Gamma^4_{4\beta}))\\ &+ ({}^{(W)}\Gamma^4_{\alpha\zeta}\,{}^{(W)}\Gamma^\alpha_{4\beta} + {}^{(W)}\Gamma^\nu_{4\zeta}\,{}^{(W)}\Gamma^4_{\nu\beta}\\ &- {}^{(W)}\Gamma^4_{\alpha\beta}\,{}^{(W)}\Gamma^\alpha_{4\zeta} - {}^{(W)}\Gamma^\nu_{4\beta}\,{}^{(W)}\Gamma^4_{\nu\zeta})\\ &- (C^4_{\zeta\beta}\,{}^{(W)}\Gamma^\alpha_{\alpha4} + C^\nu_{\zeta\beta}\,{}^{(W)}\Gamma^4_{4\nu} + C^4_{\zeta\beta}\,{}^{(W)}\Gamma^4_{44})]|_{l=l_0}.\end{aligned} \tag{13}$$

The Ricci–Weitzenböck scalar curvature can be obtained from a 5D vacuum,

$$\overbrace{{}^{(W)}R}^{5D} = \overbrace{g^{ab}\,{}^{(W)}R_{ab}}^{5D} = \overbrace{g^{\alpha\beta}\,{}^{(W)}R_{\alpha\beta}}^{5D} + \overbrace{g^{55}\,{}^{(W)}R_{55}}^{5D} = 0. \tag{14}$$

Hence, from (14) we obtain

$$\overbrace{{}^{(W)}R}^{4D} = \left.\overbrace{g^{\beta\gamma}\,{}^{(W)}R_{\beta\gamma}}^{5D}\right|_{l=l_0} = \overbrace{h^{\beta\gamma}\,{}^{(W)}R_{\beta\gamma}}^{4D}, \tag{15}$$

which means that the scalar Ricci–Weitzenböck curvature has the source

$$\overbrace{{}^{(W)}R}^{4D} = -\left.\overbrace{g^{44}\,{}^{(W)}R_{44}}^{5D}\right|_{l=l_0}, \tag{16}$$

and finally the induced Einstein–Cartan–Weitzenböck equations are

$$\overbrace{{}^{(W)}G_{\beta\gamma}}^{4D} = \overbrace{{}^{(W)}R_{\beta\gamma}}^{4D} - \frac{1}{2}\overbrace{h_{\beta\gamma}\,{}^{(W)}R}^{4D} = -8\pi G\,\overbrace{\bar{T}_{(\beta\gamma)}}^{4D}, \tag{17}$$

$$\overbrace{{}^{(W)}S_{\beta\gamma}}^{4D} = -8\pi G\, \overbrace{\bar{T}_{[\beta\gamma]}}^{4D}, \tag{18}$$
$${}^{(W)}R_{a5} = 0, \tag{19}$$
$${}^{(W)}S_{a5} = 0, \tag{20}$$

where we have taken into account in (18) that $g^{\beta\gamma}\,{}^{(W)}S_{\beta\gamma} = 0$. Furthermore, the symmetric and antisymmetric parts of the energy-momentum tensor in (17) and (18) are given by $\overbrace{\bar{T}_{(\beta\gamma)}}^{4D} = \frac{1}{2}(\overbrace{\bar{T}_{\beta\gamma}}^{4D} + \overbrace{\bar{T}_{\gamma\beta}}^{4D})$ and $\overbrace{\bar{T}_{[\beta\gamma]}}^{4D} = \frac{1}{2}(\overbrace{\bar{T}_{\beta\gamma}}^{4D} - \overbrace{\bar{T}_{\gamma\beta}}^{4D})$. Equation (17) describe the dynamics of the gravitational field using the Weitzenböck representation on the effective 4D hypersurface.

2.1 Effective 4D dynamics with the Levi-Civita representation

Now we shall intend to write the curvature and the Ricci tensors (in the Levi-Civita representation) with respect to the Weitzenböck connections and contortions. The Ricci tensor in the Levi-Civita representation is related to the Ricci tensor in the Weitzenböck representation plus additional terms that depend on contortions and structure,

$$\begin{aligned}
\overbrace{{}^{(lc)}R_{\zeta\delta}}^{4D} &= \overrightarrow{e}_{\alpha}({}^{(lc)}\Gamma^{\alpha}_{(\zeta\delta)}) - \overrightarrow{e}_{(\zeta}({}^{(lc)}\Gamma^{\alpha}_{\delta)\alpha}) \\
&\quad + {}^{(lc)}\Gamma^{\nu}_{(\zeta\delta)}\,{}^{(lc)}\Gamma^{\alpha}_{\nu\alpha} - {}^{(lc)}\Gamma^{\epsilon}_{(\delta|\alpha}\,{}^{(lc)}\Gamma^{\alpha}_{\nu|\zeta)} - C^{\nu}_{(\zeta|\alpha}\,{}^{(lc)}\Gamma^{\alpha}_{|\delta)\nu}\,|_{l=l_0} \\
&= {}^{(W)}R_{\zeta\delta} + \overrightarrow{e}_{\alpha}(K^{\alpha}_{(\zeta\delta)}) - \overrightarrow{e}_{(\zeta}(K^{\alpha}_{\delta)\alpha}) \\
&\quad + K^{\nu}_{(\zeta\delta)}K^{\alpha}_{\nu\alpha} + {}^{(W)}\Gamma^{\nu}_{(\zeta\delta)}K^{\alpha}_{\nu\alpha} + K^{\nu}_{(\zeta\delta)}\,{}^{(W)}\Gamma^{\alpha}_{\nu\alpha} \\
&\quad - K^{\nu}_{(\delta|\alpha}K^{\alpha}_{\nu|\zeta)} - {}^{(W)}\Gamma^{\nu}_{(\delta|\alpha}K^{\alpha}_{\nu|\zeta)} \\
&\quad - K^{\nu}_{(\delta|\alpha}\,{}^{(W)}\Gamma^{\alpha}_{\nu|\zeta)} - C^{\nu}_{(\zeta|\alpha}K^{\alpha}_{|\delta)\nu}.
\end{aligned} \tag{21}$$

The scalar curvature is

$$\overbrace{{}^{(lc)}R}^{4D} = -\left.\overbrace{g^{44\,(lc)}R_{44}}^{5D}\right|_{l=l_0}. \tag{22}$$

Hence, the Einstein–Cartan equations are given by

$$\overbrace{{}^{(lc)}G_{\beta\gamma}}^{4D} = \overbrace{{}^{(lc)}R_{\beta\gamma}}^{4D} - \frac{1}{2}\overbrace{h_{\beta\gamma}\,{}^{(lc)}R}^{4D} = -8\pi G\,\overbrace{T_{(\beta\gamma)}}^{4D}, \tag{23}$$
$$\overbrace{S_{\beta\gamma}}^{4D} = -8\pi G\,\overbrace{T_{[\beta\gamma]}}^{4D} = 0, \tag{24}$$

joined with ${}^{(W)}R_{a5} = 0$ and ${}^{(W)}S_{a5} = 0$, which are additional conditions. Here, $\overbrace{T_{(\beta\gamma)}}^{4D}$ and $\overbrace{T_{[\beta\gamma]}}^{4D}$ are the symmetric and antisymmetric energy-momentum tensors induced on the 4D hypersurface written in the Levi-Civita representation.[5]

3 Dual action and equations of motion

We shall consider the conditions by which we can induce curvature and currents by means of WIMT [18], on a 5D spacetime represented by cartesian coordinates. The 5D tensor metric can be written as

$$[\eta]_{AB} = \mathrm{diag}[1, -1, -1, -1, -1]. \tag{26}$$

We can construct an FB with interesting cosmological properties if we take the appropriate vielbein. The action for the gravito-electromagnetic fields in a 5D vacuum can be written in the form

$$\mathcal{S} = \int d^5x\sqrt{|\eta|}\left[\frac{R}{16\pi G} - \frac{1}{4}F_{AB}F^{AB} - \frac{\lambda}{2}(A^{B}_{;B})^2\right]. \tag{27}$$

In order to maybe render the dual currents interesting, rewrite the last action in terms of the dual tensors $\mathcal{F}_{ABC}$

$$\begin{aligned}
\mathcal{S}_1 &= \int d^5x\sqrt{|\eta|}\left[\frac{R}{16\pi G} - \frac{k}{4}\mathcal{F}_{ABC}\mathcal{F}^{ABC} - \frac{\lambda}{2}(A^{B}_{;B})^2\right], \\
\mathcal{S} &= \int d^5x\sqrt{|\eta|}\left[\frac{R}{16\pi G} - \frac{1}{4}F_{AB}F^{AB} - \frac{\lambda}{2}(A^{B}_{;B})^2\right].
\end{aligned} \tag{28}$$

It is evident that $\frac{1}{3!}\mathcal{F}_{ABC}\mathcal{F}^{ABC} = \frac{1}{3!4}\varepsilon_{ABCDE}\varepsilon^{ABCNM}F^{DE}F_{NM} = F^{NM}F_{NM}$, where we have used $\varepsilon_{ABCDE}\varepsilon^{ABCNM} = 3!\,2!\,(\delta^N_D\delta^M_E - \delta^N_E\delta^M_D)$, so that when $k = \frac{1}{3!}$, we see that both actions describe the same physical system:

$$\mathcal{S}_1 = \mathcal{S}.$$

In our case, when we use the Lorentz gauge and we deal with a 5D vacuum with $R = 0$, we have $\mathcal{S}_1 \sim \mathcal{S}$ and both actions give us the same equations of motion. In order to describe the dual sources of these equations we shall deal with the action $\mathcal{S}_1$. The dynamics of the gravito-electromagnetic fields obtained taking the extreme with the action (27)) is $A^{K\ \ ;B}_{;B} - (1-\lambda)A^{B\ \ ;K}_{;B} = 0$. On a 5D vacuum ($R = 0$), and when we take $\lambda = 1$ and the Lorentz gauge $A^{B}_{;B} = 0$, they reduce to

$$\Box A^K = \eta^{BC}A^{K}_{;BC} = 0, \tag{29}$$

[5] Equations (24) take into account the Cartan equations which describe spinor contributions:

$$\overbrace{S_{\beta\gamma}}^{4D} = -8\pi G\,\underbrace{\overbrace{T_{[\beta\gamma]}}^{4D}}_{\text{spin}}, \tag{25}$$

where $S = \sigma^{\mu\nu}S_{\mu\nu}$, $\sigma^{\mu\nu} = \frac{1}{2}[\gamma^\mu, \gamma^\nu]$ and γ^μ are the Dirac matrices.

which are the Klein–Gordon equations for massless fields. The gravito-magnetic currents come from the solutions for the fields (29). The last equations are compatible with a current which has its source in

$$F^{NB}_{;B} = -\eta^{AN}[A^M R^B_{ABM} + A^B_{;M} T^M_{BA}], \tag{30}$$

where we have used (29) and the Lorentz gauge in absence of nonmetricity. We have used the fact that

$$F^{NB}_{;B} = \eta^{AN}\eta^{BM} F_{AM;B} = g^{AN} A^B_{;AB} - \underbrace{\eta^{BM} A^N_{;MB}}_{0},$$

where clearly we have imposed the absence of currents on the 5D vacuum with respect to the Levi-Civita representation. Hence

$$^{(lc)}F^{NB}_{;B} = 0 \Rightarrow \tag{31}$$

$$\begin{aligned} ^{(W)}F^{NB}_{;B} = &-(^{(W)}F^{MB} + \eta^{RM} A^P K^B_{PR} - \eta^{RB} A^P K^M_{PR}) K^N_{MB} \\ &-(^{(W)}F^{NM} + \eta^{RN} A^P K^M_{PR} - \eta^{RM} A^P K^N_{PR}) K^B_{MB}, \end{aligned} \tag{32}$$

where $^{(lc)}F_{AB} = \vec{E}_A(A_B) - \vec{E}_B(A_A) + A_N(C^N_{AB})$ y $^{(W)}F_{AB} = \vec{E}_A(A_B) - \vec{E}_B(A_A) + A_N(^{(W)}T^N_{BA} + C^N_{AB})$.[6]

Notice that in (31) we use the covariant derivative with respect to the Christoffel symbols, but in (32) we use the derivative covariant with respect to the Weitzenböck connections. Hence, we can adopt both representations in a complementary mode to describe the 5D vacuum.

The Weitzenböck currents are related by one expression analogous to (31), but it is more complicated:

$$^{(lc)(m)}J_{AB} - {}^{(W)(m)}J_{AB} = \frac{\sqrt{|\eta|}}{2}\varepsilon_{ABCDE}\frac{1}{4}M^{[CDE]}, \tag{33}$$

where $M^{[CDE]} = \eta^{CF}\eta^{DG}\eta^{EH} M_{[FGH]}$ and we define $M_{[FGH]}$ as

$$\begin{aligned} M_{[FGH]} = &(A_M\, {}^{(W)}T^M_{[FG})_{;H]} - 2\,{}^{(W)}T^N_{[FH|}\,{}^{(W)}T^M_{N|G]} A_M \\ &-2\,{}^{(W)}T^N_{[GH}\,{}^{(W)}T^M_{F]N} A_M \\ &-{}^{(W)}T^N_{[FH|}\vec{E}_N(A_{|G]}) + {}^{(W)}T^N_{[GH|}\vec{E}_N(A_{|F]}) \\ &+{}^{(W)}T^N_{[FH}\vec{E}_{G]}(A_N) - {}^{(W)}T^N_{[GH}\vec{E}_{F]}(A_N). \end{aligned} \tag{34}$$

Once we required the gauge condition $^{(lc)}A^N_{;N} = 0$, it is preserved in the Weitzenböck representation: $^{(W)}A^n_{;n} = 0$.[7]

[6] Using the expression $^{(lc)}\Gamma^A_{BC} = {}^{(W)}\Gamma^A_{BC} + K^A_{BC}$ it is possible to obtain the following expression for both Faraday tensors: $^{(lc)}F^{NB} = {}^{(W)}F^{NB} + \eta^{RN} A^P K^B_{PR} - \eta^{RB} A^P K^N_{PR}$. If we make the derivative of this expression and we use (31), we can check the validity of (32).

[7] One can show that $^{(lc)}e^N_{n\,;m} = {}^{(W)}e^N_{n\,;m} + e^N_k K^k_{nm}$, but since the Weitzenböck connections comply with $^{(W)}e^N_{n\,;m} = 0$, it is possible to express the covariant derivative of the vielbein in the

4 Gravito-magnetic currents from WIMT

We intend to explore the flat-spacetime in order to learn under what conditions we can introduce gravito-magnetic currents using WIMT on 5D. We shall consider an orthogonal base without structure and trivial connections.

4.1 Quantization of the fields

Starting from the vacuum action (27) with $^{(lc)}R = 0$, one obtains the dynamics of the gravito-electromagnetic fields

$$\begin{aligned} \Box A^K &= \eta^{BC} A^K_{;BC} = \eta^{BC} A^K_{,BC} \\ &= A^K_{,tt} - A^K_{,xx} - A^K_{,yy} - A^K_{,zz} - A^K_{,ll} = 0, \end{aligned} \tag{35}$$

where $\vec{A} = A^K \vec{E}_K$. The differential equations (35) are separable, so that we can propose a solution of the kind

$$A^K = T^K(t) X^K(x) Y^K(y) Z^K(z) L^K(l). \tag{36}$$

The vector field can be written as a Fourier expansion,

$$\begin{aligned} \vec{A} = &\int_{K^N_x}\int_{K^N_y}\int_{K^N_z} d^3(K^N)\, e^{-K^N_l \frac{l-l_0}{l_0}} \\ &\times [A(\mathbf{K}^N, K^N_l, N) e^{i(K^N_x \frac{x}{x_0} + K^N_y \frac{y}{y_0} + K^N_z \frac{z}{z_0} - K^N_t \frac{t}{t_0})} \\ &+ B(\mathbf{K}^N, K^N_l, N) e^{-i(K^N_x \frac{x}{x_0} + K^N_y \frac{y}{y_0} + K^N_z \frac{z}{z_0} - K^N_t \frac{t}{t_0})}]\vec{E}_N. \end{aligned} \tag{37}$$

We have expanded the vector field as a function of the components of the tangent base $\{\vec{E}_N\}$. These fields comply with $\Box A^M = \eta^{BC} A^M_{;BC} = 0$, where the metric $\eta_{BC} = \vec{\vec{\eta}}(\vec{E}_B, \vec{E}_C)$ describe an inner product through the application on the elements of the tangent space. It is clear that the connections are null. The fields can be expanded with any base with the following requirement for the polarization vectors: $\xi_M(\vec{K}^N, L)\xi^M(\vec{K}^N, L') = \eta_{LL'}$, where $\vec{\xi}(\vec{K}^N, M') = \xi^M(\vec{K}^N, M')\vec{E}_M$. In general the choice of the polarization vectors is independent of $\vec{K}^N$, which is the wave vector of the N-component for the field related to the directional propagation of flat waves.

Footnote 7 continued

Levi-Civita representation as a function of the contortion of Weitzenböck: $^{(lc)}e^N_{n\,;m} = e^N_k K^k_{nm}$. If we use this on the gauge condition and we take into account that $A^N = e^N_n A^n$, we can prove that $^{(lc)}A^N_{;N} = 0 \Rightarrow {}^{(lc)}A^n_{;n} + K^m_{mn} A^n = {}^{(W)}A^n_{;n} = 0$. Hence, one can show the following equalities:

- $^{(W)}A^n_{\ ;n} = {}^{(W)}A^N_{\ ;N}$,
- $\bar{e}^k_K \eta^{BC}\, {}^{(W)}A^K_{;BC} = g^{ab}\, {}^{(W)}A^k_{;bc}$,
- $e^N_n \eta^{MC}\, {}^{(W)}F_{NM;C} = g^{mc}\, {}^{(W)}F_{nm;c}$,
- $^{(lc)}\Box A^K = e^K_k\, {}^{(W)}\Box A^k - e^K_k g^{bc}\, {}^{(W)}A^k_{;n} K^n_{bc}$. Furthermore, $g^{bc} K^n_{bc} = g^{mn}\, {}^{(W)}T^c_{cm}$.

The expression $\overrightarrow{\xi}(\overrightarrow{K}^N, M') = \xi^M(\overrightarrow{K}^N, M')\overrightarrow{E}_M$ says in a general manner that the elements $\xi^M(\overrightarrow{K}, M')$ are exactly the vielbein $e^M_{M'}$ that relate the base $\{\overrightarrow{E}_M\}$, with a certain base $\{\overrightarrow{E}'_{M'} := \overrightarrow{\xi}(\overrightarrow{K}^N, M')\}$, with the same normalization. The vector $\overrightarrow{K}^N := K^N_t \overrightarrow{E}_t + K^N_x \overrightarrow{E}_x + K^N_y \overrightarrow{E}_y + K^N_z \overrightarrow{E}_z + i K^N_l \overrightarrow{E}_l$, is a field in a 5D vacuum and complies with $|K^N|^2 = 0$, which describes the propagation of a light cone. It is usual to propose the radiation gauge $A^0 = 0$ and $A^N_{;N} = 0$. After taking into account the isotropy of the space it is obvious that (for $N, M = 1, 2, 3$) $K^N_i = K^M_j$, where $i, j = 1, 2, 3$.

The field vector $A(\mathbf{K}^N, K^N_l, N)$ depends on the component of the field which we are describing (which we rotulate with the superscript N), with the spatial part of the wave vector $\mathbf{K}^N$ and with the extra dimensional component of K^N: K^N_l. An important point is the second quantization of the field, from which is guaranteed the interpretation of this field as an hermitian operator capable to act on the Fock space through certain operators of creation and annihilation. Hence, we shall promote the elements $A(\mathbf{K}^N, K^N_l, N)$ and $B(\mathbf{K}^N, K^N_l, N)$ to operators which comply with the condition $\overrightarrow{A} = \overrightarrow{A}^\dagger$. Hence, we obtain the relationship $B^\dagger(\mathbf{K}^N, K^N_l, N)\overrightarrow{E}^*_N = A(\mathbf{K}^N, K^N_l, N)\overrightarrow{E}_N$,[8] where in our case $\overrightarrow{E}^*_N$ is merely symbolic because the bases are given by real vector fields. Hence

$$\overrightarrow{A} = \int_{K^N_x}\int_{K^N_y}\int_{K^N_z} d^3(K^N)\, e^{-K^N_l \frac{l-l_0}{l_0}} \times [A(K^N) e^{i(K^N_x \frac{x}{x_0} + K^N_y \frac{y}{y_0} + K^N_z \frac{z}{z_0} - K^N_t \frac{t}{t_0})}\overrightarrow{E}_N + A^\dagger(K^N) e^{-i(K^N_x \frac{x}{x_0} + K^N_y \frac{y}{y_0} + K^N_z \frac{z}{z_0} - K^N_t \frac{t}{t_0})}\overrightarrow{E}^*_N]. \tag{38}$$

Using (38) we obtain the canonical momentum $\Pi^N := \frac{\delta\mathcal{L}}{\delta(A_{N,0})}$, and we have

$$\Pi^N = \eta^{N(N)} A_{(N),0} = A^N{}_{,0} = i\int_{K^N_x}\int_{K^N_y}\int_{K^N_z} d^3(K^N)\, e^{-K^N_l \frac{l-l_0}{l_0}} \times \left[\frac{-K^N_t}{t_0} A(\mathbf{K}^N, K^N_l, N) e^{i(K^N_x \frac{x}{x_0} + K^N_y \frac{y}{y_0} + K^N_z \frac{z}{z_0} - K^N_t \frac{t}{t_0})}\overrightarrow{E}_N + \frac{K^N_t}{t_0} A^\dagger(\mathbf{K}^N, K^N_l, N) e^{-i(K^N_x \frac{x}{x_0} + K^N_y \frac{y}{y_0} + K^N_z \frac{z}{z_0} - K^N_t \frac{t}{t_0})}\overrightarrow{E}^*_N\right], \tag{39}$$

which complies with the algebra

$[A^N(t, \mathbf{x}, l), \Pi^M(t, \mathbf{x}', l)] = i\, a\, b\, (2\pi)^3 \eta^{NM} \delta(\mathbf{x} - \mathbf{x}')\, e^{-K_l \frac{l-l_0}{l_0}}$,
$[A^N(t, \mathbf{x}, l), A^M(t, \mathbf{x}', l)] = [\Pi^N(t, \mathbf{x}, l), \Pi^M(t, \mathbf{x}', l)] = 0$,
$[A(\mathbf{K}^N, K^N_l, N), A^\dagger(\mathbf{K}'^M, K'^M_l, M)] = a\, \delta^{NM} \delta(\mathbf{K}^N - \mathbf{K}'^M)\, \delta(K^N_l - K'^M_l)$,
$[A(\mathbf{K}^N, K^N_l, N), A(\mathbf{K}^M, K^M_l, M)] = [A^\dagger(\mathbf{K}^N, K^N_l, N), A^\dagger(\mathbf{K}^M, K^M_l, M)] = 0$,
$T_{K^N}(t)\, T^*_{K^N}(t)_{,t} - T^*_{K^N}(t)\, T_{K^N}(t)_{,t} = i\, b$.

We have used the notation $\mathbf{x} = (x, y, z)$, $A^N(t, \mathbf{x}, l)$ and $\Pi^M(t, \mathbf{x}', l)$ for the components of the corresponding fields. Furthermore, $A(\mathbf{K}^N, K^N_l, N)$ is an annihilation operator and $A^\dagger(\mathbf{K}^N, K^N_l, N)$ is a creation operator. For the constant values $b = 2\frac{K^N_t}{t_0}$ and $a = \frac{t_0}{2(2\pi)^3 K^N_t}$, we obtain

$$[A^N(t, \mathbf{x}, l), \Pi^M(t, \mathbf{x}', l)] = i\, \eta^{NM} \delta(\mathbf{x} - \mathbf{x}')\, e^{-K^N_l \frac{l-l_0}{l_0}}, \quad [A(\mathbf{K}^N, K^N_l, N), A^\dagger(\mathbf{K}'^M, K'^M_l, M)] = \delta^{NM} \frac{t_0}{2(2\pi)^3 K^N_t} \delta(\mathbf{K}^N - \mathbf{K}'^M)\delta(K^N_l - K'^M_l). \tag{40}$$

These expressions extend to the 5D Minkowski spacetime the canonical quantization obtained in a 4D spacetime.[9] Therefore, we can transform the commutators in the Fock space as second rank tensors, in the following manner:

$$[A^n(t, \mathbf{x}, l), \Pi^m(t, \mathbf{x}', l)] = i\, g^{nm} \delta(\mathbf{x} - \mathbf{x}')\, e^{-K^n_l \frac{l-l_0}{l_0}}, \quad [A(\mathbf{K}^n, K^n_l, n), A^\dagger(\mathbf{K}'^m, K'^m_l, m)] = g^{nm} \frac{t_0}{2(2\pi)^3 K^n_t} \delta(\mathbf{K}^n - \mathbf{K}'^m)\delta(K^n_l - K'^m_l),$$

where $\Pi^m = \bar{e}^m_M \Pi^M$ and $K^m = \bar{e}^m_M K^M$. These expressions provide us with the algebra in an arbitrary metric obtained from a 5D Minkowski spacetime, which is free of structure. In order to illustrate the formalism, in the following section we shall apply it to a de Sitter expansion, which describes the early inflationary universe.

[8] The adjoint operator is given by

$$\overrightarrow{A}^\dagger = \int_{K^N_x}\int_{K^N_y}\int_{K^N_z} d^3(K^N)\, e^{-K^N_l \frac{l-l_0}{l_0}} \times [A^\dagger(\mathbf{K}^N, K^N_l, N) e^{-i(K^N_x \frac{x}{x_0} + K^N_y \frac{y}{y_0} + K^N_z \frac{z}{z_0} - K^N_t \frac{t}{t_0})} + B^\dagger(\mathbf{K}^N, K^N_l, N) e^{i(K^N_x \frac{x}{x_0} + K^N_y \frac{y}{y_0} + K^N_z \frac{z}{z_0} - K^N_t \frac{t}{t_0})}]\overrightarrow{E}^*_N,$$

so that we see from (37) that $\overrightarrow{A} = \overrightarrow{A}^\dagger$; it is fulfilled when

1. $A(\mathbf{K}^N, K^N_l, N)\overrightarrow{E}_N = B^\dagger(\mathbf{K}^N, K^N_l, N)\overrightarrow{E}^*_N$,
2. $B(\mathbf{K}^N, K^N_l, N)\overrightarrow{E}_N = A^\dagger(\mathbf{K}^N, K^N_l, N)\overrightarrow{E}^*_N$.

We see that both conditions are equivalent. With respect to the interpretation of $\overrightarrow{E}^*_N$, we claim that it is $\overrightarrow{E}^*_N = (e^n_N \overrightarrow{e}_n)^* = e^{n*}_N \overrightarrow{e}_n = e^n_N \overrightarrow{e}_n$.

[9] In order to avoid any problems of the commutators, we shall use the quantization of Gupta [23] and Bleuler [24]: $\langle A^N_{;N}\rangle = 0$.

5 An example: monopoles from a 5D Minkowski spacetime with contortion

We shall study an example in which we take as an initial spacetime a 5D Minkowski described with the cartesian coordinates $\phi(p) = (t, x, y, z, l)_p$, in which the tensor metric is given by the orthonormal matrix η^{AB} (26). We choose a IB which is not coordinated, and the structure coefficients are given by $C^I_{I0} = \frac{-e^{-N}}{l^2}$, $C^I_{I4} = \frac{-e^{-N}}{l^2}$ and $C^0_{04} = \frac{-1}{l^2}$. The vielbein transforms as $\bar{e}^n_N = \mathrm{diag}(1/l, e^{-N}/l, e^{-N}/l, e^{-N}/l, 1)$, so that we obtain the metric

$$[g]_{ab} = \begin{pmatrix} \psi^2(l) & 0 & 0 & 0 & 0 \\ 0 & \psi^2(l)\, e^{2N} & 0 & 0 & 0 \\ 0 & 0 & \psi^2(l)\, e^{2N} & 0 & 0 \\ 0 & 0 & 0 & \psi^2(l)\, e^{2N} & 0 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix}, \tag{41}$$

with $\psi^2(l) = l^2/l_0^2$. The final spacetime is described by a coordinate base. This implies that the Weitzenböck torsion in the final spacetime will be nonzero. This torsion will be a possible geometrical source for the emergence of gravito-magnetic monopoles, once there has been made a constant foliation on the extra noncompact coordinate. The current components in this case are given by

$$^{(lc)(m)}J_i = C^j_{0i,\,k} A_j (1 - \delta^k_i) = 0, \tag{42}$$

$$^{(W)(m)}J_i = \epsilon_{ijk} \partial_j A_k \, e^{-Ht}, \tag{43}$$

and we have $^{(W)(m)}J_0 = ^{(lc)(m)}J_0 = 0$. Notice that the spatial components of the magnetic currents decay with time in the Weitzenböck representation. The study of the dynamics for the field fluctuations during de Sitter inflation in the framework of Gravito-electromagnetic Inflation (GEMI) was studied with detail in [25,26], and this goes beyond the scope of this paper. However, as has been demonstrated in (43), from the point of view of the Levi-Civita representation there are no currents related to gravito-magnetic sources.

6 Conclusions

We have extended the WIMT formalism to GEMI with the aim to show that gravito-magnetic currents may be obtained, at least in a Weitzenböck representation. The WIMT formalism was introduced with the idea to generalize the foliation method in the Induced Matter Theory of gravity, in which foliations which are not static as a result are very difficult to deal with. With the WIMT formalism, one can make static foliations from a 5D curved spacetime on which one defines a 5D vacuum from the point of view of a Weitzenböck representation. Once done the foliation is possible to pass to the representation of Levi-Civita. It opens a huge versatility to make static foliations to obtain arbitrary 4D hypersurfaces from 5D curved manifolds, which could be very important for quantum field theories, gravitation, cosmology, etc.

In particular, we have centered our study of the WIMT in the dual formalization of the GEMI applied to the cosmology of the early inflationary universe. We have obtained nonzero gravito-magnetic currents with the representation of Weitzenböck. The currents decrease exponentially with time with the expansion of the universe, so that at the end of inflation they become negligible. This should agree with present observations. However, these currents are null in a Levi-Civita representation because in this geometrical representation the coordinate base has no structure or torsion. In a future work we shall study an example where gravito-magnetic sources are nonzero in any representation [27].

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