International Journal of Modern Physics D
Vol. 22, No. 12 (2013) 1342028 (6 pages)

DOI: 10.1142/S0218271813420285

INFLATIONARY DARK ENERGY FROM A CONDENSATE OF SPINORS IN A 5D VACUUM*

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Received 5 July 2013
Accepted 6 October 2013
Published 12 November 2013

What is the physical origin of dark energy? Could this energy be originated by other fields than the inflaton? In this paper, we explore the possibility that the expansion of the universe can be driven by a condensate of spinors. These spinors are free of interactions on five-dimensional (5D) relativistic vacuum in an extended de Sitter spacetime. The extra coordinate is considered as noncompact. After making a static foliation on the extra coordinate, we obtain an effective four-dimensional (4D) (inflationary) de Sitter expansion which describes an inflationary universe. In view of our results, we conclude that the condensate of spinors here studied could be an interesting candidate to explain the presence of dark energy in the early universe.

Keywords: Inflation; spinor condensate; dark energy.

PACS Number(s): 98.80.Cq, 98.80.Es, 95.36.+x, 67.85.Fg

Modern versions of five-dimensional (5D) general relativity abandon the cylinder and compactification conditions used in original Kaluza–Klein (KK) theories, which caused problems with the cosmological constant and the masses of particles, and consider a large extra dimension. In particular, the Induced Matter Theory (IMT)[1,2] is based on the assumption that ordinary matter and physical fields that we can observe in our four-dimensional (4D) universe can be geometrically induced from

*This essay received an honorable mention in the 2013 Essay Competition of the Gravity Research Foundation.
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a 5D Ricci-flat metric with a spacelike noncompact extra dimension on which we define a physical vacuum. From the mathematical point of view, the Campbell–Magaard Theorem (CMT)[3] serves as a ladder to go between manifolds whose dimensionality differs by one. This theorem implies that every solution of the 4D Einstein equations with arbitrary energy–momentum tensor can be embedded, at least locally, in a solution of the 5D Einstein field equations in a relativistic vacuum: $G_{AB} = 0$.[a] Because of this, the stress-energy may be a 4D manifestation of the embedding geometry. Therefore, by making a static foliation on the spacelike extra coordinate of an extended 5D de Sitter spacetime, it is possible to obtain an effective 4D universe that suffered an exponential accelerated expansion driven by an effective scalar field with an equation of state typically dominated by vacuum.[4–7] The most conservative assumption is that the energy density $\rho = P/\omega$ is due to a cosmological parameter which is constant and the equation of state is given by a constant $\omega = -1$, describing a vacuum dominated universe with negative pressure P and energy density ρ. This is the simplest version of inflation in the IMT. However, this is not unique.

There is an intriguing problem in modern cosmology that relies to explain the physical origin of the cosmological constant, which is responsible for the exponential expansion of the early universe, as well as the present day accelerated expansion of the universe. The standard explanation for the early universe expansion is that it is driven by the inflaton field. Many cosmologists mean that such acceleration (as well as the present day accelerated expansion of the universe) could be driven by some exotic energy called *dark energy*. But what is the physical origin of dark energy? Could this energy be originated by other fields than a scalar field? Most versions of inflationary cosmology require one scalar inflaton field which drives the accelerated expansion of the early universe with an equation of state governed by the vacuum.[8] The parameters of this scalar field must be rather finely tuned in order to allow adequate inflation and an acceptable magnitude for density perturbations. The need for this field is one of the less satisfactory features of inflationary models. Consequently, we believe that it is of interest to explore variations of inflation in which the role of the scalar field is played by some other field.[9,10] Recently, it has been explored the possibility that such expansion can be explained by a condensate of dark spinors.[11] This interesting idea was recently revived in the framework of IMT. In this paper, we revise this idea, but from a 5D vacuum.

As 5D vacuum we understand a purely kinetic 5D lagrangian for fields minimally coupled to gravity which are in a 5D perfect fluid on a, at least, 5D Ricci-flat spacetime. In particular, we shall consider a 5D Riemann-flat spacetime

$$dS^2 = \left(\frac{\psi}{\psi_0}\right)^2 [dt^2 - e^{\frac{2t}{\psi_0}}(dx^2 + dy^2 + dz^2)] - d\psi^2, \tag{1}$$

[a]We shall consider that letters A, B run from 0 to 4 and Greek letters run from 0 to 3.

where t, x, y, z are the usual local spacetime coordinate system and ψ is the noncompact spacelike extra dimension. If we foliate $\psi = \psi_0 = H_0^{-1}$, the resulting 4D hypersurface describes a de Sitter spacetime. This means that a relativistic observer moving with the penta-velocity $U^{\psi} = 0$, will be on a spacetime which has a scalar curvature ${}^{(4)}R = 12/\psi_0^2 = 12\, H_0^2$, such that the constant Hubble parameter (and hence the cosmological constant: $\Lambda = \frac{3H_0^2}{8\pi G}$) is determined by the foliation from the geometrical point of view. So, after making the foliation, we get the effective 4D metric

$$dS^2 \rightarrow ds^2 = dt^2 - e^{2H_0 t} dr^2, \tag{2}$$

which describes a three-dimensional (3D) spatially flat, isotropic and homogeneous de Sitter expanding universe with a constant Hubble parameter H_0.

From the physical point of view we shall consider the action $\mathcal{S} = \int d^5x \, [\frac{\mathcal{R}}{16\pi G} + \mathcal{L}_{\rm eff}]$, where the 5D Lagrangian density for free massless spinors is $\mathcal{L}_{\rm eff} = -\frac{1}{2}(\nabla_A \overline{\Psi})(\nabla^A \Psi)$. The equation of motion for the spinor Ψ takes the form [The same procedure yields an identical equation for the field $\overline{\Psi}$.]

$$g^{AB} \nabla_A \nabla_B \Psi = 0. \tag{3}$$

A clever transformation on the spinor components is $\Psi = \left(\begin{smallmatrix}\varphi_1 \\ \varphi_2\end{smallmatrix}\right)$, where components are

$$\varphi_1 = \begin{pmatrix}\psi_1 \\ \psi_2\end{pmatrix} \quad \text{and} \quad \varphi_2 = \begin{pmatrix}\psi_3 \\ \psi_4\end{pmatrix},$$

can be followed by the conformal mapping $\Phi_+ = \varphi_1 + i\varphi_2$ and $\Phi_- = \varphi_1 - i\varphi_2$. Rewriting the equations of motion in terms of the new fields, we get decouple the equation for Φ_+, while the other coupling becomes a source for the equation for Φ_-[b]

$$\widehat{\emptyset}\Phi_+ + \frac{2\psi_0}{\psi^2}\, \frac{\partial \Phi_+}{\partial t} - \frac{4}{\psi}\, \frac{\partial \Phi_+}{\partial \psi} - \frac{5}{4\psi^2}\Phi_+ = 0, \tag{4}$$

$$\widehat{\emptyset}\Phi_- + \frac{4\psi_0}{\psi^2}\, \frac{\partial \Phi_-}{\partial t} - \frac{4}{\psi}\, \frac{\partial \Phi_-}{\partial \psi} + \frac{7}{4\psi^2}\Phi_- = \frac{i2\psi_0}{\psi^2}\, e^{-\frac{t}{\psi_0}}\, \boldsymbol{\sigma} \cdot \boldsymbol{\nabla}\Phi_+. \tag{5}$$

Notice that the equation of motion (4) for Φ_+ is homogeneous, but the Eq. (5) for Φ_- has a source where $\boldsymbol{\nabla}\Phi_+$ appears coupled to σ. Equation (4) can be factored as a product of functions $\Phi_+ \propto \Lambda_1(\psi)\, e^{i\mathbf{k}\cdot\mathbf{r}}\, \xi_k(t)$ and can be proved for cosmological scales [i.e. for $k \ll H_0\, e^{H_0 t}$], Eq. (5) also can be factored as $\Phi_- \propto \Lambda_2(\psi)\, e^{i\mathbf{k}\cdot\mathbf{r}}$

[b]Here, we have adopted the following convention [σ denotes the Pauli vector]:

$$\widehat{\emptyset}\varphi = \left(\frac{\psi_0}{\psi}\right)^2 \frac{\partial^2 \varphi}{\partial t^2} - \left(\frac{\psi_0}{\psi}\right)^2 e^{-\frac{2t}{\psi_0}}\, \nabla^2 \varphi - \frac{\partial^2 \varphi}{\partial \psi^2}.$$

$\chi_k(t)$[c]. After few calculations, the Lagrangian density $\mathcal{L}_{\rm eff}$, written in term of the new fields Φ_+ and Φ_-, results

$$\mathcal{L}_{\rm eff} = -\frac{1}{4}(\nabla_A \overline{\Phi}_+ \ \nabla^A \Phi_+ + \nabla_A \overline{\Phi}_- \ \nabla^A \Phi_-). \tag{8}$$

On the other hand, the 5D energy–momentum tensor is represented by ${}^{(5)}T_{AB} = 2\frac{\delta \mathcal{L}_{\rm eff}}{\delta g^{AB}} - g_{AB}\mathcal{L}_{\rm eff}$. This procedure take place in a 5D vacuum.

4D dynamics. Now we consider a static foliation on the 5D metric (1). The resulting 4D hypersurface after making $\psi = \psi_0$ describes a de Sitter spacetime, which rises on the metric (2). From the relativistic point of view, an observer who moves with the penta-velocity $U^\psi = 0$ on a 4D hypersurface has a scalar curvature ${}^{(4)}R = 12/\psi_0^2 = 12\, H_0^2$, such that the Hubble parameter H_0 is defined by the foliation $H_0 = \psi_0^{-1}$. He will undergo a comoving de Sitter expansion with the universe.

The effective 4D Lagrangian density (8) is expressed in terms of the fields $\Phi_\pm(x^\mu, \psi_0)$, which can be thought of as two minimally coupled bosons

$$\mathcal{L}_{\rm eff} = -\frac{1}{4}[\nabla_\mu \overline{\Phi}_+ \ \nabla^\mu \Phi_+ + \nabla_\mu \overline{\Phi}_- \ \nabla^\mu \Phi_-] + V(\Phi_+, \Phi_-).$$

Since

$$\nabla_4 \phi_+ = \frac{\partial \phi_+}{\partial \psi} = -\left(\frac{n}{\psi}\right)\phi_+ \quad \text{and} \quad \nabla_4 \phi_- = \frac{\partial \phi_-}{\partial \psi} = -\left(\frac{m}{\psi}\right)\phi_-,$$

hence the effective 4D potential

$$\begin{aligned} V(\Phi_+, \Phi_-) &= -\frac{1}{4}[\nabla_4 \overline{\Phi}_+ \ \nabla^4 \Phi_+ + \nabla_4 \overline{\Phi}_- \ \nabla^4 \Phi_-]|_{\psi=1/H_0} \\ &= \frac{H_0^2}{4}(n^2 \Phi_+ \overline{\Phi}_+ + m^2 \Phi_- \overline{\Phi}_-) \end{aligned}$$

induced by the static foliation on the fifth coordinate $\psi = \psi_0 = 1/H_0$, is the responsible to provide the dynamics of the fields $\Phi_\pm(x^\mu, \psi_0)$ on the effective 4D hypersurface, such that $\omega = P/\rho = -1$. However, after making a constant foliation we shall recover the same fields in an effective 4D de Sitter (and curved) spacetime. Some terms coming from the 5D geometry become potentials in the effective 4D

[c]The solutions on cosmological scales for the relevant functions $\Lambda_1(\psi)$, $\Lambda_2(\psi)$, $\xi_k(t)$ and $\chi_k(t)$ are $\big[M_1^2 = \frac{(n-3/2)^2-1}{\psi_0^2}$ and $[M_2(m,n)]^2 = \frac{(m-3/2)^2-(n-3/2)^2+4\sqrt{(n-3/2)^2+3}-10}{\psi_0^2}$ are separation constants that only can take certain values: $n \geq 3$ with $m > 3/2 + \sqrt{3 + (\sqrt{(n-3/2)^2+3}-2)^2}\big]$

$$\Lambda_1(\psi) = \lambda_1 \left(\frac{\psi}{\psi_0}\right)^{-n}, \quad \Lambda_2(\psi) = \lambda_2 \left(\frac{\psi}{\psi_0}\right)^{-m}, \quad \xi_k(t) \simeq C_1\, e^{(\nu-2)\,t/\psi_0} \left[\left(\frac{k}{2}\right)\psi_0\right]^{-\nu}, \tag{6}$$

$$\chi_k(t) \simeq C_2\, e^{(\mu-1)\,t/\psi_0} \left[\left(\frac{k}{2}\right)\psi_0\right]^{-\mu} + F(t), \tag{7}$$

where $F(t)|_{t\to\infty} \to C$, $\nu = \sqrt{4 + M_1^2\psi_0^2}$ and $\mu = \sqrt{\nu^2 - 4\nu + 4 + M_2^2\psi_0^2}$.

energy–momentum tensor. In turn, these terms become effective mass of particles in 4D in the equation of motion. The energy density and pressure related to these fields are obtained from the diagonal part of the energy–momentum tensor written in a mixed manner

$$\begin{aligned}\rho &= \left\langle E \left| \frac{1}{4}[(\nabla_0\Phi_+)^2 + (\nabla_0\Phi_-)^2] - \frac{e^{-2H_0t}}{4}[\boldsymbol{\nabla}\Phi_- \cdot \boldsymbol{\nabla}\bar{\Phi}_- + (\boldsymbol{\nabla}\Phi_+ \cdot \boldsymbol{\nabla}\bar{\Phi}_+)^2] \right.\right. \\ &\quad \left.\left. + V(\Phi_+,\Phi_-) + F^0{}_0 |E\rangle \right|_{\psi=1/H_0}, \right. \\ P &= \left\langle E \left| \frac{1}{4}[(\nabla_0\Phi_+)^2 + (\nabla_0\Phi_-)^2] - \frac{e^{-2H_0t}}{12}[\boldsymbol{\nabla}\Phi_- \cdot \boldsymbol{\nabla}\bar{\Phi}_- + (\boldsymbol{\nabla}\Phi_+ \cdot \boldsymbol{\nabla}\bar{\Phi}_+)^2] \right.\right. \\ &\quad \left.\left. - V(\Phi_+,\Phi_-) + F^i{}_j \delta^i_j |E\rangle \right|_{\psi=1/H_0}, \right.\end{aligned} \tag{9}$$

where

$$F^0{}_0 = \frac{C_3}{\pi}\left[\frac{H_0^7}{8k^2} + \frac{H_0^5}{k^4}\right],$$

$$F^i{}_j = \frac{C_4}{\pi}\left[\frac{15H_0^7}{32k^2} + \frac{H_0^5}{2k^4}\right]\delta^i_j.$$

The stress-energy may be a 4D manifestation of the embedding geometry. By making a static foliation on the spacelike extra coordinate, it is possible to obtain an effective 4D universe that suffered an exponential accelerated expansion driven by the fields $\Phi_\pm$. In order to obtain the equation of state $\omega = -1$ we must require that $\rho = -P = 3H_0^2/(8\pi G)$, where $|E\rangle$ is some quantum state such that $\langle E|V|E\rangle$ denotes the expectation value of the effective 4D potential V on the 3D Euclidean hypersurface. Because we are interested in the contribution of energy density on large cosmological scales, the squared gradients in P and ρ can be neglected. On the other hand, it is easy to demonstrate that $\nabla_0\Phi_\pm = \partial_t\Phi_\pm \pm \frac{1}{4\psi_0}\Phi_\pm$. Finally, we arrive one possible consistent solution for the energy density and pressure using the values $\mu = 1$ and $\nu = 2$, corresponding to the masses $M_1 = 0$ and $M_2^2 = 1/\psi_0^2 = H_0^2$ and $n = 5/2$ with $m = 7/2$. This result corresponds to the choice of fields which are constant with respect to the time and spectrums compatible with one scale invariant. The field ϕ_+ is massless and with spin 1, but the ϕ_- is a scalar boson which could be the responsible for the expansion of the universe and therefore it could be a good substitute for the inflaton field, because ϕ_- can drive the expansion of the universe. In our picture ϕ_-, is an effective field which became from a condensate of two entangled spinors that have two components. In all our analysis we have neglected the role of the inflaton field, which (in a de Sitter expansion) is freezed in amplitude and nearly scale invariant. Therefore, the condensate of spinors could be an interesting candidate to explain the presence of dark energy in the early universe.

Acknowledgment

The authors acknowledge UNMdP and CONICET Argentina for financial support.

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