Chaos in the pulse spacing of passive Q-switched all-solid-state lasers

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Received August 9, 2010; revised September 8, 2010; accepted September 20, 2010; posted September 27, 2010 (Doc. ID 133078); published October 13, 2010

We report the experimental and theoretical verification that, in a diode-pumped Nd:YAG + Cr:YAG Q-switched laser, the instabilities in the pulse spacing ("jitter") are ruled by low-dimensional deterministic chaos. From our experimental time series, we determine the embedding and fractal dimensions of the attractor, as well as the values of the Lyapunov exponents. We also present a simplified theoretical description in terms of a map of the same universality class as the logistic map, which explains the bifurcations' cascade and the period-three window of stability observed. The achieved characterization of the dynamics and its main parameters opens a door to effective ways to reduce the jitter, which is of practical interest, through mechanisms of control of chaos. Conversely, the difficulty in the prediction of the interpulse spacing makes this system attractive for high power, robust FM chaotic laser cryptography in free-space propagation. © 2010 Optical Society of America

OCIS codes: 140.1540, 140.3540, 140.3580.

Q-switching of solid-state lasers by using an intracavity saturable absorber (SA) is a technique widely used for generating giant pulses. Because of the advantages of simplicity, compactness, high efficiency, relatively low cost, and avoiding the use of electro- or acousto-optics devices and corresponding electronics and high voltage, diode-pumped *all-solid-state* passively Q-switched lasers have wide applications in radar, remote sensing, pollution detection, engine ignition, nonlinear optics, and material processing, as well as in the industry. Currently, Cr^{4+}:YAG is the most commonly used SA. However, the technique has a drawback: the pulses tend to exhibit large instabilities in the repetition rate ("jitter"), making its use inconvenient or difficult in many applications.

The model of the laser with an SA is isomorphous to that of a Bénard–Rayleigh system with two different fluids or a solute [1]. Many pulsed all-solid-state lasers are known to display complex nonlinear dynamics [2], including the response typical of an excitable system [3]. In spite of this background, the common opinion is that the observed jitter is caused by mechanical and/or thermal noise; its possible dynamical origin has remained rather unexplored. Zayhowsi and Dill [4] reported the observation of a bifurcated pulse train, but no explanation in terms of an underlying dynamical process was attempted. Tang *et al.* [5] showed a period-doubling cascade followed by a highly unstable regime with a period-three stable "island." These are typical indications that the system is ruled by nonlinear dynamics. A numerical description in terms of a set of five rate equations successfully described the observed general behavior. Wei *et al.* [6] measured the correlation dimension of the time series of the pulse amplitudes between 4 and 5, observed the existence of spatiotemporal instabilities, and described them in terms of coexisting Hermite–Gauss oscillating modes.

In this Letter, we report the experimental and theoretical verification that jitter in this system, that is, the *time* elapsed between successive Q-switched pulses, is ruled by nonlinear dynamics, including low-dimensional deterministic chaos. We find that the embedding dimension of the attractor is 5 and that there are two positive Lyapunov exponents. We also sketch a highly simplified description in terms of a one-dimensional map. This map is of the same universality class as the logistic map, which provides a simple reason for some of the more distinctive dynamical features observed.

The core of our laser prototype is a RBAG20 pump cavity ("Gold Module") from Northrop Grumman. The laser cavity is 28 cm long and it is assembled in a "V" shape, with a high-reflectivity flat end mirror, a 50 cm radius of curvature folding mirror, and a flat 80% reflectivity output coupler. The Cr:YAG crystal has an unsaturated transmission of 80%. It is mounted in a variable position inside the cavity. This allows varying the saturation parameter by changing the area of the mode. The recording system is a fast photodiode (100 ps rise time) and a 350 MHz, 5 Gigasample/s digital oscilloscope with a memory of 16 MB, which allows recording a time series of several thousand pulses with acceptable resolution. At the stable Q-switching regime, this laser typically emits a train of ≈40 ns pulses equally spaced at a rate of about 10 KHz. As the SA crystal is displaced, regions of period doubling, period four, and, eventually, fully irregular amplitude and pulse spacing are reached [Fig. 1(a)]. It is noteworthy that, inside the irregular region, there is an island of stable period three [Fig. 1(b)]. These results are in agreement with the previous observations of amplitude dynamics reported by other authors [5,6].

Now we focus on the analysis of the irregular series in the variable "time separation between pulses." To record a series as long as possible, the density of samples during the short (relative to their separation) pulses is necessarily low. In consequence, each pulse is drawn by relatively few samples. This "sampling noise" is unimportant in the case of periodic signals, but, when the goal is to calculate the dynamical properties of the chaotic time series, the peak pulse position must be known with precision. Therefore, the first step is to fit the few points that draw each pulse shape with an appropriate continuous

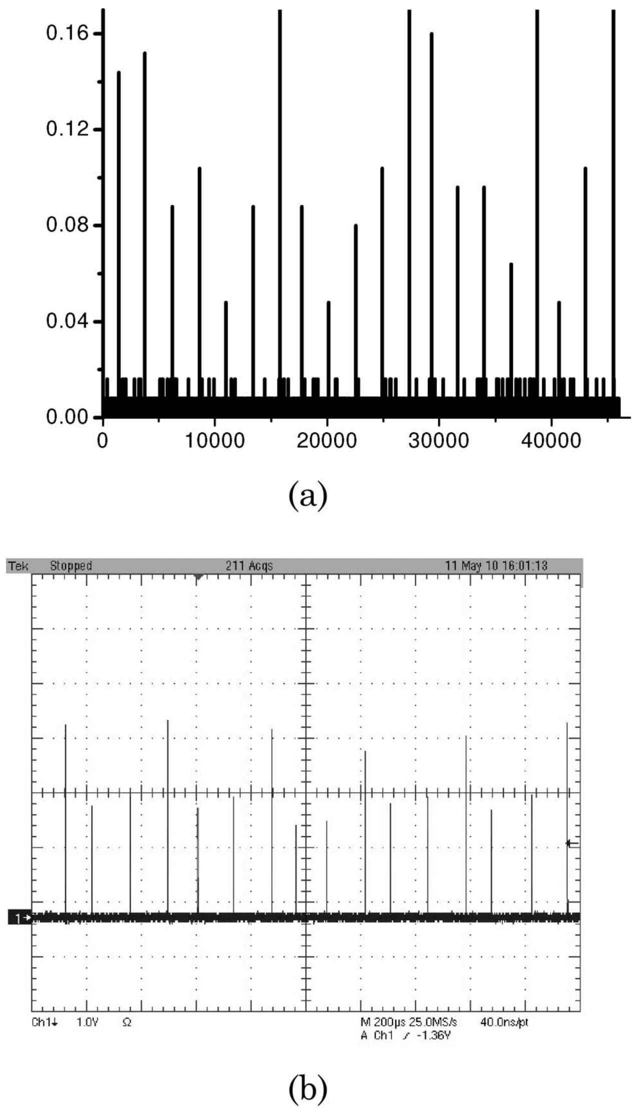

Fig. 1. (a) Zoom of one of the raw data files used to calculate d_E in the fully developed chaotic regime. The complete file has 8024 pulses. (b) Single sweep oscilloscope trace in the period-three stable region, both in amplitude (high, low, low) and in time spacing (short, long, long). Be aware that the pulse amplitudes displayed do not always correspond exactly to the real ones, because of the uneven sampling.

curve, as is described in detail in [7], in order to determine the time position of the maxima with a minimum of noise. Next, we use the TISEAN software [8] and find that the embedding dimension (d_E) is 5 by calculating, as is usual, the decay of the percentage of false nearest neighbors [9] (Fig. 2). We follow the prescription of using the value of the delay of the reconstruction (m) equal to the first minimum of the mutual information of the measured series (typically, $m = 4$ here). We repeat the calculations for $m = 1$, and no significant difference is found, which is an indication of the reliability of the results. Besides, the analogous surrogated (or time-shuffled) data curve does not decay (Fig. 2), which is the standard test that the obtained value of d_E is due to the dynamics, not a mere statistical property of the data file. As an additional test, we perform the same calculation on a complete unprocessed file, i.e., the whole set of 4 million samples.

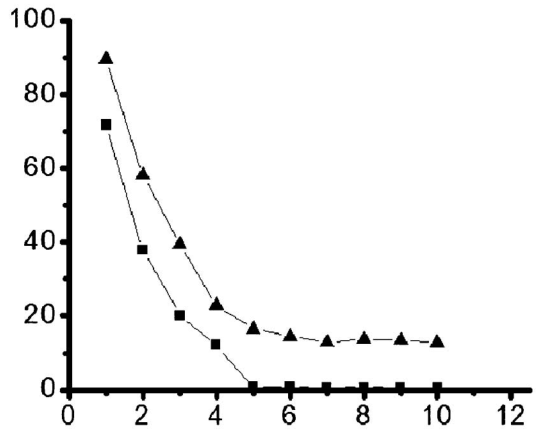

Fig. 2. Percentage of false nearest neighbors as a function of the dimension of embedding (d_E), for the complete file corresponding to Fig. 1(a), $m = 4$ (first minimum of the mutual information). Squares: from the actual series of the time separation between consecutive pulses, a value $d_E = 5$ is clearly determined. Triangles: the same from the surrogated data series. Note that this curve does not decay to zero. It indicates that the scrambled series, in opposition to the actual one, has no defined topological structure.

In spite of the high noise intrinsic to this raw file, the calculation leads to the same results.

We calculate the dimension of the attractor with the Grassberger–Procaccia algorithm, finding a value between 4 and 5, which is consistent with the value of d_E we find and also with the correlation dimension reported in [6] for the pulse amplitude dynamics. We also calculate the Lyapunov exponents. Their exact values change depending on the particular experimental run, but the pattern is always (for the irregular series) the same: two of them are clearly positive, which is a confirmation of the chaotic regime. Another one is near to zero, and the other two are clearly negative. The sum of the five exponents is always negative. The value of the highest Lyapunov exponent is close to 1, which means that the time between pulses is a variable difficult to predict [9], and, hence, is suitable for synchronous chaotic laser cryptography [10]. For this application, the message would be written by illuminating the SA with an LED slightly changing the absorption and, hence, the pulse spacing. This coding would be insensitive to attenuation (FM), and this laser system can be easily scaled in power. It appears then as an appealing alternative to perform encrypted communication at large distances through free space.

The numerical solution of a set of rate equations described well the observed amplitude dynamics [5]. Nevertheless, the period-doubling cascade and the period-three island of stability suggest that a (one-dimensional) logistic map is somehow present. In fact, a qualitative description in terms of a one-dimensional map can be achieved as follows: the pulse energy $\int \Phi dt = \Delta_i - \Delta_f$ is calculated from the implicit equation [11]

$$\Delta_i/\Delta_f - 1 = (\Delta_u/\Delta_f)\ln(\Delta_i/\Delta_f), \quad (1)$$

where Δ_i (Δ_f) is the population difference between the lasing levels; $\Delta = N_2 - N_1$ at the start (end) of the pulse, and Δ_u is its threshold value when the SA is bleached, i.e., when $N_{SA} = 0$. The values of N_2 and N_1 at the end of the pulse can be calculated assuming that, during the emission of the pulse, $N_2 + N_1 \approx$ constant. During the following recovery, there is no field present ($\Phi = 0$), which allows writing closed expressions for the $N_i(t)$. The next pulse starts at $t = \tau$, which is defined by the threshold condition:

$$\sigma \cdot l_g \cdot \Delta(\tau) = \sigma_a \cdot l_s \cdot N_{SA}(\tau) + k, \quad (2)$$

where, following the notation in [5], σ and l_g (σ_a and l_s) are the cross section and length of the gain medium (the SA), and k is the empty cavity loss. This equation also relates the pulse spacing with the population inversion and the state of the SA. If the pump is strong enough that $\Delta_i/\Delta_u \approx 5$, Eq. (1) can be approximated by $\Delta_f \approx \Delta_i \exp(-\Delta_i/\Delta_u)$ [11]. Assuming now that the lower laser level and the SA decay fast, the gain population differences at the start of the $n+1$ pulse and the n pulse become related by a map of the form

$$\Delta_{n+1} = a_1 + a_2 \Delta_n \exp(-\Delta_n/\Delta_u), \quad (3)$$

where $a_1 \approx \Delta_u + s - W \cdot a_2$ and $a_2 \approx [\sigma l_g (W - s - \Delta_u)]^{-1}$, where W is proportional to the pump rate and s to the saturation factor. This map has its maximum at Δ_u and a fixed point at $\Delta_0 > \Delta_u$. The maximum is quadratic at the lowest order, so that the map belongs to the same universality class as the logistic map [12,13]. This implies that the essential dynamics are the same: the route to chaos follows a period-doubling cascade, the accumulation of the bifurcation points converges to the Feigenbaum's constant, a period-three stability island exists in the chaotic region, etc. The first period-doubling bifurcation occurs when $d\Delta_{n+1}/d\Delta_n \le -1$, or $(1 - \Delta_0/\Delta_u)(1 - a_1/\Delta_0) \le -1$. For typical numbers, $|a_1/\Delta_0| \ll 1$, thus the stability condition is dominated by the first factor. The first period-doubling point is hence reached by increasing the pump (increasing Δ_0) or by decreasing the losses (decreasing Δ_u), in agreement with the observations and with the numerical solution of the rate equations [5]. The interpulse spacing then varies, copying the variations of Δ_n, according to Eq. (2). This is a simple explanation to understand some (not all) features of the route to chaos observed. It must be stressed that Eq. (3) is only a rough approximation. It does not take into account, for example, the effect of the slow relaxation of the SA. It cannot describe the whole dynamics of the system. This task requires, as it is shown, at least five variables.

In summary, we experimentally verify that the pulse spacing instabilities commonly observed in passively Q-switched all-solid-state lasers follow deterministic, low-dimensional dynamics. This does not mean, of course, that the influence of external perturbations in a noisy environment can be completely ignored. We measure the value of the embedding dimension and the (fractal) dimension of the attractor, as well as the Lyapunov exponents. The values obtained are consistent among them and also with the observations and the theoretical approaches reported by other authors. We present a simplified description in terms of a one-dimensional map of the same universality class as the logistic map, which makes the general features observed easy to understand. These results open the door to employing efficient and robust mechanisms of control of chaos to stabilize the Q-switch frequency. Some preliminary results obtained through the controlled bleaching of the SA with a LED can be found in [14]. Alternatively, the FM chaotic regime makes these scalable systems attractive for free-space, large distance laser cryptography.

This work was supported by the contracts PICT03-14240 and BID 1728 OC/AR from Agencia Nacional de Promocion Cientifica y Tecnica and PIP200801-02917 from Consejo Nacional de Investigaciones Cientificas. We are grateful to Prof. J. R. Tredicce (Institut Non Lineaire de Nice, France) for his advice, suggestions, and encouragement during his visit at the CEILAP, March–April 2010, and for a critical reading of the first version of this manuscript.

References

1. A. Haken, *Synergetics* (Springer-Verlag, 1978).
2. A. Hnilo and M. Larotonda, J. Opt. Soc. Am. B **18**, 1451 (2001).
3. M. Larotonda, A. Hnilo, J. Méndez, and A. Yacomotti, Phys. Rev. A **65**, 033812 (2002).
4. J. J. Zayhowsi and C. Dill III, Opt. Lett. **19**, 1427 (1994).
5. D. Y. Tang, S. P. Ng, L. J. Qin, and X. L. Meng, Opt. Lett. **28**, 325 (2003).
6. M.-D. Wei, C.-H. Chen, and K.-C. Tu, Opt. Express **12**, 3972 (2004).
7. M. Kovalsky and A. Hnilo, Phys. Rev. A **70**, 043813 (2004).
8. TISEAN software is based on M. Sano and Y. Sawada, Phys. Rev. Lett. **55**, 1082 (1985); it is freely available at http://www.mpipks-dresden.mpg.de/~tisean.
9. H. Abarbanel, *Analysis of Observed Chaotic Data* (Springer-Verlag, 1996).
10. L. Zunino, M. Soriano, A. Figliola, D. Pérez, M. Garavaglia, C. Mirasso, and O. Rosso, Opt. Commun. **282**, 4587 (2009).
11. A. Siegman, in *Lasers* (University Science, 1986), Chap. 26.
12. E. Ott, Rev. Mod. Phys. **53**, 655 (1981).
13. P. Hauser, C. Tsallis, and E. Curado, Phys. Rev. A **30**, 2074 (1984).
14. A. E. Luna, “Láser de estado totalmente sólido pulsado con técnicas de Q-switch pasivo,” M.Sc. thesis final report (Facultad de Ciencias Exactas, Universidad de Buenos Aires, 2006) (in Spanish, available upon request to the corresponding author).