24 May 2001

PHYSICS LETTERS B

Physics Letters B 508 (2001) 1–5

www.elsevier.nl/locate/npe

# Varying $c$ and particle horizons

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Received 22 December 2000; accepted 7 March 2001
Editor: J. Frieman

**Abstract**

We explore what restrictions may impose the second law of thermodynamics on varying speed of light theories. We find that the attractor scenario solving the flatness problem is consistent with the generalized second law at late time. © 2001 Elsevier Science B.V. All rights reserved.

*PACS:* 98.80.Hw
*Keywords:* Cosmology; Entropy; Particle horizons

## 1. Introduction

Recently proposals were advanced to solve the horizon and flatness problems of the standard big-bang cosmology — in a different way that inflationary picture does — as well as the cosmological constant problem by allowing the speed of light in vacuum and the Newtonian gravitational constant to vary with time [1–4]. These approaches are collectively called varying speed of light (VSL) theories. Possible variations of the fundamental physical constants in the expanding Universe are currently of particular interest because of the implications of unified theories, such as string theory and M-theory [5–7]. They predict that additional compact dimensions of space exist. The "constants" seen in our three-dimensional subspace of the theory will vary according to any variation in the scale lengths of the extra compact dimensions. While other scenarios with varying fundamental constants have been considered, like scalar–tensor theories of gravity (see, e.g., [8]) prescribing that $G$ must be function of a scalar field (the Brans–Dicke field) and the varying fine structure constant theory of Bekenstein [9], they do not touch the speed of light and respect Lorentz invariance. By contrast, certain fundamental theories, including strings, could admit spontaneous violation of CPT and Lorentz invariance [10]. For instance, within string theory, quantum aspects of the interactions between particles and non-perturbative quantum fluctuations break supersymmetry and Lorentz invariance [11]. It became an interesting issue to investigate the violation of Lorentz invariance in high energy phenomena [12]. VSL theories also break Lorentz invariance rendering their approach non-covariant. They provide simple effective models to describe these effects.

In this Letter we investigate what constraints the second law of thermodynamics may bring on the formulation of VSL theories, a point that to the best of our knowledge have received no attention so far. As it turns out these constraints are strong for homogeneous

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PII: S0370-2693(01)00400-2

and isotropic spacetimes lacking of a particle horizon. However, for spacetimes possessing a particle horizon the restrictions are much less severe — at least at late time.

## 2. Field equations and constant attractor solution

Let us consider a expanding Friedmann–Lemaître–Robertson–Walker (FLRW) universe whose source of the gravitational field is a perfect fluid and assume that the speed of light in vacuum is not really a constant but varies time in some unknown manner, i.e., $c = c(t)$. The corresponding generalized "Einstein field equations" for a homogeneous and isotropic universe can be written as

$$H^2 = \frac{8\pi G\rho}{3} - \frac{kc^2(t)}{a^2}, \tag{1}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left[\rho + \frac{3P}{c^2(t)}\right], \tag{2}$$

where $H \equiv \dot{a}/a$ is the Hubble factor, and $k$ $(= +1, 0, -1)$ denotes the curvature of the spatial sections. Eq. (1) implies that the energy density is not conserved as the universe expands

$$\dot{\rho} + 3H\left(\rho + \frac{P}{c^2}\right) = \frac{3k}{4\pi G}\,\frac{c\dot{c}}{a^2}. \tag{3}$$

It therefore looks like as though the Universe were an "open system" in the thermodynamical sense. Here we shall explore some of its consequences.

It is generally accepted that the present matter density of the universe is below the critical value [13] — though voices of dissent can be heard [14]. The density parameter $\Omega$, defined as the ratio of the energy density of the universe with the critical density, $\Omega \equiv 8\pi G\rho/(3H^2)$, is one of the best-studied cosmological parameters and its low value is indicated by a number of independent methods for the study of clusters of galaxies. They include the mass-to-light ratio, the baryon fraction, the cluster abundance and the mass power spectrum. Thus, if the energy density of our Universe were dominated by clustered matter we would find the problem that a universe with $\Omega_0 \simeq \mathcal{O}(1)$ requires extreme fine tuning of initial conditions. This is the flatness problem, and it can find a solution within the VSL framework without invoking inflationary fields. By using the above $\Omega$ expression in (1), differentiating it with respect to time and resorting to (3), the evolution equation

$$\dot{\Omega} = (\Omega - 1)\left[\left(1 + \frac{3P}{\rho c^2}\right) + \frac{2\dot{c}}{c}\right] \equiv f(\Omega) \tag{4}$$

follows. For $\dot{c} \neq 0$ it has two constant solutions, namely $\Omega = 1$, which is unstable, and $\Omega_*$. The latter arises when the square parenthesis in (4) vanishes. Our interest focuses on it because it is stable since $\partial[f(\Omega)/\partial\Omega]_{\Omega_*} < 0$ — and is an attractor of the system. For $\Omega = \Omega_*$ the speed of light obeys the law

$$c(a) = c_1 a^{\Omega_*} \exp\left[-\frac{3\Omega_*}{2}\int dt\left(1 + \frac{P}{\rho c^2}\right)H\right]. \tag{5}$$

We note that this expression leads to a decreasing speed of light provided that the dominant energy condition holds. Eq. (3) can be solved by using (5)

$$\rho(t) = \frac{\rho_1}{G}\,a^{2(\Omega_*-1)} \exp\left[-3\Omega_*\int dt\left(1 + \frac{P}{\rho c^2}\right)H\right], \tag{6}$$

hence it follows that $\rho = \rho_1 c^2/(Gc_1^2 a^2)$. By combining it with (1) a relationship between the density parameter and the integration constants can be obtained, namely

$$\Omega_* = \left[1 - \frac{3kc_1^2}{8\pi\rho_1}\right]^{-1}, \tag{7}$$

and by virtue of (1) the scale factor can be written in terms of the speed of light

$$a(t) = \sqrt{\frac{k}{\Omega_* - 1}}\int c(t)\,dt. \tag{8}$$

In the particular case of a linear barotropic equation of state $P = (\gamma - 1)\rho c^2(t)$ with constant adiabatic index $\gamma$, it follows that

$$c(a) = c_1 a^{\beta}, \tag{9}$$

$$\Omega_* = \frac{2\beta}{2 - 3\gamma}, \tag{10}$$

where use of (5) has been made.

For ordinary fluids the strong energy condition (SEC) holds. This implies $\gamma > 2/3$, thereby $\beta < 0$ for $\Omega_* > 0$. From (8) the scale factor is

$$a(t) = \left[(1-\beta)c_1\sqrt{\frac{k}{\Omega_* - 1}}\,t\right]^{\frac{1}{1-\beta}} \equiv a_1 t^{\frac{1}{1-\beta}}. \tag{11}$$

## 3. Entropy considerations

Let us assume that the number of particles in a comoving volume is conserved (i.e., $N \equiv na^3 =$ constant), then the particle number density obeys

$$\dot{n} + 3Hn = 0. \tag{12}$$

This combined with Gibbs equation

$$nT\dot{s} = \dot{\rho} - \left(\rho + \frac{P}{c^2}\right)\frac{\dot{n}}{n}, \tag{13}$$

where $s$ is the entropy per particle and $T$ the fluid temperature, leads to

$$nT\dot{s} = \dot{\rho} + 3H\left(\rho + \frac{P}{c^2}\right). \tag{14}$$

From (3) and (14) it follows that

$$nT\dot{s} = \frac{3k}{4\pi G}\,\frac{c\dot{c}}{a^2}. \tag{15}$$

Note that the entropy variation implied by last equation cannot be attributed either to dissipative processes (since the fluid is perfect) or particles production (for $N$ is a constant). We are led to conclude that the variation of $c$ entails that the entropy of the fluid must vary. This may be justified — at least naively. An increase in $c$ means a widening of the past light cone of the observers. Automatically they acquire more information and the entropy decreases accordingly [15]. That is to say, $\dot{c} < 0 \Longrightarrow \dot{s} > 0$ as well as $\dot{c} > 0 \Longrightarrow \dot{s} < 0$. This together with (15) implies that $c$ cannot increase in open universes ($k = -1$), and that flat and closed universes do not admit $\dot{c} \neq 0$.

The consequences are rather restrictive for cosmological models with varying speed of light. The only admissible FLRW model with $\dot{c} \neq 0$ is the open one. Obviously, one may always introduce some traditional source of entropy such a viscous dissipation (only that in such a case the fluid is no longer perfect), or perhaps particle production from the quantum vacuum; but this complicates matters and we wish to keep the discussion as simple as possible.

Note, by passing, that not every fluid is consistent with $\dot{c} \neq 0$. Think, for instance, in a radiation fluid — equation of state $P = \rho c^2/3$. There one has [16]

$$s = \frac{\rho + (P/c^2)}{nT} = \frac{4\rho}{3nT} = \frac{8}{45}\,\frac{\pi k_B}{\zeta(3)},$$

and accordingly $\dot{s} = 0$. Therefore, in view of (15) — barring a flat FRW — a pure radiation fluid cannot act as the gravitational source of a cosmology with $\dot{c} \neq 0$.

As is well-known, particle horizons may occur quite naturally in cosmological models and these have associated an entropy by a formula formally identical to that of event horizons (either black hole or cosmological) [17]

$$S_H = \frac{k_B}{4}\,\frac{A}{l_{\rm Pl}^2}, \tag{16}$$

where $k_B$ is the Boltzmann constant, $l_{\rm Pl} \equiv (G\hbar/c^3)^{1/2}$ the Planck's length, and $A$ the area of the horizon. The latter is given by $A = 4\pi l_H^2$, with

$$l_H = a(t)\int_0^t \frac{c(t')}{a(t')}\,dt'. \tag{17}$$

Particle horizons exist provided the integral does not diverge. The rationale behind attaching an entropy to a particle horizon is that the area is a measure of the lack of knowledge of the observer about the conditions prevailing in the universe beyond the horizon.

If a FLRW universe filled with a perfect fluid has a particle horizon, the generalized second law (GSL) of thermodynamics (firstly devised for black holes in causal contact with its environment and later extended to cosmological settings) states that the entropy in the fluid enclosed by the horizon plus the entropy of the horizon cannot decrease in time [18]

$$\dot{S}_f + \dot{S}_H \geqslant 0. \tag{18}$$

Here $S_f = (4\pi/3)l_H^3 ns$. It is natural to expect that (18) restricts the temporal dependence of $c$ less severely than the corresponding expression in the absence of horizons (i.e., when $S_H = 0$). Taking into account that $\dot{l}_H = Hl_H + c$, and that $(l_H/a)^{\cdot} = c/a$, the GSL takes the form

$$4\pi Nc\left(\frac{l_H}{a}\right)^3\left[\frac{s}{l_H} + \frac{k\dot{c}a}{4\pi NTG}\right] + \frac{\pi k_B}{G\hbar}c^2\left[(2cH + 3\dot{c})l_H^2 + 2c^2 l_H\right] \geqslant 0. \tag{19}$$

To draw specific consequences of last equation, we use the constant attractor solution $\Omega = \Omega_*$ given by (9) and (11). As we are considering just classical fluids and the horizon entropy is semiclassical in nature we may leave aside any consideration of an early quantum phase. Hence to restrict ourselves to the classical era we replace the lower index of the integral in (17) by some initial time $t_{\rm cl}$ $(> 0)$ which corresponds to the commencement of the the aforesaid era. As a consequence

$$l_H = \frac{c_1}{a_1^{-\beta}}\, t^{1/(1-\beta)} \ln \frac{t}{t_{\rm cl}} \tag{20}$$

remains finite and a horizon exists. One may held the view that by introducing a lower cutoff we illegitimately provide a horizon to an otherwise horizon-free universe. In keeping with that view the very restrictive consequences for $c(t)$ spelled above should apply. By contrast, the more liberal view that the cutoff is admissible since any observer travelling backward in time will eventually hit the quantum era (in which — presumably — the space–time ceases to be a continuum and the observer should see a foam-like structure with the light cones taking random orientations [19]), gives a reasonable chance to relax those consequences.

To obtain $\dot{S}_f$ we must must know the temperature evolution. The latter is governed by [20]

$$\frac{\dot{T}}{T} = -3H\left(\frac{\partial p/\partial T}{\partial \rho/\partial T}\right)_n + \frac{n\dot{s}}{(\partial\rho/\partial T)_n}, \tag{21}$$

therefore a positive specific entropy variation implies that in an expanding universe the temperature will decrease more slowly with a declining speed of light. Here we will consider two limiting cases at late time: monoatomic nonrelativistic mater, and radiation.

(i) In the first case, $\rho = mn + (3/2)nT$ and $P/c^2 = nT$, Eq. (21) reduces to

$$\frac{\dot{T}}{T} = -2H\left(1 - \frac{k(\Omega_* - 1)^2}{4\pi G n_1 T}\, a^2 \dot{a}\ddot{a}\right), \tag{22}$$

and its general solution is

$$T(t) = T_1 t^{-2/(1-\beta)} + T_2 t^{2(2\beta-1)/(1-\beta)}, \tag{23}$$

where $n_1$, $T_1$ are positive constants and $T_2$ depends on the previously defined parameters. In the large time limit the homogeneous part becomes dominant (i.e., $T \propto a^2$ exactly like in a constant speed of light cosmology), and combination with (15) leads to

$$s(t) = s_1 t^{(2\beta+3)/(1-\beta)}. \tag{24}$$

Then, for $t \to \infty$, $S_H \propto t^{5/(1-\beta)} \ln^2 t$, $S_f \propto t^{(2\beta+3)/(1-\beta)} \ln^3 t$ and $S_H$ dominates over $S_f$, so that (18) is satisfied.

(ii) In the second case, $\rho = c_2 T^4$ and $P = \rho c^2/3$, Eq. (21) becomes

$$\frac{\dot{T}}{T} = -\left[1 - \frac{3k(\Omega_* - 1)^2}{4c_2 G T^4}\frac{\dot{a}^2\ddot{a}}{a}\right]H, \tag{25}$$

with general solution

$$T(t) = \left[T_1 t^{4/(1-\beta)} + T_2 t^{2(2\beta-1)/(1-\beta)}\right]^{1/4}, \tag{26}$$

where $T_1$ is a positive constant and $T_2$ depends on the previously defined parameters. For $t \to \infty$ two cases arise. When $-1 < \beta < -1/2$, $T \propto 1/a$ as in standard cosmology, and when $-1/2 < \beta < 0$, one has $T \propto a^{(2\beta-1)/2}$. In the first case $S_f \propto t^{2(1+\beta)/(1-\beta)} \ln^3 t$, while in the second $S_f \propto t^{(2\beta+3)/2(1-\beta)} \ln^3 t$. Again $S_H$ dominates $S_f$ and the GSL is satisfied in both instances.

## 4. Concluding remarks

We have seen that the second law of thermodynamics implies rather severe restrictions on $c(t)$ in FLRW cosmologies free of particle horizons. Specifically, in open universes $c(t)$ cannot augment, and in flat and closed universes $c$ must stay constant. Nonetheless, the presence of a particle horizon render the situation less acute. In particular, for the constant atractor solution of Section 2, the GSL is fulfilled at late time both for non-relativistic monoatomic fluids and extreme relativistic fluids. A similar study for other cosmological solutions should be a worthy undertake.

## Acknowledgements

This work has been partially supported by the Spanish Ministry of Science and Technology under Grant PB94-0718. L.P.C. and A.S.J. thank the University of Buenos Aires for partial support under project TX-93.

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