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Atypical *k*-essence cosmologies

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We analyze the implications of having a divergent speed of sound in *k*-essence cosmological models. We first study a known theory of that kind, for which the Lagrangian density depends linearly on the time derivative of the *k*-field. We show that when *k*-essence is the only source consistency requires that the potential of the *k*-field be of the inverse square form. Then, we review the known result that the corresponding power-law solutions can be mapped to power-law solutions of theories with no divergence in the speed of sound. After that, we argue that the requirement of a divergent sound speed at some point fixes uniquely the form of the Lagrangian to be exactly the one considered earlier and prove the asymptotic stability of the most interesting solutions belonging to the divergent theory. Then, we discuss the implications of having not just *k*-essence but also matter. This is interesting because introducing another component breaks the rigidity of the theory, and the form of the potential ceases to be unique as happened in the pure *k*-essence case. Finally, we show the finiteness of the effective sound speed under an appropiate definition.

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I. INTRODUCTION

Mainstream models of accelerated expansion in the universe assume it is due to the dynamics of scalar fields evolving in a self-interaction potential. In general, the Lagrangians of the effective theories describing those fields include noncanonical kinetic terms, which might be responsible for crucial cosmological consequences like the occurrence of inflation even without a potential (purely kinetic acceleration or *k*-acceleration) [1–3]. In these models, inflation is polelike, that is, the scale factor evolves like a negative power of time. An earlier theoretical framework in which (polelike) *k*-acceleration arises naturally is the prebig bang model of string cosmology [4]. In this setup, acceleration is just due to a scalar field called the dilaton, and it will only manifest itself in the string conformal frame. Finally, for other ideas on kinetic inflation one may have a look at [5], where acceleration was put down to a dynamical Planck mass.

Coming back to *k*-essence, the noncanonical terms considered in the Lagrangian will only be combinations of the square of the gradient of the scalar field (hereafter *k*-field), because the equations of motion in classical theories seem to be of second order. Moreover, since *k*-fields can be used for constructing dark energy models it is common place to interpret them as some kind of matter called *k*-essence [2,6,7]. Nevertheless, originally *k*-fields were not introduced for the description of late time acceleration, but rather they were suggested as possible inflation driving agents [1,8]. Lately, efforts in the framework of *k*-essence have been directed toward model building using power-law solutions which preserve [3,9,10] or violate the weak-energy condition [11].

In this paper, we revisit *k*-essence cosmologies with an infinite sound speed [12], and throw in more light on their implications. In Sec. II we discuss the main features of the models and prove that for consistency the potential must be of inverse square form. In Sec. III we construct models with a divergent speed of sound and we show that the form of the Lagrangian giving rise to an infinite speed of sound is uniquely determined. We also provide an alternative view on the origin of such models which relies on how the Hubble factor depends on the *k*-field and its derivative. At the end of this section the asymptotic stability of the most appealing solutions within that framework is investigated. In Sec. IV we consider a more general model to include matter together with the *k*-essence, we study the implications of this generalization, and then we perturb the background geometry and find the effective sound speed $c_{\rm eff}$. Then, for illustration we calculate it for a simple *ad hoc* example in which the effective sound speed is just equal to the barotropic index so that $c_{\rm eff}^2 < 1$ follows from the condition for inflation. Finally, in Sec. V we draw our main conclusions.

II. MAIN FEATURES OF THE MODELS

In usual practice, *k*-essence is defined as a scalar field ϕ with noncanonical kinetic energy associated with a factorizable Lagrangian of the form

$$\mathcal{L} \equiv V(\phi)F(x), \tag{1}$$

where $x = \nabla_\mu \phi \nabla^\mu \phi$ and $F(x)$ is a function of the kinetic

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energy x. This form of the Lagrangian is suggested by the Born-Infeld one

$$\mathcal{L} = -V(\phi)\sqrt{1+x}, \tag{2}$$

which was associated with the tachyon by computations in boundary string field theory [13]. Such Lagrangian also arises in open bosonic string theory [14] and is a key ingredient in the effective theory of D-branes [15].

Using the perfect fluid analogy, the energy density and the pressure are given by

$$\rho_\phi = V(F - 2xF_x), \tag{3}$$

$$p_\phi = -VF. \tag{4}$$

We assume from now on a flat Friedmann-Robertson-Walker (FRW) spacetime with line-element

$$ds^2 = -dt^2 + a^2(t)(dx_1^2 + dx_2^2 + dx_3^2), \tag{5}$$

where $a(t)$ is the scale factor and ϕ is homogeneous so that $x = -\dot{\phi}^2$. Let $H \equiv \dot{a}/a$ be the Hubble factor, then the Einstein equations reduce to

$$3H^2 = \rho_\phi, \tag{6}$$

$$\dot{H} = xVF_x, \tag{7}$$

where we have taken units such that $8\pi G = 1$. A consequence of the latter is the conservation equation

$$(F_x + 2xF_{xx})\ddot{\phi} + 3HF_x\dot{\phi} + \frac{V'}{2V}(F - 2xF_x) = 0. \tag{8}$$

Furthermore, if we write the equation of state in the form $p_\phi = (\gamma_\phi - 1)\rho_\phi$, the barotropic index γ_ϕ will read

$$\gamma_\phi = -\frac{2\dot{H}}{3H^2} = -\frac{2xF_x}{F - 2xF_x}. \tag{9}$$

In [10] different classes of FRW k-essence cosmologies were investigated. Among them, those which lead to power-law solutions with an inverse square potential and scalar field evolving linearly with time were analyzed, and they were shown to be related by a one-to-one map (see next section) to power-law solutions arising from a theory with

$$F = \alpha + \beta\sqrt{-x}, \tag{10}$$

where α and β are arbitrary constants. Interestingly, the latter is a particular case of the F associated with the extended tachyon models considered in [3]. There solutions were found with the *a priori* assumption that the potential should be of inverse square type. In contrast, we will demonstrate below that, in fact, there is no other possibility.

Now, the cosmological models one obtains from (10) can be viewed as having an infinite sound speed if the familiar definition

$$c_s^2 = \frac{p_{\phi x}}{\rho_{\phi x}} = \frac{F_x}{F_x + 2xF_{xx}} \tag{11}$$

is used. For that reason, we will refer here to the theory arising from (10) as the divergent theory, and, by opposition, any other theory for which $c_s \neq \infty$ will be labeled as nondivergent.

As we will show immediately, another peculiarity of the theory is that compatibility requires the potential be of the inverse square form. For our choice of F the k-field Eq. (8) becomes

$$3HF_x\dot{\phi} + \frac{V'}{2V}(F - 2xF_x) = 0. \tag{12}$$

Now, using the definitions of x and γ_ϕ once and twice, respectively, one arrives at

$$\frac{\dot{V}}{V} = \frac{2\dot{H}}{H}, \tag{13}$$

which gives $V \propto H^2$. On the other hand, inserting (10) in (9), one gets

$$\gamma_\phi = -\frac{\beta}{\alpha}\dot{\phi}, \tag{14}$$

which after integration leads to

$$H\phi = \frac{1}{\Gamma_0}, \tag{15}$$

with $\Gamma_0 = -3\beta/2\alpha$, so that we can finally write

$$V = \frac{V_0}{\phi^2}, \tag{16}$$

where $V_0 = 4\alpha/3\beta^2$ is a constant. Note also that we have set the origin of the potential at $\phi = 0$. Summarizing, one can view the result as if the simultaneous requirement that $H\phi \equiv$ constant and that V be of the inverse square form characterized the solutions to the k-field Eq. (12) for an F like (10).

At this stage, we can insert (10) into the Einstein equations and use (15) and (16). The information we extract is that, necessarily,

$$F = \frac{3H^2\phi^2}{V_0} - \frac{2H\phi}{V_0}\sqrt{-x}, \tag{17}$$

where one should keep in mind that $H\phi$ must be replaced by Γ_0^{-1}.

Remarkably, since there is no evolution equation for ϕ in this theory, the time dependence of ϕ or H are not fixed by the form of the potential, and solutions belonging to this theory exist for absolutely any evolution one can imagine.

III. MODELS WITH DIVERGENT SPEED OF SOUND

Consider now theories with F functions different from (10) and their power-law solutions, which are obtained under the hypotheses

$$V = \frac{V_0}{\phi^2}, \tag{18}$$

$$\phi = \phi_0 t, \tag{19}$$

which clearly imply F is constant. Although these particular models arise from nondivergent theories, they share the property $H\phi =$ constant with all the models derived from the divergent theory, and, in particular, with the power-law ones. Thus, convenient choices of the free parameters will allow for one-to-one maps between power-law solutions of the divergent and nondivergent theories.

Specifically, if for the cosmologies arising from the nondivergent theories we set $a = a_0 t^n$ with a_0 a constant, we will have $H\phi = n\phi_0$, and the isomorphism will follow from the requirement $n\phi_0 = \Gamma_0^{-1}$, which trough the k-field Eq. (12) enforces

$$\alpha = \frac{3n^2\phi_0^2}{V_0}, \tag{20}$$

$$\beta = -\frac{2n\phi_0}{V_0}. \tag{21}$$

Nevertheless, despite this equivalence argumentation, the models arising from the divergent and nondivergent theories are not completely interchangeable in all respects. Before we go deeper into this matter, it is convenient to introduce the parameters f and f', which, respectively, stand for the function F and its first derivative evaluated at $x = x_0 = -\phi_0^2$, that is,

$$f = F(-\phi_0^2), \tag{22}$$

$$f' = F_x(-\phi_0^2). \tag{23}$$

If we substitute the latter into the Friedmann and k-field Eqs. (6) and (8) we find that the index n and the slope of the potential V_0 are given by

$$n = \frac{f + 2\phi_0^2 f'}{3\phi_0^2 f'} \tag{24}$$

$$V_0 = \frac{n}{f'}. \tag{25}$$

Now, inhomogeneous perturbations to the background FRW geometry would involve the speed of sound c_s, or equivalently the second order derivative of F, through Eq. (11). Thus, a measure of those perturbations would provide information on c_s, which could break the degeneracy of the divergent theory and be used to restrict the set of admissible F functions. This provides an adequate framework where the effective sound speed can be introduced, as it will be seen in Sec. IV.

Interestingly, there is a consistency argument that supports the validity of the above result. In [3] the first integral of the k-field Eq. (8) for any F expression was found provided the coefficient of $\ddot{\phi}$ does not vanish; it reads

$$\frac{\gamma_\phi}{\dot{\phi}} = \frac{1}{\phi}\left(\frac{2}{3H} + \frac{c}{a^3H^2}\right), \tag{26}$$

with c an arbitrary integration constant. Consistency, nevertheless, would require that (26) admitted as a particular result $H\phi =$ constant, which must otherwise hold in the limit in which the coefficient of $\ddot{\phi}$ in the k-field equation vanishes. Recalling that in such case one must have $\gamma_\phi = 2\Gamma_0\dot{\phi}/3$, the condition $H\phi = \Gamma_0^{-1}$ follows for $c = 0$.

A. Obtaining the divergent theory

Let us now try to deepen the understanding of the implications of an infinite sound speed. It is clear that an F like (10) (or (17)) is associated with a divergent sound speed, but the question that comes to mind is whether such divergence could occur for a different form of F. In order to find the answer, we are going to consider that the function F is not *a priori* of the form (10), but rather just assume that the speed of sound diverges at the point $x = x_0$, which means $(F' + 2xF'')_{x=x_0} = 0$ and $\dot{\phi} = \dot{\phi}_0$. Using the k-field equation for the inverse square potential $V = V_0/\phi^2$ recursively, we calculate below the values of F and its derivatives at that point, i.e., $F(-x_0) = F(-\dot{\phi}_0^2) = F_0$, $F'(-x_0) = F'_0$, and so on. Hence, at $x = x_0$, Eqs. (6) and (8) become

$$3H^2 = V(F_0 - 2x_0F'_0), \tag{27}$$

$$3HF'_0\dot{\phi}_0 - \frac{F_0 - 2x_0F'_0}{\phi} = 0. \tag{28}$$

Combining these equations with Eq. (7), we arrive at

$$H\phi_0 + \dot{H}\phi = 0, \tag{29}$$

and

$$F'_0 = \frac{H\phi}{V_0\sqrt{-x_0}}, \tag{30}$$

with $H\phi =$ constant, as can be seen from Eq. (27). Besides, from the vanishing of $(F' + 2xF'')_{x=x_0} = 0$, we get

$$F''_0 = \frac{H\phi}{2V_0(-x_0)^{3/2}}. \tag{31}$$

Differentiating the k-field Eq. (8) recursively and using the above results we obtain the remaining derivatives of the function F,

$$F_0''' = \frac{3H\dot{\phi}}{4V_0(-x_0)^{5/2}} \tag{32}$$

and so on. Precisely, these values of F_0, F_0', ..., coincide with those obtained from the function (17) and its derivatives evaluated at $x = x_0$. This shows that the form of F associated with an infinite speed of sound is unique, and it is necessarily given by (10). In consequence, if the speed of sound is infinite for some value of x_0, it will be so for every other value.

Although the result we just gave is quite strong, we wish to present yet one more argument to shed some more light on the origin of the divergent theory. In principle, the Hubble factor H might depend on the field ϕ and its first derivative $\dot{\phi}$ (no dependence on higher order derivatives is required because the k-field equation makes them all depend in turn on ϕ and $\dot{\phi}$). Now, let us assume for the time being that there is only dependence on ϕ, i.e.,

$$H = h(\phi). \tag{33}$$

We can then calculate the barotropic index

$$\gamma_\phi = -\frac{2h'\dot{\phi}}{3h^2}, \tag{34}$$

which by definition (see Eq. (9)) cannot depend on ϕ, so that

$$h' \propto h^2, \tag{35}$$

is required. From the latter it may be concluded that $h \propto \phi^{-1}$. Therefore, $\gamma_\phi \propto \dot{\phi}$ and $H^2 \propto \phi^{-2}$. This just means that

$$F - 2xF_x \equiv \text{constant}, \tag{36}$$

which remarkably solves to give the function $F = \alpha + \beta\sqrt{-x}$ with arbitrary α and β.

Summarizing, just by making the hypothesis that H depends on ϕ only, it is possible to obtain the divergent theory without having to use the conservation equation. We feel this clarifies why the divergent theory is atypical; it looks as if all k-essence theories would split into two classes: the divergent class which is determined by the latter F, and the class of the theories generated by all the remaining F functions.

B. Stability of the solutions

Now, in order to go a fit further, we are going to study the stability of the solution belonging to the divergent theory against changes in the initial conditions. Since it corresponds to $\gamma_\phi/\dot{\phi} =$ constant we introduce a new variable

$$\Gamma \equiv \frac{3\gamma_\phi}{2\dot{\phi}}, \tag{37}$$

so that the unperturbed solution is represented by $\Gamma = \Gamma_0$, with Γ_0 a constant. Let us write now the k-field equation in terms of Γ [3]:

$$\dot{\Gamma} + 3\left(H\Gamma + \frac{V'}{2V}\right)(1 - \gamma_\phi) = 0. \tag{38}$$

A constant solution $\Gamma = \Gamma_0$ to the last equation exists when the condition

$$H\Gamma_0 + \frac{V'}{2V} = 0, \tag{39}$$

holds, so that (38) can be cast as

$$\dot{\Gamma} + 3H\Gamma(\Gamma - \Gamma_0)(1 - \gamma_\phi) = 0. \tag{40}$$

In addition, integrating the condition (39), we get the inverse square potential (16) and the general solution of Eq. (38) is given by

$$\Gamma = \Gamma_0 + \frac{m}{a^3H^2}, \tag{41}$$

where m is an arbitrary integration constant. Therefore, the $\gamma_\phi < 1$ solutions (and, in particular, all the accelerated ones) evolve asymptotically toward $\Gamma = \Gamma_0$.

A more subtle distinction between the models, which would remove the degeneracy, could be established by perturbing these solutions. This, however, is beyond the scope of the present paper.

IV. MATTER CONTRIBUTION

We consider now a model with matter interacting with k-essence generated by the kinetic function (10). In addition, we assume the interaction happens through the geometry only, that is, the components are conserved separately. In this case the Einstein equations are given by

$$3H^2 = \alpha V + \rho_m, \tag{42}$$

$$-3\beta H + \alpha\frac{V'}{V} = 0, \tag{43}$$

$$\dot{\rho}_m + 3H(\rho_m + p_m) = 0, \tag{44}$$

where Eqs. (43) and (44) express the conservation of k-essence and matter, respectively, (Eqs. (42) and (43) follow from assuming Eq. (10)). Furthermore, if we write the equation of state of the matter in the form $p_m = (\gamma_m - 1)\rho_m$, then integrating the matter conservation Eq. (44), we get $\rho_m = \rho_0/a^{3\gamma_m}$. On the other hand, unlike in the case without matter, Eq. (43) cannot be integrated, so the proportionality rule $V \propto H^2$ found in Sec. II (See Eq. (13)) does not apply now. This is a consequence of the fact that $\gamma_\phi = -2\dot{H}/3H^2$ is no longer valid, that relation is rather satisfied by the overall barotropic index defined by

$$\gamma = \frac{\gamma_\phi\rho_\phi + \gamma_m\rho_m}{\rho_\phi + \rho_m} = -\frac{2\dot{H}}{3H^2}. \tag{45}$$

This shows the potential is no longer fixed by the theory, and it represents a crucial difference between the situation in which the universe is filled with k-essence only, or with such fluid together with matter of some other kind. Somehow matter modifies the results found in Sec. II, thus allowing for a more realistic model, because the theory is not rigid anymore as when only k-essence is present, thus matter and k-essence with an arbitrary potential jointly rule the cosmic dynamics. Constructing a model matching the observations and close enough to a Cosmological Constant should be quite easy by getting the potential sufficiently flat, and beta sufficiently small in Eq. (43).

A. The effective sound speed

A realistic model should explain recent observations, which suggest that most of the energy density of the universe consists of a dark energy component with negative pressure in addition to other ordinary components as matter and/or radiation. The most accepted candidate to describe this dark energy component is a scalar field with a negative effective pressure. We can differentiate two kinds of models, the usual of quintessence and that of k-essence with noncanonical kinetic term. These models are different in several aspects, for instance, in the dynamics of the equation of state and in the behavior of the sound speed. Precisely, we concentrate our investigations in this last issue. In quintessence models the scalar field obeys a nearly constant equation of state with a barotropic index $\gamma_\phi < 2/3$, so that $c^2_{s\phi} < 0$, but as shown in [16] this does not go against the stability of the perturbations, because what really matters is $c^2_{\rm eff}$. In contrast, if the dark energy component is described by k-essence the sound speed (11) could be $c_s^2 > 1$, even $c_s^2 = \infty$ is possible as it was seen in previous sections. This means that perturbations of the background k-field can travel faster than light as measured in the preferred frame where the background field is homogeneous. This problem should not come as a surprise, because if we calculate the adiabatic sound speed we find

$$c^2_{s\phi} = \frac{\dot{p}_\phi}{\dot{\rho}_\phi} = -1 - \frac{\beta}{\alpha}\dot{\phi} - \frac{\ddot{\phi}}{3H\dot{\phi}} \qquad (46)$$

we find it is not defined. This happens because in the field Eq. (8), the term $\ddot{\phi}/c_s^2$ vanishes (due to the fact that the speed of sound c_s^2, as usually defined in the framework of k-essence, is divergent as can be seen from (11)). In consequence, $\dot{\phi}$ and $\ddot{\phi}$ are not controlled by any field equation. In other words, the divergent theory enforces the inverse square potential but the behavior field will come from additional *ad hoc* assumptions. This makes it evident that for this particular theory, which is generated by the function (10), the value of the speed depends strongly on the definition we use to calculate it. In fact, if we had used the definition given by Eq. (46) then the speed of sound would, in principle, not have been divergent an the field Eq. (8) would have been the same as before, and the results obtained in the previous sections would have remained valid. However, things change radically when a matter component is introduced, like in the example discussed before with the help of Eqs. (42)–(44). In this case the potential is not fixed by the field Eq. (8) because $\gamma_\phi = -2\dot{H}/3H^2$ is no longer valid. In this case, the potential has to be assumed independently, and the dynamical equations will fix the field and its derivatives so that the adiabatic sound speed (46) will be perfectly defined. In this case, then, it will be the effective $c^2_{\rm eff}$ what we will have to calculate. To that end we follow the steps of [16,17].

In generalized dark matter [16] it was introduced the effective sound speed $c^2_{\rm eff}$ defined in the rest frame of the generalized dark matter component, where $\delta T^0_{j\phi} = 0$. The effective sound speed can be interpreted as a rest frame sound speed, allowing us to define a stabilization scale for a perturbation, given by the corresponding effective sound horizon. So, it was possible to show that density perturbations in the ordinary quintessence scenario are damped out below the horizon and the effective sound speed of quintessence recovers its relativistic behavior $c^2_{\rm eff} = 1$ [16]. In Ref. [17] it was shown that in extended quintessence scenarios things can be different, because the effective sound speed may be strongly affected by the nonminimally coupled scalar field.

The effect of the speed of sound on the CMB perturbation equations is such that for an effective sound speed $c^2_{\rm eff} \ll 1$ [18], k-essence energy density perturbations are enhanced by perturbations in the cold dark matter. The perturbed FRW line-element is

$$ds^2 = a^2(\eta)[d\eta^2 - (\delta_{ij} + h_{ij})dx^i dx^j], \qquad (47)$$

where δ_{ij} is the background spatial metric, $h_{ij} \ll 1$ is the metric perturbation (we consider only linear metric cosmological perturbations), h represents the trace of the spatial metric perturbation. We investigate the effects due to adiabatic perturbations to the k-essence stress-energy in the synchronous gauge for a mode with wave number k. Besides, we omit the argument k in the amplitude of the perturbation quantities in the Fourier space. For a generic component in the model investigated above, it is convenient to separate out the nonadiabatic entropy contributions. Hence, for the k-field we have

$$p_\phi \Gamma_\phi = \delta p_\phi - c^2_{s\phi}\delta\rho_\phi, \qquad (48)$$

where $c^2_{s\phi}$ is the adiabatic sound speed (46) Following Ref. [16], we can write the gauge-invariant entropy term as

$$(1 + \gamma_\phi)\Gamma_\phi = (c^2_{\rm eff} - c^2_{s\phi})\frac{\delta^{(\rm rest)}\rho_\phi}{\rho_\phi}. \qquad (49)$$

The gauge transformation into an arbitrary frame gives the density contrast in the dark energy rest frame [16]

$$\frac{\delta^{(\mathrm{rest})}\rho_\phi}{\rho_\phi} = \frac{\delta\rho_\phi}{\rho_\phi} + 3\mathcal{H}\gamma_\phi\frac{v_\phi - B}{k}, \tag{50}$$

yielding a manifestly gauge-invariant form for the non-adiabatic entropy contribution [19,20]. Here B represents the time-space component of metric fluctuations. In this equation $\mathcal{H} \equiv \dot{a}/a$, where the overdot represents (in this expression only) differentiation with respect to the conformal time $\eta = \int dt/a$.

Combining Eqs. (48)–(50) with the equation of state for the k-field $p_\phi = -(1 + \beta\dot{\phi}/\alpha)\rho$, we obtain

$$c_{\mathrm{eff}}^2 = \frac{\delta p_\phi - 3\beta\mathcal{H}V\dot{\phi}c_{s\phi}^2(v_\phi - B)/k}{\delta\rho_\phi - 3\beta\mathcal{H}V\dot{\phi}c_{s\phi}^2(v_\phi - B)/k}. \tag{51}$$

Since we are using the synchronous gauge B actually vanishes. From Eq. (51), we conclude that $c_{\mathrm{eff}}^2 \approx \delta p_\phi/\delta\rho_\phi$ on scales approaching the horizon. The overall effect is that the pressure fluctuations δp_ϕ are weak and k-essence perturbations are enhanced via gravitational instability of the matter field. Finally, for our divergent sound speed model we obtain that on subhorizon scales

$$c_{\mathrm{eff}}^2 \approx \frac{\delta p_\phi}{\delta\rho_\phi} \approx \gamma_\phi - 1 + \frac{V}{V'}\frac{d\gamma_\phi}{d\phi}, \tag{52}$$

where γ_ϕ is associated with γ_m and the geometry through Eq. (45). We have also used that the dynamical Eqs. (42) and (44) fix the derivatives of the field once the potential has been given so that $\dot{\phi} = \dot{\phi}(\phi)$, so that $\delta x = -2\dot{\phi}\delta\phi d\dot{\phi}/d\phi$.

To study the approximate expression for the finite sound speed c_{eff}^2 given by Eq. (52), we investigate a simple *ad hoc* example in which the barotropic index has the form

$$\gamma_\phi = 2\ln\frac{\phi_0}{\phi}, \tag{53}$$

where ϕ_0 is a constant, so that for an inverse square potential

$$c_{\mathrm{eff}}^2 = \gamma_\phi. \tag{54}$$

An accelerated dark energy scenario requires that $\gamma_\phi < 2/3$, consequently for this model the value of the sound speed satisfy the condition $c_{\mathrm{eff}}^2 < 1$. Combining Eqs. (14) and (53) we can investigate the solutions of the remaining equation near the value of the k-field $\phi = \phi_0$. Writing $\phi = \phi_0 + \epsilon$ with $|\epsilon| \ll 1$, we find that to first order in ϵ

$$\dot{\epsilon} = \frac{2\alpha}{\beta\phi_0}\epsilon. \tag{55}$$

Thus, whenever $\alpha(\beta\phi_0)^{-1} < 0$ the constant solution $\phi = \phi_0$ is stable, then the barotropic index and the sound speed have a vanishing limit for large cosmological times.

V. CONCLUSIONS

This work means to contribute to a better understanding of a divergent speed of sound in k-essence cosmological models. We have reviewed some known results by considering a theory of that kind, for which the Lagrangian density depends linearly on the time derivative of the k-field. In previous works, solutions were obtained under the hypothesis of an inverse square potential. In contrast, we have shown here that for the theory to be consistent the potential of the k-field cannot take any other form. Then, following with revision we have reminded the known result that the corresponding power-law solutions can be mapped to power-law solutions of theories with no divergence in the speed of sound.

After that, we have presented two important new results that reinforce our view that k-essence cosmologies with a divergent speed of sound are very special indeed. First, we have constructed a detailed argument that shows that the requirement of a divergent sound speed at some point fixes uniquely the form of the Lagrangian of the theory to be exactly the one considered earlier. After that we have shown that in the divergent theory the Hubble factor depends on the k-field only, whereas in the nondivergent ones depends on its first derivative too. On the other hand, we have proved that, from the cosmological point of view, the most interesting solutions belonging to the divergent theory are asymptotically stable.

We then have considered matter in addition of the k-essence studied in the previous section. It turns out that in this alternative model the k-essence potential is not fixed any longer, and may be freely chosen. This also brings implications on the admissible definitions of the sound speed. Following in this respect [16,17] we have studied the behavior of linear perturbations. The dark energy clustering in k-essence scenarios, where the k-field is assumed to be responsible for the cosmic acceleration today can be realized. The scalar field density perturbations can grow on subhorizon scales and the effective sound speed c_{eff}^2 may satisfy the requirement $c_{\mathrm{eff}}^2 \ll 1$ for a large set of potentials. In particular, we have introduced a cosmological model for which the effective sound speed and barotropic index have both suitable asymptotic properties.

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