

New Power Transformer Model for the Calculation of Electromagnetic Resonant Transient Phenomena Including Frequency-Dependent Losses

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Abstract—A new power transformer model for the accurate pre-determination of transient resonant processes is introduced in this paper. The accurate representation of the losses is the most important feature of the new model, since it has been developed for the accurate determination of maximal stresses during resonance phenomena within transformers.

The new equivalent circuit only contains constant lumped parameters. It represents very accurately the frequency variation of all impedances (real and imaginary parts) of the transformer, including mutual magnetic coupling. This allows carrying out transient calculations in the time domain directly using programs for the electromagnetic transients calculations like EMTP.

Several laboratory measurements on transformer winding models have been performed in order to verify the agreement of measured and calculated values of resonant overvoltages.

Index Terms—Eddy currents, electromagnetic transient analysis, losses, power transformers, resonance.

I. INTRODUCTION

A. Background

TWENTY years ago it was recognized that internal resonance phenomena in transformers have originated damages in the windings.

For the case of incoming resonant overvoltages, elevated internal voltage stresses can be expected, and in many cases these stresses could thoroughly overcome the dielectric strength of the winding insulation that was designed taking into account only lightning and switching impulsive stresses.

The origin of oscillating overvoltages has been researched and published in several works [1]–[10]. The conclusion was that the critical situations should be investigated in order to avoid the exposition of the transformers to such stresses.

It is then very important to calculate accurately the internal voltage stresses in order to evaluate if certain switching operations will be critical for the transformer.

During resonance phenomena the resistances mainly determine the maximal voltages in the windings. These resistances are strongly frequency-dependent. They represent both the copper and iron losses and for this reason they also depend on the distribution of the magnetic field. This dependence is due to the eddy currents in the winding and in the core. Therefore it

becomes necessary to introduce a representation of the losses taking these aspects into account, considering that it has not been done in an accurate form up to now.

A new transformer model is introduced in this paper. It includes an adequate representation of the frequency-dependent losses and inductances by means of an equivalent circuit including only constant lumped impedances. With this model the necessary accuracy required for the determination of the maximal overvoltages during resonance is finally reached. Once the parameters of the circuit have been determined, the calculation could be carried out with any program for transient calculations (for example the EMTP).

B. State of the Art

The damages produced in power transformers as a consequence of resonance phenomena has awakened the interest of many researchers in the last years.

The work of Müller and Buckow [11] and subsequently the Ph.D. thesis of Buckow [12] may be cited among these contributions. An attempt to introduce the frequency-dependent losses in the model was made for the first time in this work. The main assumption considered was that the iron losses can be neglected for frequencies above 10 kHz, stating that the magnetic flux would be almost completely leakage magnetic flux.

The copper losses were modeled through frequency-dependent series resistances. A recurrent solution process has been proposed for which successive electromagnetic field calculations are carried out for each considered frequency. As could be seen, this procedure is not a very practical one and it requires a great amount of calculation time. Buckow attributed the differences observed between measurement and calculation to the fact that the iron losses have been neglected.

The work of Leohold has also contributed to this topic. In his work [13], [14] he introduced a model in order to incorporate the losses within the equivalent circuit of the transformer. His model doesn't yield good results in spite of having taken into account all possible types of losses cited in the bibliography. As in the case of Buckow's model, the model of Leohold can not be used in a direct form to calculate the response of the winding in the time domain.

In the work of De Leon and Semlyen [15]–[19] a model for the calculation of the transient behavior of transformers is introduced for a wide range of frequencies. For the representation of the inductive coupling for high frequencies, a model based on leakage reactances [15] is utilized.

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The modeling of the dependence of the resistances on the frequency doesn't furnish the necessary accuracy for the case of resonance. An equivalent circuit hasn't been introduced, so that the calculation in time domain must be carried out with a program developed for this purpose.

The analysis of the previous work shows that an equivalent circuit that yields accurate results for resonance calculations has not yet been developed. The development of such a model has been investigated by the author [23]–[25], which is presented in this paper.

II. MODELING THE LOSSES

The correct modeling of the losses is the most important element in the determination of the resonant phenomena within transformers because of their impact in the highest overvoltages values.

The difficulty in finding an adequate model consists in the multiplicity of simultaneous effects. Given a system of many conductors with time-variable currents flowing within them, a mutual influence exists between these currents. The distribution of current density in each conductor of the system should be determined taking into account the simultaneous effect of all other currents. The problem should be solved in complete form and cannot be subdivided, since acceptable assumptions to make it possible do not exist. The damping of the system doesn't depend only on the frequency but also on the spatial distribution of the magnetic flux, which is produced by all the currents. This is the reason for the difficulty of introducing a model of transformer losses accounting for all the cited influences.

The feasibility of developing an adequate equivalent circuit in this case was still questioned in the past [20]. In the present work it will be demonstrated that an equivalent circuit can produce satisfactory results even in this case.

III. NEW LINEAR TRANSFORMER MODEL FOR TRANSIENT RESONANCE

The fundamental idea regarding the new model is based on the fact that a linear model can represent the resonant behavior of the transformer. The principal problem of modeling the losses is the correct consideration of all mutual influences between the several conductors of the system. The mutual influences are generated through induced voltages due to the effect of linkages of time-variable magnetic flux in the conductors. If the materials behave in linear form, the problem could be dealt with a linear model.

The direct consequence of the Faraday's law is the appearance of induced voltages in the conductors, which are modeled with the aid of inductances. A secondary consequence of the same law is that the distribution of the current density in the conductor cross-section is not constant. This effect influences directly the copper losses.

If the treatment of the multiconductor system by means of a linear model is considered an acceptable assumption, the behavior of the system could be expressed mathematically as follows:

$$\mathbf{u} = \mathbf{Z}\mathbf{i} \quad (1)$$

with

- \mathbf{u} vector of branch voltages at the inductive branches
- \mathbf{i} vector of currents in the inductive branches
- \mathbf{Z} impedance matrix.

The fact that is behind this equation is the frequency-dependence of the elements of the impedance matrix.

A new equivalent circuit of the transformer windings has been developed with the aid of the interpretation of the eddy current phenomenon in conductive media.

The main assumptions are:

- The losses in the different metallic parts of the transformer are due to the circulation of current, including the effect of eddy currents.
- A model for this purpose could be conceived through several auxiliary additional current loops.
- An induced current flows through each loop and this current gives rise to losses in the resistance of the loop. Each actual coil could be associated with certain quantity of such auxiliary circuits.
- This allows to consider an equivalent circuit formed by n main sectors and a total number of auxiliary circuits of $m = n.r$ for the whole winding. Each main inductive branch is magnetically coupled with all auxiliary circuits.

If a number of m auxiliary circuits are included in the winding model (see Fig. 1), the following matrix equation can be written:

$$\mathbf{u}_f = \mathbf{R}_f \mathbf{i}_f + \mathbf{e}_f = 0. \quad (2)$$

The relationship between total and induced voltages in the main branches can be given by (see Fig. 1):

$$\mathbf{u}_b = \mathbf{R}_b \mathbf{i}_b + \mathbf{e}_b \quad (3)$$

where

- \mathbf{u}_b vector of the voltages at the branches associated to the actual coils (main branches)
- \mathbf{u}_f vector of the voltages at the auxiliary loops
- \mathbf{e}_b vector of the induced voltages at the branches associated to the actual coils (main branches)
- \mathbf{e}_f vector of the induced voltages at the auxiliary circuits
- \mathbf{i}_b main branches current vector
- \mathbf{i}_f current vector at auxiliary circuits
- \mathbf{R}_f diagonal resistance matrix of auxiliary circuits (order $m.m$)
- \mathbf{R}_b diagonal resistance matrix of the main branches (order $n.n$).

The relationship between the currents and the induced voltages of the whole system is described by the following equation:

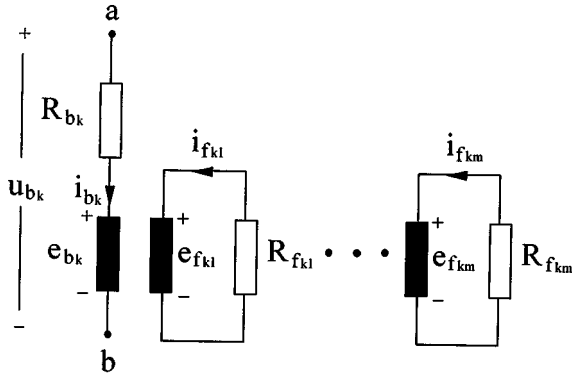
$$\mathbf{e} = j\omega \mathbf{L} \mathbf{i} \quad (4)$$

where

- \mathbf{e} branch induced voltage vector
- \mathbf{L} inductance matrix.

The matrix \mathbf{L} could be partitioned in submatrices and the following equation can be obtained:

$$\begin{bmatrix} \mathbf{e}_b \\ \mathbf{e}_f \end{bmatrix} = j\omega \begin{bmatrix} \mathbf{L}_b & \mathbf{M} \\ \mathbf{M}^T & \mathbf{L}_f \end{bmatrix} \begin{bmatrix} \mathbf{i}_b \\ \mathbf{i}_f \end{bmatrix} \quad (5)$$

Fig. 1. Equivalent circuit of a winding sector (m auxiliary circuits).

where

- \mathbf{L}_b main branches inductance matrix (order $n.n$)
- \mathbf{L}_f auxiliary circuits inductance matrix (order $m.m$)
- \mathbf{M} matrix of mutual inductances between the main branches and the auxiliary circuits (order $n.m$).

Using (2) and (3) we have for the total branch voltages

$$\begin{bmatrix} \mathbf{u}_b \\ \mathbf{u}_f \end{bmatrix} = \begin{bmatrix} \mathbf{u}_b \\ 0 \end{bmatrix} = \begin{bmatrix} \mathbf{Z}_b & j\omega\mathbf{M} \\ j\omega\mathbf{M}^T & \mathbf{Z}_f \end{bmatrix} \begin{bmatrix} \mathbf{i}_b \\ \mathbf{i}_f \end{bmatrix} \quad (6)$$

$$\mathbf{u}_b = \mathbf{Z}_b \mathbf{i}_b + j\omega\mathbf{M} \mathbf{i}_f \quad (7)$$

$$0 = j\omega\mathbf{M}^T \mathbf{i}_b + \mathbf{Z}_f \mathbf{i}_f \quad (8)$$

where

$$\mathbf{Z}_f = \mathbf{R}_f + j\omega\mathbf{L}_f \quad (9)$$

$$\mathbf{Z}_b = \mathbf{R}_b + j\omega\mathbf{L}_b \quad (10)$$

and

\mathbf{Z}_b main branches impedances matrix (order $n.n$)

\mathbf{Z}_f auxiliary circuits impedances matrix (order $m.m$).

An expression for \mathbf{i}_f can be obtained from (8):

$$\mathbf{i}_f = -j\omega\mathbf{Y}_f\mathbf{M}^T \mathbf{i}_b \quad (11)$$

where

$$\mathbf{Y}_f = \mathbf{Z}_f^{-1}. \quad (12)$$

Substituting from (11) in (7) we find

$$\mathbf{u}_b = [\mathbf{Z}_b + \omega^2\mathbf{M}\mathbf{Y}_f\mathbf{M}^T] \mathbf{i}_b. \quad (13)$$

The matrix \mathbf{Y}_f could be considered as

$$\mathbf{Y}_f = \mathbf{G}_f - j\omega\mathbf{B}_f \quad (14)$$

and (13) becomes

$$\mathbf{u}_b = [(\mathbf{R}_b + \omega^2\mathbf{M}\mathbf{G}_f\mathbf{M}^T) + j\omega(\mathbf{L}_b - \omega^2\mathbf{M}\mathbf{B}_f\mathbf{M}^T)] \mathbf{i}_b \quad (15)$$

or

$$\mathbf{u}_b = \mathbf{Z}_{eq} \mathbf{i}_b \quad (16)$$

using the following definitions

$$\mathbf{Z}_{eq} = \mathbf{R}_{eq} + j\omega\mathbf{L}_{eq} \quad (17)$$

$$\mathbf{R}_{eq} = \mathbf{R}_b + \omega^2\mathbf{M}\mathbf{G}_f\mathbf{M}^T \quad (18)$$

$$\mathbf{L}_{eq} = \mathbf{L}_b - \omega^2\mathbf{M}\mathbf{B}_f\mathbf{M}^T. \quad (19)$$

Equation (15) represents the reduced system obtained from the enlarged system, in which only the corresponding nodes of the actual coils (or winding sections) remain.

Each auxiliary circuit is loaded by a resistance. The resistances are represented by means of a diagonal matrix \mathbf{R}_f , which has order $(m.m)$. The matrix \mathbf{L}_f also has the same order, but it is in general a full matrix.

At this point of the development the first important assumption will be made:

- The matrix \mathbf{L}_f is considered to be a diagonal matrix. Since the matrix \mathbf{R}_f is diagonal, then the matrices \mathbf{Z}_f and \mathbf{Y}_f are also diagonal matrices.

This means that the mutual magnetic coupling between the auxiliary circuits are to be neglected. This assumption represents a restriction to the model. This restriction becomes necessary in order to make possible the determination of the circuit parameters, limiting their number. The capacity of the restricted model for representing the behavior of the transformer remains very high, since it allows the modeling of the frequency variation of the circuit impedances with high accuracy.

Thereby the matrices \mathbf{L}_f , \mathbf{R}_f , and therefore \mathbf{Z}_f are diagonal and the elements of the matrix \mathbf{Y}_f can be determined in a simple way.

The diagonal elements of the matrix \mathbf{Z}_f will be denoted Z_{fj} :

$$Z_{fj} = R_{fj} + j\omega L_{fj} \quad (20)$$

$$Y_{fj} = \frac{R_{fj} - j\omega L_{fj}}{R_{fj}^2 + \omega^2 L_{fj}^2}. \quad (21)$$

In accordance with (14) the elements of the matrices \mathbf{G}_f and \mathbf{B}_f are:

$$G_{fj} = \frac{R_{fj}}{R_{fj}^2 + \omega^2 L_{fj}^2} \quad B_{fj} = \frac{L_{fj}}{R_{fj}^2 + \omega^2 L_{fj}^2}. \quad (22)$$

Therefore, an equivalent circuit able to model the resonant behavior of the winding has been developed (see Fig. 2). It allows the correct representation of the copper and iron losses with high accuracy.

After a careful observation of (18) and (22), it becomes evident that \mathbf{R}_{eq} is a function of the frequency, because of the quadratic dependence on frequency and also the frequency-dependence of the matrix \mathbf{G}_f . The elements of the matrix \mathbf{L}_{eq} are

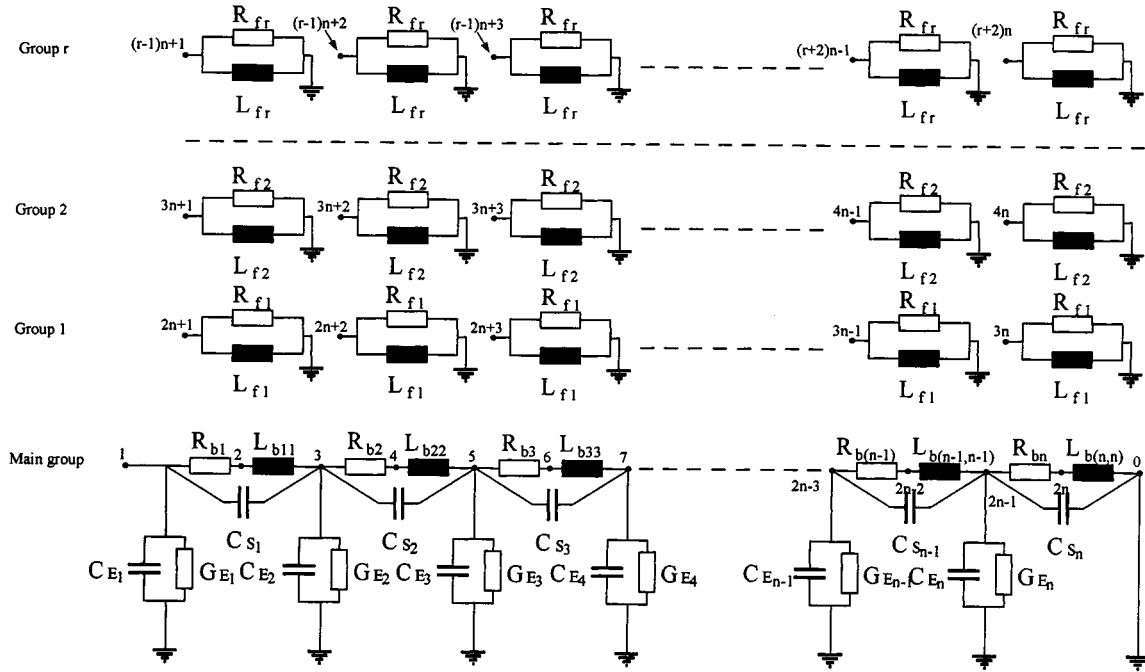


Fig. 2. Equivalent circuit of the transformer winding.

not constant, since their mathematical expression (19) also includes a frequency-dependent term.

Finally it will be mentioned that a limit to the quantity of auxiliary circuits (m) associated to the actual coils hasn't been imposed. Therefore, the capability of this model in approaching the impedance functions is high.

IV. SUBDIVISION OF THE AUXILIARY CIRCUITS IN GROUPS

In order to obtain a simple way to determine the unknown parameters of the equivalent circuit, the auxiliary circuits are divided in groups. Each group of auxiliary circuits has exactly the same number of circuits n as the main group (real coils). There are r groups, so that the total number of auxiliary circuits is $m = r \cdot n$.

Equation (6) can then be written as:

$$\begin{bmatrix} u_b \\ 0 \\ 0 \\ \dots \\ 0 \end{bmatrix} = \begin{bmatrix} Z_b & j\omega M_1 & j\omega M_2 & \dots & j\omega M_r \\ j\omega M_1^T & Z_{f1} & 0 & \dots & 0 \\ j\omega M_2^T & 0 & Z_{f2} & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ j\omega M_r^T & 0 & 0 & \dots & Z_{fr} \end{bmatrix} \begin{bmatrix} i_b \\ i_{f1} \\ i_{f2} \\ \dots \\ i_{fr} \end{bmatrix} \quad (23)$$

where

i_{fk} current vector of auxiliary circuits of group k
 M_k submatrix of mutual inductances between the main branches and the auxiliary circuits of group k (order $n \cdot n$)

Z_f impedance submatrix of auxiliary circuits of the group k (order $n \cdot n$).

The second assumption is made at this point:

- All the elements of the diagonal submatrix Y_{fk} have the same values. That means that the auxiliary circuits of the same group k have the same self inductance L_{fk} and the same resistance R_{fk} [see (21)].

After this subdivision, a new expression of the product $MG_f M^T$ can be obtained:

$$MG_f M^T = \sum_{k=1}^{k=r} G_{fk} M_k^2. \quad (24)$$

For the product $MB_f M^T$ we have:

$$MB_f M^T = \sum_{k=1}^{k=r} B_{fk} M_k^2. \quad (25)$$

Substituting from (24) in (18) and from (25) in (19):

$$\begin{aligned} R_{eq} &= R_b + \omega^2 \sum_{k=1}^{k=r} G_{fk} M_k^2 \\ L_{eq} &= L_b - \omega^2 \sum_{k=1}^{k=r} B_{fk} M_k^2. \end{aligned} \quad (26)$$

A new substitution from (22) in (26) gives:

$$\begin{aligned} R_{eq} &= R_b + \sum_{k=1}^{k=r} \frac{\omega^2 R_{fk}}{R_{fk}^2 + \omega^2 L_{fk}^2} M_k^2 \\ L_{eq} &= L_b - \sum_{k=1}^{k=r} \frac{\omega^2 L_{fk}}{R_{fk}^2 + \omega^2 L_{fk}^2} M_k^2. \end{aligned} \quad (27)$$

Equation (27) is the main equation for the determination of impedances of the equivalent circuit.

V. DETERMINATION OF PARAMETERS OF THE EQUIVALENT CIRCUIT

The problem of determination of parameters consists in finding the unknown values of the matrices \mathbf{M} , \mathbf{G}_f , and \mathbf{B}_f from the values of $\mathbf{R}(\omega)$ and $\mathbf{L}(\omega)$:

$$\begin{aligned}\mathbf{R}(\omega) &\approx \mathbf{R}_b + \sum_{k=1}^{k=r} \frac{\omega^2 R_{fk}}{R_{fk}^2 + \omega^2 L_{fk}^2} \mathbf{M}_k^2 \\ \mathbf{L}(\omega) &\approx \mathbf{L}_b - \sum_{k=1}^{k=r} \frac{\omega^2 L_{fk}}{R_{fk}^2 + \omega^2 L_{fk}^2} \mathbf{M}_k^2.\end{aligned}\quad (28)$$

The values of matrices $\mathbf{R}(\omega)$ and $\mathbf{L}(\omega)$ are considered as known, since their elements can be determined by means of electromagnetic field calculations for a given number of frequencies within the range of interest, or they can also be measured. The parameters of the equivalent circuit to be determined are to be found in matrices \mathbf{M} , \mathbf{G}_f , and \mathbf{B}_f : the matrix \mathbf{M} is a matrix of constant elements and matrices \mathbf{G}_f and \mathbf{B}_f have elements of the type described by (22), i.e., they contain a limited number of constant elements L_{fk} and R_{fk} .

The determination of the parameters has been carried out applying the strategy of minimizing the sum of quadratic errors of the approximate values [right sides of (28)], with respect to the exact values (actual values of the resistance and inductance matrices). The nonlinear optimization strategy was implemented using the method of Levenberg–Marquardt [21], [22].

The procedure of calculation of parameters comprises, in first place, the approach of one of the elements of the matrix \mathbf{Z}_{eq} . In general a diagonal element (for example Z_{11}) will be chosen. Then follows the determination of the values of L_{fk} and R_{fk} (for $k = 1, \dots, r$), which must correctly fit the frequency variation of the chosen impedance.

Once all these constants have been calculated, the matrices \mathbf{G}_f and \mathbf{B}_f can be determined. The numerical values of L_{fk} and R_{fk} calculated for Z_{11} are considered to be also representative of the dependence on frequency of any of the other elements in the matrix. Considering then the determined values of L_{fk} and R_{fk} as known, the determination of elements of the matrices \mathbf{M}_k can be carried out. This determination is performed element by element.

The process of determination of parameters L_{fk} , R_{fk} , and the elements of the matrix \mathbf{M} , implies the simultaneous optimization of real and imaginary parts of the impedances.

The resistances and inductances of experimental windings used in the investigation were measured with the aid of a HP 4192A impedance analyzer.

The real and imaginary parts of the impedance $Z_{11}(\omega)$ were modeled with six auxiliary circuits as an optimal value. The comparison between measured and calculated impedance values is shown in Figs. 3 and 4, showing an excellent agreement (maximal relative error less than 3%) for the real as well as the imaginary part.

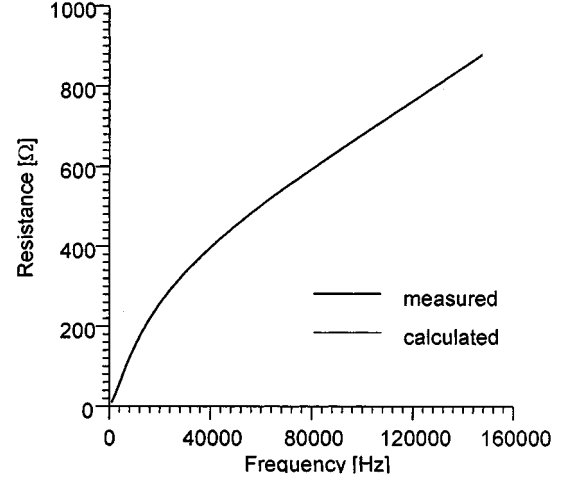


Fig. 3. Fitting of Re_{q11} with six auxiliary circuits.

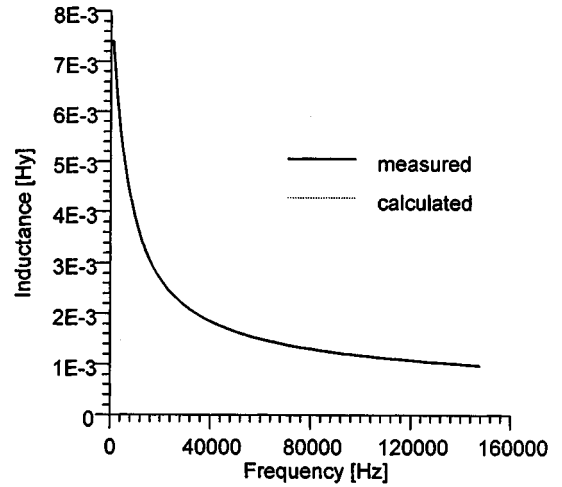


Fig. 4. Fitting of Le_{q11} with six auxiliary circuits.

VI. EXPERIMENTAL VERIFICATION

The verification of the agreement of calculated and measured values of frequency response was performed with the aid of several transformer winding models. For this purpose two winding models were used (winding I and II).

Winding I was built with 12 disc coils with 10 turns each. A winding section of this winding is one coil. The winding was earthed in its end during the tests. Winding I as well as winding II was mounted over a 40 kVA distribution transformer iron core. A set of capacitors (100 nF each) was connected between each section junction and earth potential in order to simulate the earth capacitance. The reason of this is to reach an adequate value of the total ground capacitance in order that the resonance frequencies of the test winding are within the resonance frequency range of power transformers (less than 1 MHz).

Measurements of the frequency response of voltages were carried out in the available nodes (connection between winding sectors) and they were compared with the calculated response. The frequency range used in this case comprised up to 500 kHz.

Fig. 5 shows a simplified equivalent circuit of the used windings in order to identify the nodes.

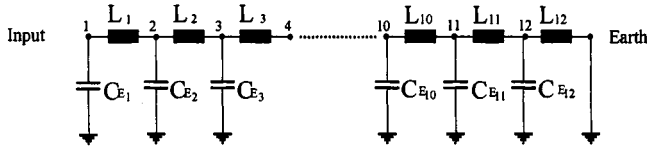


Fig. 5. Simplified equivalent circuit of winding I or II.

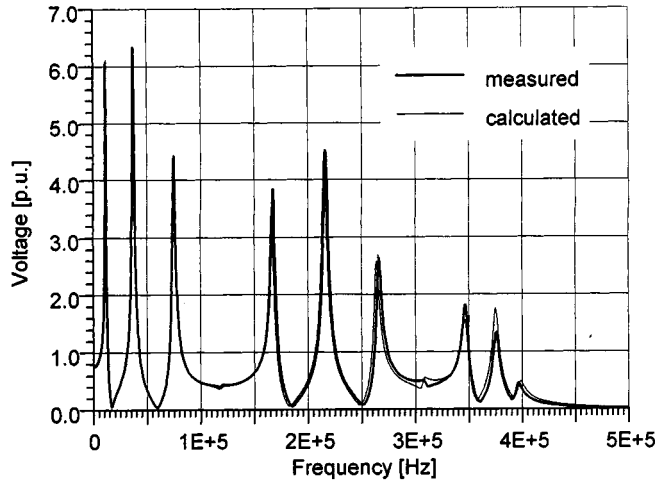


Fig. 6. Frequency response of the absolute value of node 4 voltage (winding I).

Figs. 6 and 7 show the comparison of the measured and calculated frequency responses for nodes 4 and 7 of winding I. Since it is not possible to distinguish differences in both curves of absolute values at the maximal values (resonance), a comparison using polar curves was included in Fig. 8.

Maximal voltages along the winding have been included in Table I. A notable agreement between calculated and measured values can be seen. The maximal amplitude error is 7% in the second resonance (the gray blocks have not been taken into account because there are no resonance overvoltages at these places).

The corresponding results using winding II are also shown. The winding II has a group of two disc coils with 10 turns each as winding section. The total number of sections was also 12.

Figs. 9 and 10 show the comparison of measured and calculated frequency responses for nodes 4 and 7 of winding II. Fig. 11 shows the comparison using polar curves.

In Table II it is evident that the accuracy for this case is greater than for winding I. The magnitude of maximal relative error for the first three resonances exceeds scarcely 1.2%.

The agreement between measured and calculated curves is completely satisfactory for both windings.

VII. CONCLUSIONS

A new coupled equivalent circuit has been developed for the determination of the behavior of transformer windings during resonant excitation with special consideration of the power losses representation.

Since the introduced model is an equivalent circuit with constant parameters, it can be used directly for transient calculations

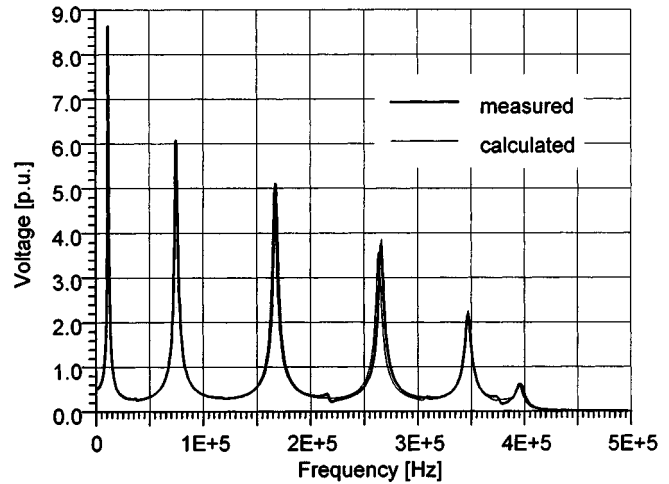


Fig. 7. Frequency response of the absolute value of node 7 voltage (winding I).

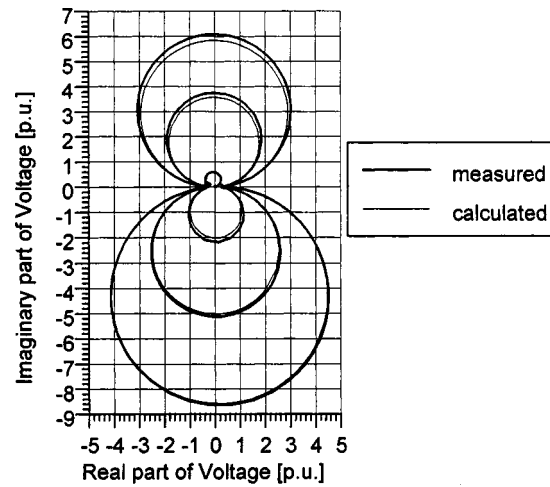


Fig. 8. Frequency response of node 7 voltage in polar form (winding I).

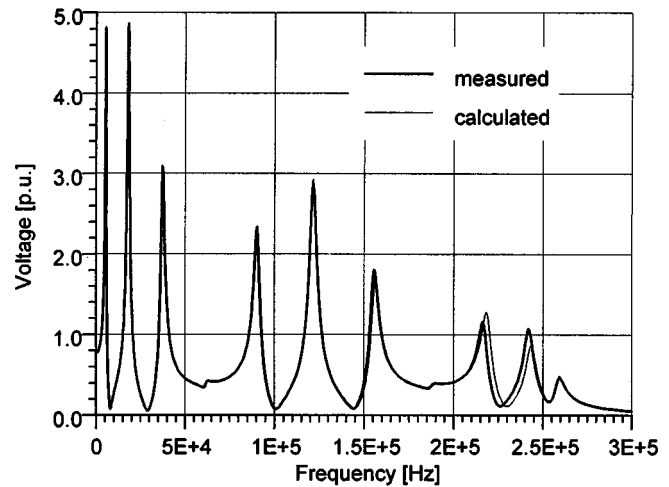


Fig. 9. Frequency response of the absolute value of node 4 voltage (winding II).

in the time domain using known mathematical tools for this purpose (for example the EMTP). Basic data for the appli-

TABLE I
MAXIMAL VOLTAGES AT FIRST, SECOND, AND THIRD RESONANCE IN WINDING I

| Reson. Order | Maximal Voltages | Winding nodes | | | | | | | | | | |
|--------------|------------------|---------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| | | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 1 | calculated | 2.39 | 4.25 | 6.01 | 7.40 | 8.28 | 8.58 | 8.27 | 7.36 | 5.93 | 4.12 | 2.08 |
| | measured | 2.42 | 4.30 | 6.07 | 7.47 | 8.36 | 8.65 | 8.34 | 7.44 | 6.01 | 4.18 | 2.10 |
| | abs. error | -0.03 | -0.05 | -0.06 | -0.07 | -0.07 | -0.07 | -0.07 | -0.08 | -0.08 | -0.06 | -0.03 |
| | rel. error [%] | -1.28 | -1.18 | -1.03 | -0.99 | -0.89 | -0.80 | -0.89 | -1.09 | -1.32 | -1.32 | -1.19 |
| 2 | calculated | 3.03 | 5.11 | 5.93 | 5.14 | 2.97 | 0.29 | 3.03 | 5.13 | 5.86 | 5.02 | 2.83 |
| | measured | 3.24 | 5.47 | 6.35 | 5.49 | 3.17 | 0.29 | 3.23 | 5.52 | 6.30 | 5.39 | 3.04 |
| | abs. error | -0.21 | -0.36 | -0.42 | -0.35 | -0.20 | 0.00 | -0.21 | -0.38 | -0.44 | -0.37 | -0.21 |
| | rel. error [%] | -6.48 | -6.61 | -6.59 | -6.42 | -6.43 | 0.15 | -6.44 | -6.97 | -7.00 | -6.83 | -6.85 |
| 3 | calculated | 4.11 | 5.89 | 4.28 | 0.40 | 4.10 | 5.85 | 4.09 | 0.31 | 4.30 | 5.91 | 4.03 |
| | measured | 4.32 | 6.15 | 4.43 | 0.33 | 4.34 | 6.09 | 4.26 | 0.29 | 4.46 | 6.14 | 4.20 |
| | abs. error | -0.21 | -0.26 | -0.15 | 0.07 | -0.24 | -0.24 | -0.17 | 0.02 | -0.17 | -0.23 | -0.17 |
| | rel. error [%] | -4.95 | -4.17 | -3.43 | 22.33 | -5.43 | -3.96 | -3.99 | 6.88 | -3.76 | -3.79 | -3.97 |

TABLE II
MAXIMAL VOLTAGES AT FIRST, SECOND, AND THIRD RESONANCE IN WINDING II

| Reson. Order | Maximal Voltages | Winding nodes | | | | | | | | | | |
|--------------|------------------|---------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| | | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 1 | calculated | 2.02 | 3.45 | 4.83 | 5.95 | 6.69 | 6.94 | 6.69 | 5.96 | 4.81 | 3.34 | 1.68 |
| | measured | 2.02 | 3.44 | 4.81 | 5.93 | 6.65 | 6.91 | 6.66 | 5.93 | 4.79 | 3.33 | 1.68 |
| | abs. error | 0.00 | 0.01 | 0.02 | 0.02 | 0.03 | 0.03 | 0.03 | 0.03 | 0.02 | 0.01 | 0.01 |
| | rel. error [%] | 0.12 | 0.33 | 0.46 | 0.41 | 0.47 | 0.43 | 0.51 | 0.46 | 0.41 | 0.37 | 0.41 |
| 2 | calculated | 2.53 | 4.17 | 4.83 | 4.21 | 2.47 | 0.28 | 2.42 | 4.17 | 4.79 | 4.12 | 2.32 |
| | measured | 2.56 | 4.21 | 4.87 | 4.24 | 2.50 | 0.29 | 2.44 | 4.21 | 4.84 | 4.16 | 2.34 |
| | abs. error | -0.02 | -0.05 | -0.04 | -0.03 | -0.03 | 0.01 | -0.02 | -0.04 | -0.05 | -0.03 | -0.02 |
| | rel. error [%] | -0.96 | -1.07 | -0.88 | -0.76 | -1.19 | 0.33 | -0.83 | -0.97 | -1.01 | -0.78 | -0.88 |
| 3 | calculated | 2.93 | 4.18 | 3.06 | 0.39 | 2.87 | 4.10 | 2.87 | 0.24 | 2.99 | 4.14 | 2.79 |
| | measured | 2.97 | 4.20 | 3.08 | 0.40 | 2.91 | 4.14 | 2.90 | 0.25 | 3.02 | 4.15 | 2.81 |
| | abs. error | -0.03 | -0.02 | -0.02 | 0.01 | -0.04 | -0.04 | -0.03 | 0.01 | -0.03 | -0.01 | -0.02 |
| | rel. error [%] | -1.13 | -0.58 | -0.67 | 2.10 | -1.22 | -0.87 | -0.97 | 1.03 | -0.93 | -0.27 | -0.69 |

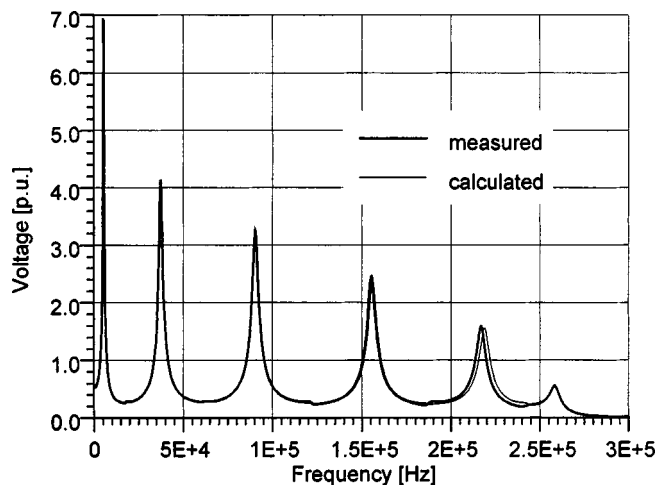


Fig. 10. Frequency response of the absolute value of node 7 voltage (winding II).

cation of the model are values of the impedances of the winding over a range of frequencies. These impedance values could be obtained through calculation or by measurements.

The obtained results demonstrate that the presented transformer losses representation allows an accurate calculation of winding voltages, overcoming clearly the limitations of existing models.

VIII. APPLICATION AND USE

The new equivalent circuit presented is a valuable tool that can be used in general problems of circuit synthesis to model coupled inductive networks for high frequencies. In the particular case of transformers, it can be applied to the design as well as the analysis of the effects of transient (especially resonant) overvoltages coming from the network.

The model allows the analysis of transformer transient behavior during failures under two possible viewpoints. On the one hand it allows an accurate calculation of the inner transformer overvoltages, in order to evaluate the possibility of insulation damage. On the other, the transformer equivalent circuit can be applied together with other impedances and models of the entire network, obtaining an accurate representation of the transformer-network interaction. In the last case, it is advantageous to use a reduced equivalent circuit.

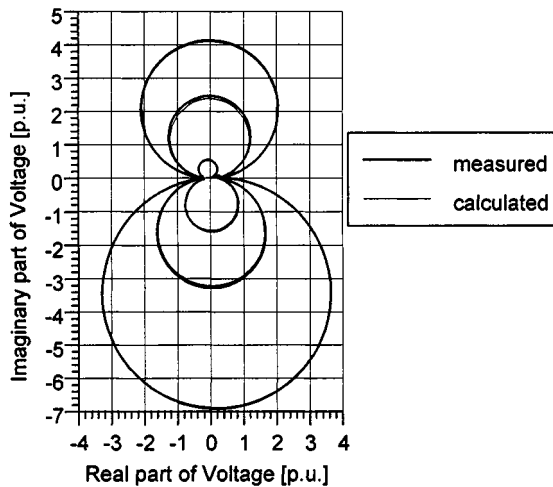


Fig. 11. Frequency response of node 7 voltage in polar form (winding II).

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