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Review

Buckling of vertical oil storage steel tanks: Review of static buckling studies

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ABSTRACT

Research on the structural behavior and buckling of vertical, aboveground tanks employed to store oil and fuels have significantly increased during the past two decades. Interest in this shell form is related not just to the cost of the infrastructure, but also because failures in cases of accidents or natural disasters may cause huge economic, environmental and social losses. This review concentrates on buckling problems of such tanks under static or quasi-static loads, including uniform pressure, wind, settlement of foundation, and fire. In all cases, buckling is considered as a static process. Attention is given to the load definition in each case, followed by buckling studies under previously defined pressures or temperatures. The structural configuration of tanks is first described in order to understand what is specific about this structural form. Next, the theoretical framework for stability and buckling is briefly described to place each contribution in a wider context. Each loading case is first explained, experiments or case-studies are briefly described, and computational analytical modeling is reviewed; finally, efforts towards improving design are mentioned. Most papers published in the literature have been motivated by wind effects on tanks, but the review shows that other areas, such as thermal buckling under an adjacent fire, are currently receiving increasing attention.

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1. Introduction

Vertical aboveground tanks are employed in several industries to store water, oil, fuel, chemicals, and other fluids. Depending on the specific industry and on the fluid stored, different materials are employed in the fabrication of tanks: in the oil industry, metals have been used almost exclusively. Basically oil tanks are thin-walled short cantilever shells, and have geometrical and structural differences with other storage shells, such as silos or pressure vessels which tend to be taller. Oil storage tanks are constructed using curved steel sheets, commonly known as courses, with dimensions depending on the local steel industry, and which are welded together to form the cylinder. Because of their geometric slenderness, tanks are prone to fail by buckling, and frequently this failure initiates in a form of elastic buckling.

Considerations about buckling of tanks under various loading conditions should not be restricted to new designs, because most tanks employed at present were fabricated before the 1980 s and efforts are continuously made to extend their service life in order to avoid interruptions in the operation of a plant. Recently there is a trend to fabricate new tanks which are increasingly larger and thinner than old ones [126], with the consequence that they have geometries which are more slender than those used in the aerospace or in the off-shore industries.

There have been several reviews of shell buckling which may be related to this one. The most influential review of computational buckling analysis of shells was published by Bushnell in 1981 [25] as a summary to his book on the topic. The aim in Bushnell's work was to provide a large number of illustrative examples in order to foster a feeling for buckling behavior in engineers. Emphasis was on understanding buckling by intuition rather than from theoretical formulations. By means of carefully chosen examples, Bushnell illustrated cases in which eigenvalue bifurcation analysis provides inadequate information; effects due to boundary conditions, nonlinearity in the fundamental equilibrium path, plasticity, mode interaction; and problems in which deep knowledge is required in order to fully understand the behavior in design or in a failure investigation. Because many shell forms and applications were covered in Ref. [25], Bushnell could bring experience to illustrate a wide variety of unstable behavior. However, the possibilities of nonlinear behavior and buckling are considerably reduced when one specifies a shell configuration, as in the present case.

A thorough review paper on shell buckling by Teng [161] was published in 1996. Teng and Rotter [162] compiled a book on buckling of metal shells. A review of the developments of stability of shells in the XX Century was written by Elishakoff [43]. Thompson has recently revisited his views on the mechanics of shell buckling from the perspective of bifurcation and chaos [164].

Schmidt [153] summarized work on shell buckling research that was subsequently adopted in the European Recommendations [147]. Failure modes in silos and tanks have been summarized by Rotter [144,146], with emphasis on silos. Recently, Zingoni [187] published a comprehensive review of strength, stability and dynamics of tanks, including vertical and horizontal designs, as well as tanks used to store water which are supported by a central column. This is the closest review to the present one, but because a wider scope of tank configurations and mechanics of behavior were considered by Zingoni, only his Section 2 overlaps with the present review.

Many buckling problems in steel oil or fuel storage tanks may be modeled as a quasi-static problem, including the buckling under uniform external pressure, wind load, thermal loads, and foundation settlement, and those are reviewed in this paper. Dynamic buckling problems arise under time-dependent loads due to earthquakes and suddenly applied loads associated with explosions, and require using dynamic buckling criteria to assess the stability of the shell; those are not considered in this paper.

In all cases reviewed in this paper there is a sequential analysis, in the sense that pressures are evaluated at a first stage (either from lab tests or from computer simulations) and the structural response under such pressures is computed at a second stage.

2. Structural configuration of tanks

A summary of the main characteristics of tanks as fabricated for the oil industry is given in this section; further details are given in books on tanks, such as Myers [121] and Refs. [54,40]. Design recommendations are given in Refs. [7,20,45], among others. Comparisons of design methods for tank thicknesses acceptable to API standards [7] are reported by Azzuni and Guzei [11].

Oil storage tanks are fabricated with a cylindrical shell having a stepped thickness with a bottom circular plate at the base and some form of roof at the top. Less frequently, there are some small tanks fabricated with uniform thickness. There are two main classes of roof: a fixed roof welded to the cylinder, or a floating roof; however, some tanks have both a fixed roof on top and a floating roof inside.

The buckling strength of the cylindrical shell of tanks largely depends on two geometric parameters: the aspect ratio, as defined by the ratio between the height H and the diameter D of the cylinder (H/D), and the slenderness (R/t) calculated as the ratio between the radius R of the shell and its minimum thickness t. As mentioned before, tanks fabricated with a rather short cantilever cylinder with $1{,}000 < R/t < 2{,}000$, and $H/D < 1$. Diameters have been increased in the recent decade, in a trend to build fewer tanks with larger capacity. Increasing sizes are reported in China, reaching $D = 100$ m, with volumes of fluid storage in the order of 100,000m^3 [186]. Similar trends are informed in France, with tanks reaching $D = 80$ m; to illustrate different sizes, volume capacities have been classified as 100,000m^3, 10,000m^3, and 1,000m^3 [125]. An increase in volume capacity is accompanied by an increase in D and a decrease in the aspect ratio H/D.

Because results for one tank geometry may not be directly applicable to other geometric configurations, then care is taken in

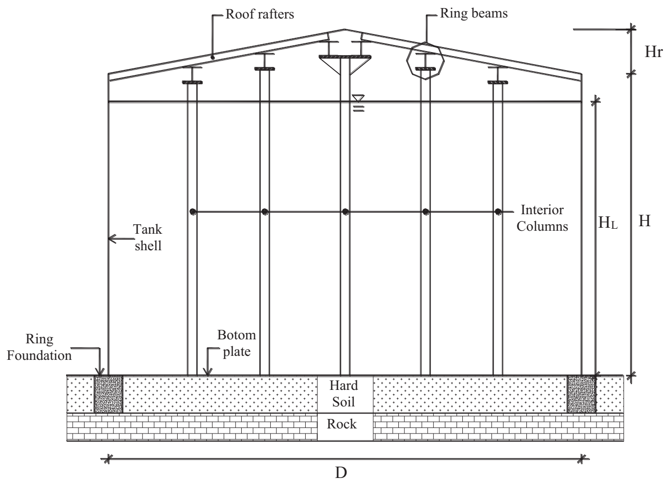

Fig. 1. : Typical structural configuration of a tank with conical roof.

Fig. 3. : Group of tanks in CAPECO tank farm in Bayamon, Puerto Rico. Each tank is located in a dike.

this review to inform about the geometric relations considered in each case.

In their most usual configuration, tanks are clamped at the base by means of various devices: anchor bolt chair attachments are discussed by Rondon and Guzeij [143]. Tanks may be opened at the top, but in many cases they have a fixed roof with a conical or a dome-type shape. Very large tanks, with 40 m < D < 80 m are usually fabricated without a fixed roof but have a wind girder at the top in order to reduce the flexibility of the shell. Small tanks (D < 20 m) are constructed with shallow or deep dome roof or with a conical roof, and in most cases the roof is self-supported. For intermediate cases, say 20 m < D < 40 m, the preference is on conical or flat roof, but this alternative requires the use of an additional structure because the roof cannot support its own weight. The roof support system typically consists of a set of radial rafters, ring rafters, and columns, evenly distributed in the conical roof, as illustrated in Figs. 1 and 2. In a typical design there may be over 100 radial rafters supported by three ring rafters, which in turn are supported by columns. Supported cone roofs are considered in API 650, Section 5.10.4 [7]; the document states that, in case of overpressure or support settlement, the column should be allowed to have vertical displacements relative to the bottom without restraint, and should not have lateral displacements.

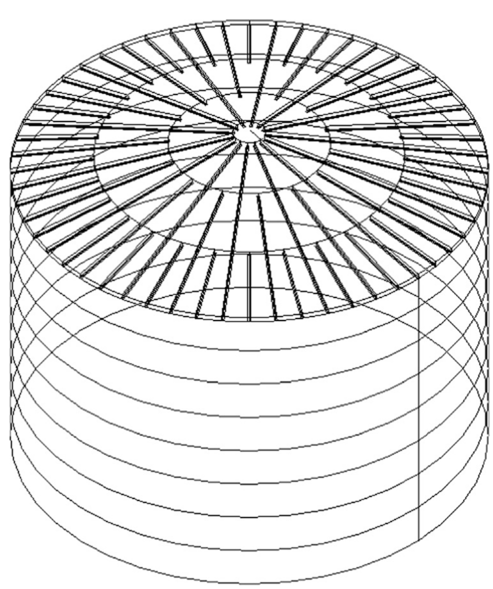

Fig. 2. : Typical model of a tank with rafters and showing shell courses.

It has been pointed out [170] that the actual characteristics of the roof play an important role in the buckling process, changing both the critical load and the associated buckling mode, because the supporting system stiffens the structure and provides further constraints to displacements. This stiffening is of great importance in problems in which displacement constraints play a significant role, such as in tanks under fire and under settlement of the foundation. Thus, it seems that realistic representations of the roof need to be made in an analysis in order to avoid errors due to oversimplified models [27].

Most research on buckling of tanks considers isolated tanks in a flat terrain. But this is not a common situation in practice because other factors need to be taken into account, such as group or topographic effects. Aerial views of refineries and oil and fuel depots indicate the existence of a large number of tanks in one facility: these groups of tanks are known as tank farms. For example, the aerial view in Fig. 3 shows a small tank farm with some 40 tanks of different sizes and characteristics. In most plants of this kind, growth has occurred over decades of expansion, so that new tanks are located in the available space, leading to irregular patterns of dissimilar tanks. The different characteristics of tanks also respond to different needs when storing fuel products: some fuels need to be stored for longer times than others, depending on the local demand, so that it becomes convenient to have several storage possibilities in terms of volume capacity. Group effects normally do not affect buckling under uniform pressure, but may have a significant influence on the buckling under wind and under temperature caused by fire. Thus, in those cases there is a need to investigate not just the behavior of individual tanks, but also the response of tanks in groups.

Other factors may also affect the response of the structure. Tanks and tank farms are frequently located in coastal areas to facilitate transfer from/to ships, or because of the proximity to offshore oil production. This may cause exposures to wind which are more severe than those occurring inland. Further, topographic effects may be relevant: a tank is usually placed inside a dike to avoid oil spilling in case of an accident; dikes of this kind affect wind and may cause different flow patterns that need to be taken into account. Other topographic factors may be the location of a tank in a hill, or on top of an elevated terrain.

In the cases covered under this review, buckling occurs under increasing lateral pressures or temperature; problems in which buckling is due to axial loads have been studied for example in Refs. [131,132] to account for self-weight of the roof and snow. However, tank roofs are frequently fabricated with a supporting structure so that the loads are not transferred to the cylindrical shell. Axial loads have also been considered in Ref. [77].

3. Framework for understanding stability problems

Buckling is here considered as a nonlinear phenomenon, in which the structure cannot take further load with the same geometry and changes its shape in order to find alternative equilibrium configurations [56]. Engineers design structures with a well-defined shape thoughtfully chosen to fulfill some purpose, but under certain critical conditions the structure cannot withstand further load with the same geometry and changes its shape in a slow or sometimes in a violent way.

The view of buckling as the development of a new form is relatively recent and has emerged as a consequence of the work of W. T. Koiter [101], who developed both a theoretical framework for the understanding and for the classification of the mechanics of buckling, and also a tool to carry out the computations. Koiter's theory was the first to provide explanations to the anomalies between classical buckling theory (based on eigenvalue analysis) and laboratory tests, especially for cases of cylinders under axial loads and spherical shells under uniform pressure. The key was to be found in the post-buckling behavior of shells (addressed by Koiter by means of perturbation expansions leading to an asymptotic post-buckling theory) and the role of geometric imperfections in eroding the buckling capacity of shells with respect to geometrically "perfect" models. For problems with a single buckling mode, Koiter classified the behavior into bifurcations (either stable symmetric, unstable symmetric, or asymmetric) and limit points; for interacting modes the scenario became more complex and has been the subject of much research. According to Bushnell, "the purposes of Koiter's theory are to (1) Determine the stability of equilibrium at the lowest bifurcation point on the equilibrium path, (2) Ascertain the sensitivity of the maximum load-carrying capacity of the structure to initial geometric imperfections ([25], pp. 1193). There have been many reviews of the General Theory of Elastic Stability, and our own account is given in Ref. [56]. How Koiter's theory is employed in a computational environment has been discussed in Refs. [8,61] and several others.

The predictive side of Koiter's approach relies on perturbation analysis; its main limitation is the quality of the predictions that can be obtained from this technique: for shells, the results are only accurate up to displacement amplitudes less than the wall-thickness. Thus, a perturbation analysis can only carry out computations close to the critical state, and as such it gives information limited to the initial post-critical behavior of the shell. But while Koiter's approach has almost no competitors at present for understanding the buckling process, only in a few cases a tool has been implemented to perform the computations.

Efforts to implement Koiter's theory in the context of finite element analysis have been carried out at various academic centers, notably by Arbocz in Delft [7], and Casciaro and co-workers at the University of Calabria in Italy [31]. Flores and Godoy [50,51] developed a semi-analytical finite element formulation based on the General Theory of Elastic Stability for systems with multiple degrees-of-freedom [163].

Koiter's success has been highest regarding its power to explain the phenomena of shell buckling. Here the main point was the discovery that small imperfections (either in the geometry or in the loads) destroy a bifurcation behavior leading to a nonlinear fundamental path with a different critical state. For unstable post-critical behavior this means that there is a reduction in the critical load, and this reduction may be significant in shell problems. A review of imperfection-sensitivity studies in structural stability was published by Calladine [29].

Deviations from the "perfect" or "as-designed" geometry of tanks may arise for several reasons, including constructional defects due to welded seams between shell courses, errors in the fabrication of the geometry (local deviations or ovalization of the circumference at different levels), and damage produced during the service life of the structure, such as dents [141] caused by impact with objects or with equipment operating in the facility, geometric deviations due to buckling under wind, and others [55].

The buckling and post-buckling analysis of tanks requires the use of advanced software with geometrical nonlinear capabilities: such analyses have been made in most cases using general purpose programs like ABAQUS [1] and ANSYS [2], but special purpose programs have also been employed in many investigations. A powerful semi-analytical finite element for shells of revolution considering post-buckling analysis and the influence of geometric imperfections was reported by Hong and Teng [85].

Following the notation adopted by the European Design Recommendations on Buckling of Steel Shells [147,153], the alternatives for buckling analyses are Linear Bifurcation Analysis (LBA), Geometrically Nonlinear Elastic Analysis (GNA), Materially Nonlinear Analysis (MNA), Geometrically and Materially Nonlinear Analysis (GMNA), Geometrically Nonlinear Elastic Analysis with Imperfections (GNIA), and Geometrically and Materially Nonlinear Elastic Analysis with Imperfections (GMNIA). This notation will be employed in this review to identify the type of analysis carried out in each case.

The simplest form of buckling analysis is designated as LBA, in which bifurcation instability is assumed to occur and the bifurcation state is identified by means of the solution of an eigenvalue problem. The eigenvalues represent the critical states, and the lower one is the classical critical load. The eigenmodes are the deflected shapes at the associated eigenvalue, and they are crucial to identify the shape of the structure at buckling. The eigenmode associated with the lowest eigenvalue provides the most likely critical shape of the geometric imperfection. The critical loads obtained via LBA are an upper bound to buckling loads, but no indication is obtained regarding how far they are from more realistic estimates. Further, there is no indication of the post-buckling behavior of the shell. Most authors start their investigations by using LBA, in order to have an initial approximation to the solution.

For many problems, such as shells under lateral pressure, there are only minor differences in critical load between LBA and GNA. For thin-walled tanks, in which elastic buckling occurs well below load levels required for plasticity, GNIA and GMNIA lead to virtually the same results.

A less evident aspect of imperfection sensitivity is the shape of the imperfection to be considered in GNIA. In buckling problems, not any shape of imperfection has a full participation in triggering buckling modes: For a given mode of buckling, an imperfection only participates through its component in the geometry of such mode.

Following Koiter's asymptotic formulation, in the vicinity of a critical state the most detrimental shape of imperfection is the eigenmode associated with the lowest eigenvalue; this is called the "eigenmode-affine imperfection". Greiner and Derler [74] concluded that short cylindrical shells, such as those typically employed in the oil industry, are most sensitive to eigenmode-affine imperfections, so that local shapes and ovalization do not constitute the most critical imperfection shapes. Expected reductions due to imperfection sensitivity (known as Knock-Down Factor, η) under pressure and under wind were found to be around $\eta = 0.7$. Weld-induced imperfections, on the other hand, have been studied in Refs. [84,88,133], and [148], being a localized form of geometric deviation. Guggenberger [78] performed GNIA analysis using ABAQUS to explain the effect of a deep single longitudinal dent. Related studies on dent imperfections were reported by Rathinam and Prabu under uniform pressure [141]. Because tanks subject to lateral pressures tend to buckle in modes which involve displacements on a large zone of the shell, then very local imperfections are not the most critical ones because their component in the mode required to trigger an overall mode is small [186]. Circumferential weld imperfections have been addressed in Ref. [175]. Finally, non-homogeneous random fields have been used in simulating imperfections in tanks by Gorski and Mikulski [70] in order to define an envelope of imperfections.

Much research has been done using other computational strategies not embraced by the European recommendation: one strategy is the lower bound theory based on Reduced Energy (or Reduced Stiffness) approach, developed by Croll and collaborators [39]. In this approach it is important to identify the energy components of the shell in the classical eigenmodes, including membrane and bending components as well as load potentials. Depending on the shell and load system, some of the contributions to the second variation of the total potential energy are positive and others are negative, which means that they are stabilizing or destabilizing components. The main hypothesis is that stabilizing components may be lost in the shell in the presence of imperfections. Thus, the approach uses a simplified energy version in which some stabilizing components are eliminated from the initial post-critical condition and a modified eigenproblem is solved. Application to shells under lateral load [12,176] have been validated with experiments. The advantage of this approach is that only a modified bifurcation problem needs to be solved, instead of performing several GNIA studies. A recent interpretation of the Reduced Energy Approach as a penalty method has been proposed [60].

4. Buckling of tanks under uniform pressure

The buckling of tanks under uniform external pressure is usually caused by operational problems during the discharge of the liquid contents in such a way that partial vacuum is produced. These events are usually classified as accidents, and occur in individual tanks in a tank farm, rather than affecting many tanks in the same event, such as those produced by a natural disaster. A short review of cylinder buckling under pressure is given in Ref. [72].

Reports of buckling of tanks under internal vacuum seldom reach the open literature but can be commonly observed during visits to tanks farms. Examples of this kind of failure are shown in Fig. 4: This form of collapse tends to be catastrophic and frequently leads to the destruction of the tank.

Because of symmetry in geometry and load, the loss of symmetry due to bifurcation leads to a buckling mode having a regular wavy pattern with the same maximum amplitude around the circumference. There are cases in which secondary components (such as ladders or openings) are present and the buckling mode is modified by this localized constraint; this is shown in the photographs of Fig. 4.

Fig. 4. : Examples of buckling of liquid storage tanks under uniform pressure. This form of buckling tends to be catastrophic. (a) Penuelas, Puerto Rico, (b) Penuelas, Puerto Rico. (Photographs by the author).

4.1. Experiments conducted to evaluate shell buckling

Early experimental work in this field was reported in Ref. [115] for small scale models under pressure. An unusual situation in this engineering problem is that full-scale experiments of various tanks (some of them with stepwise variable thickness) have been reported in the literature by Hornung and Saal [86]. The tests were performed on four cylindrical tanks with fixed shallow dome roof, with diameters ranging between $D=10$ m and $D=70$ m, by producing internal vacuum. The tests were continued until collapse; however, the authors also identified the load levels at which the first signs of buckling were observed and also estimated the number of circumferential waves in the deflected mode at failure. Information of the measured imperfections is also available for some of the tested tanks. This is the most valuable empirical evidence in this problem and constitutes a benchmark for computational work addressing the buckling and collapse of tanks under uniform external pressure.

Experiments on small scale models of tanks with a roof under uniform pressure were recently reported in Refs. [48] and [123]. Both tank and roof had the same uniform thickness, and the roof was self-supported. Weld-induced imperfections were taken into account. Because no structural support was provided to the roof (as in real cases) buckling occurred on the roof and deflections in the cylinder were a consequence of a roof failure.

Fakhim et al. [46] carried out tests on cylinders ($H/D=1$, $R/t=500$ and 600) with a conical roof (roof slope 13.5°) under uniform pressure to evaluate buckling and post-buckling behavior. Their special interest was investigating thickness changes and effect of imperfections on small scale models; however, both effects are difficult to relate to full scale structures. Initially the shells had outwards displacements, whereas large inward displacements occurred at buckling. Buckling modes had large displacements in the top region, where the thickness is small, with non-symmetric deflections in the circumferential direction.

4.2. Computational buckling analysis

The direct application of Koiter's theory to model the critical and post-critical behavior of tanks under uniform pressure was implemented by Jorgensen in the 1980 s [97] using a special purpose finite difference program. Results in this study refer to a shell with stepwise variable thickness, with $H/D=0.43$ and $R/t=2{,}333$. An unspecified roof is modeled as a boundary condition at the top.

The results confirmed that a linear fundamental path is an adequate representation for this problem, even if the effect of the fluid stored in the tank is taken into account. The fluid has a stabilizing effect on the critical load, and a relation is found between imperfection sensitivity and fluid level: by increasing the level of fluid, sensitivity to imperfections also increases, until a maximum is reached (at fluid level approximately $h = 0.9\,H$), after which, sensitivity decreases. Severe imperfection sensitivity is predicted in this case; this, however, may be unrealistic on account that asymptotic theory employed is restricted to the neighborhood of the critical state and is not accurate for imperfections of the order of the thickness of the shell. It was found that consideration of self-weight or snow does not modify the results by more than 1.5%.

Early finite element modeling of buckling of silos and tanks under pressure and wind was reported by Ansourian [4] with reference to case studies in which the author was involved.

Hornung and Saal [86] modeled their tests on real tanks by means of finite elements using a GNIA approach. They showed that adoption of eigenvalue-affine geometric imperfections does not necessarily represent the buckling process which occurs in a real situation. The authors were able to obtain better agreement with experiments by adopting the geometry of real imperfections measured in the tanks. Although this work makes a severe warning to the use of GNIA approach with the imperfection in the form of the critical eigenmode when modeling a specific shell with specific very large imperfections, it is not clear how that information can be used with advantage in more general cases for which measurements are not possible. Using a special purpose program, Jaca et al. [91] modeled those experimental results using a Reduced Stiffness approach, in which details of imperfections are not represented.

The influence of welding-induced deviations in the geometry was investigated by Hubner et al. [88] for steel cylinders with welded patterns. In this case, a two-stage procedure was employed: first, the welding process was simulated by means of an imposed shrinkage strain to obtain weld depressions and residual stresses in the shell; whereas geometrically and material nonlinear analysis was performed as a second stage to investigate buckling of the shell, with bifurcation checks along the equilibrium path. A case study ($H/D = 0.62$, $R/t = 1000$) was conducted on a shell panel supported at both ends under axial compression, axial compression and external pressure, axial compression and internal pressure, and global shear. Under external pressure, the study concluded that residual stresses mainly have a beneficial effect and can be neglected in design, and meridional deviations caused the largest reduction in buckling capacity of the panel.

Paor et al. [128] investigated tanks usually employed in the food, pharmaceutical, and biotechnology industries, in which case vacuum may occur by condensation of steam in the tank, leading to a sudden pressure loading. Their work included testing five small-scale commercially available steel cans under uniform pressure in which geometric imperfections induced by manufacturing were measured. Test results of buckling and collapse were compared with nonlinear GNIA finite element analysis, and the authors were able to model mode jumping along the nonlinear equilibrium path.

Effects of two different thicknesses in a cylinder on the buckling and post-buckling has been studied by Aghajari et al. [2], by means of experiments and finite element modeling with ANSYS. Four specimens were tested up to collapse, having $H/D = 0.5$ and 0.25, $R/t = 500$ and 750, and thickness reduction in the second course of 25% and 50%. Numerical results show significant differences with experiments in the post-buckling path. The authors recommend using shells with less thickness changes.

In all previous studies, external pressure has been considered producing compression in most of the tank. However, there are also studies of bifurcation buckling of tanks with a fixed roof under internal pressure by Yoshida [180,181], in which pressure is associated with an internal explosion due to ignition of gasses inside the tank. Buckling modes under such loads may affect the junction between cylinder and roof, or the junction between the cylinder and the bottom plate.

4.3. Design oriented work

Lower bounds to obtain buckling loads under uniform pressure based on Reduced Stiffness/Reduced Energy approaches were developed since the 1980s (see, for example, Refs. [12,176,39]). The application of Reduced Energy to tanks under uniform pressure was reported in Refs. [159,157], whereas Reduced Stiffness methodologies were discussed in Ref. [91]. In both strategies, reliable lower bounds have been achieved for tanks under pressure by means of elimination of the meridional membrane components. Opened and closed tanks with stepped thickness were considered in Ref. [91], with excellent agreement with respect to GNIA. Two strategies were considered: a homogeneous reduction affecting the complete shell, and a selective reduction affecting just the upper courses where the thickness is a minimum. Differences between both strategies are not to be found in eigenvalues but in eigenvectors.

Chen et al. [34,35] considered a simplified, design-oriented, method to account for step-wise variable thickness in cylindrical shells in the buckling under uniform pressure. The methodology, known as "Weighted Smeared Wall" method, computes an effective thickness associated with the buckling mode of the problem. Depending on the wall thickness distribution, the authors identified cases in which buckling affects the thinner top courses of the shell, the complete wall, or mixed cases. Finally, analytical approximations to elasto-plastic instability have been proposed by Yang et al. [177].

5. Buckling of tanks under wind loads

Wind buckling of aboveground tanks in the oil industry is of great concern in many parts of the world, mainly those located in areas where hurricanes or typhoons may occur. The list of examples during the last 25 years in the Caribbean region and the east coast of the United States (at least those which are more familiar to the author) includes damage to tanks in the US Virgin Island of St. Croix due to Hurricane Hugo in 1989; in St. Thomas due to Hurricane Marilyn in 1995 [49]; in Puerto Rico due to Hurricane Georges in 1998; and in the Gulf Coast of the US due to Hurricanes Katrina and Rita in 2005 [57,124], among others. A more complete list could be compiled if effects due to typhoons in the Pacific region in Asia were included. Fig. 5 shows an example of tanks that buckled under strong hurricane winds.

There are also examples of tanks that failed under regional winds; notable examples have been observed in tanks under construction, in which a tank which had not achieved its full structural integrity failed under low intensity winds. Examples of such failures have been reported in Argentina [90], United States [18], Brazil and Iran, showing the need to assure the stability of the shell at each step during construction and not just in their final configuration.

The buckling of cylinders under wind was reviewed in Ref. [76], with emphasis on tall cylinders rather than short tanks.

5.1. Methodologies for the evaluation of pressure coefficients

Wind pressures on aboveground steel storage tanks may be

Fig. 5. : Example of buckling of tanks during Hurricane Hugo in the US Virgin island of St. Croix, 1989.

evaluated by means of boundary layer wind tunnel (BLWT) simulations, or by Computational Fluid Dynamics (CFD). Both methodologies have experimented considerable improvements over the past decade, so that advances in computing power, electronic instrumentation and computer-based rapid prototyping have improved the efficiency of BLWT simulations, whereas CFD models have benefited from computing power and improved software. There are still several advantages in using BLWT over CFD simulations, especially whenever the flow is dominated by turbulence, as in the present case. On the other hand, BLWT facilities are not available in most academic laboratories (even less so in developing countries) and those institutions that have the facilities tend to be overloaded.

Wind pressures are characterized by a stagnation pressure on the windward meridian, with a region of positive pressures covering some 90°. At windward meridian, pressures are higher at the top of the cylinder (0.6 to 0.9 H) than at the base due to a non-uniform wind profile. Negative pressure follow, with a maximum located at approximately 90° from windward, and smaller negative pressures on the leeward zone. Flow separation occurs at about 130° from windward, and pressures remain constant at leeward. Positive pressures induce hoop compressions in the windward region which are responsible for buckling of the shell. It is now customary to provide contour maps of pressure coefficients Cp as an outcome of wind tunnel or CFD studies. Cp values depend on the ratio H/D; the characteristics of the roof (open, wind girder, cone roof, dome roof); group effects (the location of the tank with respect to other tanks or buildings); topographic factors (location in a hill, inside a dike); and terrain conditions. Other factors affecting the flow need to be considered, such as Reynolds number.

Some recent studies employ Computational Fluid Dynamic (CFD) tools to assess the pressure coefficients in tanks under wind. This is in part due to the current availability of CFD general purpose programs, and also to the rising cost of performing wind tunnel experiments and lack of accessible facilities. One of the advantages of CFD with respect to wind tunnel tests is that it is possible to visualize the flow and thus better understand wake effects and flow separation detaching from a surface.

In CFD the planetary boundary layer near ground is simulated under the assumptions of stationary mean flow, viscous, incompressible, isothermal, and turbulent flow. To solve the problem, the system of equations includes continuity conditions and Navier-Stokes. In the classical approach, averaged equations according to Reynolds (known as RANS, Reynolds-Averaged Navier-Stokes) are solved, in which each component of the velocity vector is modeled as a mean plus a fluctuating value, together with Boussinesq hypothesis concerning proportionality between stresses and strain rates. A model of turbulence needs to be chosen, such as the k-ε or Large Eddy Simulation models. Instead of Finite Elements, Finite Volume Methods are employed in CFD, and the resulting equations are solved in an Eulerian system. Software to perform CFD analysis is currently available to the researcher or designer, such as FLUENT [5], COMSOL [38], OPEN FOAM [127], and several others.

5.2. Wind studies for isolated tanks

5.2.1. Wind pressures in isolated tanks

The classical way to investigate wind effects in tanks and cylindrical shells has been through small scale models in a BLWT. Although it is possible to study aero-elastic models in a wind tunnel, in which the fluid and structural problems are coupled, most research has concentrated on the evaluation of pressure coefficients and those are subsequently used in a computational model of the structure.

The complexity in wind tunnel studies arises as a consequence of the need to extend small-scale results to full-scale problems. This requires achieving supercritical Reynolds numbers Re, for which pressures are no longer dependent on Re. It was found that little change is observed for $Re > 1 \times 10^5$ [110]. Other factors to be considered are turbulence intensity profile and surface roughness (in both, model surface and wind tunnel floor). Turbulence intensity is frequently in the order of 15% measured at the top of the cylinder [150,110]. Vorticity generators are used to build the velocity profile which is expected to occur in an atmospheric boundary layer. Unfortunately, limited results are available on full scale measurements for tanks [139] so that meaningful comparisons are extremely difficult to achieve.

There are also limitations in extrapolating results from silos to tanks, not so much on the cylinder but mainly on the roof. In cylinders with a conical roof the slope is typically large for silos (about 35°), whereas smaller values are used in tanks (10° to 15°). Roof damage is more frequently observed in tanks than silos due to such slope differences.

In the 1960s, Maher [113] tested domes supported by cylinders or inverted cones as sub-structures, aiming to provide data for preliminary design. The geometries tested were representative of aeronautical interests. This effort at Virginia Tech in the USA was continued by Purdy et al. [140] who made experiments on cylinders with flat roof, more closely related to tanks with $H/D > 1$. The authors concluded that the pressure distributions heavily depend on the aspect ratio H/D.

In 1975, Prabhu et al. [138] tested shells with $0.12 < H/D < 1.25$ and $250 < R/t < 500$. Interest in the work of Megson et al. 1987 [120] focused on shells with $0.16 < H/D < 1.2$ and $1030 < R/t < 2960$.

In the 1980 s and 1990 s, Resinger and Greiner [142], Johns [94], and Greiner and Derler [74], in Europe, and Uematsu and Uchiyama in Japan [165], reported test on long cylinders, with interest in the range of application to silos rather than to storage tanks.

In a series of three papers, Holroyd studied wind pressure loads on an open-topped oil storage tank without a wind girder. First, the quasi-static (mean) pressure was evaluated from wind tunnel tests [79] for a model with $H/D = 0.2$ in scale 1/250. Turbulence intensity was 14%, $v = 6.15$ m/s, and the tests were performed at $Re = 1.65 \times 10^5$; the paper describes in detail the flow and vortex generated by the flow around the tanks model. For the same geometry, a flat roof was added to obtain pressures. Next, the author studied the structural response under the quasi-static pressure, with special interest in displacements and vibrations of the shell [80]. Finally, the third part of the study dealt with a qualitative strategy to identify unstable vibrations of the shell; however, the author could only delineate a methodology without implementing it [81].

Several authors in Australia measured wind pressures in silos, including silos with conical roof by Sabransky and Melbourne [150]; and open top, flat roof, and conical roof by Macdonald et al. [110,111,112]. Sabransky and Melbourne considered conical roof structures with 27° slope (typical of silos), and compared pressures with 15° and 45° roof slopes. They also compared their wind tunnel tests with independent full scale measurements under natural wind, with good agreement in positive pressures and differences of 20% in mean negative pressures at 90° from windward. They concluded that this is a three-dimensional problem, and that two-dimensional models would not be acceptable.

Geometries tested by Macdonalds et al. [110] covered H/D=0.5, 1.0, and 2.0, with $Re=1.3\times10^5$ being measured in the tests performed in an aeronautical wind tunnel. An increase in aspect ratio caused an increase in negative pressures at 90° from windward: for example, pressure -1.0 was measured for H/D=0.5, whereas for H/D=2.0 a value -1.8 was reached. High negative pressures were measured at the roof apex and at the junction with the cylinder, for a roof with 25° slope (typical of silo structures rather than tanks). Satisfactory comparisons were achieved with full scale tests.

Pasley and Clark [129] performed CFD modeling of wind flow on opened-top tanks with an internal floating roof. The study aimed to evaluate volatile organic compounds emitted from oil-storage facilities, for which they concentrated on the flow past the tank, rather than on pressures acting on the tank.

Kebeli [100] and Kebeli et al. [99] reported wind tunnel and full-scale testing of silos with deep conical roof, in order to evaluate contour pressures in the cylinder and roof of the shell. Several parameters were considered in the wind tunnel tests, including the roof slope (25°, 30°, 35°, and 40°), aspect ratio of the cylinder (H/D=1.0, 1.5, and 2.5), and type of roof surface (smooth or rough surface). Cp mean values were found to be higher for smooth surface. An increase in roof slope induced higher negative Cp values. A novelty of the study was that a full-scale silo was fabricated, instrumented and tested under natural wind flow; this is perhaps the only case in the literature for which full scale results are available. The dimensions of the silo were D=H=3.5 m, with 30° roof slope. There were significant differences between full scale and wind tunnel results, mainly attributed to differences in Reynolds number, which was five times higher in the wind tunnel. Turbulence intensity of 20% was present in wind tunnel, whereas it was 40% in natural wind.

Wind tunnel tests performed in a BLWT were reported by Portela and Godoy [134], for tanks with conical roof (H/D=0.43, roof slope of 11°) and tanks having a shallow dome roof with H/D=0.48 with Hr/H=0.318 and deep dome roof with Hr/H=0.475 [135], where Hr is the height of the roof measured from the junction with the cylinder. Results were presented as contour maps of pressure coefficients on the cylindrical part and on the roof. The details of the geometric transition between the cylinder and the dome roof are of great importance because this transition changes the flow separation, with the consequence that shallow conical roofs yield higher pressure levels than smooth dome roofs. The slope in a conical roof is responsible for an increase in pressures at the center of the roof; this effect should perhaps be considered while designing the roof.

Lin and Zhao [106] tested fixed roof tanks representative of very large containers, which are short (H/D=0.275), considering flat roof and dome roof. The authors found that roof configuration had little influence on pressures on the cylinder, but pressures on the roof are considerably influenced by details of the junction between cylinder and roof. The structure of the pressure field was investigated by POD, similar to what was done in [167].

For isolated tanks, pressure coefficients calculated with CFD have been compared with those obtained by wind tunnel tests, with good agreement found between them [47].

To understand the significance of differences in pressure patterns obtained from wind tunnel experiments performed at different institutions, two of them have been compared in Ref. [64], considering pressures on a short tank with a conical roof of Ref. [110] with those of Ref. [134]. Wind tunnels have variations in equipment, data collection systems, type of flow reached during the studies, and scale of the model; with the consequence that it is not expected to find exact duplication of results in two experiments carried out at different laboratories. Furthermore, such pressures are not expected to occur exactly in any real situation, so that they are indicators of patterns of pressure for design. The two patterns are taken as variations in load in order to assess the influence of pressures on the buckling loads and post-buckling paths, for tanks with a roof. The critical loads reached in both cases have differences of the order of only 6%; this is considered a small sensitivity, on account of the significant differences between the two wind-tunnel load systems.

For open top tanks, design wind loads are commonly specified as the product of the mean wind force coefficient and a gust effect factor, in which the wind force coefficient is the difference between mean external and internal pressure coefficients. Considerable effort has been made at Tohoku University in Japan to evaluate pressures in tanks which are opened at the top via wind tunnel tests. Pressure measurements in isolated opened top tanks were reported by Yasunaga et al. [178], followed by buckling studies on flexible models [179]: the authors found that the shell response to buckling was dominated by the positive pressure in the windward region. Design pressure proposals may be found in Uematsu et al. [166], including internal and external pressure coefficients for opened-top tanks, and proposed pressure coefficients to be used in design. One of their recommendations is that one could use the same pressure patterns on the cylinder for both open and closed cylinders.

The influence of various wind pressure assumptions in elevation on buckling and post-buckling has been studied for open topped [166,167]. There are small changes in buckling behavior associated with changes in vertical wind pressures.

5.2.2. *Buckling of isolated tanks*

Early work on tanks employed uniform pressure distributions which were assumed to be equivalent to wind effects. However, wind induces a non-axisymmetric distribution of pressures and their effects do not approximate those due to uniform pressure. Greiner discussed the state of the art for silos under wind and the ideas behind using an equivalent uniform pressure instead of a more detailed non-axisymmetric pressure variation due to wind [73].

Analytical studies of cylindrical shells under wind loads concentrated on tanks that are open at the top, which are simpler to model because there is no need to represent structural details of the roof, and also because wind pressure coefficients on the roof were lacking at that time. Research in this field started in the early 1960 s with evaluation of critical loads via LBA for simple open cylinders with uniform thickness [102,154,104,69] or with corrugated walls [93]. Wang and Billington in 1974 [174] reported an analytical solution to solve the LBA problem in open cantilever cylinders. LBA studies to evaluate critical wind pressures in vertical cylinders were presented in Ref. [172] for simply supported conditions at both ends by using Galerkin method; uniform vertical pressure distribution was assumed. This is an extension of similar analysis for uniform external pressure [171].

Schmidt et al. [154] investigated bifurcation and post-critical behavior of open cantilever cylinders stiffened at the top with wind girders via computational models, which were validated by means of small-scale experiments on patch, non-axisymmetric pressures expected to be similar to wind pressures. The analyses were oriented to provide design recommendations which were subsequently reflected in Ref. [147]. LBA and non-linear analyses defined in the European Recommendations were investigated by Chen and Rotter [33] for stocky, intermediate, and slender cylinders under wind. One of the conclusions was that the relation between stagnation pressure at buckling under wind and uniform pressures are only valid using Donnell's shell theory, but do not hold under a more general theory.

Numerical studies were conducted by Saal and Shrufer [149] at the beginning of the 1990 s. Buckling and post-buckling analyses of cylinders using a finite element formulation based on Koiter's asymptotic theory, were reported in Ref. [49] for a tank with $R/t=100$ and $H/D=1$.

A tank without roof that buckled during hurricane Marilyn in 1995, with $H/D=0.2$ and $R/t=1900$, was taken as a case study in Ref. [52]. Analyses included LBA, GNLA, GNIA, and nonlinear dynamic analysis, to show that buckling occurs in the form of a bifurcation with small displacements at the critical state. Post-critical states were identified as unstable, leading to decreasing post-critical equilibrium path in a load-displacement plot. The maximum load that the shell attains is a function of the assumed geometric imperfection, with values of some 70% of the critical bifurcation load for eigenmode-affine imperfections of the order of the thickness of the shell. The presence of the liquid inside the tank has a stabilizing effect, but the increase in the bifurcation load for a liquid level of H/2 is of only 14%.

Rather than looking at the buckling behavior, as in most other studies, Holroyd investigated the vibration response; the hypothesis is that turbulence-induced vibrations are responsible for triggering the collapse of tanks under high winds [80]. However, the numerical study of Ref. [52] concluded that static and dynamic analyses provided virtually the same results, so that this may be considered as a quasi-static problem. This was subsequently confirmed in Refs. [53,156].

Imperfection-sensitivity was further investigated in Ref. [59] for empty tanks under wind using GNIA models considering four geometries of open cantilever tanks, covering geometries with $0.16<H/D<1$ and $1250<R/t<2000$. The imperfection sensitivity of this class of tanks under wind is similar to what is obtained in cylinders under uniform lateral pressure, with maximum load at the critical state between 60% and 90% of that computed for the perfect shell. Regions of stable behavior based on imperfection amplitude equal to twice the thickness, $\xi=2t$, were established for a range of tank geometries: very short tanks, with $H/D=0.17$ display high imperfection sensitivity (with 40% drop in maximum load), whereas taller tanks with $H/D=1$ are almost insensitive to imperfection (only 5% drop in maximum load).

Portela and Godoy investigated buckling, post-buckling, and imperfection-sensitivity (LBA and GNIA analyses) using ABAQUS, for tanks with conical roof supported by rafters and girders [134], and with self-supported dome roof [135], using the pressure coefficients from wind tunnel tests obtained as part of the same research. The tank with conical roof had $H/D=0.43$, $R/t=1929$, and roof slope of 10.6°, which were the slenderness and aspect ratio tested in wind tunnel. Details of the rafters and girder were included as part of the computational model. A simplified uniform pressure was also considered in the vertical direction and differences in buckling loads with those obtained from the more refined C_p distributions recorded from wind tunnel tests were of less than 2%.

For an eigenmode-affine imperfection with amplitude $\xi<t$, the shell displays a maximum followed by an unstable post-critical behavior and the path regains stability at large displacements; whereas for $\xi>t$, the maximum in the equilibrium path and the minimum in the post-buckling path approach each other and coalesce in the limit, so that an inflection point is obtained rather than a maximum. As an example, knock-down factors of $\eta=0.77$ were obtained for $\xi=0.5$ t. It was found that, independently of the amplitude of the initial imperfection, all equilibrium paths approach the path for the perfect shell at large displacements.

The buckling behavior of the conical roof tank was compared with a tank of similar geometry but without a fixed roof in Ref. [134]. Critical values computed with LBA and GNIA are approximately twice those in the tank which is open at the top. Because of the large flexibility of open tanks, they are usually reinforced with a wind girder. On the other hand, it is the conical roof tank that is more sensitive to imperfections. Effects due to thickness reductions (possibly due to corrosion) were also performed. A similar study was carried out for tanks with shallow and deep dome roof [135], and higher buckling capacity was found in comparison with conical roof tanks.

Zhao et al. [184] considered large silos with a conical roof under wind, performing LBA, GNA, GMNA, GNIA, and GMNIA using ANSYS [5]. Differences between silos and tanks concern the conical roof, which is self-supported in silos and requires a supporting structure in tanks; however, the results of silos are often illustrative of the behavior of tanks. The range of roof geometries investigated covered $0.5<H/D<1.1$ and $500<R/t<1000$, with tapered walls of the cylinder. Buckling loads are reduced with a decrease in H/D. Buckling was identified to occur in the elastic range.

For open tanks, Ref. [186] addressed six geometries representative of tanks in the oil industry, and discussed the differences of their behavior under wind. The geometric ranges investigated are tanks with $12.9<H/D<21.8$ (with $14.5\ m<D<80\ m$) and $1833<R/t<3375$. Zhao and Lin found that GNA provides results which are very close to LBA, with differences in the order of 1%. Smaller tanks with higher H/D ratios have a higher buckling capacity than larger tanks. For GNIA studies, the authors found that eigenmode-affine imperfections produce significantly larger reduction due to imperfection-sensitivity than weld-induced imperfections. For large diameter tanks, the equilibrium path does not show a maximum for $\xi=2t$, in which case the problem becomes one of limiting displacements rather than buckling. Imperfection-sensitivity is more severe in large tanks with smaller H/D ratios (knock-down factor $\eta=0.55$), but $\eta=0.7$ is computed for the smaller tanks. Liquid stored in the tank increases stability, provided a large liquid volume is stored. Similar conclusions were obtained in Refs. [97, 52].

Adding a wind girder produced an increase in buckling capacity of 25-30%, but for tanks of intermediate capacity the change is even more important, reaching approximately 50% [186].

It is interesting to notice that virtually identical results are obtained by Zhao and Lin by using the complete wind pressures around the circumference or by employing only the positive pressures in the windward region [186]. As stated by Zingoni, the current evidence indicates that "…buckling behavior of the tank is mainly governed by the magnitude and distribution of the positive wind-pressure coefficients on the windward surface of the tank" (pp. 104, [187]). For some time there have been conflicts between wind engineers (who attempt to refine procedures and measurements in wind tunnel tests without paying attention at the subsequent use of that information) and structural engineers (who are mainly interested in what the structure can "feel" of the applied load). This case shows that neglecting the negative pressure due to wind (which would be anathema for a wind engineer) is in fact a perfectly acceptable estimate in terms of buckling behavior.

5.3. Group effects on the wind buckling of tanks

Interest in group effects in tanks under wind reflect the trends to build large tank farms and refineries in which individual tanks may be in a first line of exposure with respect to the prevailing winds, or may be shielded by others if they are located in a second or third row from the perimeter of the facility. On the one hand, it is known that structures that are closely spaced between them may induce an increase in pressure coefficients; on the other hand, depending on the configuration, there may be shielding effects between them thus reducing the pressures with respect to isolated structures. What effect prevails depends on the geometry of the tanks involved and on their separation.

Research on group effects is more complex because of the dimensions of wind tunnel sections required to avoid interference between the flow and the walls of the tunnel. Modern wind tunnels, on the other hand, do not have this limitation.

A scheme of tank arrangements for two and three tanks is shown in Fig. 6, indicating the target tank T_1, in which measurements are taken in the wind tunnel experiment, and one or two blocking tanks identified as Tank(s) 2, for which usually no measurements are made. The variables in this problem are the dimensions H and D of Tank 1 and Tank 2, their separation S_1, the wind direction β, the roof configuration of the tanks in the experiment, and the separation S_2 between Tanks 2 in three-tank arrangements. All experiments assume flat terrain. Such a long list of variables indicates that mainly case-studies can be performed.

The 2008 European Recommendations for shell buckling [147] include only marginal information on group effects on wind buckling of shells. The only reference given on this topic is an early wind tunnel work on a row of silos perpendicular to the wind direction carried out in Australia [169]. Regarding tank farms, this would be equivalent to considering tanks located in the perimeter of a facility.

Early wind tunnel experiments on two identical tall silos under different angles of wind incidence were reported by Esslinger and coworkers in 1971 [44]. Most subsequent wind tunnel studies on

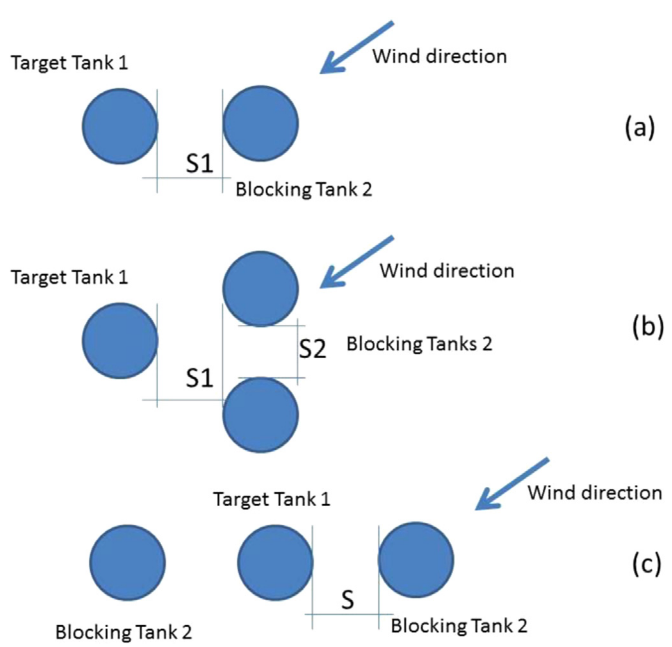

Fig. 6. : Group effects in tank farms investigated to obtain Pressure Coefficients due to wind. (a) Two tanks; (b) Three tanks; (c) Tanks aligned in a row.

group effects in silos and tanks in the 1980 s and 1990 s took place in Australia [169,150,111]. Studies concerning wind pressure distributions in two tall circular cylinders ($H/D > 10$) having spacing $S = 0.325$ D between them, were reported by Zdravkovich in 1980 [182].

The importance of the separation between cylinders was studied by Taniguchi et al. [160], who found that maximum interference between two cylinders of finite length and equal diameter in a turbulent boundary layer occurred for $S = 0.2$ D.

Important wind tunnel testing was conducted in Australia in the 1980-1990; geometries considered were representative of silo structures, in which case the conical roof is considerably steeper than in oil-storage tanks. Such a difference has important consequences on roof pressures, but even more important, they completely change the pressure pattern that is received by other tanks located in a group. A crucial consequence of this is that results obtained for silo geometries should not be directly employed to estimate group effects in tanks.

Sabransky and Melbourne [150] tested identical silos in 1987, (with $H/D = 1.16$ and roof slope 27°) arranged in a line of three silos with very small separation $S = 0.25$ D. Group effects in this case induces an increase in pressures, as the flow passes between two cylinders at $\beta = 90°$. Amplification factors of two were recorded for $\beta = 75°$.

In 1990, Macdonald et al. [111] published a set of experiments of tanks with a flat roof in a line of five tanks, thus having a configuration similar to that studied by Vickery [169]. The tanks in [111] had all the same geometry ($H/D = 1$), with separations between tanks of $S = 0.125$ D, 0.25 D, and 0.5 D. Wind direction of $\beta = 0°$ was adopted, but in the first case $S = 0.125D$, directions of $\beta = 20°$, 45°, and 90°, were also investigated. Measurements were taken at 0.81 H_1 in the target tank (located at the center of the arrangement). For closer tanks (i.e. decreasing S), a small decrease in the maximum positive pressures (windward meridian) was found; whereas increasing S has the consequence of increasing the maximum suction at 90° from windward. This last observation is attributed to a "Venturi" effect.

Group effects in silos were investigated in wind tunnel by Kebeli [100] for three configurations: three silos in a line; six silos arranged in two lines of three each; and six silos arranged in a circle. Separation between silos was $S/D = 0.25$, 0.5, and 1.0; with wind direction $\beta = 0$, 45, and 90°. The roof slope considered was clearly representative of silos rather than tanks, and this has a major effect on pressures in a target structure.

Portela and Godoy [136] reported wind tunnel studies in arrays formed by two and three tanks with fixed roof, in which the blocking tanks are located in the perimeter of a facility and the target tank T_1 is in a second line and is shielded by the others. For the two-tank arrangements, the target tank T_1 had a conical roof with $H_1/D_1 = 0.43$, whereas the blocking tank T_2 (with $H_2/D_2 = 0.43$ and also with $H/D = 0.30$) had a flat roof; separations $S_1 = 1\ D_1$ or 1.5 D_1 were investigated, whereas the angle of wind incidence was $\beta = 0°$. For the three-tank arrangements, $H_1/D_1 = 0.43$, $H_2/D_2 = 0.39$, $S_1 = 1\ D_2$, and 0.5 D_2, $S_2 = 0.5\ D_2$, were considered. In general terms, a flat roof tank T_2 in front of another tank T_1 blocks the flow and can reduce the pressures in some zones of the target tank T_1. However, this effect depends on the relative geometric features and on the separation the between the tanks involved. For a configuration of closely spaced tanks with $S_1 = 0.5\ D_2$, with $D_2 < D_1$ and $H_1 > H_2$, there are differences between the results showing an increase in pressures of the order of 20% and a change in pressure distribution in T_1 with respect to an isolated tank. In this case the effect is due to flow separation from the first tank T_2 which subsequently impacts the target tank T_1. This effect does not occur for tanks with $H_1 = H_2$.

LBA and GNA results for the pressures measured in the tests were also reported in Ref. [136]. One would expect to find buckling loads directly related to the changes in pressure coefficients; however, pressures distributions are also modified due to group effects and this has an effect on the buckling results. In other words, it is not possible to estimate changes in wind-induced buckling based on changes in peak pressures. For two tanks with the same dimensions, located at S=D, the critical load (LBA) is twice that of the single tank case; whereas an increase of 44% was found for a closer separation S=0.5D. In understanding the results, one has to consider that the maximum Cp values change their location along the meridian from an area between 0.5 H and 0.75 H (isolated tank) to an area between 0.8 H to 1.0 H (tank in tandem array). As the peak pressures approach the junction between the cylinder and the roof, their effect is reduced with respect to patterns in which maximum pressures are located at the center in elevation.

Increasing the height H_1 of the target tank, so that H1/D1=0.6 for H2/D2=0.43, the LBA critical loads are of the same order as in the isolated tank. The behavior for three tanks of equal size is similar to what was described for two tanks. It is speculated that "… if a lower obstacle is located in front of the tank studied in tandem configuration, it would develop high external wind pressures at a region far enough from the roof. Under this behavior, the buckling load would be significantly reduced" ([136], pp. 14).

A case of a group of six tanks arranged in an irregular pattern and having different geometries has been presented in Reference [137], for which buckling was reported to occur in Puerto Rico under Hurricane Georges in 1998 in a large tank with a conical roof. Most tanks blocking the target tank are shorter in this case. For one of the configurations, the critical pressure is 4% lower than in the corresponding isolated tank; however, the imperfection sensitivity in the group is less pronounced than in a single tank. Because the eigenvectors in the isolated and group configurations are similar, then it is possible to adopt eigenmode-affine imperfections from the isolated tank to be used in the group configuration. The conclusion of this case-study is that "… an isolated tank yields critical wind speeds that are lower than those computed in more complex group arrays" ([137], pp. 44).

Iamandi et al. [89] reported an accident in a small chemical storage station in Romania involving a four-tank configuration. The tanks had similar aspect ratios with H/D=1.1, and were placed in a diamond configuration with respect to the direction of wind, in which case the target tank was the last one in the configuration and was shielded by the other three. Separation between tanks and roof conditions were not specified. The wind tunnel tests showed asymmetric pressure distributions with overpressure zones on the front of the cylinder that are less developed than in the case of an isolated tank.

Wind tunnel and finite volume simulations on tall cylinders (H/D=2.56) with flat roof in tandem configuration were investigated by Said et al. [151]. It was found that the flow pattern depends on the separation between tanks: for the shortest separation considered, S/D=1.28, the flow accelerates on the roof of the first cylinder and impacts on the top part of the second cylinder. The wake of the first cylinder modifies the pressure field on the target tank and reduces the pressures on the windward region. This effect decreases as the distance increases to S=5.12D, with the consequence that the target tank becomes subjected to a flow pattern that is similar to that in the isolated tank.

A comprehensive study of group effects in tanks which are open at the top was recently published by Zhao et al. [185]. The arrangements investigated include two, three, and four tanks having all the same geometry, with H/D=0.275. The models represent very large open tanks with D=80 m and H=22 m in real scale. Reynolds number $Re=2.16\times10^5$ was obtained. For configurations of two tanks aligned with the wind direction ($\beta=0°$), pressures on external and internal surfaces were measured. There is a significant reduction in pressures in the windward region in T_1 for S=1.5D (24% of the C_p values measured in the isolated tank), and the maximum C_p values increase with S. There are also reductions in negative pressures at 90° from the windward meridian, ranging from 56% to 75% depending on the separation S between tanks. The authors also investigated two tanks perpendicular to the wind direction ($\beta=90°$), and intermediate configurations with changes in angle separation of 10°. For three tank configurations, there is a large incidence of S_1 and a less pronounced influence of S_2 [185]. Four tank arrangements were investigated to represent tanks located at a corner in a tank farm. Those pressures have not yet been employed in subsequent buckling analyses to evaluate changes due to group effects.

For tanks opened or closed at the top, Uematsu et al. [167] performed wind tunnel experiments in configurations of two and three tanks in a line, with different separation between them and several angles of wind incidence. The aspect ratios considered were H/D=1, 0.5, and 0.25 and spacing of $0.125<S/D<1.0$ in a scale 1/400. A turbulent boundary layer was simulated with power law exponent 0.15 and $Re=1.7\times10^5$, with turbulence intensity ranging between 0.18 and 0.14 depending on height. For small spacing S/D=0.125, wind flow perpendicular to the line of tanks seems to be critical in terms of the magnitude and extent of the zone of positive pressures.

The configurations considered at Tohoku [167] are similar to those tested at Zhejiang [185], i.e. two tanks in a line, three tanks in a line (of which the central one and one of the end tanks are of interest), two tanks shielding a third one, and four-tank array, for various angles of wind incidence. The aspect ratios of the tanks considered were H/D=1.0, 0.5, and 0.25. Gaps between tanks are critical in these cases: for $S/D>1$, the pressures approach those obtained for an isolated tank. In this case, the turbulent boundary layer has a power law exponent 0.15. External pressure coefficients were similar to those in an isolated tank, and larger differences were obtained for internal pressures. Buckling was investigated on the small scale geometries tested in wind tunnel. The differences in buckling (GNA) between measured wind pressures and idealized pressure uniform in elevation were found to be of only 7%, with uniform pressures yielding lower buckling states [167]. A recommendation emerging from this study is that one should pay attention to pressure distributions on the last tank in a tandem configuration, and on the center tank when $\beta=0°$, at least for short separations S.

Burgos et al. [26] performed wind tunnel tests on two equal tanks with conical roof to investigate the influence of separation S and angle of wind incidence β on buckling loads. The tanks tested had H/D=0.52 (taller than those of Ref. [134]), with angles $\beta=0$ (tandem configuration), 22.5°, 45°, 67.5°, and 90°, and with separations S =1.0D, 1.5D, and 1.8D. For a tandem array, an increase in pressures was measured in the zone close to the roof. The location of maximum pressures lowers as angle β increases. For the geometries considered, the maximum reductions in LBA occurred for $\beta=0°$ and S=1.0D, in which case the critical load is reduced by 30% of that obtained in a single tank; for S=1.5D, the reduction is 20%; and for S=1.8D there is a 15% reduction. The comparison is the same for LBA or for GNIA, the percentages being similar.

The overall conclusion is that there are significant differences between pressure distributions due to group effects versus isolated tanks; however, in most cases investigated the latter seem to yield an upper bound in C_p values to the former; shielding seems to dominate over other wind turbulent effects. However, depending on the type of roof of the blocking tank, pressures on the target tank may even increase with respect to an isolated tank. For the purpose of design recommendations, current research should perhaps concentrate on isolating those cases in which increase (rather than decrease) in pressures is likely to occur due to group effects.

5.4. Topographic effects

The importance of taking into account the influence of topography to estimate wind pressures has been recognized for some time in the context of civil engineering structures. Current codes of practice, such as the ASCE [9] provisions, include factors to account for the location of a structure with respect to hills and escarpments. For complex situations, the ASCE document recommends the use of engineering judgment, expert advice, or wind tunnel studies. An alternative is to perform a computer simulation via CFD.

Wind pressures in a cylindrical tank (H/D=1) with a conical roof (roof slope 25°) were investigated in Ref. [47] using CFD [122]. The tank was assumed to be rigid, and the specific feature of interest was the influence of topography on pressures. The results indicate that a hill obstructing the flow has a strong influence on the pressure profile affecting structures located in the hill. This was also predicted by the ASCE provisions [9]; however, it was found that the pressures depend on the specific geometry of the hill, including the dimensions and radius of curvature of the top flat part of the hill, and not just on the slope and height of the hill. Furthermore, the current provisions do not represent an upper bound to the pressures.

For a tank located on top of a hill, pressure coefficients are of the order of three times those acting on an isolated tank in flat terrain. For other locations at the base and mid-height of the hill, a decrease in pressure coefficients is observed with respect to the isolated tank. The reason for this is that the flow on the sides of the tank at mid-height of the hill has much less energy than in the isolated tank, because of the shielding effect produced by the flat surface on which the tank is supported. However, the maximum pressures in the windward meridian in the structure located at mid-height in the hill (Cp=0.94) are larger than those in the isolated tank (Cp=0.75), because there is no shielding for that part of the tank. Not just the values but the pressure distributions depend on the location. This shows that the results cannot be easily extrapolated for other topographic conditions, tank locations, and sizes. Further research in this area is expected to quantify such influences by means of parametric studies.

A second line of research has been the influence of dikes on the pressure distributions in tanks. Most tanks in a tank farm are not directly placed in flat terrain, but are located inside dikes which are part of a containment system that provides safety to the plant or tank farm. Dikes are designed to store the fluid contents of a tank in such a way that no harm is produced to the underground soil in case of fuel spilling. Earth dikes are commonly built with slopes, leaving space for vehicle circulation between dikes; less common, depending on the available space in a plant, they may be built with concrete walls. In their usual configuration, a dike has just one tank, but in special cases they may house two or three tanks.

A CFD study has been recently published [155] on the influence of dikes on pressure coefficients due to wind. Results were reported for dikes with a slope and for protecting walls with variations in height. As expected, there is a reduction in C_p values due to interference of the flow with the dike, with also a decrease in the area affected by positive pressures. There is a slight increase in pressures as the separation between dike and tank increases.

5.5. Buckling of tanks under construction

Many structures fail during their construction stages, for which calculations were made by assuming properties of the completed structure and not enough provisions were taken to withstand wind loads before completion. A case of tank failure during its construction is mentioned (together with a photograph) in a 2001 book by Noon (see Fig. 2.2 in Ref. [125]).

Ref. [90] reports failures of aboveground steel tanks at two different locations in Patagonia, Argentina (General Roca, 2001 and Rincón de los Sauces, 2006), in which the structures were designed with a top roof but failure occurred under moderate wind loads before the cylindrical part was completed and the roof was added to the structure.

Nonlinear analyses GNIA and GMNA were performed to represent the buckling process in these cases in Ref. [90]; it was found that the continuous welding between the shell and the bottom plate had not been completed even at the stage of finishing the construction of the complete cylinder.

Borgersen and Yazdani [18] reported wind buckling of a tank with H/D=1 under construction in which the conical roof had not yet been installed but the cylindrical part was completed. Buckling occurred at wind speed of 100 Km/h. CFD models were used to obtain pressure coefficients and snap-through buckling was modeled by means of a finite element analysis.

The main conclusion is that continuous welding between the base plate and the first course should be completed before other courses are added on top; this would provide a much higher stiffness to the shell and the wind speeds required to buckle the shell would be within acceptable levels. Regarding the structural analysis of the tank, it is recommended [90] that models should be considered to account for the construction stages, thus assuring that the structure is stable before it is completed and with the roof set in place. The models should carefully follow the construction stages, so as to provide temporary buckling strength for the stages in which the final configuration has not been completed.

5.6. Design oriented work

Design aspects of wind buckling of tanks following American and European codes has been reported by Maraveas et al. [118]. To carry out the comparisons, two case studies were investigated, one with H/D=0.43 and conical roof, whereas a second case was an opened top tank with H/D=0.22 and wind girders. Both tanks had variable thickness. The foundations were modeled by line vertical springs. Clamped conditions at the base led to critical pressures under wind which were 10% higher than those computed using elastic foundation assumptions, but had no effect under uniform pressures. Imperfections were taken into account following code provisions, which result in large amplitudes (between 2 and 7.5 time the minimum shell thickness). Limiting displacements seems to be a reasonable design constraint in such cases, but these are outside the recommendations of American and European provisions.

On the other hand, comparisons limited to wind loads on silos and tanks have been reported in Ref. [87], with emphasis on Australian/New Zealand standards [6]. Code changes have been recently proposed to ASCE 7 to take into account wind pressures in circular bins, silos, and tanks [10]. This is a much needed addition to ASCE recommendations; however, the proposed implementation is basically the same as in the Australian/New Zealand code, which is based on the early wind tunnel testing on silo geometries done in Australia around 1990 [150,110,111,112]. This is most unfortunate because the proposal ignores a wealth of contributions made in the XXI Century on isolated and grouped silos and tanks [23], for which BLWT testing were carefully performed and constitute the current state of the art, as shown in this review.

Various lines of research attempted to provide more simplified ways of estimating buckling loads of tanks under wind. Reduced stiffness methods have been investigated by Jaca et al. [92] for tanks without a fixed roof, whereas Reduced Energy Methods were explored by Sosa and Godoy [158], in both cases via elimination of membrane energy in the meridional direction. The most successful lower bound strategy for wind loaded tanks has been the Reduced Stiffness approach, in which there is no assumption regarding the mode shape at the critical state, and this mode is calculated as part of the modified bifurcation analysis. It is found that the lower bound mode may be quite different from the classical eigenmode. A special purpose program [49] was modified for this purpose, and the strategy was applied to opened tanks with $0.2 < H/D < 1$ and $1250 < R/t < 2000$. The results are a lower bound to GNIA studies using ABAQUS, and also to the collection of wind tunnel data from various sources.

In the Reduced Energy approach using ABAQUS, the assumption that the shape of the eigenmode in the classical LBA is the same as in the lower bound is adopted but this is not valid under wind loads. Challenges still remain in order to arrive at reliable lower bounds via reduction of energy in general purpose software [158].

The influence of wind girders as a way to improve buckling performance has been the subject of recent attention. The use of a light ring stiffener near the bottom of the shell has been explored by Chen et al. [36] as a way to improve the shell capacity with respect to “elephant foot” buckling modes. The strengthening achieved depends on the location and size of the stiffener: it is shown that the use of a larger than optimal ring decreases (rather the increase) the buckling capacity. Bu and Quian [21] discussed the localization of stiffeners by means of case studies, whereas the optimal location has been the subject of work by Lewandowski et al. [105]. Results in this latter work indicate that current provisions for stiffener sizing are conservative and more sophisticated methods are required to obtain an economy in the design.

Design equations for cylinders as used in tanks were derived by Chen and Rotter [33] to estimate bifurcation buckling, effects due to geometric nonlinearity, and influence of imperfections. The authors found that for the range of geometries employed in tanks, plasticity did not develop at macroscopic level.

6. Buckling of tanks under foundation settlement

6.1. Problems of settlement

Thin-walled metal tanks may be supported in various forms, including compact soil foundation, ring walls, slabs, or pile-supported foundations. The support may be lost in some part of the base circumference affecting the cylindrical shell and the tank bottom. The causes of such differential settlements may include “non-homogeneous geometry or compressibility of the soil deposit, non-uniform distribution of the load applied to the foundation, and uniform stress acting over a limited area of the soil stratum” [119]. But heavy rains, such as those that happen during tropical storms and typhoons, may aggravate the situation.

For tanks, Myers [121] indicates a possible mechanism of settlement at the base but does not provide information regarding actual displacements in the shell.

Failure of tanks under settlement of the foundation has been repeatedly reported in the literature in the 1960 s [19,71,37], and in the 1980 s [119,15,41], among others. Notably is the report of the failure of a shell with R=26.15 m storing hot-oil in Japan in 1974. The consequences of this failure were manifold: “The contents flooded much of the refinery property and flowed into the adjacent inland sea causing severe damage to the fishing industry. As a result, the 270,000 bbl/day refinery was shut down for about nine months, largely because of public reaction. By the time the refinery was permitted to resume operation, the accident had cost the refinery more than $ 150,000,000” [15].

The differential settlements in tanks may have several consequences: (a) Out-of-plane displacements are induced in the shell in the form of buckling under a vertical displacement-controlled mechanism; (b) High stresses develop at the base of the shell and in the region of the settlement; and (c) High stresses develop in the tank bottom.

Only localized settlement is likely to produce severe damage to the shell in the form of radial displacements and buckling. There are many reasons to be concerned about such stresses and distortions: First, tanks are not isolated from other parts in an industrial plant, and have pipes and connections to other facilities that may be damaged due to the vertical displacements. Second, excessive displacements in the cylindrical shell affect the normal operation of a floating roof. Third, a geometric distortion greatly affects the buckling resistance of the shell under wind. Fourth, plasticity may occur in parts of the shell wall.

According to D'Orazio and Duncan [41], “…examination of the settlement measured for the tanks … shows one fact clearly: Steel tank bottoms can undergo a wide variety of types of distortion as they settle”. However, most analytical studies concentrate on just one type of distortion: a vertical displacement pattern at the base of the shell that follows a harmonic shape. In another paper, the same authors state: “Because their walls have significant stiffness and ability to span local soft spots, the settlement profiles of tank walls tend to be smooth and free of sharp variations. Through examination of measured settlement profiles and approximate theoretical analysis, the writers have concluded that for the tanks studied, which are typical floating-roof oil-storage tanks, significant distortion will not occur over circumferential distances shorter than about 20 to 30 m” ([42], pp. 875). For a tank with R=25.6 m, as they considered, the central angle associated to 20 m is 45° or 1/8 of the circumference.

6.2. Experiments to evaluate buckling under foundation settlement

Experiments on small scale models were published by D'Orazio et al. [42] for an open cylinder supported on eight points around the perimeter, and the settlement in the laboratory model are related to wall movements in real tanks by linear expressions which are independent of the slenderness of the shell.

Tests reported by Jonaidi and Ansourian [95] were performed on steel open cylinders with H/D=0.26, variable thickness R/t=375, and a simulation of the top ring. The tests recorded low amplitude as well as large deflection settlements, the main purpose being the evaluation of stress mechanisms.

Most studies in this field refer to open tanks, and “little data and few analyses exist to set a criterion for the validity of coned-roof tanks” ([119], pp. 1024). More recent experiments carried out on flat roof acetate tanks with settlement over an area with a central angle of 30° [62] indicate that there is a strong nonlinearity in the response, even for values of vertical settlement of the order of the thickness of the shell. A typical buckling mode recorded during tests is shown in Fig. 7.

6.3. Simplified linear models

The settlement of the foundation in large, thin-walled shells has been of great concern in the past. Studies on large reinforced concrete cooling towers shells constructed in the form of hyperboloids of revolution indicate ratios of maximum amplitude of the out-of-plane displacement versus the vertical settlement of the support between 3.5 and 6.0.

Studies for thin-walled tanks are also found in the technical literature in the 1960s [19,71,37] and in the 1980s [15]. However, all the computer simulations carried out by those authors

Fig. 7. : Buckling of tanks under settlement at the base: Testing. (Photograph by the author)

employed a geometrically linear formulation, in spite the fact that significant displacements were identified. An interesting observation in the change from reinforced concrete cooling towers to steel tanks is that there is a shift of interests from the evaluation of stresses to the assessment of radial deflections.

In the early 1980s, Marr et al. ([119], pp. 1028) stated: “We assume that buckling resulting from differential settlement would occur in the top course, would not rupture the shell and would not result in loss of oil. However, failure by buckling requires more studies”. The situation has not significantly changed since that paper was published, so that at present, the usual criteria for settlement do not consider buckling of the shell.

To evaluate the distortions in the cylindrical part of the tank, various models have been proposed, notably a harmonic shape to account for the vertical displacement at the base in terms of the wave number. Malik et al. [114] used an inextensional theory of shells and derived a relation of the out-of-plane displacement in terms of the settlement. This equation has several limitations: it is independent of the thickness of the shell, it does not account for localized settlements, and it is based on a simplified linear shell theory.

Radial displacements of open-topped tanks due to localized settlements were evaluated by Kamyab and Palmer [98], who derived another expression based on linear membrane shell theory instead of inextensional theory. Fourier analysis was used to produce charts, which were validated by tests. However, the simplified formulation was restricted to linear displacements and stresses.

Jonaidi and Ansourian [95] argued that errors in the range of 10-18% are obtained from the use of this membrane model.

More refined analysis used finite element models for the shell and assumed harmonic settlement [95] including more realistic features such as tapering wall thickness and the influence of the top ring stiffener (wind girder). These authors used ABAQUS to evaluate out-of-plane displacements, bending and membrane stress resultants as a function of the wave number. Their numerical results showed that there is a critical value of wave number close to 8, or central angle of 45°, for which the radial displacements reach a maximum value.

6.4. Computational buckling analysis

A review of foundation settlement effects on cylindrical shells was published by Holst and Rotter [82] in the context of metal shell buckling. The authors reviewed contributions to elucidate the different forms that vertical displacements at the base may take and the stress fields in tanks under those displacements. The discussion in Ref. [82] highlights that, although shell buckling in this problem is dominated by axial stresses, the shell behavior is not as catastrophic as in axially loaded cylinders, and the response moves quickly into a stable post-buckling path. The potential of settlement at the base to induce geometric deviations (mainly local uplift distortions) has been studied in Ref. [82] as a way to reduce the buckling capacity of the shell under axial loads. This problem has also been considered in Ref. [83].

Rather than dealing with local settlements at the base, several authors described the vertical displacements in terms of Fourier components. Jonaidi et al. [96] performed LBA and GNA for shells with displacement constraints at both ends and harmonic settlement, and proposed simple expression to estimate critical stresses. The influence of internal pressure was also investigated, to find elephant-foot buckling mode at failure for $n=2$.

Treatment of shell response using such components is useful to identify the most damaging shape, but because the shell response leading to buckling and post-buckling is non-linear, a coupled solution cannot be predicted from the response to individual harmonics.

Local settlement of the foundation of tanks was identified as a buckling problem in Refs. [62] and [63]. The “loading” parameter in this problem is the amplitude of the vertical displacement caused by settlement, which is associated with a function describing the settlement around the circumference at the bottom of the tank. Typical equilibrium paths are plotted as out-of-plane (radial) displacements of the shell versus vertical displacements caused by settlement. The computer analysis, as well as the tests performed on a small scale model [62], shows that the deflection patterns in thin-walled shells associated to localized settlements of the foundation is due to a highly non-linear behavior of the shell. The patterns of displacements identified in the shell are different from those found in buckling of the same shells under wind load or internal vacuum.

For a shell with $H/D=0.4$ and $R/t=1700$, the equilibrium path displays a non-linear behavior with a plateau, which is a clear sign of instability. The tangent to the equilibrium path becomes zero for a vertical displacement equal to half the shell thickness, and then it increases for higher settlement values. The results suggest that the shell buckles for a small value of the control parameter, and then deflects into a post-buckling mode.

LBA analyses have been shown to give a reasonable qualitatively approximation to the problem. A linear fundamental equilibrium path is seen to occur, before buckling develops into a new shape for the shell. In the new stable configuration, the shell can withstand further vertical displacements with an increase in the amplitude of the post-buckling mode.

Regarding the engineering importance of this effect, one has to look at the displacement amplitudes: the out-of-plane displacements computed using GNIA methodology are much larger than the linear values, so that it does not seem wise to establish tolerance criteria for settlements based on linear shell models.

Interest in this topic emerged in China [183,30,65–68]. For tanks opened at the top with a floating roof, Zhao et al. [183] reported parametric studies considering the stress fields in prebuckling states; interest concentrated on the number of waves in the harmonic model that dominates the amplitude of radial displacements. Buckling studies were reported in Ref. [30], using LBA, GNA, and GNIA, together with imperfection sensitivity for eigenvalue-affine imperfections. Parametric studies were performed by taking the harmonic wave number, R/t ratio and amplitude of imperfection ξ/t into account. The results confirm a strong nonlinearity in the equilibrium path. Buckling modes obtained via LBA are found to be a good representation of modes at limit points computed via GNIA. The authors classify modes as shear buckling for low wave numbers ($n=2$, 4) and elephant foot buckling for modes with higher number n.

Gong et al. [67] investigated tanks that are opened at the top, and similar ones that are reinforced with a wind girder at the top [66], in both cases under settlement described as a harmonic function, with the number of harmonics being a variable of the problem. Results include LBA and GNIA studies. The results for a tank with H/D=0.4 and R/t=1700 indicate that the critical displacement in the vertical direction is reduced with the addition of a ring at the top. The location of buckling modes depending on the harmonic considered is studied. The authors also considered changes in imperfection-sensitivity as a consequence of the addition of the ring. In evaluating imperfection-sensitivity, Gong et al. tried eigenmode-affine shapes of imperfection, as well as local weld depressions located in the zone where buckling is likely to occur. The influence of the ring stiffener is noticeable on the buckling problem for small wave number n in the harmonic settlement, but is not significant for large harmonic numbers.

For tanks with a conical roof, Gong et al. [65] assumed harmonic settlement to perform computational analysis using ABAQUS. For small wave number, buckling occurred on the roof, whereas cylinder buckling was identified for large wave numbers. The actual value of critical harmonic settlement is a function of the wave number and is very sensitive for small wave numbers.

Most works consider foundation settlement of a tank which is empty. The influence of the fluid stored in a tank with a conical roof has been investigated in Ref. [68]; the results show that the fluid has a stabilizing effect on buckling, so that it would be justified to establish limits based on empty tanks. Depending on the angle of settlement, buckling may even not occur if the tank has large fuel content.

Finally, a related problem of having local flexible discrete supports has been considered in Ref. [75] under axial load and lateral pressure.

(a)

(b)

Fig. 8. : Examples of buckling of tanks caused by thermal effects due to fire. (a) Aerial photograph of CAPECO oil plant in Bayamon, Puerto Rico, during the 2009 fire. (Photograph by US Chemical Safety Board) (b) Tank with a conical roof (partially filled with fluid) that was affected by a fire in the US isle of Guam, in 2002.

7. Buckling under fire

7.1. Background

Explosions and fires in oil refineries and storage facilities have been responsible for damage and failure of storage tanks, causing significant human, environmental, and financial losses. Dramatic examples of oil tanks involving fire are shown in Fig. 8.

An investigation on the causes of 242 accidents of hydrocarbon storage tanks that occurred between 1960 and 2003 [32] concluded that fire is the most frequent accident type and (excluding explosions as primary causes) comprise 60% of the cases. North-America hosted approximately 47% of the cases (of which, 43% of the cases occurred in United States), Asia and Australia have 30%, Europe 16%, Africa 4% and South-America 4%.

A review of 480 accidents involving fire in oil storage tanks between 1951 and 2003 shows that the number of accidents increases by at least 20% each decade, reporting an average of 16 yearly accidents in the decade of 1990 [130], with half of the cases registered occurred in the United States. Persson and Lönnermark [130] mentioned that the number of tanks involved in the fire events is considerably larger than the number of accidents, meaning that many tanks were affected in each accident. For cases in which the number of tanks under fire is known, the average of those tanks per incident is six, but the largest incident involved 200 tanks. The tank diameters vary between 8 m and 90 m and most of the tanks considered were identified as floating roof tanks.

A common feature in the investigation of this type of accidents is that there are significant difficulties in reconstructing the sequence of events leading to fire propagation in a tank farm because the fire tends to destroy most evidence. In a fire investigation, the tanks affected by fire have a level of destruction at the end of the process so that it might be extremely difficult to use that final information as indicative of the source of the accident. On the other hand, there may be photographic records (as in the case of the Puerto Rico accident of 2009 [13]) of tanks with various levels of deformation due to fire and at various times, and such evidence may be useful for the reconstruction of events. However, the links between distorted shapes and their cause is not self-evident and it necessitates a careful study of the deflected shapes of shells in tanks under various fire scenarios, in order to be able to understand the deflected patterns observed in real situations.

Efforts to understand the effects of fire in structural systems made by the scientific community in the past three decades have concentrated on the behavior of individual beam and frame structures (see, for example, Buchanan [22], Wang [173], and Usmani et al. [168]). However, studies of thin-walled oil storage steel tanks under fire that account for the specific behavior of shells have only been addressed in the last decade.

Although fire accidents in tank farms are such catastrophic events, it is surprising to find little information in the technical literature on the behavior of steel tanks under heat induced by fire. Modeling the structural behavior of tanks due to fire requires modeling fire as a thermal load, modifying the properties of steel as a function of temperature, and computing the nonlinear behavior of the structure. This could be done in a coupled (multi-physics) thermo-mechanical model, but the most common approach is to separate the thermal and mechanical problems and solve them in a sequential way.

7.2. Observations

Case studies of fire on individual tanks have been reported in the literature [16]. But the most dramatic and enlightening cases are those affecting a plant or refinery, in which case many tanks are involved in fire.

A large fire accident occurred in 2002 in the US island of Guam [17]. In July 2002 Typhoon Chata'an produced damage to a fuel storage facility, causing buckling of a tank with fixed and floating roofs. Because of the buckling deflections, the floating roof mechanism was blocked and could not operate properly. Typhoon Pongsona affected the same facility in December 2002, and the buckled tank received sand transported by wind causing friction on the metal wall and inducing a static charge: This started ignition and fire propagated to other tanks in a domino effect.

One of the best-known cases of fire in an oil storage facility occurred in Buncefield, UK, in December 2005, affecting 20 tanks [24]. The Buncefield fire was attributed to failure of a sensor that monitors the fluid level in a tank while the tank was being filled. The fluid level pushed the floating roof causing fluid spilling through the roof ventilation. Ignition was most likely due to sparks produced by engines.

Four years later, an accident of similar proportions occurred at the facilities of the Caribbean Petroleum Corporation (CAPECO), a large oil storage plant located in Bayamon, Puerto Rico (west of the city of San Juan), in October 2009, causing the destruction of 21 tanks [13]. The plant hosted 76 containers of which 40 were used to store gasoline, diesel, gasoil, aviation fuel, liquated petroleum gas, oil fuel, and crude oil. Those were cylindrical tanks, fabricated with steel or aluminum, with conical or dome roofs, and with internal floating roofs. The largest tank had D=74 m, and typical tanks had diameters of approximately D=30 m and H=12 m. Some tanks were supported on reinforced concrete rings, whereas others were directly placed on consolidated soil.

As described in Ref. [13], the accident involved more than half of the tanks in the farm, causing significant damage to the environment and to the population in the disaster zone, and generating a huge economic loss. It is believed that the initial source of the fire was associated with the failure of the liquid level gauges in one of the steel tanks, which was then overfilled with gasoline pumped from a ship docked in the San Juan Bay. Failure in the security system devices triggered a chain of events that resulted in the catastrophe. Notice that an almost identical cause was also present in the Buncefield accident in the UK [24].

7.3. Temperatures due to an adjacent fire

In a fire accident, heat may be transferred to a target tank from an adjacent fire; this may be the source of the accident or from another tank in a domino effect in which fire propagates from one tank to another.

The most complete work on effects due to fire on tanks is the 2011 doctoral dissertation of Liu [107] at the University of Edinburgh, which was motivated by the Buncefield accident. This work involved modeling the source of fire and the heat transfer from the source to the target tank. Liu adopted a semi-empirical solid flame model, in which fire is simplified as a cylinder (the flame is geometrically represented by its diameter and height) which radiates heat at a given rate by assuming a surface emissive power. Rather than describing details of the source of fire, solid flame models concentrate on their effects in the form of thermal radiation as a function of several flame parameters. Next, one, two, and three dimensional heat transfer analyses (including radiation, convection and conduction) were performed to evaluate the temperature reaching the tank; for this, a view factor is required which depends on the distance and angle from the solid flame to the target tank.

The outcome is a pattern of temperature distribution on the surface of the structure, in much the same way as pressures on the structure are obtained for wind effects. High temperatures are obtained at the meridian of "incidence of fire", with values decreasing at meridians that are further around in the circumferential direction. Non-uniform temperatures were obtained in the vertical direction, with a maximum at the center and lower values at the bottom and top of the tank. For fire that starts from ground level and is not so close to the target tank, the vertical distribution tends to be uniform. Thus, a simplified temperature distribution was proposed by Liu [107] as a cosine square function of the circumferential coordinate over a sector of the shell, with uniform or variable distribution in the vertical direction and employed in Refs. [108,109]. This simplified model has been adopted by other authors as the starting point in their buckling analyses [14,58,28,118].

Regarding the effect of liquid stored in the tank, Liu found that an empty tank receives higher temperatures (by a factor of five) than a tank that is completely filled with fluid. For intermediate cases, the computations indicate that there is a temperature jump between the low values below the level of fluid to the high values corresponding to an empty tank above that level. In a simplified version, the temperature below the level of fluid may be considered equal to the ambient temperature and thermal effects affect only the region above the level of liquid. This effect can be seen in Fig. 9.b for a tank affected by fire during the Guam accident in 2001, in which the top part has large deflections and the lower part (below liquid level) is almost unaffected.

Thesis work by Mansour [116] addressed models to evaluate the radiant heat flux to a specific target tank and compared the results with tests, to find that the solid flame model was the most appropriate to evaluate safety. For a tank with a conical roof, the solid flame model was used to predict temperatures on the target tank. The thermal response of the fluid stored in the target tank was also modeled. The study emphasized the required separation between tanks to avoid fire propagation effects, from the perspective of Chemical Engineering.

The evaluation of temperatures in a target tank caused by an adjacent fire at a source tank has been revisited by Santos and Landesmann [152]. The authors perform a two-stage analysis, first a semi-empirical solid flame model is used to determine the temperature on the source tank. Wind is taken into account through the inclination of the cylinder representing the flame. Second, a heat transfer analysis is performed using ABAQUS by considering an open cavity radiation; the 3D space between source and target tanks is discretized. Rather than using a cosine square circumferential variation of temperatures on the target tank, as in Ref. [107], Santos and Landesmann propose using an cosine with an exponent which depends on the material of the target tank (either concrete or steel), wind speed and burning fuel. A temperature variation in the vertical directions was not identified. The goal of this study was in establishing the best location of tanks in order to limit the effects of fire and the minimum separation between tanks for performance-based analysis.

7.4. Computational buckling analysis

The mechanics of behavior of simple components under fire has been described in Ref. [167]. Fire acts as a thermal loading on the structure and typically the shape becomes distorted due to thermal elastic buckling before significant changes in material properties take place. In essence, fire affects material properties (i.e., stiffness and strength parameters) and induces severe shape changes. For buckling problems of tanks under temperature-dependent loads there is no need to establish *a priori* a temperature at the most heated meridian because incremental temperature values are used in LBA and GNIA strategies.

Under thermal loads, tanks are expected to have large radial displacements leading to buckling, and this process depends on the displacement constraints associated with the boundary conditions at the base and by the roof at the top, and also by the constraints produced by other parts of the shell that have not yet been affected by fire.

The first work in this field was perhaps due to Landucci et al. [103] in the context of chemical engineering, with emphasis on predicting domino effects in cases of fire. In a sequential analysis, time to failure was correlated with radiation intensity; part of the data in this correlation was empirical and part was computational based on ANSYS finite element analysis of tanks. Then the authors developed a simplified methodology for evaluation of damage probability of tanks. Damage probability was calculated "by a site-specific probabilistic function that takes into account the calculated time to failure with respect to the time required for effective mitigation" (Ref. [103], pp. 1206). However, the structural analyses were limited to stress evaluations and no buckling analyses were carried out.

Liu [107–109] considered the structural analysis of tanks under the predicted temperature distributions reviewed in the last section, using the finite element package ABAQUS to evaluate shell buckling. LBA, GNA, and GMNIA analyses were performed on a tank with H/D=1 and R/t=1000. The roof was assumed as self-supported with 10° roof slope. The LBA results show bifurcation at low temperature levels in the order of 130 °C; consideration of temperature due to fire on the roof does not affect the eigenvalues of the tank. The non-linear equilibrium path followed with GNA has a maximum and then there is a sharp drop to the post-buckling equilibrium path. The shell recovers stability at a temperature of about 70% of the critical value. For a roof with a low stiffness, the buckling mode is identified in the lower part of the shell, close to the bottom boundary condition, and the meridional compression is identified as triggering the buckling mechanism. Material degradation due to temperature seems to be a factor affecting advanced post-buckling states, rather than changing the critical temperature at buckling.

A number of parametric studies were performed, including the influence of the central angle of the heated zone (central angles of 30°, 60°, 90°, and 120°), showing an increase in the critical temperature as the extent of the heated zone is increased. This effect was also observed in Ref. [58]. Inclusion of plasticity in GMNA leads to higher values of critical temperature, in the order of 150°. An eigenvalue-affine imperfection has been employed in the GNIA study, but the results showed small imperfection-sensitivity. Finally, increasing the roof thickness has the consequence of reducing the critical temperature [107,58].

As a consequence of the 2009 accident in Bayamon, Puerto Rico, research was performed to understand the deflection patterns and buckling of tanks with [58] and without [14] a fixed conical roof. The simplified temperature distributions on the shell reported by Liu [107] were adopted as a starting point in the analysis, and buckling analyses under thermal load were performed using ABAQUS. Temperature-dependent material properties were adopted in GMNA, and results were compared with those of LBA.

For tanks with a fixed conical roof, two cases that failed during the Bayamon fire were investigated: one with a self-supporting roof (H/D=1, R/t=1150, roof slope 11°, and roof thickness twice the minimum shell thickness); and a tank with a supporting structure for the roof, with H/D=0.4, R/t=1930, roof slope 11°, and variable thickness [58].

In the tank with self-supporting roof, the buckling mode consists of vertical bulges in the cylinder, deflections in the form of waves at the base and zones of upward and downward displacements on the roof. The zone affected by fire has been taken as a variable in the study: an interesting observation is that for central angles between 135° and 225°, buckling occurs in the "hot" zone affected by an increase in temperature, but for central angle of 270° the shell expands and induces hoop compression in the region not directly affected by heat, thus producing buckling in that "cold" zone. This effect was seen to occur in one of the tanks in Bayamon. Other topics reported were the influence of a thermal gradient in the shell thickness, the fluid stored, and the shell thickness.

For the tank with a structural support system, roof and cylinder buckling occur at different temperatures: a roof mode is first detected at 173 °C, followed by cylinder buckling at 230 °C. Again, the critical temperatures are relatively low and elastic behavior is identified. Outward displacements in the cylinder are predicted along the fundamental equilibrium path, which change to inward displacements at the critical state; in advanced post-critical states the shell recovers stability. The influence of substituting the structural support system with an equivalent thickness has been investigated: the conclusion is that even if the equivalence is established in terms of equivalent stiffness, there are significant differences in critical loads and modes [27].

A tank opened at the top, having H/D=0.38 and R/t=1525, which failed during the Bayamon accident has been studied in Ref. [14] using LBA and GMNA in ABAQUS. The temperature at buckling (in the order of 500 °C) is much higher than in tanks with a roof, on account that the tank is now free to deform at the top and is only restricted by boundary conditions at the base. The thermal load develops low stresses in the cylinder and stresses concentrate at the base where there are restrictions to displacements. The influence of thermal gradient through the thickness, liquid level, shell thickness, circumferential zone affected by fire, has also been investigated. Unlike tanks having a roof, no cases were found in which buckling modes occurred in the "cold" zone.

A sequel to previous research was presented by Maraveas [117] for a tank with conical roof, in which parametric studies of region affected by temperature, roof thickness and filling level were investigated. Using LBA and GNA with ABAQUS, the author confirmed previous results, and found that by increasing the roof thickness (and consistently scaling the roof weight) there is a very small reduction in critical temperatures.

Effects due to the geometry of the tanks on the buckling modes caused by temperature due to fire have been reported in Ref. [28] for a large tank with D/H=1 and a smaller one with D/H=0.5. GNA results allow identification of the critical temperature but do not provide information on the post-critical states, because the path remains along the fundamental equilibrium path. Post-critical states need using GNIA; they may be stable or slightly decreasing, but there are no signs of imperfection-sensitivity in this problem.

Other studies are not specific of tanks but related to thermal buckling and post-buckling of composite cylindrical shells, such as in Ref. [3], in which imperfection sensitivity was investigated for clamped-clamped shells. The shell radius was identified as a crucial geometric parameter, but the shell length did not have a significant influence on results.

8. Conclusions

This review highlights that a wealth of information based on research has been published around the world; however, little of that permeates into design codes or design recommendations. There are inherent difficulties in transferring knowledge from academia to industry in this field, due to accident and insurance issues, with the consequence that recommendations tend to specify as little as possible and leave much of the technical responsibility to the designer. Designers, on the other hand, have limited knowledge of advances in structural analysis and in load determination.

Much has been achieved in this field in the last 20 years, but it would be difficult to produce a fair summary embracing all topics covered in this review. Some highlights may be summarized as follows:

- Wind pressures for isolated tanks are now a well-established topic, and our current understanding of wind pressures for open and closed tanks seems to be adequate for design purposes thanks to a number of wind tunnel tests.
- Ten years ago there was little knowledge concerning thermal buckling due to fire. The situation has now changed thanks to temperature models established based on heat transfer considerations, and buckling is known to occur under low temperatures, in the order of 100 °C to 200 °C. Buckling initially occurs under elastic behavior and changes in material properties associated to temperature effects are expected to be present in post-buckling states.
- The need to include geometric nonlinearity as part of settlement of the foundation of tanks and other thin-walled shells has been clearly established during the last decade. Thanks to LBA and GNIA investigations, is has been possible to quantify central angles affected by settlement that induce lower buckling states. This problem depends on the existence of displacement constraints in the shell, such as wind girder or fixed roofs.
- Computational buckling analysis of shells is currently the standard way to perform buckling analysis of vertical tanks, with preference for LBA and GNIA approaches, i.e. bifurcation and geometrically nonlinear analysis with geometric imperfections. Although the European Recommendations on shell buckling advocate for a combines LBA-MNA approach [145], this has never been employed in the analysis of oil storage tanks in the literature.

  There are a number of areas opened for future research, and a few from a personal perspective, may include:
- Wind buckling: Voids in the literature may include effects of tank components on wind pressures, such as ring stiffeners or ladders, which modify the flow around the axisymmetric shell. Limited information is available on topographic effects, such as those produced by buildings in a refinery, secondary containment dikes, or hills. Finally, most studies on wind pressures have been conducted in wind tunnel environments. Confirmation that this is observable in full scale tanks requires measurements in real structures, which would be a much needed validation considering differences in Reynolds numbers between full and reduced scale tanks.
- Thermal buckling caused by fire: Current studies are mainly limited to elastic buckling of the shell, but forensic studies may also be much interested in collapse mechanisms in order to reproduce failure modes observed in the field. Thus, the inclusion of plasticity and large deformations in the post-buckling analysis of a tank under thermal loads would be needed if realistic failure modes are to be reproduced. Material nonlinearity in his case should be temperature-dependent to account for material softening as temperature increases. Finally, thermal models are currently established for a single source of fire which is adjacent to the target tank. Cases in which there are two or three sources, such as those found in tank farms during domino effects, are still to be considered.
- The most damaging effects caused by settlement seem to be blocking of the internal floating roof. Further studies are needed to identify levels of deflection affecting the floating roof mechanism for the most common types of such roofs employed in practice.
- Evidence of failure of tanks in real situations has been addressed in the literature in a small number of cases. This would help in establishing emblematic cases for which failure mode is known and could serve as cases to be modeled as part of validation procedures in research. Of course, failure cases are not frequently available in the open literature because of insurance issues and litigation processes, but they are extremely important to identify what failure modes are observable in practice and verify if they are covered by current design codes.

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