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Classical images as quantum entanglement: An image processing analogy of the GHZ experiment

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ABSTRACT

In this paper we present an optical analogy of quantum entanglement by means of classical images. As in previous works, the quantum state of two or more qbits is encoded by using the spatial modulation in amplitude and phase of an electromagnetic field. We show here that bidimensional encoding of two qbit states allows us to interpret some non local features of the joint measurement by the assumption of "astigmatic" observers with different resolving power in two orthogonal directions. As an application, we discuss the optical simulation of measuring a system characterized by multiparticle entanglement. The simulation is based on a local representation of entanglement and a classical interferometric system. In particular we show how to simulate the Greenberger–Horne Zeilinger (GHZ) argument and the experimental results which interpretation illustrates the conflict between quantum mechanics and local realism.

1. Introduction

Multiparticle entanglement is an essential resource for quantum communication schemes, quantum error correction, quantum key distribution and quantum computation. However, one of the first applications of entanglement of more than two particles was motivated by the discussion on whether quantum mechanics is or not a complete theory of nature [1]. Most of the experiments with two entangled particles testing Bell inequalities, contradict the local realism hypothesis only from statistical predictions [2,3]. In this sense, Bell inequalities are widely used to show the contradiction between quantum mechanics and the Einstein–Podolsky–Rosen (EPR) local realism hypothesis. Experiments involving Bell inequalities require bipartite entanglement and the results are obtained in an statistical way, after several local measurements. In contrast, GHZ showed that by making only one measurement over a certain set of observables it is possible to contradict the local realism hypothesis where three entangled particles are needed. [4,5]. As only one measurement is required, the GHZ experiment is a concise way to test the validity of the local realism hypothesis. In addition, assuming the validity of the quantum mechanics predictions, this kind of experiment is also used to test the possibility of generating entangled states. In this way, if we are able to produce the classic analogy of three entangled states, we will also be able to test them by using the classical analogy of the GHZ argument as is shown in this paper.

Recently it has been proved that optics provides an elegant tool for simulating quantum mechanics problems shedding light to the interpretation of the counterintuitive results of quantum mechanics. A variety of classical optics analogies of quantum information processing was recently published [6–13]. Most of the cited papers make the distinction between "true" entanglement between two or more spatially separable particles and a weaker form of local entanglement between different degrees of freedom of a single particle. Only the second type of entanglement has a classical analogy. In [8], an experimental set-up representing measurements on a three classical bits entangled GHZ state is also proposed.

In this paper we propose a simple and illustrative way of simulating quantum entanglement. We will show that although "true" entanglement has no classical counterpart, some non local features of quantum measurements arise even in a classical local representation from the assumption of observers with "anisotropic resolving power". In addition, we exploit the analogy in order to design an optical setup for simulating measurements on multiparticle entangled states. This could be useful for experimental testing of any quantum information protocol involving multiparticle entanglement. In particular, we show how the designed architecture works in the fundamental measurement of GHZ states. We show that our optical setup allows us to achieve experimental results which are in good agreement with the quantum interpretation.

2. Optical representation of pure quantum states

Classical encoding of qbits has been used to simulate different quantum information processes, such as quantum random walks or quantum teleportation [9–13]. The main idea is encoding quantum

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states by dividing the plane of an optical scene in zones representing a basis state of a given Hilbert space. For example, if A is the area of the scene, we could define a one to one mapping between each localized domain of size $A/2^N$ of the plane and a N length binary string which represents a certain state of the basis of N qbits. Let us suppose that a two qbits system is represented with the convention

$$\begin{array}{ll} l\rightarrow|0\rangle_1 & r\rightarrow|1\rangle_1 \\ u\rightarrow|0\rangle_2 & d\rightarrow|1\rangle_2 \end{array} \quad (1)$$

where l, r and u, d means left–right and up–down respectively and the subindices 1, 2 denote the qbit number, where this spatial organization can be seen in Fig. [1.a]. In this way, the most general two qbits state

$$|\Psi\rangle = C_{00}|0\rangle_1|0\rangle_2 + C_{01}|0\rangle_1|1\rangle_2 + C_{10}|1\rangle_1|0\rangle_2 + C_{11}|1\rangle_1|1\rangle_2 \quad (2)$$

could be represented by means of the spatial modulation of the transverse amplitude of a wavefront as suggested in Fig. [1.b]. The four coefficients C_{ij}; i, $j=0$, 1 are the complex amplitudes of the electromagnetic field in each of the four domains lu, ld, ru, and rd of the full plane. The normalized complex coefficients $\tilde{C}_{ij} = C_{ij} / \sum_{ij}\left|C_{ij}\right|^2$ are interpreted as probability amplitudes such that $\left|\tilde{C}_{ij}\right|^2$ is the probability of measuring the state $|i\rangle_1|j\rangle_2$ in a projective measurement in the computational basis.

3. Non local features of the joint measurement with bipartite local entanglement

Let us suppose that the information contained in a pure state of a bipartite two level quantum system is encoded in an image as in Fig. [1]. Fig. [1.d] represents local entanglement in the sense that the two subsystems sharing entanglement cannot be spatially separated. However, some further assumptions about the observers, namely "astigmatic resolving power" in up–down and left–right directions, allows a non local interpretation. In fact, non locality is equivalent to consider that the observers 1 and 2 can resolve two slices of the image only in complementary orthogonal directions as we schematize in Fig. [2]. For example, according to Fig. [2], observer 1 can clearly distinguish two different slices in the left–right direction of the plane. On the contrary, the information encoded in the up–down direction appears to be diffuse in a way that the intensity pattern distributed along the left–right direction arises from the integrated intensity in the up–down direction. Observer 2

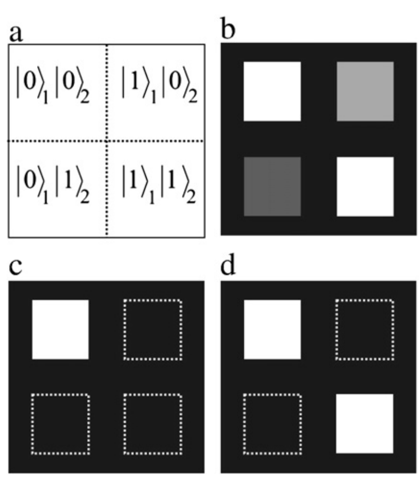

Fig. 1. Schematic picture of the representation of two qbit states by using optical scenes. (a) Spatial organization of the input plane. (b) Optical representation of the general pure two qbits state. Coefficients C_{ij} represented in gray level scale correspond to different amplitudes and phase modulations of the classical wavefront. (c) Optical representation of the separable $|0\rangle_1|0\rangle_2$ state. (d) Optical representation of the maximally entangled $(|0\rangle_1|0\rangle_2 + |1\rangle_1|1\rangle_2)/\sqrt{2}$ Bell state.

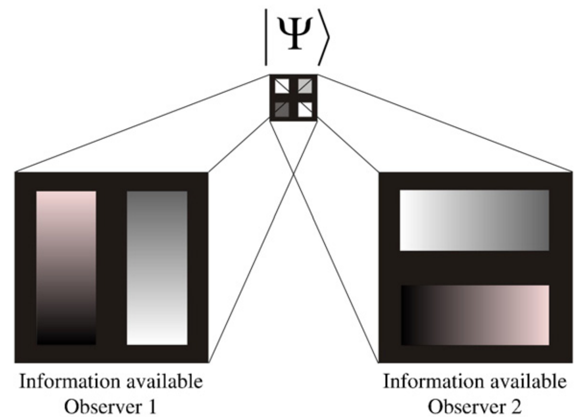

Fig. 2. Two observers with complementary "astigmatic resolving power" as an analogy of two spatially separated systems sharing a two qbits state. Dicotomic information available for subsystem 1 encoded in left–right direction is unavailable for subsystem 2 and vice versa.

has the same limitation with respect to the complementary direction. Entanglement is related to the problem to establish whether or not a local measurement in one subsystem affects the state of the other. As we will see, this problem is well represented and can be easily understood from our optical analogy.

For a deeper understanding of the following discussion, it could be useful to consider the *Schmidt representation* of the state $|\Psi\rangle$ [14]. According to it, the general two qbits state can be written as

$$|\Psi\rangle = \sum_{i=0}^{1} \lambda_i|\phi(i)\rangle_1|\varphi(i)\rangle_2 \quad (3)$$

where the local unitary operators U and V define the two orthonormal *Schmidt* basis

$$\left\{|\phi(i)\rangle_1 = \sum_{j=0}^{1} U_{ij}|j\rangle_1, i=0,1\right\} \quad \left\{|\varphi(i)\rangle_2 = \sum_{j=0}^{1} V_{ij}|j\rangle_2, i=0,1\right\} \quad (4)$$

and the non-negative *Schmidt coefficients* $\lambda_{i}, = 0, 1$ satisfy $\sum_{i=0}^{1}\lambda_i^2 = 1$. The number of non-vanishing Schmidt coefficients of a given state is the so called *Schmidt number S*. For instance, if the state is separable, it will be a tensor product of some two basis vectors, as in the case labeled as $S=1$ in Fig. [3]. In this case, a local measurement in the *Schmidt* basis does not affect neither the quantum state nor the outcoming measurement on the other subsystem since the states before and after measurement are exactly the same. Also, local measurement in other basis will not affect the complementary subsystem, since the Schmidt number of a given state is preserved under local unitary transformations. On the other hand, it is clear that if the state is entangled as in the case labeled as $S=2$ in Fig. [3], the states before and after a local measurement in the *Schmidt* basis (or any other) will be different. In this way, any local measurement modifies the state and of course the outcoming measurement on the complementary subsystem. In quantum mechanics it is said that a two qbits state is separable if there exists a unitary transformation mapping that state into a representation in the Schmidt basis with $S=1$ $|\phi(0)\rangle_1|\varphi(0)\rangle_2$. The optical analogy presented here consists in transforming an image that represents a general state (such as the one shown in Fig. [1.b]), by means of optical operations that simulate quantum unitary operators and having as a result an image as the one shown in Fig. [1.c]. In a recent work, we showed the simulations of Bell inequalities violation with the use of optical processors and this kind of classical optical scenes [16]. In the following section we will extend the results to the case of multiparticle entanglement.

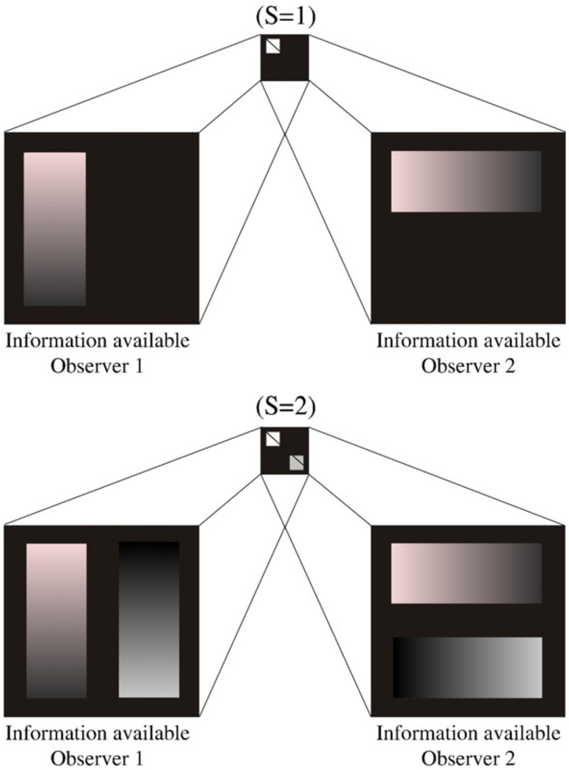

Fig. 3. Optical representation of the Schmidt decomposition of a two qbits state. ($S=1$) Separable state. ($S=2$) Entangled state.

4. Simulating measurements on multiparticle entanglement and the GHZ argument

In this section we briefly describe the GHZ experiment. Let us consider the following four observables defined on the Hilbert space of three qbits:

$$\begin{aligned} O_1 &= \sigma_x^1\sigma_y^2\sigma_y^3 \\ O_2 &= \sigma_y^1\sigma_x^2\sigma_y^3 \\ O_3 &= \sigma_y^1\sigma_y^2\sigma_x^3 \\ O_4 &= \sigma_x^1\sigma_x^2\sigma_x^3 \end{aligned} \tag{5}$$

where σ_x^i and σ_y^i are the Pauli matrices in the directions x and y for each observer $i=1, 2, 3$ respectively. It is easy to verify that

$$|\Psi_{GHZ}\rangle = (|0\rangle_1|0\rangle_2|0\rangle_3 - |1\rangle_1|1\rangle_2|1\rangle_3)/\sqrt{2} \tag{6}$$

is eigenstate of those four operators in such way that

$$O_{1,2,3}|\Psi_{GHZ}\rangle = |\Psi_{GHZ}\rangle \tag{7}$$

$$O_4|\Psi_{GHZ}\rangle = -|\Psi_{GHZ}\rangle. \tag{8}$$

According to EPR local realism hypothesis one could assign the realistic value ± 1 driven by some hidden variables to each local observable. There are eight assignations compatible with the statement $O_{1,2,3}|\Psi_{GHZ}\rangle = |\Psi_{GHZ}\rangle$ and all of them give the value $+1$ for the observable O_4 contradicting the quantum mechanical prediction $O_4|\Psi_{GHZ}\rangle = -|\Psi_{GHZ}\rangle$. Such assignations are detailed in Table 1.

As we mentioned before, if we are able to i) represent entangled states of three qbits and ii) represent the corresponding products of Pauli matrices; we will be able to optically simulate the GHZ experiment.

Table 1
Local realism prediction for observable compatible with $O_{1,2,3}=1$.

σ_x^1	σ_x^2	σ_x^3	σ_y^1	σ_y^2	σ_y^3	O_4
1	1	1	1	1	1	1
1	1	1	−1	−1	−1	1
1	−1	−1	−1	1	1	1
−1	1	−1	1	−1	1	1
−1	−1	1	1	1	−1	1
1	−1	−1	1	−1	−1	1
−1	1	−1	−1	1	−1	1
−1	−1	1	−1	−1	1	1

Three or more qbit states could be easily represented following the analogy described above. In Fig. [4.a], one possible spatial organization of the input plane in order to give an optic representation of the GHZ state is shown. In order to perform a fully optical simulation of the GHZ proposal, we use a coherent optical processor. The optical architectures for simulating unitary operators acting on a single qbit are completely described in previous works [11–13,17]. The main idea is to use an optical processing architecture where the spatial filter is represented in an SLM (spatial light modulator). In order to represent each $U(2)$ operators, certain phase gratings are represented in the SLM which is located in the Fourier plane [17]. Here we will briefly discuss the method that we used in ref.[16]. We have demonstrated that spatial filtering in the Fourier plane performed by a square wave phase modulation of width $p=1/4d$, amplitude $0<\phi<2\pi$ and spatial period

$$2p = 1/2d. \tag{9}$$

In this case, the input output relationships for the complex amplitudes α and β of two thin slices in the input plane separated by a distance $2d$ are:

$$\begin{pmatrix}\alpha\\ \beta\end{pmatrix} \rightarrow \begin{pmatrix}\alpha'\\ \beta'\end{pmatrix} = \begin{pmatrix}\cos\frac{\phi}{2} & \frac{2}{\pi}\sin\frac{\phi}{2}e^{i\gamma-} \\ \frac{2}{\pi}\sin\frac{\phi}{2}e^{i\gamma+} & \cos\frac{\phi}{2}\end{pmatrix}\begin{pmatrix}\alpha\\ \beta\end{pmatrix} \tag{10}$$

where we have defined the phase constants

$$\gamma^{\pm} = \frac{\pi}{2} \pm \frac{\pi}{p}f_c \tag{11}$$

being f_c the position of the center of the phase pulse in the frequency domain. This imaging architecture is capable of representing any $U(2)$

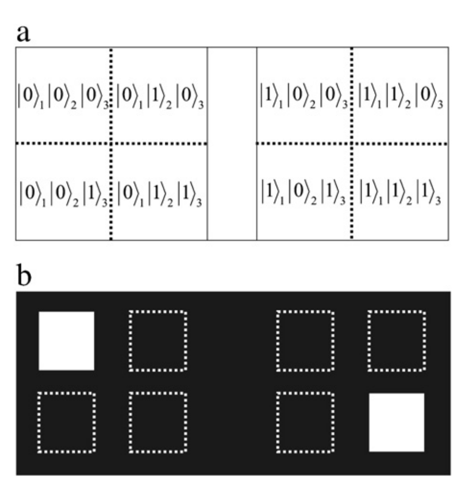

Fig. 4. (a) Spatial organization of the input plane used to represent three qbits. (b) Optical representation of the GHZ state $|\Psi_{GHZ}\rangle$. A π phase shift in the right bottom is needed.

operator acting on a single qbit $\alpha|0\rangle+\beta|1\rangle$. This can be accomplished by means of an adequate setting of the experimental parameters (p, ϕ, $f_c(p)$) and a phase shift against the input plane. For instance, by setting $\phi = 2\ Arctan\left(\frac{\pi}{2}\right)$ and $f_c = -p/2$ we obtain the Hadamard operator U_x that satisfies $U_x\sigma_x U_x^{\dagger} = \sigma_z$. Instead, by setting $\phi = 2\ Arctan\left(\frac{\pi}{2}\right)$ and $f_c = 0$ we obtain the analog of Hadamard U_y for measuring σ_y which satisfies $U_y\sigma_y U_y^{\dagger} = \sigma_z$. In both cases a phase shift representing σ_z must be included. This phase shift has a value of π in the region corresponding each $|1\rangle_i$ state. The choice of the parameter p and the orientation of each grating are related, according to Eq. (9), to the separation and orientation of the two slices associated to the qbit which is modified by the simulated U(2) operator.

In Fig. [5] the optical set-up used to perform the simulated GHZ experiment is shown. An Argon laser ($\lambda = 477$ nm) is spatially filtered and then collimated. The collimated beam impinges onto a binary mask which represents the GHZ state $|\Psi_{GHZ}\rangle$. A first optical processor of type *4f* [17] (focal length $f_1 = 26$ cm) with the SLM1 in the Fourier plane of the scene, simulates the direct product of the two local unitary operators $U^2_{x\vee y}U^3_{x\vee y}$. According to the previous discussion, this can be done with the composition of two orthogonal almenary phase gratings [16]. A second optical processor of the same kind (focal length $f_2 = 70$ cm) takes as input the output of the previous process $U^2_{x\vee y}U^3_{x\vee y}|\Psi_{GHZ}\rangle$. By means of the SLM2 in the Fourier plane, the remaining operator $U^1_{x\vee y}$ is simulated. The gratings were programmed in two identical SLMs. These devices consist in a Sony liquid crystal display (LCD) that combined with two polarizers and two quarter wave plates, acts as a mostly phase modulator [15]. The LCD (model LCX012BL) was extracted from a commercial video-projector and is a VGA resolution panel (640×480 pixels) with square pixels of 34 μm size separated by a distance of 41.3 μm. The final image in the output plane $U^1_{x\vee y}U^2_{x\vee y}U^3_{x\vee y}|\Psi_{GHZ}\rangle$ is captured by a videocamera (CCD). It should be mentioned that the same process could be simulated with a single processor only. This could be done by a mod(2π)-addition of three almenary phase gratings with adequate parameters and orientations in the Fourier plane. We used two *4f* optical processors with different focal distances because of an easier adjustment between the sizes of our qbits slices and the spatial resolution of our SLMs. On the other hand, as we have mentioned above, the simulation of each $U^1_{x\vee y}$ operator acting on a single qbit $i = 1, 2, 3$ needs a phase shift σ_z in the input scene. This introduces a π phase shift in the $|1\rangle_i$ slice. From Fig. [4] it is easy to understand that for encoding the $|\Psi_{GHZ}\rangle$ state (uniform illumination of the left top and the right bottom) the only effect of adding a π phase shift in each of the corresponding zones where $|1\rangle_1$ is encoded, is the same as a π phase shift in the right bottom, as is the only region illuminated in the scene that needs it. Since the codification of the state $|\Psi_{GHZ}\rangle$ needs an additional π phase shift in the right bottom, no phase shift is needed in this case, because it is a total phase shift of $\pi+\pi = 2\pi$.

5. The GHZ argument, experimental results

In Fig. [6] experimental results for the optical simulation of the GHZ argument are shown. Fig. [6.a], [b] and [c] are the images and *3D* profiles corresponding to the measurements of the three observables $O_1 = \sigma_x^1\sigma_y^2\sigma_y^3$, $O_2 = \sigma_y^1\sigma_x^2\sigma_y^3$, and $|\Psi_{GHZ}\rangle O_3 = \sigma_y^1\sigma_y^2\sigma_x^3$ respectively; meanwhile Fig. [6.d] shows the result for the measurement of $O_4 = \sigma_x^1\sigma_x^2\sigma_x^3$. From the images and *3D* profiles we can observe that the expected value of any of the three observables $O_{1,2,3}$ for the GHZ state are close to be $+1$ since only the computational states with an even number of $|1\rangle_i$ factors have significant amplitudes. On the other hand (and for similar reasons) the expected value of $O_4 = \sigma_x^1\sigma_x^2\sigma_x^3$ is close to be -1. These results are nearly close to the quantum mechanics prediction namely $O_{1,2,3} = 1$ and $O_4 = -1$ with certainty for the state. It has to be mentioned that the SLM was used in its maximal resolution, i.e. the spatial period of the grating represented on it was 2 pixels. The phase modulation of the SLM in high resolution has a diminished range, not reaching the highest value, so some differences between experimental amplitudes and the theoretical ones are expected. However, although the experimental results differ slightly from the theoretical predictions, a good qualitative agreement is obtained.

6. Discussion and remarks

Summarizing, we investigated some possibilities of classical optics representation of quantum entanglement, by means of encoding two or more qbit states in the spatial modulation of the complex amplitude of a plane wavefront. We show how the non-local properties of the joint measurement can be interpreted in this classical context, from the assumption of selective resolving power in orthogonal directions of the observers. Moreover, we show that with an optical representation of local unitary operations, this scheme conduces to a pictorical

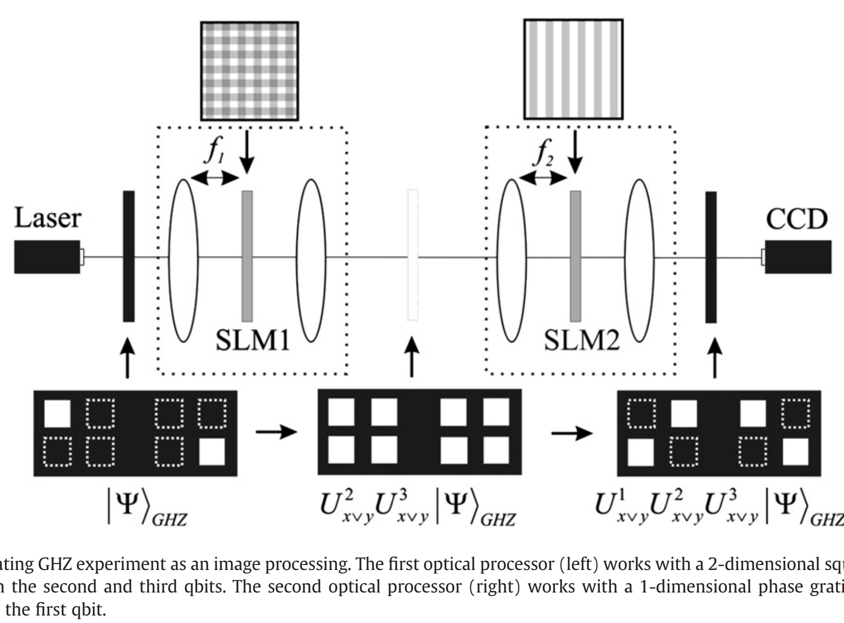

Fig. 5. Schematic set-up for simulating GHZ experiment as an image processing. The first optical processor (left) works with a 2-dimensional square phase grating at the filter plane to simulate unitary operations on the second and third qbits. The second optical processor (right) works with a 1-dimensional phase grating at the filter plane to simulate a $O_2 = \sigma_y^1\sigma_x^2\sigma_y^3$ unitary operation on the first qbit.

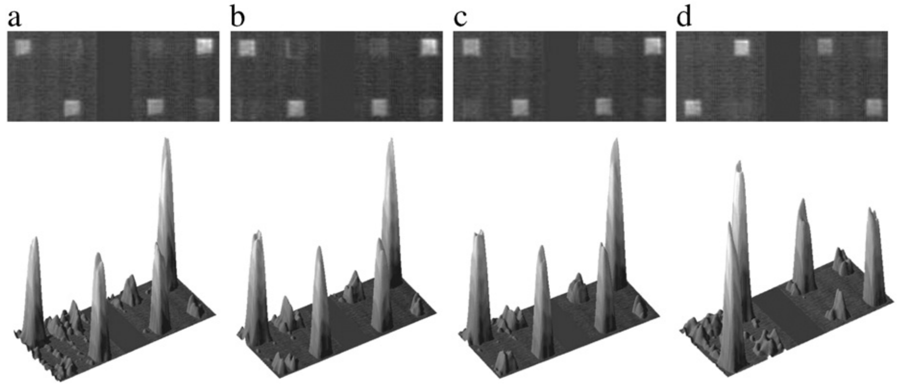

Fig. 6. Experimental results of the optical simulation of the GHZ proposal. Output images and *3D* profiles corresponding to the four measurements of $O_{1,2,3,4}$ are shown. a) $O_1 = \sigma_x^1\sigma_y^2\sigma_y^3$, b) $O_2 = \sigma_y^1\sigma_x^2\sigma_y^3$, c) $O_3 = \sigma_y^1\sigma_y^2\sigma_x^3$ and d) $O_4 = \sigma_x^1\sigma_x^2\sigma_x^3$.

description of the Schmidt representation and consequently of the formal understanding of bipartite quantum entanglement. We investigated an optical setup to simulate measurements on multiparticle entanglement. With this interpretation, we have a classical quantum analogy and we use it to test the GHZ argument, just like it is done in the real quantum case. The obtained experimental results constitute a nice example where optical simulation provides a new pictorical way to look in a fundamental problem allowing a deeper understanding of it, as is the contradiction of EPR local-realism hypothesis in a classical system. These results allow us to offer this technique as an accessible experimental testing of quantum information protocols involving multiparticle entanglement of a reduced number of qbits.

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