A Measurement of *g* Using Alexander's Diving Bell

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A Measurement of g Using Alexander's Diving Bell

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This paper describes a very simple exercise using an inverted test tube pushed straight down into a column of water to determine the free-fall acceleration g. The exercise employs the ideal gas law and only involves the measurement of the displacement of the bottom of the "diving bell" and the water level inside the tube with respect to the water column surface. The experiment is ideal for students in introductory physics courses.

Probably the first recorded mention of a diving bell is found in Aristotle's *Problemata*,[1] where it is stated that by letting down a cauldron, it would be possible for sponge-divers to breathe the air enclosed inside because it does not fill with water. French chronicles of the 13th century credit Aristotle's pupil, Alexander the Great, as the first person ever to use a diving bell, which according to legend consisted of some sort of glass bell pushed straight down into the water, during the siege of Tyre in 332 B.C. This rudimentary bell was not supported by fresh air from the outside, restricting the duration of the diving expedition. Chronicles are scarce and do not specify whether by those days some sort of relation between the depth and the water level inside the bell was established or not. However, this is highly improbable due to the technical difficulties involved.

Most ultrasonic experiments for measuring g in freshman physics laboratories are mainly based on the use of manually operated stopwatches, motion detectors, or optical sensors employed as start/stop switches for electronic counter timers.

Here we propose an alternative way to determine g that does not involve the measurement of time. The most elementary diving bell is modeled in terms of the physical principles of hydrostatic pressure and the ideal gas law.

Figure 1 shows the setup. In our experiment, a test tube (0.73 ± 0.02 cm internal diameter and length l_0 87.1 ± 0.1 cm) is pushed down into a column of water. Several measurements of the depth h and the l_1 level are performed by means of a cathetometer[2] for increasing depths starting at a depth of about 12 cm in order to avoid any significant errors that could arise from capillarity effects.

The first step in determining the value of g is to find the pressure p_1 at the water level inside the bell,

$$p_1 = p_0 + \rho g l_1, \quad (1)$$

where ρ is the density of the water. Then, making use of the ideal gas law,

$$\begin{aligned} p_0 V_0 &= nRT_0, \\ p_1 V_1 &= nRT_1, \end{aligned} \quad (2\text{–}3)$$

where T_0 and T_1 are the air and water temperature, respectively. Now assuming that the cross-sectional area A of the tube is uniform, we write $V_0 = Al_0$ and $V_1 = A[l_0 - (h - l_1)]$. From Eqs. (2) and (3) we get the following expression for the air pressure inside the tube:

$$p_1 = \frac{T_1}{T_0}\frac{l_0}{[l_0-(h-l_1)]}p_0. \quad (4)$$

Combining Eqs. (1) and (4), we get

$$\frac{T_1}{T_0}\frac{l_0}{[l_0-(h-l_1)]}p_0 = p_0 + \rho g l_1, \quad (5)$$

which can be easily rearranged as follows,

$$\left(\frac{T_1}{T_0}\frac{l_0}{[l_0-(h-l_1)]}-1\right)p_0 = \rho g l_1. \quad (6)$$

Comparing Eqs. (1) and (6) shows that the left-hand side (lhs) of Eq. (6) is equivalent to $p_1 - p_0$. A plot of the lhs of Eq. (6) versus l_1 should be linear with a slope of ρg.

Once the container shown in Fig. 1 was filled with water, it was allowed to rest for a few hours in order to minimize the temperature difference between the air and the water. These temperatures were measured with thermocouples, yielding the following values, which remained constant throughout the experiment: T_{air} = 21.5°C and T_{water} = 21.0°C. The atmo-

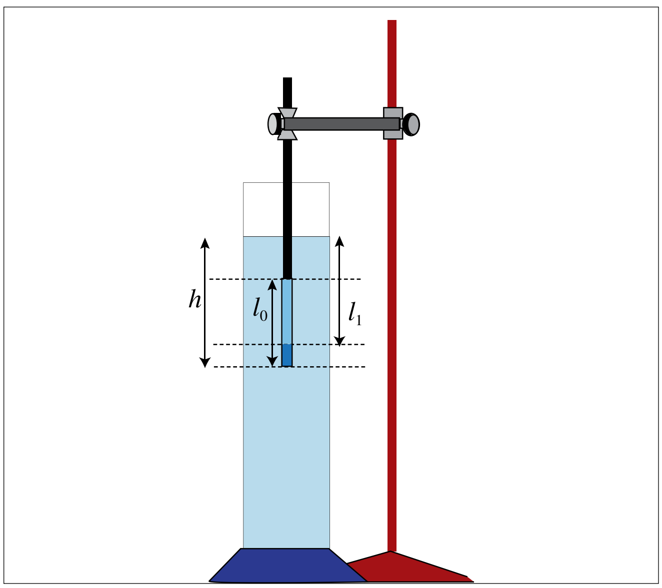

Fig. 1. Experimental setup used to measure the free-fall acceleration *g*. A cathetometer[2] placed at a proper distance has been used to get the l_0 and l_1 values measured at different depths *h*.

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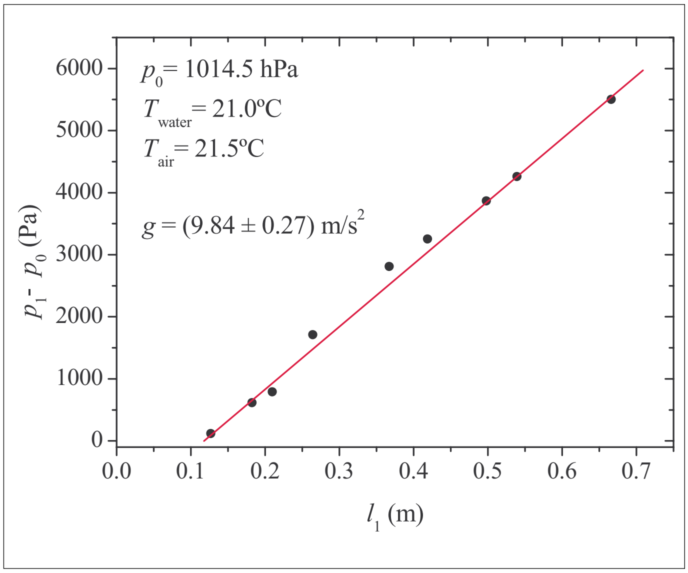

Fig. 2. Right-hand side of Eq. (6) vs l_1. The dots are the values resulting from our measurements while the straight line represents the corresponding linear fit.

spheric pressure value p_0 = 1014.5 hPa in Bahía Blanca at the time the experiment took place was obtained from the website of the National Meteorological Service of Argentina.[3] Finally, we have assumed that the density of the water was ρ = 1.00 g/cm^3.

Using measured values for h and l_1, the lhs of Eq. (6) is plotted versus l_1 in Fig. 2. The line shown is the result of a least square fit of the data shown. The obtained value for the free-fall acceleration is $g = (9.84 \pm 0.27)$ m/s^2.

We recall that in spite of the very simple nature of our device, we obtained a g value of 9.80 m/s^2. The overall uncertainty taking into account the scatter in the data is less than 3%. It is worth noting that the best results are obtained at the higher depths, where the capillarity effect and thermal expansion or contraction of the air enclosed are negligible.

To summarize, the simple experiment we describe involves a number of important physical concepts and is nicely suited to the typical introductory physics course.

Acknowledgments

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1. W. D. Ross, *The Works of Aristotle, Vol. VII. Problemata* (Oxford University Press, Oxford, 1927), p. 960b 30.
2. Alternatively, the distances can be measured using a ruler placed inside the cylinder.
3. National Meteorological Service (Argentina); www.smn.gov.ar.

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