

Positive parity pentaquark towers in large N_c QCDDan Pirjol¹ and Carlos Schat^{2,3}¹*Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA*²*CONICET and Departamento de Física, FCEyN, Universidad de Buenos Aires, Ciudad Universitaria, Pab.1, (1428) Buenos Aires, Argentina*³*Departamento de Física, Universidad de Murcia, E-30071 Murcia, Spain*

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We construct the complete set of positive parity pentaquarks, which correspond in the quark model to $\bar{s}q^{N_c+1}$ states with one unit of orbital angular momentum $L = 1$. In the large N_c limit they fall into the $\mathcal{K} = 1/2$ and $\mathcal{K} = 3/2$ irreps (towers) of the contracted $SU(4)_c$ symmetry. We derive predictions for the mass spectrum and the axial couplings of these states at leading order in $1/N_c$. The strong decay width of the lowest-lying positive parity exotic state is of order $O(1/N_c)$, such that this state is narrow in the large N_c limit. Replacing the antiquark with a heavy antiquark $\bar{Q}q^{N_c+1}$, the two towers become degenerate, split only by $O(1/m_Q)$ hyperfine interactions. We obtain predictions for the strong decay widths of heavy pentaquarks to ordinary baryons and heavy $H_Q^{(*)}$ mesons at leading order in $1/N_c$ and $1/m_Q$.

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I. INTRODUCTION

The last few years witnessed a renewed interest in hadronic physics, originated in part by many new findings in hadron spectroscopy, the most conspicuous being the narrow pentaquark states reported by more than ten independent experimental groups [1–7]. The narrow state predicted by a chiral soliton model [8] provided the initial motivation for the search of the Θ^+ pentaquark and its possible partners. After the reports of null results started to accumulate [9–11] the initial optimism declined, and the experimental situation remains ambiguous to the present. The increase in statistics led to some recent new claims for positive evidence [7], while the null result [10] by CLAS is specially significant because it contradicts their earlier positive result [2], suggesting that at least in their case the original claim was an artifact due to low statistics. All this experimental activity spurred a great amount of theoretical work in all kinds of models for hadrons and a renewed interest in soliton models. There is a great amount of uncertainty in model calculations that could be reduced with more experimental input, like the spin and parity of the reported exotic states [12] or the possible existence of spin-flavor partners [5,13]. Lattice QCD calculations are constantly improving but the situation also remains ambiguous [14,15], in part by the extrapolations to light masses and the difficulty to disentangle scattering states from bound states in a finite volume. Given the difficulties still faced by first principle QCD calculations, the $1/N_c$ expansion [16] of QCD, where N_c is the number of colors, provides a very useful analytical tool (for a recent account see [17]). It is based on the fundamental theory of the strong interactions, and relates the chiral soliton model to the more intuitive quark model picture [18–21], where the pentaquarks correspond to states with quark content $\bar{q}q^4$.

In the large N_c limit, QCD has a contracted spin-flavor symmetry $SU(2F)_c$ [22,23] in the baryon sector, also

known as \mathcal{K} -symmetry, that constrains the mass spectrum and couplings of the baryons. The breaking of the spin-flavor symmetry can be studied systematically in a $1/N_c$ expansion. This approach has been applied to the ground-state $[56, 0^+]$ baryons [23–26], and to their orbital excitations [27–33]. The large N_c expansion has also been applied to hybrid baryons [34] and more recently to exotic baryons containing both quarks and antiquarks [35–38].

In this paper we assume the existence of these exotic states, and investigate their properties in the case that they have positive parity. A partial subset of these states were considered in the $1/N_c$ expansion in Ref. [35]. Negative parity exotic states have been studied in [37–39]. A brief report of some results presented here has been given in [40]. We start by constructing the color singlet $\bar{s}q^{N_c+1}$ states by coupling the spin-flavor, orbital and color degrees of freedom, all constrained by Fermi statistics. The light exotic states we obtain are members of the $\mathcal{K} = \frac{1}{2}$ and $\mathcal{K} = \frac{3}{2}$ irreducible representations of the contracted spin-flavor symmetry. This extends the analysis of [35], which considered only the first irreducible representation.

Similar states with one heavy antiquark exist, that can be labeled in the large N_c limit by the conserved quantum number \mathcal{K}_ℓ associated with the light degrees of freedom. In our case of the positive parity pentaquarks, there is only one tower with $\mathcal{K}_\ell = 1$, containing all nonstrange states for which the isospin I and spin of the light degrees of freedom J_ℓ satisfy $|I - J_\ell| \leq \mathcal{K}_\ell \leq I + J_\ell$. An important difference with respect to the treatment of the strong decay amplitudes done in Ref. [35] is that we will keep the orbital angular momentum explicit in the transition operator, which is required to get the correct N_c scaling of the relevant couplings.

Although the existence of pentaquarks is not yet established or completely ruled out by experiments, one thing that seems to be fairly well established is that if they exist they should be narrow, with a width of $\Gamma \leq 1$ MeV, other-

wise they would have been observed long before [41]. Explanations for the uncanny narrow width vary. Cancellations between coupling constants have been invoked in the context of the original chiral soliton model [8]. This cancellation has been argued to be exact in the large N_c limit [42]. However, a detailed comparison of different versions of the chiral soliton model suggests that there is only one coupling constant to leading order and this cancellation cannot take place [43]. Recently, it has been argued [44] that heavy pentaquark states $\bar{Q}q^4$ should be narrow in the combined $1/m_Q$ and $1/N_c$ limit. Unfortunately, the experimental situation for the heavy pentaquark states $\bar{Q}q^4$ is also inconclusive. The charmed pentaquark initially reported by the H1 Collaboration [6] has not yet been confirmed [11,45].

We agree with the conclusions of Refs. [35,43] about the existence of a single operator mediating $\Theta \rightarrow NK$ transitions at leading order, but we find an overall suppression factor of $1/\sqrt{N_c}$. This gives the N_c scaling of the strong widths $\Gamma(\Theta \rightarrow NK) \sim O(1/N_c)$ for the positive parity pentaquarks. The corresponding pion widths among different Θ states scale like $\Gamma(\Theta \rightarrow \Theta'\pi) \sim O(N_c^0)$ if the transition $\Theta \rightarrow \Theta'$ is allowed by phase space. Any states for which the pion modes are not allowed, which include the lowest lying pentaquark, are thus predicted to be narrow in the large N_c limit.

The paper is organized as follows: In Sec. II we construct the complete set of the positive parity pentaquarks and give their tower structure in the large N_c limit. In Sec. III we discuss the strong couplings of the light pentaquarks in the language of quark operators. Section IV derives the large N_c predictions for the ratios of strong couplings from a consistency condition for $\pi + \Theta \rightarrow K + B$ scattering. In Sec. V we discuss the heavy pentaquarks in the combined large N_c and heavy quark symmetry limit. Finally, Sec. VI summarizes our conclusions.

II. CONSTRUCTING THE STATES

We start by discussing the construction of the exotic states, using the language of the constituent quark model in the large N_c limit. The quantum numbers of a $\bar{q}q^{N_c+1}$ hadron are constrained by the fact that the $N_c + 1$ quarks have to be in the fundamental representation of the color $SU(N_c)$ group. Fermi symmetry requires their spin-flavor-orbital wave function to be in the mixed symmetric representation $\mathcal{M}S_{N_c+1} = [N_c, 1, 0, \dots]$, where $[n_1, n_2, \dots]$ give the number of boxes in the first, second, etc. row of the corresponding Young tableau. This spin-flavor-orbital wave function can be decomposed into products of irreps of $SU(2F) \otimes O(3)$, i.e. spin-flavor wave functions with increasingly higher orbital angular momentum

$$\mathcal{M}S_{N_c+1} \rightarrow (MS_{N_c+1}, L = 0) \oplus (S_{N_c+1}, L = 1) \oplus \dots \quad (1)$$

The first term corresponds to negative parity exotic states. Their properties have been considered in the $1/N_c$ expansion in Refs. [37,38]. The second term corresponds to states with a symmetric $SU(2F)$ spin-flavor wave function for the q^{N_c+1} system, carrying one unit of orbital angular momentum $L = 1$. After adding in the antiquark, this produces positive parity exotics. A subset of these states were studied using the $1/N_c$ expansion in Ref. [35]. We reconsider them here, including all the states dictated by the contracted $SU(6)_c$ symmetry.

Adopting a Hartree description, one can think of the q^{N_c+1} system as consisting of N_c quarks in s -wave orbitals, plus one excited quark in a p -wave orbital. We write this schematically as

$$\Theta = \bar{q}q^{N_c}q^*, \quad (2)$$

with q^* denoting the orbitally excited quark. The spin-flavor of the excited quark is correlated with that of the q^{N_c} system such that the total system is in a symmetric representation of $SU(2F)$, with F the number of light quark flavors.

For $F = 3$ the minimal set of these states contains two irreducible representations of the contracted $SU(6)_c$ symmetry, with $\mathcal{K} = 1/2$ and $\mathcal{K} = 3/2$. The first few states in each of these representations are [38] (see Fig. 1)

$$\mathcal{K} = \frac{1}{2}: \bar{\mathbf{10}}_{1/2}, \mathbf{27}_{1/2,3/2}, \mathbf{35}_{3/2,5/2}, \dots, \quad (3)$$

$$\mathcal{K} = \frac{3}{2}: \bar{\mathbf{10}}_{3/2}, \mathbf{27}_{1/2,3/2,5/2}, \mathbf{35}_{1/2,3/2,5/2,7/2}, \dots \quad (4)$$

We use the \mathcal{K} label to denote an entire $SU(6)_c$ representation by the $SU(4)_c$ multiplets containing the states with maximal strangeness, sitting at the top of the corresponding weight diagrams of $SU(3)$. For the case considered in (3) and (4) these are the strangeness $\mathcal{S} = +1$ states with quark content $\bar{s}q^{N_c+1}$. We recall here that an irreducible representation of $SU(4)_c$ (tower with fixed strangeness) is labeled by $\mathcal{K} = 0, 1/2, 1, \dots$ and contains all states with

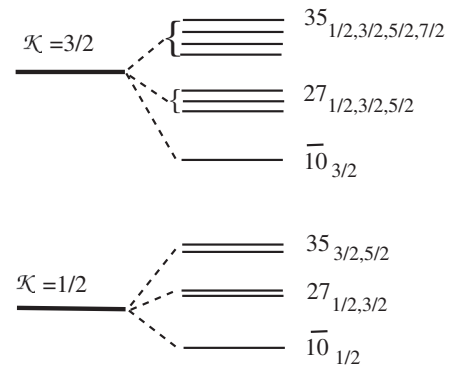


FIG. 1. Schematic representation of the mass spectrum of the positive parity pentaquarks. In the flavor symmetric large N_c limit, all these states are degenerate into two irreducible representations of the contracted symmetry, labeled with $\mathcal{K} = 1/2, 3/2$. The splittings within each tower are of order $\sim 1/N_c$.

spin J and isospin I satisfying $|I - J| \leq \mathcal{K} \leq I + J$. The first set of $\mathcal{K} = 1/2$ states has been considered in Ref. [35]. The treatment adopted here can describe both towers. In this paper we will also consider the $\mathcal{K} = 3/2$ tower in detail.

As the antiquark mass m_Q is increased, the two towers become closer in mass, split only by effects of order $O(1/m_Q)$ as a consequence of heavy quark symmetry [46]. This corresponds to the charmed or bottom exotic states $\bar{Q}q^{N_c+1}$, with $Q = c, b$. A more appropriate description for these states is given [38] in terms of one tower for the light degrees of freedom with $\mathcal{K}_\ell = 1$

$$\mathcal{K}_\ell = 1: \bar{\mathbf{6}}_1, \mathbf{15}_{0,1,2}, \mathbf{15}'_{1,2,3}, \dots \quad (5)$$

where the subscript denotes the spin of the light degrees of freedom J_ℓ . Each of these multiplets corresponds to a heavy quark spin doublet, with hadron spin $J = J_\ell \pm 1/2$, except for the singlets with $J_\ell = 0$. The properties of these states are studied below in Sec. V.

Next we discuss the relation between the different coupling schemes when constructing the pentaquark states $|\Theta; JI\rangle$ in terms of spin and orbital states. They are obtained by combining the system of $N_c + 1$ light quarks in a spin-flavor symmetric state $|S_q = I\rangle$ with the orbital angular momentum $|L = 1\rangle$ and the antiquark $|S_{\bar{q}} = \frac{1}{2}\rangle$

$$|\Theta; JI\rangle \in |q^{N_c+1}; S_q = I\rangle \otimes |L = 1\rangle \otimes |\bar{q}; S_{\bar{q}} = \frac{1}{2}\rangle. \quad (6)$$

$$|[L, S_{\bar{q}}]\mathcal{K}, S_q]JI; m\alpha\rangle = (-)^{I+1/2+L+J} \sum_{S=I\pm 1/2} \sqrt{[S][\mathcal{K}]} \begin{Bmatrix} I & \frac{1}{2} & S \\ L & J & \mathcal{K} \end{Bmatrix} |[L, (S_{\bar{q}}, S_q)S]JI; m\alpha\rangle, \quad (8)$$

where $[S] = 2S + 1$, etc. This recoupling relation fixes the mixing matrix of the pentaquark states in the large N_c limit. This is analogous to a result found for the $\mathbf{70}^-$ orbitally excited baryons, for which the mixing angles are determined in the large N_c limit, up to configuration mixing effects [27].

Finally, another possible choice for the pentaquark states combines first the light quark spin with \vec{L} into the angular momentum of the light degrees of freedom $\vec{J}_\ell = \vec{L} + \vec{S}_q$, which is then coupled with the antiquark spin to the total

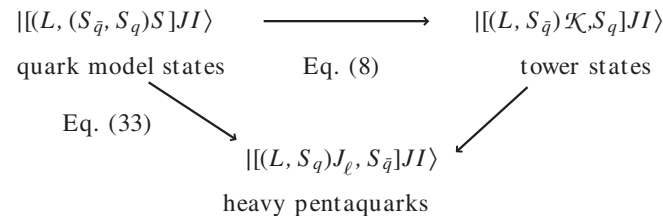


FIG. 2. The three possible couplings of the angular momenta in a pentaquark state with orbital excitation. The connection between the basis states is given by recoupling relations, given in the text.

The total hadron spin \vec{J} is thus given by

$$\vec{J} = \vec{S}_q + \vec{S}_{\bar{q}} + \vec{L}. \quad (7)$$

The three angular momenta $\vec{S}_q, \vec{S}_{\bar{q}}, \vec{L}$ can be coupled in several ways, which give different pentaquark states (see Fig. 2). The large N_c QCD states are obtained by coupling these three vectors in the order $\vec{L} + \vec{S}_{\bar{q}} = \vec{\mathcal{K}}, \vec{\mathcal{K}} + \vec{S}_q = \vec{J}$, with \mathcal{K} taking the two possible values $\mathcal{K} = 1/2, 3/2$. These states will be denoted $|[L, S_{\bar{q}}]\mathcal{K}, S_q]JI; m\alpha\rangle$, with $I = S_q$, and can be identified in the large N_c limit with the two towers corresponding to $\mathcal{K} = 1/2, 3/2$.

Another possible choice for the pentaquark states involves coupling first $\vec{S}_{\bar{q}} + \vec{S}_q = \vec{S}$, with \vec{S} the total spin of the quark-antiquark system. In a second step, the spin \vec{S} is coupled with the orbital angular momentum \vec{L} as $\vec{L} + \vec{S} = \vec{J}$, with \vec{J} the total spin of the hadron. We will denote these states as $|[L, (S_{\bar{q}}, S_q)S]JI; m\alpha\rangle$, and they are the most convenient choice for quark model computations of matrix elements. Note that this coupling scheme is arbitrary since S is not a good quantum number. On the other hand \mathcal{K} is the right quantum number that labels the physical states in the well defined large N_c limit of QCD.

The connection between the tower states and the quark model states is given by the usual recoupling formula for 3 angular momenta

spin of the baryon $\vec{J} = \vec{J}_\ell + \vec{S}_{\bar{q}}$. The corresponding states will be denoted as $|[L, S_q]J_\ell, S_{\bar{q}}]JI; m\alpha\rangle$ and are appropriate in the heavy antiquark limit, when the spin of the light degrees of freedom is a conserved quantum number. A detailed discussion of these states is given in Sec. V.

III. LIGHT PENTAQUARKS

We start by discussing the mass spectrum of the positive parity states. The formalism is very similar to that used for the $L = 2$ baryons in the $\mathbf{56}^+$ [30] and p -wave orbitally excited charm baryons [47]. The mass operator is a linear combination of the most general isoscalar parity even operators constructed from the orbital angular momentum L^i and the generators of the $SU(6)_q \otimes SU(6)_{\bar{q}}$ spin-flavor algebra [35], the quark operators S_q, G_q^{ia}, T_q^a and the antiquark operators $S_{\bar{q}}, G_{\bar{q}}^{ia}, T_{\bar{q}}^a$.

For simplicity we restrict our consideration here to the $S = +1$ pentaquarks with quark content $\bar{s}q^{N_c+1}$ and assume isospin symmetry. To leading order in the $1/N_c$ expansion, the mass operator acting on these states reads

$$\hat{M} = c_0 N_c \mathbf{1} + c_1 L^i S_{\bar{q}}^i + O(1/N_c), \quad (9)$$

where c_0 and c_1 are unknown constants. Spin-flavor symmetry is broken at leading order in $1/N_c$ by only one operator, describing the spin-orbit interaction of the anti-quark. This operator is diagonal in the \mathcal{K} basis given by Eq. (8) and is responsible for the $O(N_c^0)$ splitting of the two towers with $\mathcal{K} = 1/2$ and $3/2$. The situation is analogous to the one we have for the nonstrange members of the $\mathbf{70}^-$ excited baryons, where three irreducible representations of the \mathcal{K} -symmetry have different masses because there are three operators in the expansion of the mass operator up to $O(N_c^0)$ [32].

The states at the top of the weight diagram of a given $SU(3)$ representation (with maximal strangeness) can decay into a nonstrange ground-state baryon and a kaon $\Theta \rightarrow BK$. These transitions are mediated by the strangeness changing axial current and can be parametrized in terms of an operator $Y^{i\alpha}$ defined as

$$\langle B | \bar{s} \gamma^i \gamma_5 q^\alpha | \Theta \rangle = (Y^{i\alpha})_{B\Theta}, \quad (10)$$

with $B = N, \Delta, \dots$. The operator $Y^{i\alpha}$ can be expanded in $1/N_c$ as $Y^{i\alpha} = Y_0^{i\alpha} + Y_1^{i\alpha}/N_c + \dots$, where the leading term scales like $O(N_c^0)$, as will be shown below in Sec. IV.

At leading order in $1/N_c$, the explicit representation of $Y_0^{i\alpha}$ in the quark operator expansion gives only one operator

$$Y^{i\alpha} = g_0 \bar{s} \xi^i q^{*\alpha} + O(1/N_c), \quad (11)$$

where $\alpha = \pm 1/2$ denotes the flavor of the orbitally excited light quark $q^* = u, d$ and g_0 is an unknown constant that stands for the reduced matrix element of the QCD operator. In addition to the $SU(4)_q \otimes O(3)$ generators we now need as another basic building block an isoscalar vector operator acting on the orbital degrees of freedom, which we denote ξ^i .

In the following we compute the $\Theta \rightarrow N, \Delta$ matrix elements of the axial current operator, Eq. (11), and show that they can be expressed in terms of a few reduced matrix elements whose expressions are already known for arbitrary N_c . The matrix elements of the operator in Eq. (11) take the simplest form when expressed using the quark model states on the right-hand side of Eq. (8). They can be computed straightforwardly with the result

$$\begin{aligned} & \langle I' m' \alpha' | \bar{s} \xi^i q^{*\beta} | [L, (\frac{1}{2})S] J I; m \alpha \rangle \\ &= \frac{1}{\sqrt{N_c + 1}} \langle 0 | \xi^i | L \rangle \delta_{L,1} \delta_{S,I'} \sqrt{\frac{[J]}{[I']}} \\ & \times t(I', I) \begin{pmatrix} J & I & I' \\ m & i & m' \end{pmatrix} \begin{pmatrix} I & \frac{1}{2} & I' \\ \alpha & \beta & \alpha' \end{pmatrix}, \end{aligned} \quad (12)$$

where $t(I', I)$ is the reduced matrix element of the $\bar{s} q^\beta$ operator on spin-flavor symmetric states of the q^{N_c+1} system, defined as

$$\langle I' m' \alpha' | \bar{s} q^\beta | S I; m \alpha \rangle = t(I', I) \delta_{S I'} \delta_{m' m} \begin{pmatrix} I & \frac{1}{2} & I' \\ \alpha & \beta & \alpha' \end{pmatrix}. \quad (13)$$

The reduced matrix elements $t(I', I)$ for arbitrary N_c can be obtained easily using the occupation number formalism as described in Ref. [48]. Explicit results for $t(I', I)$ for all pentaquarks with quantum numbers of interest are tabulated in Ref. [48]. For completeness, we reproduce here the expressions needed in the following.

$$\begin{aligned} t\left(\frac{1}{2}, 0\right) &= \frac{1}{2} \sqrt{N_c + 1}, & t\left(\frac{1}{2}, 1\right) &= \frac{\sqrt{3}}{2} \sqrt{N_c + 5}, \\ t\left(\frac{3}{2}, 1\right) &= \frac{1}{2} \sqrt{\frac{3}{2}} \sqrt{N_c - 1}, & t\left(\frac{3}{2}, 2\right) &= \frac{1}{2} \sqrt{\frac{5}{2}} \sqrt{N_c + 7}. \end{aligned} \quad (14)$$

The explicit suppression factor of $O(1/\sqrt{N_c})$ in Eq. (12) arises because we have to annihilate the excited quark q^* carrying one unit of angular momentum. The detailed derivation of this factor is given below in Sec. IV. This suppression factor is absent in the case of negative parity pentaquarks where the $N_c + 1$ quarks are all in s -wave orbitals, and the axial current can annihilate any of them. The reduced matrix element in Eq. (13) scales like $t(I', I) \sim N_c^{1/2}$ [48], which implies that the $\Theta \rightarrow B$ matrix element of the axial current scales like $\langle Y^{i\alpha} \rangle \sim N_c^0$. This means that the $\Theta \rightarrow BK$ partial widths of these exotic states are suppressed as $1/N_c$ in the large N_c limit.

Finally, the matrix elements of the axial current on the tower (physical) pentaquark states are obtained by substituting Eq. (12) into the recoupling relation Eq. (8). Because of the Kronecker symbol $\delta_{S,I'}$, only one of the two terms on the right-hand side of Eq. (8) gives a nonvanishing contribution. The result is given by

$$\begin{aligned} & \langle I' m' \alpha' | Y_0^{i\beta} | [L, (\frac{1}{2})\mathcal{K}, S_q] J I; m \alpha \rangle \\ &= g_0 T(I', I J \mathcal{K}) \begin{pmatrix} J & 1 & I' \\ m & i & m' \end{pmatrix} \begin{pmatrix} I & \frac{1}{2} & I' \\ \alpha & \beta & \alpha' \end{pmatrix} \\ &= \bar{g}_0 t(I', I) \sqrt{[\mathcal{K}][J]} \begin{Bmatrix} I & \frac{1}{2} & I' \\ 1 & J & \mathcal{K} \end{Bmatrix} \begin{pmatrix} J & 1 & I' \\ m & i & m' \end{pmatrix} \begin{pmatrix} I & \frac{1}{2} & I' \\ \alpha & \beta & \alpha' \end{pmatrix}, \end{aligned} \quad (15)$$

with \bar{g}_0 an overall constant of order N_c^0 into which we absorbed the unknown orbital overlap matrix element.

We list in Tables I and II the reduced axial matrix elements $T(I', I J \mathcal{K})$ following from Eq. (15), corresponding to the two towers with $\mathcal{K} = 1/2$ and $\mathcal{K} = 3/2$, respectively. These tables show also the ratios of the p -wave $\Theta \rightarrow BK$ partial widths of these states. They are obtained as usual, by summing over final states and averaging over

TABLE I. Large N_c predictions for the p -wave strong decay amplitudes of the positive parity light pentaquarks in the $\mathcal{K} = 1/2$ tower. The last column shows the ratios of the partial p -wave rates, normalized to the $\overline{\mathbf{10}}_{1/2} \rightarrow NK$ width, and with the phase space factor $|\vec{p}|^3$ removed.

Decay	$T(I', IJ \frac{1}{2})$	$\frac{1}{p^3} \Gamma_{N_c=3}^{(p\text{-wave})}$	$\frac{1}{p^3} \Gamma_{N_c \rightarrow \infty}^{(p\text{-wave})}$
$\overline{\mathbf{10}}_{1/2} \rightarrow NK$	$\frac{1}{\sqrt{N_c+1}} t(\frac{1}{2}, 0)$	1	1
$27_{1/2} \rightarrow NK$	$\frac{1}{3\sqrt{N_c+1}} t(\frac{1}{2}, 1)$	$\frac{2}{9}$	$\frac{1}{9}$
$\rightarrow \Delta K$	$-\frac{2}{3\sqrt{N_c+1}} t(\frac{3}{2}, 1)$	$\frac{4}{9}$	$\frac{8}{9}$
$27_{3/2} \rightarrow NK$	$-\frac{2\sqrt{2}}{3\sqrt{N_c+1}} t(\frac{1}{2}, 1)$	$\frac{8}{9}$	$\frac{4}{9}$
$\rightarrow \Delta K$	$\frac{\sqrt{5}}{3\sqrt{N_c+1}} t(\frac{3}{2}, 1)$	$\frac{5}{18}$	$\frac{5}{9}$
$35_{3/2} \rightarrow \Delta K$	$\frac{1}{\sqrt{5(N_c+1)}} t(\frac{3}{2}, 2)$	$\frac{1}{2}$	$\frac{1}{5}$
$35_{5/2} \rightarrow \Delta K$	$-\frac{2}{\sqrt{5(N_c+1)}} t(\frac{3}{2}, 2)$	$\frac{4}{3}$	$\frac{8}{15}$

initial states

$$\Gamma_{p\text{-wave}} = g_0^2 \frac{[I']^2}{[J][I]} |T(I', IJ\mathcal{K})|^2 |\vec{p}|^3. \quad (16)$$

The predictions for the ratios of the decay amplitudes for the $\mathcal{K} = 1/2$ states agree with those of Ref. [35] after taking $N_c = 3$. The results for arbitrary N_c and the ratios for the $\mathcal{K} = 3/2$ states are new.

In the large N_c limit the ratios of strong decay widths satisfy sum rules. These sum rules express the equality of the widths of each tower state in each partial wave, and are a consequence of the contracted $SU(4)_c$ symmetry, which

TABLE II. Large N_c predictions for the p -wave strong decay amplitudes of the positive parity light pentaquarks in the $\mathcal{K} = 3/2$ tower. The last column shows the ratios of the partial p -wave rates, normalized to the $\overline{\mathbf{10}}_{3/2} \rightarrow NK$ width, with the phase space factor $|\vec{p}|^3$ removed.

Decay	$T(I', IJ \frac{3}{2})$	$\frac{1}{p^3} \Gamma_{N_c=3}^{(p\text{-wave})}$	$\frac{1}{p^3} \Gamma_{N_c \rightarrow \infty}^{(p\text{-wave})}$
$\overline{\mathbf{10}}_{3/2} \rightarrow NK$	$-\frac{\sqrt{2}}{\sqrt{N_c+1}} t(\frac{1}{2}, 0)$	1	1
$27_{1/2} \rightarrow NK$	$-\frac{2\sqrt{2}}{3\sqrt{N_c+1}} t(\frac{1}{2}, 1)$	$\frac{16}{9}$	$\frac{8}{9}$
$\rightarrow \Delta K$	$-\frac{1}{3\sqrt{2(N_c+1)}} t(\frac{3}{2}, 1)$	$\frac{1}{18}$	$\frac{1}{9}$
$27_{3/2} \rightarrow NK$	$\frac{\sqrt{10}}{3\sqrt{N_c+1}} t(\frac{1}{2}, 1)$	$\frac{10}{9}$	$\frac{5}{9}$
$\rightarrow \Delta K$	$\frac{2}{3\sqrt{N_c+1}} t(\frac{3}{2}, 1)$	$\frac{2}{9}$	$\frac{4}{9}$
$27_{5/2} \rightarrow \Delta K$	$-\sqrt{\frac{3}{2(N_c+1)}} t(\frac{3}{2}, 1)$	$\frac{1}{2}$	1
$35_{1/2} \rightarrow \Delta K$	$\frac{1}{\sqrt{2(N_c+1)}} t(\frac{3}{2}, 2)$	$\frac{5}{2}$	1
$35_{3/2} \rightarrow \Delta K$	$-\frac{2}{\sqrt{5(N_c+1)}} t(\frac{3}{2}, 2)$	2	$\frac{4}{5}$
$35_{5/2} \rightarrow \Delta K$	$\frac{1}{2} \sqrt{\frac{14}{5(N_c+1)}} t(\frac{3}{2}, 2)$	$\frac{7}{6}$	$\frac{7}{15}$

relates all tower states in the large N_c limit. They are given by

$$\Gamma(\Theta_{R_J} \rightarrow NK) + \Gamma(\Theta_{R_J} \rightarrow \Delta K) = \Gamma(\Theta_{\overline{\mathbf{10}}} \rightarrow NK), \quad (17)$$

where $R_J = 27_{1/2}, 27_{3/2}, 27_{5/2}, 35_{1/2}, \dots$. These sum rules can be checked explicitly using the results listed in the last column of Tables I and II.

These sum rules are not apparent in the results of Ref. [35], which correspond to the case of finite $N_c = 3$, for which the contracted symmetry is broken.

Walliser and Weigel [43] discussed the pentaquark strong width in the chiral soliton model. They found that only one operator contributes to the $\Theta \rightarrow NK$ coupling at leading order in the $1/N_c$ expansion and give the prediction $\Gamma(\Theta_{27_{3/2}})/\Gamma(\Theta_{\overline{\mathbf{10}}_{1/2}}) = 4/9$, which is in agreement with our model independent result in Table I.

IV. LARGE N_c POWER COUNTING AND CONSISTENCY CONDITIONS

The results of the preceding section on the $\Theta \rightarrow BK$ strong couplings take a particularly simple form at leading order in the $1/N_c$ expansion. This follows in a model-independent way from a consistency condition satisfied by the matrix elements of $Y^{i\alpha}$, similar to a consistency condition constraining kaon couplings to ordinary baryons [23].

We start by deriving in some detail the $Y^{i\alpha} \sim N_c^0$ scaling of the leading term, which follows from the special structure of the exotic states with positive parity considered in this work. In particular, we show that taking into account the nonzero orbital angular momentum $L = 1$ of these states is crucial in order to obtain the correct N_c scaling.

As explained in Sec. II, the exotic state can be written schematically as $\Theta = \bar{q} q^{N_c} q^*$, where the q^* quark carries one unit of angular momentum. The $N_c + 1$ quarks are in a completely symmetric spin-flavor wave function and a completely antisymmetric color-orbital wave function. This state can be described as a linear combination of terms with given occupation numbers for one-particle states

$$|q^{N_c} q^*\rangle = \sum_{\{n_i\}} c_{\{n_i\}} |\{n_1, n_2, n_3, n_4\}\rangle \otimes |[n_s^1, n_s^2, \dots, n_s^{N_c}; n_p^1, n_p^2, \dots, n_p^{N_c}]\rangle. \quad (18)$$

The first factor denotes the occupation numbers of the four spin-flavor one-quark states $\{u_\uparrow, u_\downarrow, d_\uparrow, d_\downarrow\}$ as defined in Ref. [48]. The second factor denotes the occupation numbers of the $2N_c$ possible orbital-color one-quark states. These are s - and p -wave orbitals, times the N_c possible color states $\phi_s(x) \otimes |i\rangle$ and $\phi_p(x) \otimes |i\rangle$, respectively. $\{n_i\}$ denotes the set of all occupation numbers. We consider in this paper only states with one quark in a p -wave orbital, and denote the color-orbital wave function of such states as $[N_c; 1]$.

The axial current is given by the operator in Eq. (11)

$$Y_0^{i\alpha} = g_0 \bar{s} \xi^i q^{*\alpha}, \quad (19)$$

where q^* annihilates the spin-flavor state of the orbitally excited quark and ξ^i acts on the orbital wave function of that state. When acting on the state, Eq. (18), this operator annihilates one quark in a p -wave orbital. Taking for definiteness $q^* = u_\uparrow^*$, the matrix element of the axial current reduces to evaluating expressions of the form

$$u_\uparrow^* |q^{N_c} q^*\rangle = \sum_{\{n_i\}} c_{\{n_i\}} u_\uparrow^* |n_1, n_2, n_3, n_4\rangle \otimes |[N_c; 1]\rangle. \quad (20)$$

The action of the annihilation operator u_\uparrow^* on the symmetric spin-flavor state can be computed using the methods of Ref. [48]. However, one subtle point is that this operator can only annihilate the excited quark, but not the other N_c s -wave quarks.

The spin-flavor state of the excited quark can be made explicit with the help of the identity

$$\begin{aligned} \{n_1, n_2, n_3, n_4\} &= \sqrt{\frac{n_1}{N_c + 1}} (u_\uparrow^{*\dagger}) \{n_1 - 1, n_2, n_3, n_4\} \\ &+ \sqrt{\frac{n_2}{N_c + 1}} (u_\uparrow^{*\dagger}) \{n_1, n_2 - 1, n_3, n_4\} \\ &+ \sqrt{\frac{n_3}{N_c + 1}} (d_\uparrow^{*\dagger}) \{n_1, n_2, n_3 - 1, n_4\} \\ &+ \sqrt{\frac{n_4}{N_c + 1}} (d_\uparrow^{*\dagger}) \{n_1, n_2, n_3, n_4 - 1\}, \end{aligned} \quad (21)$$

where $n_1 + n_2 + n_3 + n_4 = N_c + 1$. Using this identity, Eq. (20) can be evaluated explicitly with the result

$$u_\uparrow^* |q^{N_c} q^*\rangle = \sum_{\{n_i\}} c_{\{n_i\}} \sqrt{\frac{n_1}{N_c + 1}} |n_1 - 1, n_2, n_3, n_4\rangle \otimes |[N_c; 0]\rangle \quad (22)$$

and similarly for other spin-flavor states of the excited quark. These relations generalize the relations given in Ref. [48] for the action of one-body operators in the occupation number formalism to the case of more complicated orbital wave functions. Note the additional suppression factor $1/\sqrt{N_c + 1}$, which would not be present if all $N_c + 1$ quarks were in s -wave orbitals. Together with Eq. (12) given in the previous section, this completes the proof of the N_c scaling of the $\Theta \rightarrow B$ matrix elements of the axial current.

This scaling implies that the decay amplitude $A(\Theta \rightarrow BK)$ scales like $N_c^{-1/2}$, which in turn predicts that the corresponding strong decay widths are parametrically suppressed by $1/N_c$. This suppression may be obscured in the total widths of the Θ states by two possible mechanisms. First, the pion modes $\Theta \rightarrow \Theta' \pi$, whenever allowed by phase space, have widths of order $O(N_c^0)$. Second, mixing of the exotic states with radially excited nucleon states,

such as the Roper resonance, could enhance the N_c scaling of the decay amplitude as $A(\Theta \rightarrow \pi N) \sim O(N_c^0)$. None of these mechanisms applies to the lowest lying pentaquark state(s), for which the $1/N_c$ expansion offers thus another possible explanation for their small widths.

Accounting explicitly for the $L = 1$ orbital momentum of these states is crucial for obtaining the $Y^{i\alpha} \sim N_c^0$ scaling. This can be contrasted with the approach of Ref. [35], where the orbital angular momentum is not explicit. Instead, the angular momentum $L = 1$ is coupled with the antiquark spin $S_{\bar{q}}$ to a fixed value $\mathcal{K} = L + S_{\bar{q}} = 1/2$, and \mathcal{K} is effectively identified with $S_{\bar{q}} = 1/2$. The ξ^i operator acting on the orbital part does not appear in any of the operators describing physical quantities, such as masses, axial currents, etc. In this approach, the axial current operator mediating the $\Theta \rightarrow B$ transition is identified with [35]

$$Y_{(\text{no } \xi)}^{i\alpha} = g_0 \bar{s} \sigma^i q^\alpha + O(1/N_c) \quad (23)$$

and its matrix elements scale like $N_c^{1/2}$ [48].

We turn next to derive the leading behavior of $Y^{i\alpha}$ in a hadronic language. The matrix elements of the leading order piece $Y_0^{i\alpha}$ satisfy a consistency condition from $\pi^\alpha + \Theta \rightarrow K^\alpha + B$ scattering and can be obtained in a model independent way in the large N_c limit. The pion couplings to ordinary baryons and pentaquarks are parametrized by

$$\langle B' | \bar{q} \gamma^i \gamma_5 \tau^a q | B \rangle = N_c (X^{ia})_{B'B}, \quad (24)$$

$$\langle \Theta' | \bar{q} \gamma^i \gamma_5 \tau^a q | \Theta \rangle = N_c (Z^{ia})_{\Theta'\Theta}. \quad (25)$$

These operators have a $1/N_c$ expansion of the form $X^{ia} = X_0^{ia} + X_1^{ia}/N_c + \dots$, and similarly for Z^{ia} , where the leading order terms X_0^{ia} and Z_0^{ia} scale as $O(N_c^0)$.

After including the meson decay constants, the overall scaling of the direct and crossed diagrams separately is $O(N_c^0)$. The calculation of the scattering amplitude at the quark level gives a $1/\sqrt{N_c}$ scaling for the $\pi^\alpha + \Theta \rightarrow K^\alpha + B$ amplitude. This leads to the consistency condition

$$Y_0^{j\alpha} Z_0^{ia} - X_0^{ia} Y_0^{j\alpha} = 0. \quad (26)$$

The derivation is similar to the one given for the consistency condition of meson couplings to ordinary and hybrid baryons in Ref. [34].

The leading order matrix element $\langle X_0^{ia} \rangle$ is given by the model-independent expression [22,23]

$$\begin{aligned} \langle X_0^{ia} \rangle &= g_X (-)^{J+I'+\mathcal{K}+1} \sqrt{[I][J]} \begin{Bmatrix} I' & 1 & I \\ J & K & J' \end{Bmatrix} \\ &\times \begin{pmatrix} J & 1 & J' \\ J_3 & i & J'_3 \end{pmatrix} \begin{pmatrix} I & 1 & I' \\ I_3 & a & I'_3 \end{pmatrix} \end{aligned} \quad (27)$$

and similarly for $\langle Z_0^{ia} \rangle$ with g_Z instead of g_X . The $\Theta \rightarrow K^\alpha + B$ vertex is parametrized by

$$\begin{aligned} & \langle I' I'_3; J' J'_3; \mathcal{K}' | Y_0^{i\alpha} | I I_3; J J_3; \mathcal{K} \rangle \\ & = \sqrt{[I][J]} \mathcal{Y}_0(I' J' \mathcal{K}'; I J \mathcal{K}) \begin{pmatrix} J & 1 & J' \\ J_3 & i & J'_3 \end{pmatrix} \begin{pmatrix} I & \frac{1}{2} & I' \\ I_3 & \alpha & I'_3 \end{pmatrix}. \end{aligned} \quad (28)$$

$$\begin{aligned} & \sum_{\bar{I}, \bar{J}} (-)^{-\bar{I}-(1/2)} [\bar{I}][\bar{J}] \begin{Bmatrix} 1 & I' & \bar{I} \\ \mathcal{K}' & \bar{J} & J' \end{Bmatrix} \begin{Bmatrix} 1 & \bar{J} & J' \\ 1 & H & J \end{Bmatrix} \begin{Bmatrix} \frac{1}{2} & \bar{I} & I \\ 1 & T & I' \end{Bmatrix} \mathcal{Y}_0(\bar{I} \bar{J} \mathcal{K}'; I J \mathcal{K}) \\ & = (-)^{H+\mathcal{K}+\mathcal{K}'-I'-J} \delta(J' 1 H) \delta\left(I' \frac{1}{2} T\right) \frac{g_Z}{g_X} \begin{Bmatrix} 1 & T & I \\ \mathcal{K} & J & H \end{Bmatrix} \mathcal{Y}_0(I' J' \mathcal{K}'; T H \mathcal{K}), \end{aligned} \quad (29)$$

where $\delta(J' 1 H) = 1$ if $|J' - 1| \leq H \leq J' + 1$ and zero otherwise, etc. The most general solution of this equation implies $g_X = g_Z$, and depends on two arbitrary couplings c_y with $y = 1/2, 3/2$

$$\mathcal{Y}_0(I' J' \mathcal{K}'; I J \mathcal{K}) = \sum_{y=1/2, 3/2} c_y \begin{Bmatrix} \frac{1}{2} & 1 & y \\ I & J & \mathcal{K} \\ I' & J' & \mathcal{K}' \end{Bmatrix}, \quad (30)$$

up to an arbitrary phase $(-1)^{2n+2mJ}$ with n, m integers. This is the main result of this section and gives the most general solution for the $\Theta \rightarrow BK$ coupling at leading order in $1/N_c$. For decays to nonstrange baryons $\mathcal{K}' = 0$, and this equation gives the asymptotic form for $T(I', IJ\mathcal{K})$ in the large N_c limit

$$\lim_{N_c \rightarrow \infty} T(I', IJ\mathcal{K}) \propto (-)^{1+I+I'+\mathcal{K}} \sqrt{\frac{[I][J]}{[I'][\mathcal{K}]}} \begin{Bmatrix} \frac{1}{2} & 1 & \mathcal{K} \\ J & I & I' \end{Bmatrix}. \quad (31)$$

This agrees with the large N_c limit of the reduced matrix element obtained by the quark operator calculation in Sec. III.

V. HEAVY PENTAQUARKS

Taking the antiquark to be a heavy quark $Q = c, b$, the quantum numbers of the $\bar{Q}q^{N_c+1}$ states are simply related

$$|[(L, S_q)J_\ell, S_{\bar{q}}]JI; m\alpha\rangle = (-)^{I+1/2+L+J} \sum_{S=I\pm 1/2} \sqrt{[S][J_\ell]} \begin{Bmatrix} I & \frac{1}{2} & S \\ J & L & J_\ell \end{Bmatrix} |[(L, (S_q, S_{\bar{q}})S]JI; m\alpha\rangle. \quad (33)$$

A similar recoupling relation can be written which expresses the heavy pentaquark states $|[(L, S_q)J_\ell, S_{\bar{q}}]JI; m\alpha\rangle$ in terms of tower states $|[(L, S_{\bar{q}})\mathcal{K}, S_q]JI; m\alpha\rangle$, appropriate for the light pentaquark states. Such a relation makes explicit the correspondence between light and heavy pentaquarks, as the mass of \bar{Q} is gradually increased. We do not write this relation explicitly, but just mention one of its implications: any given heavy pentaquark state is related to states in both $\mathcal{K} = 1/2$ and $3/2$ towers. Any treatment which neglects one of these towers will therefore be difficult to reconcile with a quark model picture.

For $\mathcal{K}' = 0$ this expression is equivalent to Eq. (15), with the identification $g_0 T(I', IJ\mathcal{K}) = \sqrt{[I][J]} \mathcal{Y}_0(I' I' 0; IJ\mathcal{K})$. Taking the matrix elements of Eq. (26) between $B(I' J' \mathcal{K}')$ and $\Theta(IJ\mathcal{K})$, and projecting onto channels with total spin H and isospin T in the s -channel we obtain

to those of the $N_c + 1$ quarks, as was discussed earlier in Sec. II. These states belong to one large N_c tower with $\mathcal{K}_\ell = 1$ and are shown in Eq. (5).

We pause here to compare these states with the positive parity heavy pentaquarks considered in [35]. The light quarks in those states belong to a $\mathcal{K}_\ell = 0$ tower and include the $SU(3)$ representations

$$\mathcal{K}_\ell = 0: \bar{\mathbf{6}}_0, \mathbf{15}_1, \mathbf{15}'_2, \dots \quad (32)$$

where the subscript denotes the spin of the light degrees of freedom J_ℓ . Each of these multiplets corresponds to a heavy quark spin doublet, with hadron spin $J = J_\ell \pm 1/2$, except for the singlets with $J_\ell = 0$. Note that they are different from the states constructed here in Eq. (5), which in addition to the more complex mass spectrum also have very different strong couplings, as will be seen below.

The heavy pentaquark states require a different recoupling of the three angular momenta $S_q, S_{\bar{q}}, L$. Neither S_q nor L is a good quantum number for a heavy pentaquark, but only their sum, the angular momentum of the light degrees of freedom $J_\ell = S_q + L$, is. The states with good J_ℓ are expressed in terms of the quark model states by a recoupling relation analogous to Eq. (8)

A generic positive parity heavy pentaquark state $\Theta_{\bar{Q}}$ can decay strongly into the four channels

$$\Theta_{\bar{Q}} \rightarrow NH_{\bar{Q}}, NH_{\bar{Q}}^*, \quad (34)$$

$$\Theta_{\bar{Q}} \rightarrow \Delta H_{\bar{Q}}, \Delta H_{\bar{Q}}^*, \quad (35)$$

where $H_{\bar{Q}}$ is a pseudoscalar $J^P = 0^-$ heavy meson with quark content $\bar{Q}q$, and $H_{\bar{Q}}^*$ is its heavy quark spin partner with $J^P = 1^-$. Heavy quark symmetry gives relations among the amplitudes for these decays [46]. However,

these relations alone are in general not sufficient to predict the ratios of the decay widths of the two modes in Eq. (34), and of the two modes in Eq. (35). On the other hand, large N_c relates the $NH_{\bar{Q}}$ and $\Delta H_{\bar{Q}}$ modes. We will show that in the *combined* large N_c and heavy quark limits it is also possible to make predictions for the ratios of the $NH_{\bar{Q}}$ and $NH_{\bar{Q}}^*$ modes.

We start by considering first the large N_c predictions for the amplitude ratios into $NH_{\bar{Q}}$ and $\Delta H_{\bar{Q}}$. These decays are mediated by the heavy-light axial current, which at leading order in $1/N_c$ is given by a single operator in the quark operator expansion, analogous to that mediating kaon decay

$$\langle B | \bar{Q} \gamma^i \gamma_5 q^\beta | \Theta_{\bar{Q}} \rangle = g_Q (\bar{Q} \xi^i q^\beta)_{B\Theta} + O(1/N_c), \quad (36)$$

with $B = N, \Delta, \dots$ and g_Q an unknown constant.

The matrix elements of this operator can be parametrized in terms of a reduced matrix element, defined as

$$\begin{aligned} & \langle I' m' \alpha' | \bar{Q} \xi^i q^\beta | [(L, S_q) J_\ell, \frac{1}{2}] J I ; m \alpha \rangle \\ &= T(I', I J J_\ell) \begin{pmatrix} J & 1 & I' \\ m & i & m' \end{pmatrix} \begin{pmatrix} I & \frac{1}{2} & I' \\ \alpha & \beta & \alpha' \end{pmatrix}. \end{aligned} \quad (37)$$

At leading order in $1/N_c$, the amplitudes $T(I', I J J_\ell)$ can be expressed in terms of the amplitudes $t(I', J I)$ given in Eq. (14). To show this, recall that the matrix elements of the axial current for the transitions between the quark model states and the ground state baryons were given in Eq. (12). The corresponding matrix elements taken on the physical heavy pentaquark states are found by substituting this result into the recoupling relation Eq. (33). We find

$$\begin{aligned} & \langle I' m' \alpha' | g_Q \bar{Q} \xi^i q^\beta | [(L, S_q) J_\ell, \frac{1}{2}] J I ; m \alpha \rangle \\ &= \bar{g}_Q \frac{1}{\sqrt{N_c+1}} t(I', I) \sqrt{[J_\ell][J]} \begin{Bmatrix} I & \frac{1}{2} & I' \\ J & 1 & J' \end{Bmatrix} \\ & \times \begin{pmatrix} J & 1 & I' \\ m & i & m' \end{pmatrix} \begin{pmatrix} I & \frac{1}{2} & I' \\ \alpha & \beta & \alpha' \end{pmatrix}, \end{aligned} \quad (38)$$

where the unknown orbital overlap matrix element has also been absorbed in the order N_c^0 unknown constant \bar{g}_Q .

We summarize in Table III the reduced matrix elements $T(I', I J J_\ell)$ of the leading order operator in the expansion of the heavy-light axial current, for heavy pentaquarks with positive parity. We denote the pentaquark states as $\Theta_{\bar{Q}J_\ell}^{(R)}(J)$, with $R = \bar{\mathbf{6}}, \mathbf{15}, \mathbf{15}'$ the $SU(3)$ representation to which they belong. These results give predictions for the ratios of the partial widths of p -wave strong decays $\Theta_{\bar{Q}J_\ell}^{(R)}(J) \rightarrow NH_{\bar{Q}}, \Delta H_{\bar{Q}}$.

TABLE III. Large N_c predictions for the p -wave heavy pentaquark decay amplitudes $\Theta_{\bar{Q}J_\ell}^{(R)} \rightarrow NH_{\bar{Q}}$ and $\Theta_{\bar{Q}J_\ell}^{(R)} \rightarrow \Delta H_{\bar{Q}}$. In the last column we show the ratios of the partial p -wave rates, with the phase space factor p^3 removed.

Decay	$T(I', I J J_\ell)$	$\frac{1}{p^3} \Gamma_{N_c=3}^{(p\text{-wave})}$	$\frac{1}{p^3} \Gamma_{N_c \rightarrow \infty}^{(p\text{-wave})}$
$\Theta_{\bar{Q}1}^{(\bar{\mathbf{6}})}(\frac{1}{2}) \rightarrow NH_{\bar{Q}}$	$\frac{1}{\sqrt{N_c+1}} t(\frac{1}{2}, 0)$	1	1
$\Theta_{\bar{Q}1}^{(\bar{\mathbf{6}})}(\frac{3}{2}) \rightarrow NH_{\bar{Q}}$	$-\sqrt{\frac{2}{N_c+1}} t(\frac{1}{2}, 0)$	1	1
$\Theta_{\bar{Q}0}^{(\mathbf{15})}(\frac{1}{2}) \rightarrow NH_{\bar{Q}}$	$\frac{1}{\sqrt{3(N_c+1)}} t(\frac{1}{2}, 1)$	$\frac{2}{3}$	$\frac{1}{3}$
$\rightarrow \Delta H_{\bar{Q}}$	$-\frac{1}{\sqrt{3(N_c+1)}} t(\frac{3}{2}, 1)$	$\frac{1}{3}$	$\frac{2}{3}$
$\Theta_{\bar{Q}1}^{(\mathbf{15})}(\frac{1}{2}) \rightarrow NH_{\bar{Q}}$	$-\sqrt{\frac{2}{3(N_c+1)}} t(\frac{1}{2}, 1)$	$\frac{4}{3}$	$\frac{2}{3}$
$\rightarrow \Delta H_{\bar{Q}}$	$-\frac{1}{\sqrt{6(N_c+1)}} t(\frac{3}{2}, 1)$	$\frac{1}{6}$	$\frac{1}{3}$
$\Theta_{\bar{Q}1}^{(\mathbf{15})}(\frac{3}{2}) \rightarrow NH_{\bar{Q}}$	$-\frac{1}{\sqrt{3(N_c+1)}} t(\frac{1}{2}, 1)$	$\frac{1}{3}$	$\frac{1}{6}$
$\rightarrow \Delta H_{\bar{Q}}$	$\sqrt{\frac{5}{6(N_c+1)}} t(\frac{3}{2}, 1)$	$\frac{5}{12}$	$\frac{5}{6}$
$\Theta_{\bar{Q}2}^{(\mathbf{15})}(\frac{3}{2}) \rightarrow NH_{\bar{Q}}$	$\sqrt{\frac{5}{3(N_c+1)}} t(\frac{1}{2}, 1)$	$\frac{5}{3}$	$\frac{5}{6}$
$\rightarrow \Delta H_{\bar{Q}}$	$\frac{1}{\sqrt{6(N_c+1)}} t(\frac{3}{2}, 1)$	$\frac{1}{12}$	$\frac{1}{6}$
$\Theta_{\bar{Q}2}^{(\mathbf{15})}(\frac{5}{2}) \rightarrow \Delta H_{\bar{Q}}$	$-\sqrt{\frac{3}{2(N_c+1)}} t(\frac{3}{2}, 1)$	$\frac{1}{2}$	1
$\Theta_{\bar{Q}1}^{(\mathbf{15}')}(\frac{1}{2}) \rightarrow \Delta H_{\bar{Q}}$	$\frac{1}{\sqrt{2(N_c+1)}} t(\frac{3}{2}, 2)$	$\frac{5}{2}$	1
$\Theta_{\bar{Q}1}^{(\mathbf{15}')}(\frac{3}{2}) \rightarrow \Delta H_{\bar{Q}}$	$\frac{1}{\sqrt{10(N_c+1)}} t(\frac{3}{2}, 2)$	$\frac{1}{4}$	$\frac{1}{10}$
$\Theta_{\bar{Q}2}^{(\mathbf{15}')}(\frac{3}{2}) \rightarrow \Delta H_{\bar{Q}}$	$-\frac{3}{\sqrt{10(N_c+1)}} t(\frac{3}{2}, 2)$	$\frac{9}{4}$	$\frac{9}{10}$
$\Theta_{\bar{Q}2}^{(\mathbf{15}')}(\frac{5}{2}) \rightarrow \Delta H_{\bar{Q}}$	$-\frac{1}{\sqrt{10(N_c+1)}} t(\frac{3}{2}, 2)$	$\frac{1}{6}$	$\frac{1}{15}$
$\Theta_{\bar{Q}3}^{(\mathbf{15}')}(\frac{5}{2}) \rightarrow \Delta H_{\bar{Q}}$	$\sqrt{\frac{7}{5(N_c+1)}} t(\frac{3}{2}, 2)$	$\frac{7}{3}$	$\frac{14}{15}$

In the case of heavy pentaquarks there is a second sum rule

$$\sum_{J_\ell} \Gamma(\Theta_{\bar{Q}J_\ell}^{(R)}(J) \rightarrow NH_{\bar{Q}}) = \Gamma(\Theta_{\bar{Q}1}^{(\bar{\mathbf{6}})}(J) \rightarrow NH_{\bar{Q}}), \quad (39)$$

in addition to the one already discussed in Eq. (17). Both hold in the large N_c limit and can be checked explicitly using the results in the last column of Table III.

Finally, we will also use heavy quark symmetry to make predictions for modes containing a $H_{\bar{Q}}^*$ heavy meson in the final state. To that end, we first describe the heavy quark symmetry relations and then we combine them with the large N_c predictions. For simplicity, we restrict ourselves to the $NH_{\bar{Q}}^{(*)}$ modes in a p -wave.

The specification of the final state in $\Theta_{\bar{Q}J_\ell} \rightarrow [NH_{\bar{Q}}^*]_{p\text{-wave}}$ must include in addition to the total angular momentum $\vec{J} = \vec{S}_N + \vec{J}_{H_{\bar{Q}}^*} + \vec{L}$, also the partial sum of two of the three angular momenta. We denote here with \vec{S}_N the spin of the nucleon, $\vec{J}_{H_{\bar{Q}}^*}$ the spin of the vector heavy meson, and \vec{L} the orbital angular momentum. The heavy quark symmetry relations take a simple form when this

partial sum is chosen as $\vec{J}_N = \vec{S}_N + \vec{L}$. The decay amplitude for $\Theta_{\bar{Q}}(IJJ_\ell) \rightarrow [NH_{\bar{Q}}^{(*)}(J'J'_\ell)]_{J_N}$ is given by [46]

$$A(\Theta_{\bar{Q}}(IJJ_\ell) \rightarrow [NH_{\bar{Q}}^{(*)}(J'J'_\ell)]_{J_N}) = \sqrt{[J_\ell][J]} \begin{Bmatrix} J_\ell & J'_\ell & J_N \\ J' & J & \frac{1}{2} \end{Bmatrix} F_{J_\ell J'_\ell J_N}^I, \quad (40)$$

where we denoted as usual with J_ℓ and $J'_\ell = 1/2$ the spins of the light degrees of freedom in the initial and final heavy hadrons. $F_{J_\ell J'_\ell J_N}^I$ are reduced matrix elements, which in general also depend on S_N , although this dependence was omitted for simplicity.

The predictions from these relations are shown in explicit form in Table IV, from which one can read off the number of independent hadronic amplitudes parametrizing each mode. The $\Theta_{\bar{Q}0}(\frac{1}{2})$ decays are parametrized in terms of one reduced amplitude f_0 , the decays of the $\Theta_{\bar{Q}1}(\frac{1}{2}, \frac{3}{2})$ depend on two independent amplitudes $f_{1,2}$, and the decays of the $\Theta_{\bar{Q}2}(\frac{3}{2}, \frac{5}{2})$ contain one independent amplitude f_3 . From this counting it follows that heavy quark symmetry does not relate, in general, all modes with pseudoscalar and vector heavy mesons.

Such a relation becomes possible however in the large N_c limit, where all modes with a pseudoscalar heavy meson in the final state $NH_{\bar{Q}}$ are related. In the language of the reduced amplitudes in Table IV, this amounts to a relation among the amplitudes f_{0-3}^I . These predictions can be obtained by comparing the amplitudes listed in Tables III and IV. We find

$$f_0^{I=1} \equiv F_{0,1/2,1/2}^{I=1} = -\sqrt{\frac{N_c + 5}{N_c + 1}}, \quad (41)$$

$$f_1^{I=0} \equiv F_{1,1/2,1/2}^{I=0} = \frac{1}{\sqrt{3}}, \quad f_2^{I=0} \equiv F_{1,1/2,3/2}^{I=0} = \sqrt{\frac{2}{3}}, \quad (42)$$

TABLE IV. Heavy quark symmetry predictions for the reduced decay amplitudes $\Theta_{\bar{Q}J_\ell} \rightarrow [NH_{\bar{Q}}^{(*)}]_{p\text{-wave}}$ for final states with given $\vec{J}_N = \vec{S}_N + \vec{L}$. The zero entries denote amplitudes forbidden in the heavy quark limit and suppressed as $\sim O(\Lambda_{\text{QCD}}/m_Q)$.

Decay	$J_N = 1/2$	$J_N = 3/2$	Decay	$J_N = 1/2$	$J_N = 3/2$
$\Theta_{\bar{Q}0}(\frac{1}{2}) \rightarrow NH_{\bar{Q}}$	$-\frac{1}{\sqrt{2}}f_0$	—	$\Theta_{\bar{Q}0}(\frac{1}{2}) \rightarrow NH_{\bar{Q}}^*$	$\frac{1}{\sqrt{2}}f_0$	0
$\Theta_{\bar{Q}1}(\frac{1}{2}) \rightarrow NH_{\bar{Q}}$	$\sqrt{\frac{5}{2}}f_1$	—	$\Theta_{\bar{Q}1}(\frac{1}{2}) \rightarrow NH_{\bar{Q}}^*$	$\frac{1}{\sqrt{6}}f_1$	$-\sqrt{\frac{2}{3}}f_2$
$\Theta_{\bar{Q}1}(\frac{3}{2}) \rightarrow NH_{\bar{Q}}$	—	$-\sqrt{\frac{5}{2}}f_2$	$\Theta_{\bar{Q}1}(\frac{3}{2}) \rightarrow NH_{\bar{Q}}^*$	$-\frac{2}{\sqrt{3}}f_1$	$\sqrt{\frac{5}{6}}f_2$
$\Theta_{\bar{Q}2}(\frac{3}{2}) \rightarrow NH_{\bar{Q}}$	—	$\sqrt{\frac{5}{2}}f_3$	$\Theta_{\bar{Q}2}(\frac{3}{2}) \rightarrow NH_{\bar{Q}}^*$	0	$\frac{1}{\sqrt{2}}f_3$
$\Theta_{\bar{Q}2}(\frac{5}{2}) \rightarrow NH_{\bar{Q}}$	—	—	$\Theta_{\bar{Q}2}(\frac{5}{2}) \rightarrow NH_{\bar{Q}}^*$	—	$-\sqrt{2}f_3$

TABLE V. Combined large N_c and heavy quark symmetry predictions for the ratios of strong decay rates for heavy pentaquark decays $\Theta_{\bar{Q}} \rightarrow NH_{\bar{Q}}^{(*)}$. In the last line we show for comparison also the corresponding predictions for the pentaquark states with positive parity considered in Ref. [35].

$I = 0$	$\Theta_{\bar{Q}}(\frac{1}{2}) \rightarrow (NH_{\bar{Q}}):(NH_{\bar{Q}}^*)$	$\Theta_{\bar{Q}}(\frac{3}{2}) \rightarrow (NH_{\bar{Q}}):(NH_{\bar{Q}}^*)$
$\mathcal{K}_\ell = 1$	$\frac{1}{2}:\frac{3}{2} (J_\ell = 1)$	$\frac{1}{2}:\frac{3}{2} (J_\ell = 1)$
$\mathcal{K}_\ell = 0$	$1:3 (J_\ell = 0)$	—
$I = 1$	$\Theta_{\bar{Q}}(\frac{1}{2}) \rightarrow (NH_{\bar{Q}}):(NH_{\bar{Q}}^*)$	$\Theta_{\bar{Q}}(\frac{3}{2}) \rightarrow (NH_{\bar{Q}}):(NH_{\bar{Q}}^*)$
$\mathcal{K}_\ell = 1$	$\frac{1}{6}:\frac{1}{2} (J_\ell = 0)$	$\frac{1}{12}:\frac{7}{12} (J_\ell = 1)$
	$\frac{1}{3}:\frac{1}{3} (J_\ell = 1)$	$\frac{5}{12}:\frac{1}{4} (J_\ell = 2)$
$\mathcal{K}_\ell = 0$	$1:11 (J_\ell = 1)$	$4:8 (J_\ell = 1)$

$$f_1^{I=1} \equiv F_{1,1/2,1/2}^{I=1} = -\sqrt{\frac{2}{3}}\sqrt{\frac{N_c + 5}{N_c + 1}}, \quad (43)$$

$$f_2^{I=1} \equiv F_{1,1/2,3/2}^{I=1} = \frac{1}{\sqrt{3}}\sqrt{\frac{N_c + 5}{N_c + 1}},$$

$$f_3^{I=1} \equiv F_{2,1/2,3/2}^{I=1} = \sqrt{\frac{N_c + 5}{N_c + 1}}. \quad (44)$$

The corresponding predictions for the partial decay rates are shown in Table V. For comparison, we also show in this table the results found in Ref. [35] for the $\mathcal{K}_\ell = 0$ states.

Note that the predictions are different for the states considered here and in Ref. [35]. This difference can be traced back to the different \mathcal{K}_ℓ quantum numbers of the two sets of states and can be used to distinguish between them experimentally.

VI. CONCLUSIONS

In this paper we studied the complete set of light and heavy pentaquark states with positive parity at leading order in the $1/N_c$ expansion. We discussed the structure of the mass spectrum and the strong decays of these states. Both are strongly constrained by the contracted spin-flavor

$SU(4)_c$ symmetry emerging in the large N_c limit, leading to mass degeneracies and sum rules for their decay widths.

The exotic states which are composed of only light quarks ($\bar{s}q^{N_c+1}$) belong to two irreducible representations (towers) of this symmetry, with $\mathcal{K} = \frac{1}{2}$ (containing a $J^P = \frac{1}{2}^+$ isosinglet), and $\mathcal{K} = \frac{3}{2}$ (containing a $J^P = \frac{3}{2}^+$ isosinglet), respectively. The strong transitions between any members of a pair of towers are related by the contracted symmetry. The states with $\mathcal{K} = \frac{1}{2}$ are identical to those considered in the large N_c expansion in Ref. [35], and we find complete agreement with their $N_c = 3$ predictions for these states. The more general results for arbitrary N_c and the $\mathcal{K} = \frac{3}{2}$ tower states are new.

Taking the antiquark to be a heavy quark, the two irreducible representations with $\mathcal{K} = \frac{1}{2}, \frac{3}{2}$ are split only by $O(1/m_Q)$ hyperfine interactions. In the heavy quark limit they become degenerate and the spin of the light degrees of freedom is a conserved quantum number. When this is combined with the large N_c limit a new good quantum number emerges: \mathcal{K}_ℓ , the tower label for the light quarks.

We find that the heavy pentaquarks with positive parity belong to one tower with $\mathcal{K}_\ell = 1$. These are different from the heavy exotic states considered in Ref. [35], which belong to $\mathcal{K}_\ell = 0$. Both sets of states are legitimate

from the point of view of the large N_c symmetry, although explicit realizations of these states are more natural in different models: the $\mathcal{K}_\ell = 0$ states are obtained in a Skyrme model picture, while the $\mathcal{K}_\ell = 1$ states considered here appear naturally in the constituent quark model picture. The predictions for the strong decays of the two sets of states differ, as shown in Table V, which can be used to discriminate between them.

There is an important difference between our treatment of the transition operators and that given in Ref. [35], due to the fact that we keep the orbital angular momentum explicit. In Sec. IV we show, by explicit computation of the $\Theta \rightarrow B$ axial current matrix element in the quark model with N_c colors, that the strong width of the lowest-lying positive parity exotic state scales like $1/N_c$. This provides a natural explanation for the existence of narrow exotic states in the large N_c limit.

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