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A Three-dimensional orbit for the binary star Alpha Andromedae

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ABSTRACT
Stars that are both spectroscopic and optical binaries present a means to determine simultaneously the masses of the components and the distance of the system independent of trigonometric parallax. Alpha Andromedae (Alpheratz) represents such a system and, moreover, the primary is the brightest of the mercury-manganese stars. An orbit, based on 42 interferometric observations and 378 radial velocities, is calculated to solve for 10 parameters: the 6 coefficients of the apparent ellipse, the constant of areal velocity, the systemic velocity, and the semi-amplitudes. From these one calculates the orbit of the binary, its period and time of periastron passage, the masses of the components, and the distance of the system. The dynamical parallax does not differ greatly from the trigonometric parallax found from *Hipparcos.*

Key words: binaries: spectroscopic; binaries: visual; methods: data analysis

1 INTRODUCTION

Binary stars, double stars that are also physically connected, are of fundamental importance to astronomy. Alpha Andromedae (BD +28 0004, HD 358, HIP 677), also known as Alpheratz , "The Horses Navel" according to Allen (1963) or Sirrah, represents a particularly intriguing binary. Not only is it the brightest star of the constellation Andromeda, but the primary is also the brightest of the mercury-manganese stars, stars whose atmospheres contains abnormally high levels of mercury, manganese, and other elements. Slipher (1904) discovered the spectroscopic binary nature of Alpheratz. Subsequent radial velocity determinations treated the system as single line, but in 1995 Tomkin, Pan, and McCarthy (1995) detected spectra for the secondary. The system became a double line spectroscopic binary, which allows one to estimate the masses of the components as well as the distance of the system independent of trigonometric parallax. In 1992 Pan et al. (1992) were able to resolve the components using interferometry. Thus Alpheratz represents a combined double line spectroscopic binary plus optical binary. Techniques that simultaneously adjust for the orbital elements and the radial velocity parameters have been developed to study such systems. Pourbaix (1998) uses simulated annealing and the Nelder-Mead simplex algorithm to solve for the parameters. I prefer an approach based on semi-definite programming (SDP), summarized in Calafiore (2002), and applied by me to 24 Aquarii (2005, 2007) and Capella (2008). Alpheratz, however, presents a complication not found in either of those two binaries: the radial velocity observations cover more than a century whereas the interferometric observations only a little more than a year. The combination of the two types of observations becomes nontrivial.

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Table 1. Sources for radial velocities

Observatory	Number of velocities; year(s)	Reference
Lick	12, 1901-1903	Campbell and Curtis (1905)
Lick, Cerro San Cristobel	13, 1901-1903	Campbell and Moore (1928)
Lowell	13, 1902-1904	Slipher (1904)
Allegheny	94, 1907-1908	Baker (1910
Yerkes	4, 1922-1926	Frost, Barrett, and Struve (1929)
Dominion	3, 1923-1924	Harper (1937)
Yerkes	40, 1930-1935	Luyten, Struve, and Morgan (1939)
Berlin-Babelsberg	53, 1931	Kohl (1937)
Kitt Peak	13, 1965-1967	Abt and Snowden (1973)
Dominion	15, 1970-1974	Aikman (1976)
Crimea, Dominion	50, 30 (sec.), 1990-1998	Ryabchikova, Malanushenko, and Adelman (1999)
Palomar	35, 8 (sec.), 1991-1994	Tomkin, Pan, and McCarthy (1995)
Catania, Italy	12, 2001-2001	Catanzaro and Leto (2004)

Many orbits have been published for this binary, the *9th Catalogue of Spectroscopic Binary Star Orbits* (Pourbaix et al. 2004) being the most recent. The catalog contains five orbits of which I chose for comparison Pourbaix (2000), which uses recent radial velocities, from 1970 to 1994, representing 50 for the primary and 8 for the secondary. I use radial velocities from 1901 (JD 2415609.5) to 2002 (JD 2452534.5549), 340 for the primary and 38 for the secondary. One must, of course, judge if the early radial velocities are of sufficient quality to justify inclusion; error may compromise their contribution to an extent that merits rejection. It is true, however, that the observers have attempted to render compatible the measurements. Campbell and Moore (1928) state that great efforts were taken to place on the same system the Mills spectrograph observations made on Mt. Hamilton with those made when the spectrograph was installed at Cerro San Cristobel in Santiago, Chile. My study of Capella (2008) showed that it is preferable to incorporate all of the radial velocities rather than just the best ones even if the final mean errors are higher, although for Alpheratz this must be demonstrated rather than assumed.

2 THE OBSERVATIONS OF ALPHA ANDROMEDAE AND THE REDUCTION MODEL

Information for this binary can be found on the Simbad database (simbad.u-strasbg.fr/simbad/): $\alpha_{2000} = 00^h08^m23.^s259$, $\delta_{2000} = +29°05'25.''56$. The visual magnitude and spectral type of the primary are 2.22 B8IVpMnHg, for the secondary 4.21 A3V. Van Leeuwen (2007) gives for the parallax ϖ of the system 33.63±0.35 mas. This serves to check the quality of the final orbit. Pan et al. (1992) list the 42 optical, interferometric observations made for Alpheratz, which cover the period 8 Sept. 1988 (JD 2447412.91323) to 7 Oct. 1989 (JD 2447806.79042). Because the period of the system is ≈ 96.7 days, this represents slightly over 4 revolutions. The radial velocities, on the other hand, cover 382 revolutions. It seems evident that the period and time of periastron passage will be determined basically by the radial velocities.

The radial velocities come from a potpourri of sources. The ICATE catalog (http://icate-conicet.gob.ar), maintained by the CASLEO Observatory of San Juan, Argentina, provides a useful starting point. Then the indispensable ADS database (adswww.harvard.edu) furnished as many radial velocities as could be found. Table 1 list the sources for the radial velocities in chronological order of the velocities. Some velocities were duplicates and when eliminated resulted in 340 independent velocities for the primary and 38 for the secondary. The times of velocities before 1925 must be corrected for the difference between the civil and the astronomical day.

A previous publication (Branham 2008) has shown how the two types of measurement, optical and radial velocity, can be reduced simultaneously. But because the interferometric observations and radial velocity data represent two distinct types of measurement and there is great disparity in the number of each type of observation, there is reason to assume that the best fit orbit for each class of data, reduced independently, will not necessarily be the same. Such in fact is true for Alpheratz. For details of how to perform a simultaneous reduction see Branham (2007). Visual and speckle observations of a binary involve two measurements, the separation ρ, usually given in arc-sec or milli-arc-sec (mas), and the position angle θ. We can convert ρ and θ to rectangular coordinates, $x = \rho\cos\theta$ and $y = \rho\sin\theta$. Not only must the apparent orbit be an ellipse, but the observations must also satisfy the areal velocity law,

$$\rho^2\dot{\theta} = 2C, \tag{1}$$

where C is constant. From these measurements one calculate the coefficients a, b, c, d, f, l of the apparent ellipse. Having the coefficients of the apparent orbit, we can calculate the true orbit from a number of methods, of which I prefer Kowalsky's (Smart 1962, Sec. 191). The orbital elements are: the period of revolution P; the time of periastron passage T_0; the eccentricity e; the semi-major axis a'; the node Ω; the inclination i; and the perihelion ω.

For double-line spectroscopic binaries the radial velocities $\dot{r}_1$ and $\dot{r}_2$ involve the systemic velocity V_0, e the eccentricity of the true orbit, ω the argument of perihelion, v the true anomaly, and the semi-amplitudes K_1 and K_2. One must remember that the periastron for spectroscopic binaries is displaced 180° from that used with optical binaries. The semi-amplitude coefficient K_1 is given by

$$K_1 = \frac{2\pi \cdot 149.598 \cdot 10^6 a' \sin i}{86400 \cdot 365.2422 \varpi P \sqrt{1-e^2}}\kappa, \tag{2}$$

where ϖ is the parallax of the system in mas and a' is also measured in mas, P the period in years, and κ the mass ratio, $K_1/(K_1 + K_2)$. If we use Eq. (2) with K_2 in lieu of K_1, replace κ by $(1 - \kappa)$.

Given that the visual and interferometric observations generally include discordant data, one should implement the adjustment criterion by use of the robust L_1 norm, minimize the sum of the absolute values of the residuals, for which the semidefinite programming method (SDP) becomes ideal. The SDP algorithm, moreover, calculates a unique ellipse that corresponds to a global minimum of the objective function; see Calafiore (2002) for details. The function to be minimized becomes $F = F_1 + F_2 + F_3$, F_1 corresponding with the interferometric observations, F_2 with the areal velocity constant, and F_3 with the elements corresponding with the radial velocities. The flexibility of the SDP approach permits mixing the norms in the objective function, for example least squares with F_3 and L_1 with F_1, although for this research I use L_1 for all of the $F's$.

3 SOLUTION FOR COMBINED DATA

SDP needs the rectangular coordinates and also $d\theta/dt$, found by numerical methods, for use in Eq. (1). Having the data for ρ, θ, $\dot{\theta}$, and the radial velocities SDP calculates values for $a, b, c, d, f, l, C, V_0, K_1, K_2$. I used the robust L_1 criterion for the first solution, then calculated weights. Although an L_1 solution is robust and little influenced by large residuals, previous experience has shown that the solution is nevertheless improved by use of weighting factors; see Branham (2005). The impersonal weighting factors come from Welsch weighting, which I have used with success with Galactic kinematics: scale the post-fit residuals by the median of the absolute values of the residuals and calculate weighting factors w_i by:

$$w_i = \exp(-r_i/2.985)^2, \tag{3}$$

where r_i represents an individual residual. For radial velocities I also, to maintain consistency, use the L_1 criterion and again use of Eq. (3) weighting. The second solution applies weights to both the interferometric observations and the radial velocities.

Weighting schemes other than impersonal can be used. Double star observer van den Bos (1962) writes, "A valuable asset [presumably for determining weights] . . . is familiarity with the reliability ... of observations made by different observers, using different telescopes and measuring apparatus on double stars of various degrees of separation, brightness, difference of magnitude, and altitude above the observer's horizon." Although this point of view has merit, it can be compromised by an observer's subjectivity. As Sherrill (1999) remarks about noted double star observer T.J.J. See, " ... there were some astronomers who began to accuse See privately of carelessness as an observer ... they felt that the young man's overconfidence frequently led him to mischaracterize results ...." Aside from being impersonal another advantage of Welsch weighting recognizes an important fact from the central limit theorem of statistics, smaller residuals are more probable than larger ones, and assigns higher weight to small residuals.

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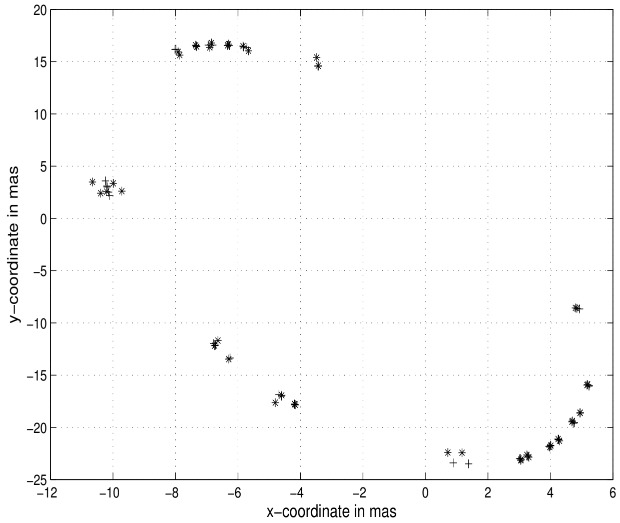

Figure 1. Observations (*); interferometric solution (+). East up, North right.

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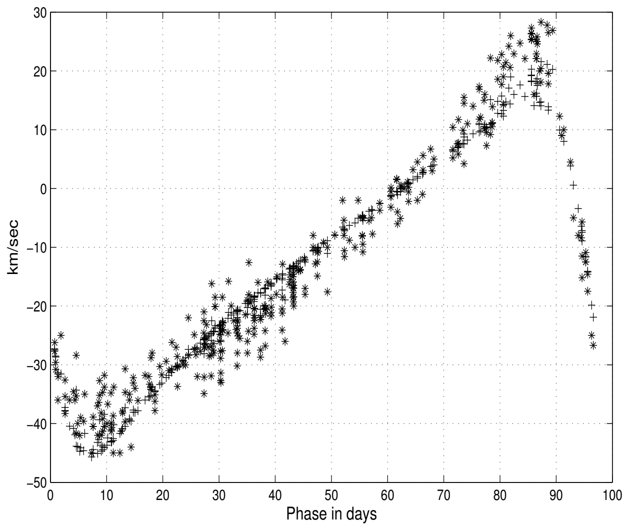

Figure 2. Radial velocities of primary; observed (*), predicted (+).

Fig.1 shows the orbit for the interferometric observations, Fig. 2 the fit for the radial velocities of the primary, and Fig. 3 for the secondary.

The goodness of the solution can be tested by a runs test on the residuals. This test measures how often a variable, distributed about the mean, changes sign from plus to negative or negative to positive, the runs, which have a mean for n data points of $n/2+1$ and a variance of $n(n-2)/4(n-1)$. An advantage of the runs test over other tests for randomness resides in its being nonparametric, making no assumption about the normality of the data, although to actually calculate probabilities for the observed runs one does assume

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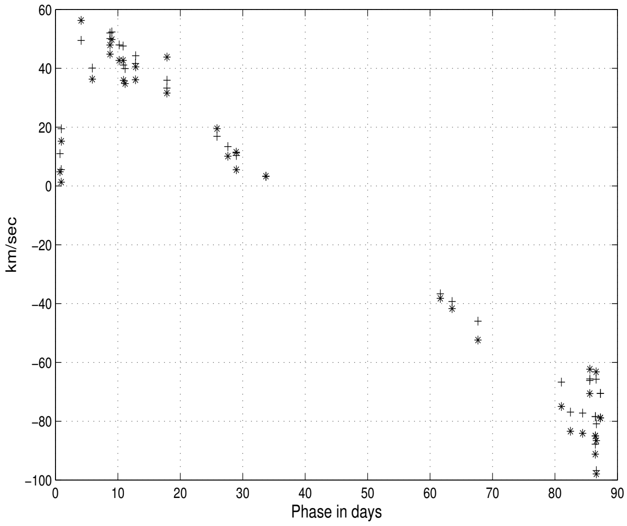

Figure 3. Radial velocities of secondary; observed (*), predicted (+).

approximate normality. For a detailed description of the runs test see Wonnacott and Wonnacott (1972, pp. 409-411).

The period can be found from the areal velocity constant, but as Branham (2007) shows, this is not a good idea. It generally is better calculated from Kepler's equation by use of the optical observations. See Branham (2007) for the details. When both the optical and the radial velocity data cover about the same time period the polynomial fitted for θ can be used to calculate the true anomaly needed in the radial velocities. This procedure, however, performs poorly here, unlike the situation with Capella (Branham 2008), because of the disparity between the length of time of the optical observations and that of the radial velocities, a factor of 100 to 1. If we use just the 42 interferometric observations we calculate $P = 96.^{d}944$ and $T_0 = 2447760.852$, values that differ significantly from those presented in Table 3.

To overcome this difficulty I calculated P and T_0 using the e and ω found from just the interferometric observations, the V_0 , K_1, and K_2 from the catalog of spectroscopic binaries (Pourbaix 2004), and the Nelder-Mead simplex algorithm (Branham 1990, pp. 185-190). Once these two quantities, P and T_0 and only these quantities, were calculated, their values were held fixed and the other orbital elements computed from the SDP algorithm. The mean errors for P and T_0 were calculated from a 2×2 covariance matrix derived from the partial derivatives for corrections to the epoch and mean daily motion that Petrie gives in his Eq. (7) (Petrie 1962). For the true orbital elements let $\mathbf{\Sigma}$ represent the covariance matrix for the elements $a, b, \cdots, l, C, V_0, K_1, K_2$. Then the error in an element such as Ω can be found from:

$$\begin{array}{l} \mathbf{V} = (\ \partial\Omega/\partial a \quad \partial\Omega/\partial b \quad \cdots \quad \partial\Omega/\partial C \quad \cdots\ ); \\ d\Omega^2 = MAD \cdot \mathbf{V}^T \cdot \mathbf{\Sigma} \cdot \mathbf{V}. \end{array} \qquad (4)$$

Table 2 shows this solution from the combined observations and Table 3 the elements of the true orbit. Table 2 gives the mean absolute deviation (MAD), the sum of the absolute values of the residuals divided by the degrees of freedom, for the optical observations and for the radial velocity data. The MAD is a more appropriate measure of dispersion for L_1 solutions than $\sigma(1)$, although the errors themselves are better calculated from a normal covariance matrix. Table 3 also includes the parallax derived from Eq. (2) and the mass of the system, calculated from

$$\mathsf{M}_1 + \mathsf{M}_2 = (a'/\varpi)^3/P^2, \qquad (5)$$

with the period calculated in years and the masses in solar masses. The individual masses come from multiplication of the sum of the masses by κ and (1-κ). When calculating the error in the dynamical parallax and

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Table 2. Orbit from combined observations

Quantity	Value	Error
$MAD(optical)$ $(dimensionless)$	4.751610e-013	···
$MAD(radialvelocity)$ $(km\ s^{-1})$	2.433	···
$MAD(radialvelocity, primary)$ $(km\ s^{-1})$	2.376	···
$MAD(radialvelocity, secondary)$ $(km\ s^{-1})$	3.978	···
a (mas^{-2})	2.634522e-007	5.714455e-009
b (mas^{-2})	3.997286e-008	1.141479e-009
c (mas^{-2})	1.096284e-007	1.750711e-009
d (mas^{-1})	1.725851e-009	6.514068e-011
f (mas^{-1})	5.594740e-010	5.356510e-011
l $(dimensionless)$	-8.314407e-012	2.968851e-013
C $(arc - sec^2\ degree\ day^{-1})$	2.513589e-004	4.119213e-005
V_0 $(km\ s^{-1})$	-9.95	0.73
K_1 $(km\ s^{-1})$	31.77	0.19
K_2 $(km\ s^{-1})$	61.48	0.95
κ	0.34069	0.00373

in the sum of the masses one must keep in mind that P in Eq. (5) has been determined independently. Thus the error in the dynamical parallax and in the sum of the masses comes from the square root of the sum of the errors given by Eq. (5) and the error calculated for P.

Calculation of a covariance matrix $\mathbf{C}_v$ for the values obtained is far from trivial because a different number of data go into the various unknowns. Both the interferometric observations and the radial velocities contribute to calculating a, b, c, d, f, l; only the optical observations contribute to C; all of the radial velocities determine V_0, only the velocities of the primary enter into K_1 and of the secondary into $K2$. See Branham (2007) for a discussion of how to calculate the matrix. It suffices to say that the data matrix assumes the size of 420×10, with the first six columns full, the unknowns $a - l$, (378 radial velocities plus 42 interferometric observations), the seventh column with 42 components that correspond to C, the eighth column the 378 components of V_0, the ninth column of the 340 components of K_1, and the tenth column the 38 components of K_2. The data matrix itself has a condition number of $3.8{\cdot}10^{13}$, not excessive when compared with the machine ϵ of $5{\cdot}10^{-20}$ for the Intel processor used for the computations, but the covariance matrix must be calculated with care to avoid numerical instability. This is accomplished by column scaling of the data matrix, reducing its condition number to $3.2{\cdot}10^{3}$; there is then no problem with the computation of the covariance matrix. Because the covariance matrix is used only to calculate mean errors its importance for the computations is minimal.

Of more import is the correlation matrix:

$$\begin{pmatrix}
1.000 & 0.898 & 0.667 & 0.889 & 0.447 & -0.860 & 0.544 & 0.071 & 0.064 & -0.009 \\
0.898 & 1.000 & 0.707 & 0.914 & 0.465 & -0.834 & 0.714 & 0.064 & 0.058 & -0.008 \\
0.667 & 0.707 & 1.000 & 0.520 & 0.336 & -0.538 & 0.711 & 0.047 & 0.043 & -0.006 \\
0.889 & 0.914 & 0.520 & 1.000 & 0.269 & -0.880 & 0.611 & 0.232 & 0.010 & -0.199 \\
0.447 & 0.465 & 0.336 & 0.269 & 1.000 & -0.009 & 0.339 & 0.199 & 0.048 & -0.152 \\
-0.837 & -0.834 & -0.538 & -0.880 & -0.009 & 1.000 & -0.574 & 0.142 & -0.067 & -0.194 \\
0.712 & 0.714 & 0.711 & 0.611 & 0.339 & -0.574 & 1.000 & 0.050 & 0.044 & -0.006 \\
0.071 & 0.064 & 0.047 & 0.232 & 0.199 & 0.142 & 0.050 & 1.000 & 0.319 & -0.890 \\
0.064 & 0.058 & 0.043 & 0.010 & 0.048 & -0.067 & 0.046 & 0.319 & 1.000 & -0.003 \\
-0.009 & -0.008 & -0.006 & -0.199 & -0.152 & -0.194 & -0.006 & -0.890 & -0.003 & 1.000
\end{pmatrix}$$

Notice that some correlations among the unknowns a, b, c, d, f, l are generally high, with C low to moderate, and with the unknowns V_0, K_1, K_2 low. Among V_0, K_1, K_2 the only high correlation is between V_0 and K_2. The interferometric observations and radial velocity data divide into two almost, but not quite, independent subsystems, reason for a simultaneous reduction of all the data.

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Table 3. True orbit

Quantity	Value	Error
P	$96.^{d}69342$	$0.^{d}05757$
T_0	$2447763.^{d}097745$	$0.^{d}02785$
a'	23.917 mas	0.127 mas
e	0.525855	0.012815
Ω	$104.^{\circ}456$	$0.^{\circ}478$
i	$105.^{\circ}799$	$0.^{\circ}172$
ω	$77.^{\circ}320$	$0.^{\circ}383$
ϖ	32.64 mas	0.66 mas
κ	0.341	0.004
$M_1 + M_2$	5.505 $M_\odot$	0.275 $M_\odot$
M_1	1.875 $M_\odot$	0.096 $M_\odot$
M_2	3.630 $M_\odot$	0.201 $M_\odot$

4 DISCUSSION

We have obtained a solution. How good is it? Despite its mean error the value for C is consistent with a period of $P = 94.^{d}862 \pm 0.^{d}280$, not too close to the period determined by the radial velocities. One can say that C should be included in the reduction model because it represents a fundamental constant of the dynamical system, but the period itself is better determined by other means. A runs test applied to just an orbit calculated from the interferometric observation, minimization of $F_1 + F_2$, gives 21 runs out of an expected 21, excellent. But the fit to the radial velocities is bad. This is hardly surprising because the difference in the period between the interferometric solution and the combined solution is $0.^{d}25$ and in T_0 $2.^{d}25$, which wreaks havoc when the true anomalies needed for the radial velocities are calculated 88 years from epoch.

The orbit of Table 2 and shown in Fig. 1 gives 25 runs out of 21. Not as good, but acceptable. To be precise there is a 20.7% chance of these residuals being random. Applied to the radial velocities the runs test yields 132 runs out of an expected 189 for all of the velocities, 119 out of 170 for the primary, and 12 out of 19 for the secondary. The corresponding MAD's are 2.376 $km\ s^{-1}$and 3.978 $km\ s^{-1}$.

The fit between the ellipse defined by the radial velocities and the interferometric observations is good, as Fig. 4 shows. To calculate the rectangular coordinates I used Eq. (39) of Aitken (1935, p. 91) and took Ω and i from Table 3. The centres of the two ellipses differ by only 0.010 mas.

It is true that one can obtain a better fit to the radial velocities by use of a radial velocity only solution, minimize F_3, but then the fit to the interferometric observations becomes bad. One can say that the interferometric observation solution gives the best fit to only those data and a radial velocity fit only solution agrees better with the velocities. To combine the two types of data, therefore, becomes necessary to fit both and obtain good results. The dynamic parallax derived from Eq. (5) and given in Table 2 differs, but not greatly, from the *Hipparcos* determination: 32.64±0.66 mas versus 33.63±0.35 mas. Although a classical comparison of means test (Wonnacott and Wonnacott 1972, p. 164) would reject at the 95% confidence level that the two means are statistically the same, there is nevertheless a 13.4% chance that the *Hipparcos* parallax falls within the range given by the dynamical parallax, although only a 0.4% chance that the dynamical parallax falls within the *Hipparcos* range. Values found in Table 6 of Pan et al. (1992) plus the K_1 and K_2 of Pourbaix (2004) would give 33.03 mas. Tomkin et al. (1995) present orbital elements consistent with 29.4 mas. The dynamical parallax found here, therefore, cannot be considered discrepant.

To examine whether it is a good idea to incorporate all of the radial velocities rather than just the latest ones, I performed an analysis of two groups, using the Matlab routine *fminsearch* but this time applying no weighting factors, the velocities up to JD 2442396.838 and the velocities after JD 2448116.454 for the primary; there are no velocities for the secondary before JD 2442396.838. These dates were chosen because of the 15.6 year gap between the two sets, although the grouping is admittedly arbitrary. The first group of earlier velocities, with 240 velocities, exhibits 103 runs out of an expected 120 with $\sigma(1) = 3.100$ km s^{-1}. The second group of 100 velocities shows 27 runs out of an expected 50 with $\sigma(1) = 3.169$ km s^{-1}. A criterion for outlier rejection, such as Pierce's (Branham 1990, pp. 79-80), would reject no outliers for either group. There

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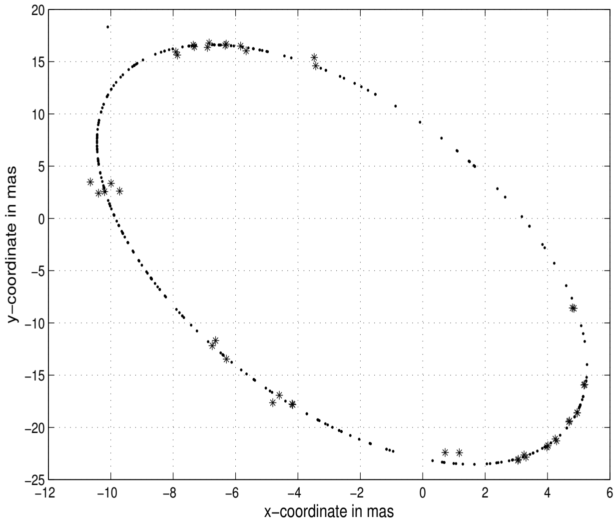

Figure 4. Observations (*) versus radial velocity elipse (.). East up, North right.

remains, therefore, no reason to arbitrarily reject the earlier velocities, which seem to be of the same quality as the later observations.

Regarding the lack of runs in general among the various solutions from radial velocities, not only here but also in work on other binary systems, this seems common. I found the same phenomenon in 24 Aquarii and Capella. So have others. The 9th *Catalogue of Spectroscopic Binary Orbits (*Pourbaix et al. 2004) gives 15 runs out of an expected 25 for the 50 radial velocities of the primary contained in the catalog. The most likely explanation arises from systematic differences among the various observatories that make the measurements, but differences that nevertheless fall below the overall noise level of the data.

5 CONCLUSIONS

It is possible to link the 42 interferometric observations, made over a little more than a year, with the 378 radial velocities, made over a century. Although separate solutions, one for the interferometric observations and one for the radial velocities, produce better results for their respective types of data, the interferometric solution gives poor residuals for the radial velocities and the radial velocity solution poor residuals for the interferometric observations. A combined solution, with the period and time of periastron passage determined from just the radial velocities, produces good results for each type of data, although not as well when measured by a runs test as when each type of data is reduced separately. The parallax calculated from the combined solution, however, shows good, although not perfect, agreement with the *Hipparcos* parallax, Because the two parallaxes come from completely different types of measurement, this implies that the combined solution approximates reality and definitely remains superior to the separate solutions.

5.1 Acknowledgment

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