

# Tracking the Dynamics of the Supply Chain for Enhanced Production Sustainability

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One of the key components of enterprise-wide optimization (EWO) is decision-making coordination and integration at all decision levels. In this paper, a supply chain design–planning model, which translates a recipe representation to the supply chain environment, is coupled with a scheduling formulation so that decision levels integration is achieved. This approach enabled us to assess the impact of considering scheduling aspects of process operations in the design of a supply chain network. A comparison of the proposed scheme and the traditional hierarchical approach shows the significance of such integration. Moreover, the scheduling details enable the dynamics of the supply chain to be tracked. We show the degree to which a holistic decision-making model within a model predictive control framework is able to react to incidents occurring in the supply chain components, including disturbances arising from local monitoring, control, and diagnosis of incidents in real time. Finally, a decomposition technique is applied to reduce the computational burden associated with the monolithic model solution. Validation of the proposed approach and the resulting potential benefits are highlighted by a case study. Moreover, the results obtained from this particular case study are examined and discussed with respect to future work.

## Introduction

The trend toward globalization has significantly increased the scale and complexity of current businesses. Businesses have become global networks that are made up of a number of business units and functions. Operational functions include research and development (RD), production networks (continuous, discontinuous, and discrete), and supply networks. These functions are buttressed by financial planning and marketing strategy functions. Businesses are subject to internal and external uncertainties. Examples of internal uncertainties include the success rate of RD projects, given the technological risks involved, and disruptions to production, such as production failures and unforeseen stoppages. External uncertainties include those related to the cost of raw materials and products (unless they are subject to monopoly conditions), fluctuations in the exchange rate, and uncertainties in market size and demand, due to competition and macroeconomic factors. Businesses control their operations through the decisions they make about their capital expenditure, the financing of the company, growth strategies, and operations. Strategic decisions about capital expenditure and planning include the technology used, the choice of RD projects, and decisions about infrastructure and supply chain management (SCM). Financial decisions are made by identifying the assets and liabilities required from the working capital for larger projects and operations, and by assessing and protecting the company from the risk of change. Examples of tactical production decisions include the following: planning activities in plants that are run on a discontinuous basis, to respond to anticipated demand; making decisions about the sources of energy used, in accordance with market prices; and increasing the production capacity in response to the pressures of demand. Current studies and solutions in the field of process systems engineering (PSE) and operations research tend only to consider subsets of such decisions, even though a business must act as a cohesive body in which its various functions are

to a certain extent coordinated. Therefore, from a company's viewpoint, overall performance will be below optimum if strategic and tactical decisions are taken separately, as has been the practice to date. However, it is significantly more complex for a company to make decisions that involve its overall interests than to make decisions about specific functions. This explains why integral modeling that reflects the overall running of companies has been virtually unheard of to date.

The PSE community faces an increasing number of challenges, while enterprise and SCM remain subjects of major interest that offer multiple opportunities. Further progress in this area is thought to bring with it a unique opportunity to demonstrate the potential of the PSE approach to enhance a company's value. As previously mentioned, one of the key components of SCM and EWO is decision-making coordination and integration at all levels. Most of the recent contributions offer models that separately address problems arising in the three standard supply chain (SC) hierarchical decision levels (i.e., strategic, tactical aggregate planning, and short-term scheduling).

As stated by Grossman,<sup>1</sup> the major pending research problem is the integration of planning, scheduling, and control, whether at plant or SC level. The nature of the SC planning problem is quite similar to the production scheduling problem. Both problems usually seek answers to the questions of in what amount, when, and where to produce each of the products comprising the business portfolio so as to obtain financial returns. However, planning brings into play a broader, aggregated view of the problem. The time periods used in planning problems are usually longer than task processing times; thus, the sequencing/timing decisions in a scheduling problem are transformed into rough capacity decisions in a tactical planning problem. In fact, equipment capacity modeling is a highly sensitive aspect that must be taken into account in order to ensure consistency and feasibility when problems are being integrated across SC hierarchical decision levels. Furthermore, at the strategic level, designing an SC network does not just involve selecting the type and size of the equipment, but also allocating this equipment to the different potential SC echelons.

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Therefore, we require an SC modeling approach that (i) considers equipment capacity similarly at strategic and operational levels, so that this capacity can be aggregated and disaggregated in a straightforward and transparent manner; (ii) is able to handle strategic decisions associated with processes/equipment allocation to sites and not merely site locations; and (iii) easily represents the transport material and financial flows among SC sites at the scheduling level.

The external and internal dynamics of real businesses have not been represented by current models in a way that achieves desired consumer satisfaction levels and acceptable financial returns. Fluctuating demand patterns, increasing customer expectations and competitive markets, coupled with internal disturbances, mean that today's supply networks are not reliable in such an environment unless their external and internal dynamics are appropriately incorporated into the SC model. The ability to capture the dynamics of the SC has become a matter of survivability. The more survivable a network, the more reliable it will be. As a result, another challenge is to characterize the dynamics in SCs to improve responsiveness. Robust formulations have been proposed using stochastic optimization techniques.<sup>2–4</sup> Such modeling approaches explicitly incorporate uncertainty into the model, by means of estimated scenarios or probability distribution functions for the random parameters, in order to optimize the expected value of a given performance metric. Alternatively, predictive control technologies have been used to deal with uncertainties in SCs.<sup>5–8</sup> Particularly, some studies have developed approaches that focus on limiting a phenomenon known as the “bullwhip effect”, which is the increase in fluctuation of demand upstream in the SC.<sup>9,10</sup> However, incorporating low-level decisions (local scheduling, supervisory control and diagnosis, incident handling) and the implications of incorporating these decisions for the dynamics of the entire SC (production switching between plants, dynamic product portfolios) have not yet been fully studied.

In general terms, the satisfactory management of a company's business matters requires the direct appraisal of the results of the decisions taken at various levels. This requires significant integration of a problem's multiple planning facets in nonconventional manufacturing networks and in multisite systems. Financial matters are typically disregarded when SC operational decisions are addressed. Nevertheless, it is widely recognized that financial assets bear a strong and direct relation to core aspects of SCs, such as inventories, capacity expansion, and allocation and purchases of raw material and services. Most SC modeling approaches account for fixed assets when the economic performance of the available alternatives is assessed in the design phase. However, in the planning formulation, they usually ignore the net working capital (NWC), which represents the variable assets associated with the daily SC operations. NWC is constituted by material inventories, accounts receivable (physical distribution), accounts payable (procurement), and cash. All of these components are directly affected by decisions regarding SC operations. NWC can be understood as the capital tied up within the cash conversion cycle, which measures how efficiently an enterprise converts its inputs into cash through final product sales (Figure 1). The less capital that is tied up by SC operations, the better the performance will be in terms of the business's bottom line. The NWC is not a static figure; indeed, it may change from period to period throughout the planning horizon, in accordance with tactical SC decisions. Recently, Laínez et al.<sup>11</sup> addressed the SC design problem by considering financial issues. A capital budgeting formulation, which enables a firm's value to be assessed quantitatively, is

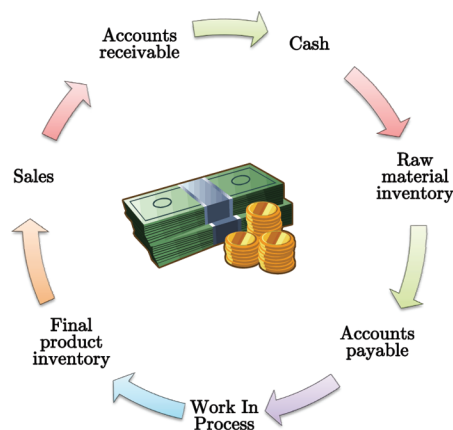


Figure 1. Cash conversion cycle.

embedded in their model. The corporate value (CV) of the firm is proposed as the objective function, which is calculated using the discounted free cash flow (DFCF) method. In brief, free cash flow is a function of net operating profit after taxes, fixed assets, and NWC for any given period.<sup>12</sup>

A major disadvantage of discounted cash flow methods is that they do not account for the managerial flexibility needed to be able to alter the course of an investment over time as uncertain factors unfold. Real options analysis has been proposed as an alternative valuation approach that would overcome this drawback. Traditionally, discounted cash flow methods assume a single decision pathway with fixed outcomes. In contrast, the real options approach considers multiple decision pathways as a consequence of management flexibility; thus, midcourse strategies can be corrected to deal with future uncertainty.<sup>13</sup> Monte Carlo sampling has been coupled with real options modeling to account for different uncertain scenarios. From another standpoint, stochastic optimization models assume that decision-makers take some actions in a first stage, after which a random event occurs and affects the outcome of those first-stage decisions. Recourse decisions can then be made in the following stages to compensate for any negative effects that might have occurred as a result of the first-stage decisions. Multistage stochastic optimization models and real options analysis show significant similarities. In fact, the stochastic optimization solution consists of a map in which different decisions are proposed, depending on the scenario that arises. Consequently, a stochastic optimization model that is formulated by extending the deterministic mixed integer linear programming (MILP) proposed by Laínez et al.<sup>11</sup> will render a stochastic DFCF model with the same features as real options approaches. Furthermore, a stochastic DFCF model offers more realistic solutions, since it considers the so-called nonanticipativity conditions, whereas real options approaches typically disregard these conditions. For this reason, real options usually lead to “wait-and-see” solutions (i.e., complete knowledge of information is assumed during the decision-making process).

The aim of this paper is to demonstrate that the integration of decision-making in businesses leads to significant added value. This paper continues and extends the recently presented enterprise-wide model<sup>14</sup> to more realistic and complex situations, which demonstrate its ability to handle incidents that arise in the SC with visibility at both the SC and plant levels in an integrated formulation that accounts for the optimization of a suitable financial performance indicator.

In this paper, we present a stochastic model in which it is possible to integrate the three classical SC hierarchical decision

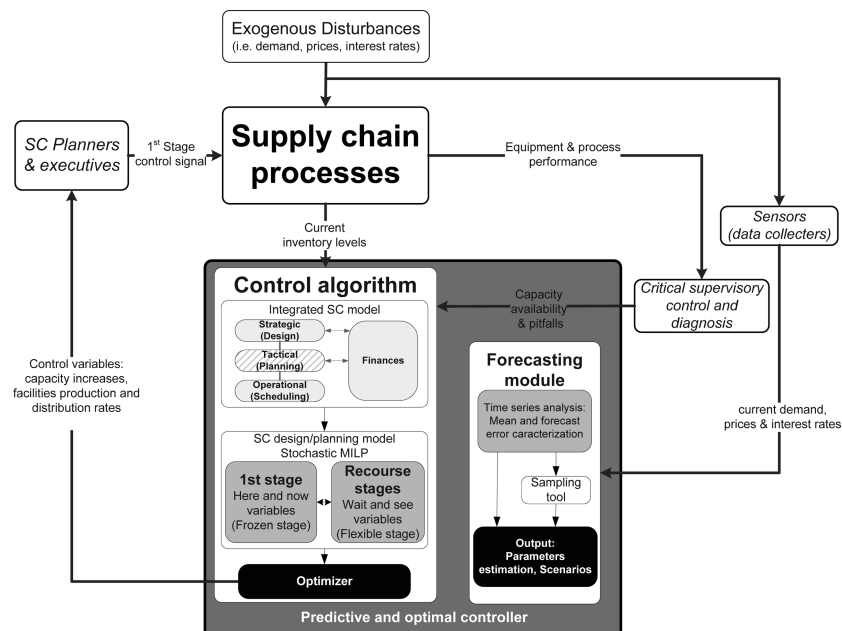


Figure 2. Control strategy for SCM.

levels. For this purpose, we use the SC design–planning model proposed in the work of Lafnez et al.<sup>15</sup> This model translates a recipe representation to the SC context, thus facilitating the integration of scheduling decisions during SC design. Here, expected CV is the objective function to be maximized using a stochastic DFCF formulation. As stated previously, such an approach should be more effective than real options analysis. Moreover, the challenge of solving large multiscale optimization problems becomes evident when decision level integration is considered. Therefore, a decomposition technique is applied to reduce the computational burden associated with the model solution. Finally, the scheduling details about production equipment enable the dynamics of SC to be tracked. By considering information from the equipment supervisory module, which we can incorporate into the scheduling formulation, it is possible to handle the incidents that may arise in the SC in low-level decisions. The main advantages of the integrated approach are highlighted by a case study in which our strategy is compared with the traditional sequential, hierarchical approach.

### SCM as a Control Strategy

It is noteworthy that SC planning is a dynamic activity. Firms are in the need of a closed-loop planning approach in order to preserve competitiveness. Such an approach should be capable of revising planned activities, updating uncertain parameters (e.g., lead times, market demand, and interest rates), and considering the effects of incidences; so that future plans are adapted to enhance SC performance under the current highly dynamic business environment. A model predictive control (MPC) framework can be used as an appropriate approach to continuously improve the SC planning. Indeed, SCM can be conceived as an “onion-shell” comprised of different control loops, each corresponding to one hierarchical level. At the top level, the strategic decision making may integrate the midterm planning, then midterm planning may incorporate short-term decision making. Decisions regarding each hierarchical level should be revised at different intervals depending on the nature of the problem being addressed so that the most recent information about SC state and uncertainties is taken into account. The strategic decisions of capacity expansion could

be analyzed every year, while planning is usually revised every month or week. What is more, information from low-level equipment control and supervisory modules can be used to feed the SC planning and scheduling decision-making process. Such information can be utilized to account for equipment pitfalls and breakdowns, so that actual capacity availability is considered when planning and scheduling SC operations.

Briefly, an MPC framework attempts to optimize a performance criterion that is a function of the future control variables. By solving the optimization problem associated to the control algorithm, all elements of the control signal are defined. However, only a portion of the control signal is applied to the system. Next, as new control input information and disturbance forecasts are collected, the whole procedure is repeated, which produces a feed-forward effect and enables the SC system to follow-up the process dynamics.

In Figure 2, a general schematic of the proposed MPC framework for SCM is shown. It follows a description of the control strategy. When the SC process is disturbed, data required to describe the current SC state are captured and sent to the controller. This information includes the actual SC state (e.g., current inventory levels, new historic demand data, capacity availability). The information related to capacity can be collected by a supervisory system. Such a system may be able to collect information about the critical equipment capacity which is then given as input data to the SC control algorithm. On the other hand, information about those external parameters regarded as uncertain in the mathematical model is sent to the forecasting module. That module computes the mean value and does the forecast error characterization for each uncertain parameter so as to define the different scenarios to be considered in the predictive model. This consists of a multistage stochastic mathematical model. The variables associated to first-stage decisions are the control signal that is implemented in the SC processes. In fact, first-stage variables are associated to next period decisions which are made prior to uncertainty realization.

Notice that for the actual period the proposed algorithm is considering the detailed scheduling, therefore disturbances, can be contemplated as frequent as the time bucket considered by the scheduling formulation. It is also important to point out that



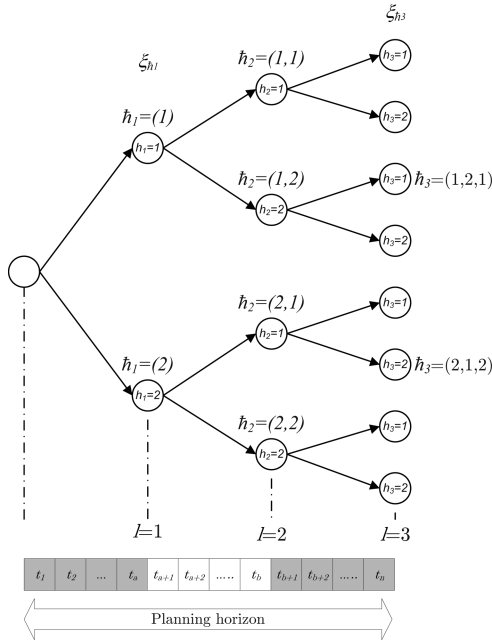


Figure 3. Scenario tree.

we are integrating the three standard hierarchical decision levels; however, more decision levels may exist in an organization. The disturbances can be considered as frequent as the time bucket of the lower decision level allows when using discrete time SC formulations. Continuous time SC formulations should overcome this drawback.

It is important to point out that the control strategy presented allows us to handle uncertainty and incidences by combining reactive and preventive approaches. A proactive treatment of uncertainty is included by means of stochastic programming. The review and update process that is required to tackle incidences and changes in random factors is performed by introducing the SC stochastic holistic model into a MPC framework.

As shown in Figure 2, the predictive controller entails a stochastic multiperiod design/planning/scheduling MILP model of a multiechelon SC with financial considerations. The model assumes that different technological equipment is available to be installed in potential sites and assists in their selection. Regarding the financial area, the mathematical program endeavors to evaluate the CV by using a DFCF method.

### Predictive Model

**Scenario Tree.** The exogenous uncertainty is represented by a scenario tree (see Figure 3). We follow the notation introduced by Puigjaner and Laínez.<sup>14</sup> The reader is referred to this work for details. It will be useful to recall sets  $T_l$ ,  $L_t$ , and  $AH_{l^*h_l}$ . Here, it is assumed that there are  $|L|$  events in which uncertainty ( $\xi$ ) unfolds over the planning horizon. The value that random factors can take in event  $l$  can be identified by index  $h_l$  which belongs to set  $H_l$ . Then,  $\tilde{h}_l$  is the combination of event realizations identified by  $(h_1, h_2, \dots, h_l)$ .  $T_l$  is the subset of planning periods  $t$  that are associated to uncertain event  $l$ .  $L_t$  is the reciprocal set of  $T_l$ .  $L_t$  is the event  $l$  which is related to period  $t$ . Finally,  $AH_{l^*h_l}$  is defined to be given by  $AH_{l^*h_l} = \{\tilde{h}_{l^*} | \tilde{h}_{l^*} \subseteq \tilde{h}_l\}$ , that is  $AH_{l^*h_l}$  denotes the event combination related to  $l^*$  ( $\tilde{h}_{l^*}$ ) which is an ancestor of  $\tilde{h}_l$ . Notice that it is not necessary for  $\tilde{h}_{l^*}$  to be a proper subset of  $\tilde{h}_l$  in order to allow the case when  $l = l^*$ .

**Operations Model.** Here, the SC operations model presented in the work of Laínez et al.<sup>15</sup> is extended to a stochastic version.

That work proposes a flexible SC design–planning formulation whose distinctive features are that it (i) considers all feasible links and material flows among the potential SC components inherently and (ii) does not need any pre-established process network superstructure so that the subtrains (if any) in which the production process is decoupled and their location are determined by the model. Consequently, the model does merely require as input the SC production process recipe representation. Furthermore, a more appropriate description of manufacturing processes at the SC level is achieved by translating a recipe representation to the SC environment. Finally, it is worth mentioning that given that the SC model is a translation of a classical multipurpose plant scheduling formulation, it facilitates the consideration/integration of scheduling decisions while designing a SC.

The SC operations model can be divided into four parts: mass balance equations, constraints related to design and capacity modeling, market and suppliers limitations, and those equations that allow integrating scheduling implications within strategic–tactical SC decision making.

**Mass Balances.** Mass balances must be satisfied for each material in every facility that integrates the SC network. Equation 1 represents the material balance for each facility  $f$  and state  $s$  in every period  $t$  and every combination of events  $\tilde{h}_l$ . In this equation, the inventory of the previous period ( $S_{sft-1}^{*h_{l^*}}$ ) related to the combination of events that is the ancestor of  $\tilde{h}_l$  ( $l^* \in L_{t-1}^*$ ,  $\tilde{h}_{l^*} \in AH_{l^*h_l}$ ) is taken into account. Change in inventory ( $S_{sft}^{l\tilde{h}_l} - S_{sft-1}^{*h_{l^*}}$ ) must be equal to the difference between the material produced/transported by tasks whose destination is facility  $f$  ( $P_{ijf\tilde{h}_l}^{l\tilde{h}_l}$ ) and material consumed by tasks whose origin is facility  $f$  ( $P_{ijf\tilde{h}_l}^{l\tilde{h}_l}$ ). Here,  $\alpha_{sij}$  and  $\bar{\alpha}_{sij}$  represent the mass fraction coefficient of material  $s$  for task  $i$  performed in equipment  $j$ .

$$S_{sft}^{l\tilde{h}_l} - S_{sft-1}^{*h_{l^*}} = \sum_f \sum_{i \in T_s} \sum_{j \in (J_f \cap \hat{J}_f)} \alpha_{sij} P_{ijf\tilde{h}_l}^{l\tilde{h}_l} - \sum_f \sum_{i \in \bar{T}_s} \sum_{j \in (J_f \cap \hat{J}_f)} \bar{\alpha}_{sij} P_{ijf\tilde{h}_l}^{l\tilde{h}_l} \quad \forall s, f \notin (\text{sup} \cup M), \tilde{h}_l, t \in T_t, l^* \in L_{t-1}^*, \tilde{h}_{l^*} \in AH_{l^*h_l} \quad (1)$$

**Design and Capacity Constraints.** Equation 2 is to control the changes in the facilities capacity over time. These constraints include binary variables  $V_{jft\tilde{h}_{l-1}}$ , which take a value of 1 if the facility being represented is expanded in capacity; otherwise, it is set to zero. The capacity increments are bounded in the range  $[FJE_{jft}^L, FJE_{jft}^U]$ , which represents the realistic interval where they must fall. Equation 3 is added to update the total capacity ( $F_{jft\tilde{h}_l}^l$ ) by the amount increased during planning period  $t$  ( $FE_{jft}^{l\tilde{h}_l}$ ).

Equation 4 forces the total production/distribution rate in each facility to be greater than a minimum desired capacity utilization and lower than the available capacity. In this equation,  $\theta_{ijf}$  represents the capacity utilization rate of equipment  $j$  by task  $i$ . To go on,  $\beta_{jf}$  expresses the minimum percentage of utilization of equipment  $j$  at site  $f$ . It is noteworthy that the model considers that task  $i$  is to be performed in equipment that is installed on the facility of “origin”.

$$V_{jft\tilde{h}_{l-1}}^l FE_{jft}^L \leq FE_{jft\tilde{h}_{l-1}}^l \leq V_{jft\tilde{h}_{l-1}}^l FE_{jft}^U \quad \forall f \notin (\text{sup} \cup M), j \in \hat{J}_f, l, \tilde{h}_{l-1}, t \in T_l \quad (2)$$

$$F_{jft\hat{h}_l}^l = F_{jft-1\hat{h}_l}^* + FE_{jft\hat{h}_l}^l \quad \forall f \notin (\text{sup} \cup M),$$

$$j \in \hat{J}_p, l, \hat{h}_l, t \in T_l, l^* \in L_{t-1}^*, \hat{h}_{l^*} \in AH_{l^*, \hat{h}_l}, \hat{h}_{l-1} \in AH_{l-1, \hat{h}_l} \quad (3)$$

$$\beta_{jft} F_{jft\hat{h}_l}^l \leq \sum_f \sum_{i \in I_j} \theta_{ijff} P_{ijff\hat{h}_l}^l \leq F_{jft\hat{h}_l}^l \quad (4)$$

$$\forall f \notin (\text{sup} \cup M), j \in \hat{J}_p, j \notin J_{\text{stor}}, l, \hat{h}_l, t \in T_l$$

In the same way, total inventory in facility  $f$  is constrained to be equal to or lower than the available capacity ( $F_{jft\hat{h}_l}^l$ ) in each period  $t$  and combination of events  $\hat{h}_l$  (eq 5). In this equation,  $v_s$  holds for specific volume of material  $s$ .

$$\sum_s v_s S_{sft\hat{h}_l}^l \leq F_{jft\hat{h}_l}^l$$

$$\forall f \notin (\text{sup} \cup M), j \in \hat{J}_p, j \in J_{\text{stor}}, l, \hat{h}_l, t \in T_l \quad (5)$$

**Markets and Suppliers.** Equation 6 is used to compute the sales of state  $s$  executed at each market. Equation 7 forces the sales of state  $s$  carried out in markets during time period  $t$  to be less than or equal to the demand. While, eq 8 imposes a minimum target for the demand satisfaction (minCSL), which must be attained in all periods  $t$  and event combinations  $\hat{h}_l$ .

$$\sum_{f \in M} \sum_{i \in (T_s \cap Tr)} \sum_{j \in (J_f \cap \hat{J}_p)} \alpha_{sij} P_{ijff\hat{h}_l}^l = \text{sales}_{sft\hat{h}_l}^l$$

$$\forall s \in FP, f \in M, l, \hat{h}_l, t \in T_l \quad (6)$$

$$\text{sales}_{sft\hat{h}_l}^l \leq \text{dem}_{sft\hat{h}_l}^l \quad \forall s \in FP, f \in M, l, \hat{h}_l, t \in T_l \quad (7)$$

$$\frac{\sum_{f \in M} \text{sales}_{sft\hat{h}_l}^l}{\sum_{f \in M} \text{dem}_{sft\hat{h}_l}^l} \geq \text{minCSL} \quad \forall s \in FP, l, \hat{h}_l, t \in T_l \quad (8)$$

The model assumes a maximum availability of raw materials. Therefore, eq 9 forces the amount of raw material,  $s \in RM$ , purchased from location  $f \in \text{sup}$  at each period  $t$  to be lower than an upper bound  $A_{sft}$ . In this expression,  $R_f$  denotes the set of raw materials that can be provided from location  $f$ .

$$\sum_{f \in \text{sup}} \sum_{i \in (\bar{T}_s \cap Tr)} \sum_{j \in J_i} \bar{\alpha}_{sij} P_{ijff\hat{h}_l}^l \leq A_{sft}$$

$$\forall f \in \text{sup}, s \in R_f, l, \hat{h}_l, t \in T_l \quad (9)$$

**Integration of SC Scheduling.** The scheduling formulation is an extension of STN representation.<sup>16</sup> Here, the formulation permits scheduling in multiple facilities. The proposed model divides the planning horizon into  $H$  periods of length  $H1$  where aggregated production is planned using the previous model. Time buckets at this level are represented by index  $t$ . The first planning period ( $t = 1$ ) of the time horizon is divided into time intervals of lower length  $H2$  where detailed scheduling is executed as depicted in Figure 4. Time buckets at the scheduling level are represented by index  $t_k$ . When the proposed control strategy is applied, the model is to be rerun every planning period  $t$  as new information regarding disturbances and SC state are updated (i.e., the model is to be applied following a rolling horizon approach).

The equations concerning the short-term decision level can be classified into two groups; namely, the detailed scheduling constraints and the integrating equations. We gather into the set of detailed scheduling equations, the ones that account for

the mass balance, the assignment, and the batch size restrictions. The latter group includes those equations that ensure the consistency between planning and scheduling models.

**Detailed Scheduling Equations.** Equation 10 is the mass balance applied at each time interval ( $t_k$ ). It can be noticed that this equation is very similar to the mass balance in the planning formulation (eq 1). The basic assignment constraint is stated by eq 11.

$$S_{\text{sched}}^l_{sft_k\hat{h}_l} - S_{\text{sched}}^l_{sft_k-1\hat{h}_l} = \sum_{i \in T_s} \sum_{j \in (J_i \cap \hat{J}_p)} \alpha_{sij} B_{ijff\hat{h}_l}^l - \sum_{i \in \bar{T}_s} \sum_{j \in (J_i \cap \hat{J}_p)} \bar{\alpha}_{sij} B_{ijff\hat{h}_l}^l + \text{rawM}_{sft_k\hat{h}_l}^l$$

$$\forall s, f \notin (M \cup \text{sup}), t_k \in L_{t_k}, \hat{h}_l \quad (10)$$

$$\sum_{i \in J_i} \sum_{t_k=t_k-p_{t_i}+1}^{t_k=t_k-p_{t_i}+1} W_{ijff\hat{h}_l}^l \leq 1$$

$$\forall f \notin (M \cup \text{sup}), j \in (J_{\text{batch}} \cap \hat{J}_p), t_k, l \in L_{t_k}, \hat{h}_l \quad (11)$$

The capacity limits for equipment can be approximated as follows:

$$B_{ijff\hat{h}_l}^l \leq \left( \frac{pt_i}{\theta_{ijff}} \right) W_{ijff\hat{h}_l}^l \quad \forall i, j, f \notin (\text{sup} \cup M), t_k, l \in L_{t_k}, \hat{h}_l \quad (12)$$

Let us notice that the supervisory control is providing to the mathematical model updated information about installed capacity. Once a equipment failure is diagnosed, its implications on capacity availability are passed to the model. Specifically, the batch sizes ( $B_{ijff\hat{h}_l}^l$ ) associated to activities to be performed in a failed equipment  $j$  are restricted to be equal to zero while the failure is repaired. Then, one can observe that capacity turns out to be the essential factor integrating the different hierarchical levels; even more, the supervisory and control module is linked to the predictive model by means of capacity.

**Integrating Equations.** The integration between design—planning and scheduling models is carried out through eqs 13–15. Equation 13 states that production allocated in equipment  $j$  is identical in both hierarchical levels. In eq 14, the availability of raw material is computed from received materials according to the planning formulation. Raw material availability ( $RM_{sft\hat{h}_l}^l$ ) is then included in the scheduling mass balance (eq 10). Scheduling equations may be applied in more than one planning period. The appropriate equations for incorporating scheduling in first planning period ( $t = 1$ ) are presented next.

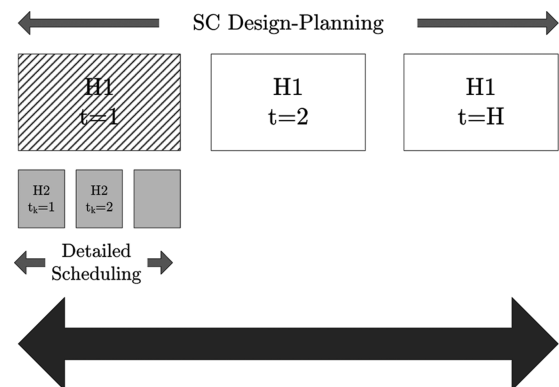


Figure 4. Rolling horizon strategy.

$$P_{ijfth_l}^l = \sum_{t_k} B_{ijfth_l}^l \quad \forall f, j \in (J_{\text{batch}} \cap J_j), i \in I_j, t = 1, l \in L_r, h_l \quad (13)$$

$$\text{rawM}_{sft_k h_l}^l = \sum_{f \neq f} \sum_{i \in \bar{T}_s} \sum_{j \in J_i} \bar{\alpha}_{sij} P_{ijfth_l}^l \quad \forall s, f, t_k = 1, t = 1, l \in L_r, h_l \quad (14)$$

Finally, eq 15 is included to rectify capacity availability in the planning model. This correction is done based on the scheduling model task assignment ( $W_{ijfth_l}^l$ ). Equation 15 should be merely applied to those equipments which are production bottlenecks. Additionally, it is worth mentioning that it must be checked that market demand is not actually the bottleneck process in the planning period, where scheduling is performed.

$$\sum_{i \in I_j} \theta_{ijfth_l} P_{ijfth_l}^l \leq \sum_{i \in I_j} \sum_{t_k} W_{ijfth_l}^l P_{t_i} \quad \forall f, j \in (J_{\text{bottle}} \cap \hat{J}_j), t > 1, l \in L_r, h_l \quad (15)$$

As it can be noticed, eqs 10–15 can be easily unplugged from the whole operations model in case the SC manager decides not to consider scheduling issues.

**Integration between Operations and Financial Model.** The integration between the operations and financial formulations is carried out through the following: the sales of products, the purchases of raw materials, transport services and utilities to final providers, the fixed cost associated with SC network operation, and the total capital investment.

**Operating Income.** Revenue is calculated by means of net sales which are the income source related to the normal SC activities. Thus, the total income incurred in any period  $t$  can be easily computed from the sales of products executed in period  $t$  as it is stated in eq 16.

$$\text{Esales}_{lth_l}^l = \sum_{s \in FP} \sum_{j \in M} \text{sales}_{sftth_l}^l \text{price}_{sftth_l}^l \quad \forall l, h_l, t \in T_l \quad (16)$$

**Operating Cost. Indirect Cost.** The total fixed cost of operating a given SC structure can be computed using eq 17.  $FCFJ_{jft}$  is the fixed unitary capacity cost for production and distribution equipment.

$$\text{Fcost}_{lth_l}^l = \sum_{j \in (\text{sup} \cup M)} \sum_{j \in \hat{J}_f} \text{FCFJ}_{jft} F_{jftth_l}^l \quad \forall l, h_l, t \in T_l \quad (17)$$

**Direct Cost.** The cost of purchases from supplier  $e$ , which is computed through eq 18, includes purchases of raw materials, transportation, and production resources. Let us notice that  $e$  refers to supplier entity and not to supplier location  $f$  ( $f \in \text{sup}$ ).

$$\text{Epurch}_{eth_l}^l = \text{purch}_{eth_l}^{\text{rm},l} + \text{purch}_{eth_l}^{\text{tr},l} + \text{purch}_{eth_l}^{\text{prod},l} \quad \forall l, h_l, t \in T_l \quad (18)$$

The purchases ( $\text{purch}_{eth_l}^{\text{rm},l}$ ) associated to raw materials made to supplier  $e$  can be computed through eq 19. Here,  $\psi_{est}$  is the cost associated to raw material  $s$  purchased from supplier  $e$ . Since it is assumed that several suppliers of raw materials may exist, eq 20 expresses that the total quantity of raw materials purchased in period  $t$  must be equal to the sum of the amounts purchased from each supplier  $e$ .

$$\text{purch}_{eth_l}^{\text{rm},l} = \sum_{s \in RM_e} \sum_{f \in \text{sup}} \text{purch}_{esfth_l}^l \psi_{est} \quad \forall e, l, h_l, t \in T_l \quad (19)$$

$$\sum_{f'} \sum_{i \in (\bar{T}_s \cap Tr)} \sum_{j \in J_i} \bar{\alpha}_{sij} P_{ijfth_l}^l = \sum_{E_s} \text{purch}_{esfth_l}^l \quad \forall s \in RM, f \in \text{sup}, l, h_l, t \in T_l \quad (20)$$

Otherwise, for the sake of simplicity, external transportation services as well as production resources are assumed to be “acquired” each of them from one unique supplier (i.e.,  $|\hat{E}_{\text{tr}}| = |\hat{E}_{\text{prod}}| = 1$ ). This assumption can be easily relaxed to address more general cases. Production and transportation costs are determined by eqs 21 and 22, respectively. Here,  $\rho_{eff}^{\text{tr}}$  denotes the unitary transportation cost associated with sending products from location  $f$  to location  $f'$ ; while  $\tau_{jfe}^{\text{ut1}}$  represents the unitary production cost associated to perform task  $i$  in processing equipment  $j$ , and  $\tau_{sfe}^{\text{ut2}}$  represents the unitary inventory costs.

$$\text{purch}_{eth_l}^{\text{tr},l} = \sum_{i \in Tr} \sum_{j \in J_i} \sum_f \sum_{f'} P_{ijfth_l}^l \rho_{eff}^{\text{tr}} \quad \forall e \in \hat{E}_{\text{tr}}, l, h_l, t \in T_l \quad (21)$$

$$\text{purch}_{eth_l}^{\text{prod},l} = \sum_f \sum_{i \in Tr} \sum_{j \in (J_f \cap \hat{J}_j)} P_{ijfth_l}^l \tau_{jfe}^{\text{ut1}} + \sum_s \sum_{f \in (\text{sup} \cup M)} S_{sftth_l}^l \tau_{sfe}^{\text{ut2}} \quad \forall e \in \hat{E}_{\text{prod}}, l, h_l, t \in T_l \quad (22)$$

**Capital Investment.** Finally, the total investment on fixed assets is computed through eq 23. This equation includes the investment made to expand the equipment  $j$  capacity in facility site  $f$  at period  $t$  ( $\text{price}_{jft}^{FJ} FE_{jftth_l}^l$ ), plus the investment required to open a manufacturing plant in location  $f$ , in case it is opened at period  $t$  ( $I_{jft}^l JB_{jftth_l}^l$ ), plus the investment required to set a distribution center if it is opened at period  $t$  ( $I_{jft}^l SB_{jftth_l}^l$ ). Here,  $JB_{jftth_l}^l$  and  $SB_{jftth_l}^l$  are binary variables which take value of 1 in case the facility being represented, processing site or distribution center, starts construction in period  $t$ .

$$\text{Fasset}_{lth_l}^l = \sum_f \sum_j \text{price}_{jft}^{FJ} FE_{jftth_l}^l + \sum_f (I_{jft}^l SB_{jftth_l}^l + I_{jft}^l JB_{jftth_l}^l) \quad \forall l, h_l, t \in T \quad (23)$$

Equations 24 and 25 are to force definition of variable  $JB_{ft}$ , while eqs 26 and 27 restrict variable  $SB_{ft}$ .

$$\sum_{j \in (J_f \cap J_{\text{prod}})} \left( \sum_{t' \leq t} \sum_{l^* \in L_r} \sum_{h_{l^*} \in AH_{l^*} h_l} JB_{ft}^{l^*} - V_{jftth_{l-1}}^l \right) \geq 0 \quad \forall f \notin (\text{sup} \cup M), l, h_{l-1}, t \in T_l \quad (24)$$

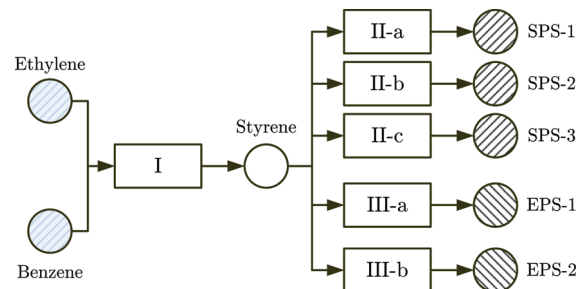


Figure 5. Polystyrene production process.

$$\sum_t \sum_{l^* \in L_t} \sum_{h_{l^*} \in AH_{l^*} h_l} JB_{fth_{l^*}}^{l^*} \leq 1 \quad \forall f \notin (\text{sup} \cup M), t = T, l \in L_t, h_l \quad (25)$$

$$\sum_{j \in (J_f \cap \text{stor})} \left( \sum_{t' \leq t} \sum_{l^* \in L_{t'}} \sum_{h_{l^*} \in AH_{l^*} h_l} SB_{fth_{l^*}}^{l^*} - V_{fth_{l-1}}^l \right) \geq 0 \quad \forall f \notin (\text{sup} \cup M), l, h_{l-1}, t \in T_l \quad (26)$$

$$\sum_t \sum_{l^* \in L_t} \sum_{h_{l^*} \in AH_{l^*} h_l} SB_{fth_{l^*}}^{l^*} \leq 1 \quad \forall f \notin (\text{sup} \cup M), t = T, l \in L_t, h_l \quad (27)$$

**Financial Model.** This financial model is similar to the one presented in the work of Puigjaner and Laínez.<sup>14</sup> The financial model aims at computing the expected corporate value ( $E[CV]$ ) which is the objective function to be maximized. For clarity and completeness, such formulation can be found in the Appendix.

The stochastic problem embedded in the control algorithm can be thus mathematically posed as follows:

$$\max_{\mathcal{X}, \mathcal{Y}} E[CV]$$

subject to eqs 1–27; 32–66

$$\mathcal{X} \in \{0, 1\}; \quad \mathcal{Y} \in \mathbb{R}$$

Here  $\mathcal{X}$  denotes the model binary variables set, while  $\mathcal{Y}$  represents the model continuous variable set. This model considers demand as an uncertain parameter; however, it can be easily extended to consider other parameters' uncertainties. The only change to be done is to add the indexes  $l$  and  $h_l$  to the new uncertain parameter. These indexes identify and locate the parameter inside the scenario tree. Problems considering prices' and interest rates' uncertainties have been tackled using a similar approach.<sup>14</sup>

## Case Study

This example was first introduced by You and Grossmann.<sup>17</sup> This case study was motivated by a real world application concerning a polystyrene SC design. The polystyrene production process is shown in Figure 5. Styrene monomers are produced from ethylene and benzene, then styrene is processed to obtain five final products: three different types of solid polystyrene (SPS) and two types of expandable polystyrene (EPS). We assume that only one type of reactor can be installed for each production task (i.e., I, II, III). The equipment capacity may be increased in a discrete manner which corresponds to the quantity of installed reactors. Additionally, it is considered that a cleaning task must be performed when shifting to a different polystyrene production. Potential benzene suppliers are located in Texas (TX), Louisiana (LA), and Alabama (AL); while ethylene suppliers are located in Illinois (IL), TX, and Mississippi (MS). Customers are aggregated into nine sale regions according to their geographical proximity. Distribution centers and processing plants may be established in eight different states which are Michigan (MI), TX, California (CA), LA, Nevada (NV), Georgia (GA), Pennsylvania (PA), and Iowa (IA). Figure 6 shows the SC components locations.

An horizon of 4 years is considered. Each year is composed of 12 monthly planning periods. In this example, market demand is regarded as an uncertain factor which unfolds every year. It is considered that demand may develop into 3 different events leading to a scenario tree which contains 81 scenarios. Given that the considered demand pattern does not show any trend

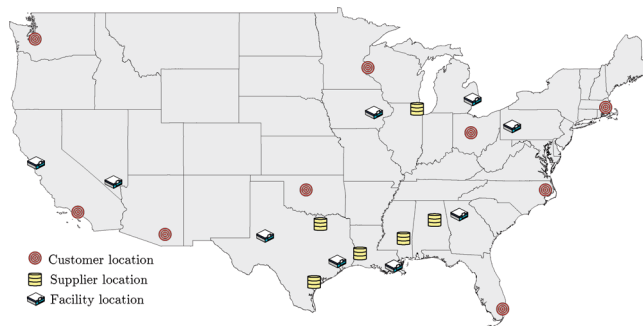


Figure 6. Potential SC echelons location map.

and seasonal component, simple exponential smoothing techniques are used in the forecasting module in order to determine the mean values and the forecast error characterization. A methodology to generate scenarios from demand forecasting has been proposed in the work of Puigjaner and Laínez.<sup>14</sup> Such methodology has been applied in this work. Basically, the forecast errors' distributions depend on previous errors and how many periods ahead the forecast is being done. If the correct forecasting model has been chosen and if the statistical procedure used to estimate parameters in the model yields unbiased estimates, then the expected forecast error will be zero and its standard deviation ( $\sigma$ ) can be easily calculated.<sup>18</sup> Once the standard deviation error is calculated, a Monte Carlo sampling method can be applied to the error probability distribution described by  $N(0, \sigma)$  in order to obtain error scenarios. Later on, error is added to the mean demand to get their corresponding demand scenarios. To approximate the multistage stochastic problem solution, the two-stage shrinking horizon (SHT) approach presented in the work of Balasubramanian and Grossmann<sup>19</sup> is used.

The case study has been also solved using a sequential manner in order to emphasize the benefits that may be gained by using the proposed integrated approach. In the sequential approach, the scheduling is not considered when dealing with the design of the SC network. Under this scheme, decisions are made following a hierarchical decision-making process. First the SC design decisions are made in an isolated manner. Then, once the SC network configuration has been obtained, the planning and scheduling decisions are determined. Otherwise, the integrated approach considers planning and scheduling decisions when designing the SC network. The integrated approach is deployed by using the MILP previously described.

The case study results are divided in three subsections. First, the SC design problem is tackled. From this first step, the optimal SC network configurations for both approaches (sequential and integrated) are obtained. Second, both solutions are tested using the MPC loop; so that operations are scheduled for every period following a rolling horizon procedure. Here, it is demonstrated how well SC configurations work when deployed for daily operations use (scheduling details). As a matter of fact, the results from this step show a more adequate performance assessment of the optimal SC network configurations. Finally, it is shown how equipment failures are resolved using the MPC algorithm.

**Design Problem.** In the first step of the control strategy, an SC design problem is solved. The scheduling model is taken into consideration in the first month for the integrated approach. As shown in Figure 7, the  $E[CV]$  and financial risk for the traditional sequential approach seem to yield better values. Here, financial risk can be defined as the probability of not meeting a certain CV level. For this case study, the integrated approach



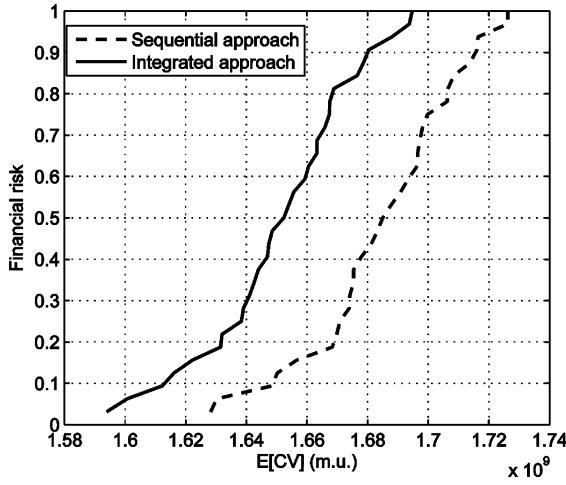


Figure 7. Risk curve for corporate value (sequential approach  $E[CV] = 1.68 \times 10^9$  mu; integrated approach  $E[CV] = 1.65 \times 10^9$  mu).

Location	Equipment (h/period)			Distribution center (c.u.)
	I	II	III	
TX (Houston)	18000.0	3600.0		1080.0
CA	1440.0	1440.0		
LA	18720.0		2880.0	832.0
MI	1440.0		1440.0	
TX (Midland)				633.1
GA				576.3
IA				287.6

Installed   
Not installed

Figure 8. Optimal network configuration obtained using the sequential approach.

results in a solution 2.05% lower than the sequential one. However, it should be noted that SC performance may worsen when executing detailed scheduling for the sequential approach. Evidently, capacity aggregation considered at the design level results in higher nominal plant productivity, since idle times introduced by task sequencing and changeovers are disregarded at the strategic level. The integrated approach avoids this situation by incorporating the scheduling formulation in the design problem. Therefore, no proper comparison nor conclusions can be drawn from these preliminary results.

The optimal configuration obtained by addressing the SC design–planning problem using the sequential approach is shown in Figure 8; while the optimal SC network configuration resulting from the integration of the three hierarchical decision levels is shown in Figure 9. As it can be noticed, the SC network configurations proposed by each approach are quite different. The integrated approach installs equipment technology II in three additional sites, namely, LA, MI, and TX (Midland); while equipment technology III is also installed in three additional sites, namely, TX (Houston), GA, and IA. In those figures, it is shown that more capacity is installed for equipment technologies II and III by the integrated approach.

**Lagrangian Decomposition.** The optimal condition decomposition (OCD), which is a particular case of the Lan-

Location	Equipment (h/period)			Distribution center (c.u.)
	I	II	III	
TX (Houston)	18000.0	2880.0	720.0	957.0
CA	720.0	1440.0		
LA	18000.0	720.0	2880.0	938.8
MI	1440.0	720.0	2160.0	536.2
TX (Midland)		720.0		501.5
GA			720.0	606.7
IA			720.0	

Installed   
Not installed

Figure 9. Optimal network configuration obtained using the integrated approach.

grangean relaxation procedure, is applied to overcome the computational cost of solving the monolithic problem which integrates the design, planning, and scheduling formulations. One advantage of OCD is that it provides information to update multiplier estimates in each subproblem iteration; therefore, no master problem exists and the algorithm converges in fewer iterations.

The variables which are complicating the integrated mathematical model are the first-stage, design variables ( $V_{jft\bar{h}_l}^l$ ,  $FE_{jft\bar{h}_l}^l$ ). We duplicate these two variables so that one copy exists for every combination of events ( $\bar{h}_l$ ). The following constraints have been added in order to do so.

$$\tilde{V}_{jft\bar{h}_l}^l - V_{jft\bar{h}_{l-1}}^l = 0 \quad \forall f \notin (\text{sup} \cup M), j \in \hat{J}_p, l, t \in T_p, \bar{h}_l, \bar{h}_{l-1} \in AH_{l-1, \bar{h}_l} \quad (28)$$

$$\tilde{FE}_{jft\bar{h}_l}^l - FE_{jft\bar{h}_{l-1}}^l = 0 \quad \forall f \notin (\text{sup} \cup M), j \in \hat{J}_p, l, t \in T_p, \bar{h}_l, \bar{h}_{l-1} \in AH_{l-1, \bar{h}_l} \quad (29)$$

Now, the affected eqs 2 and 3 are rewritten in terms of the proper duplicated variables.

$$\tilde{V}_{jft\bar{h}_l}^l FE_{jft}^L \leq \tilde{FE}_{jft\bar{h}_l}^l \leq \tilde{V}_{jft\bar{h}_l}^l FE_{jft}^U \quad \forall f \notin (\text{sup} \cup M), j \in \hat{J}_p, l, \bar{h}_l, t \in T_l \quad (30)$$

$$F_{jft\bar{h}_l}^l = F_{jft-1\bar{h}_{l-1}}^{l*} + \tilde{FE}_{jft\bar{h}_l}^l \quad \forall f \notin (\text{sup} \cup M), j \in \hat{J}_p, l, \bar{h}_l, t \in T_l, l^* \in L_{l-1}^*, \bar{h}_{l-1} \in AH_{l-1, \bar{h}_l} \quad (31)$$

The main difference between the OCD and the classical Lagrangean decomposition is that the OCD does not dualize all the complicating constraints. Instead, a subproblem is obtained by dualizing all the complicating constraints of other subproblems and maintaining its own complicating variables. Therefore, the OCD does not need any procedure to update the Lagrange multipliers since this updating process is automatic and results from keeping the complicating constraints in every subproblem. More details about the convergence properties and the procedure of this decomposition technique can be found in the work of Conejo et al.<sup>20</sup> Here, a subproblem is constructed for every scenario ( $\bar{h}_L$ ). Let us define  $\bar{h}_L$  as the scenario that is



Table 1. Optimal Condition Decomposition Algorithm

<b>Initialization</b> Each sub-problem initialize its complicating variables and multipliers
<b>Set</b> $gap = +\infty$
<b>While</b> $gap \leq 0.03$ <b>do</b>
<b>For</b> all scenarios
<b>Fix</b> complicating variables and multipliers of other sub-problems using previous iteration solutions (initial values in case of first iteration)
<b>Solve</b> subproblem $h_L^l$
<b>Save</b> complicating variables and multipliers of this sub-problem
<b>End For</b>
<b>Compute</b> $gap \leftarrow$ relative complicating variables change in two consecutive iterations
<b>End While</b>

Table 2. Decomposition Subproblem’s Complexity

problem	iterations	equations	variables	discrete variables	time (CPU sec)
decomposed	9	80 256	780 612	14 724	49 885

being evaluated in each subproblem. Then, the Lagrangean decomposition is applied by dualizing those eqs 28 and 29 that belong to other subproblems (scenarios). Notice that the following subproblem is decomposable when the Lagrange multipliers ( $\pi^l$ ,  $\pi^u$ ) and the duplicated variables for other scenarios are fixed to a given value. Also, it is noteworthy that constraints 28 and 29 related to the scenario being evaluated ( $h_L^l$ ) are left into the subproblem, so that their corresponding dual variables (Lagrange multipliers) are obtained and updated using the subproblem solution.

$$\begin{aligned} \max \quad & P_{h_L^l}^{L'} CV_{h_L^l}^{L'} + \sum_{f \in (\text{sup} \cup M)} \sum_{j \in J_f} \sum_l \sum_{i \in T_l} \sum_{h_i \in AH_{li} h_L^l} \times \\ & \sum_{h_{l-1} \in AH_{l-1} h_l} \pi_{f,j,l,h_l,h_{l-1}}^l (\tilde{V}_{jffh_l}^l - V_{jffh_{l-1}}^l) + \\ & \sum_{f \in (\text{sup} \cup M)} \sum_{j \in J_f} \sum_l \sum_{i \in T_l} \sum_{h_i \in AH_{li} h_L^l} \times \\ & \sum_{h_{l-1} \in AH_{l-1} h_l} \pi_{f,j,l,h_l,h_{l-1}}^{ll} (\tilde{F}E_{jffh_l}^l - FE_{jffh_{l-1}}^l) \end{aligned}$$

subject to eqs 1; 4–66  $\forall h_l \in AH_{li} h_L^l$ .

The decomposition algorithm is described in Table 1. The decomposition subproblem’s complexity for this case study is presented in Table 2. As shown, this kind of problem can be solved with reasonable computational cost by using the OCD strategy. The problems were solved in an Intel 2 Core Duo, 2.0 GHz, 2 GB RAM with a 3% integrality gap.

**Testing Solutions Using the MPC Framework.** To demonstrate the benefits of using the integrated model, the algorithm has been repeated also during the 48 planning periods contemplated in the whole planning horizon. Each period represents 1 month. Here, the uncertainty is assumed to change every month. In order to model the SC process for this case, the production rates and acquisition of production resources ( $P_{ijfth_l}^l$ ) become first stage variables. Both SC optimal configurations (integrated and sequential) have been tested. As previously stated, the results from this step allow us to make a fair and real comparison between the integrated and sequential approach.

Figure 10 shows how average corporate value behaves along the planning horizon for both approaches. The values presented

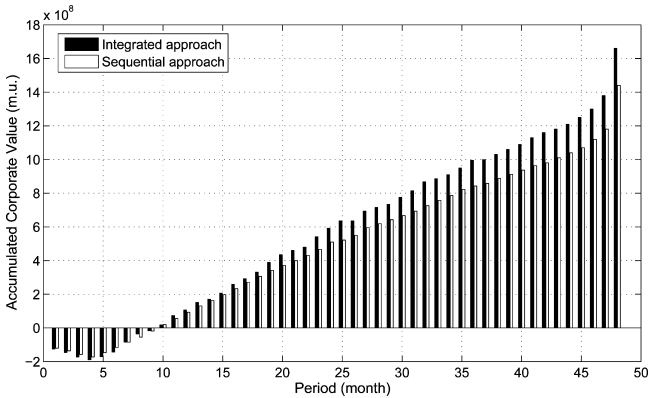


Figure 10. Corporate value behavior by simulating operations scheduling for both approaches.

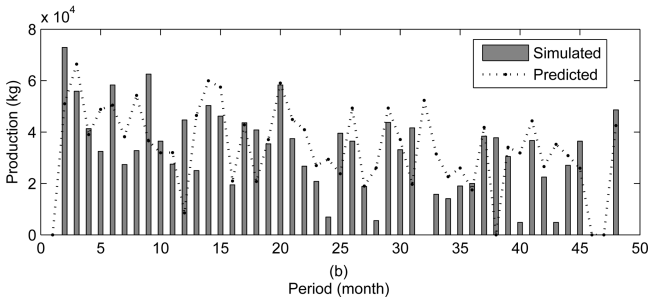
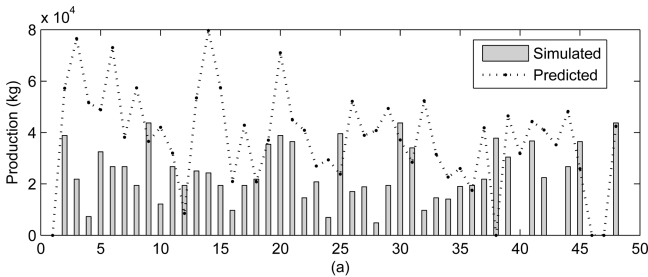
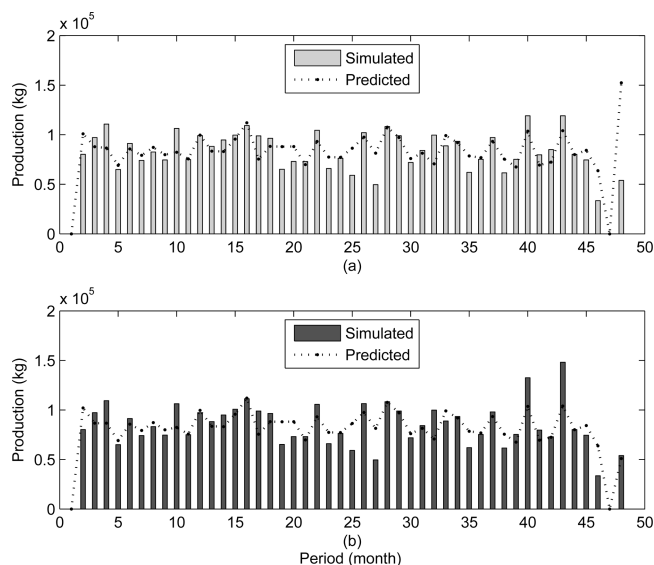


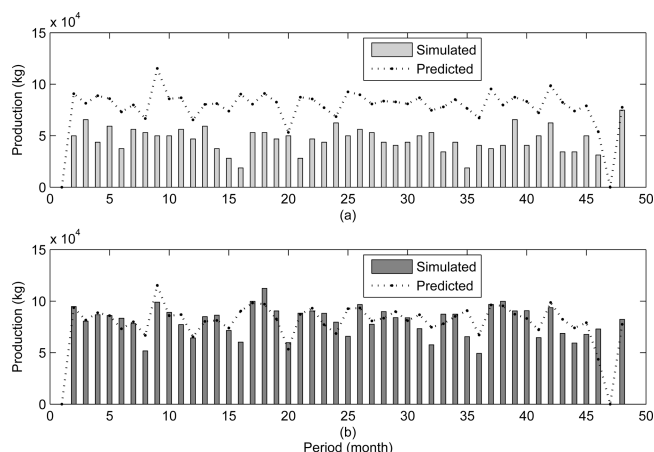
Figure 11. Production level obtained by the design phase vs production level obtained by simulating scheduling operations for final product s5 with the (a) sequential approach and (b) integrated approach.

in that figure are obtained by simulating production scheduling for each period using the MPC framework. The execution of the MPC loop is equivalent to applying a rolling horizon procedure. At the end of 48th period using the integrated approach, the SC system yields to a CV of  $1.66 \times 10^9$  mu which is 0.79% higher than the one predicted during the design phase ( $1.65 \times 10^9$  mu). On the other hand, the sequential approach yields a CV equal to  $1.44 \times 10^9$  mu at the end of the planning horizon which is 14.47% lower than the one predicted during the design phase ( $1.69 \times 10^9$  mu). The production levels for final products are depicted in Figures 11–14. It can be seen from these figures that the sequential approach SC configuration does not have enough capacity to reach the s5 and s6 production levels that were predicted during the design phase. Table 3 compares the final product total production levels predicted during the SC design step and the ones obtained during the production scheduling simulation. The overall total production deviation is –1.54% for the integration approach, while –22.86% for the sequential one.

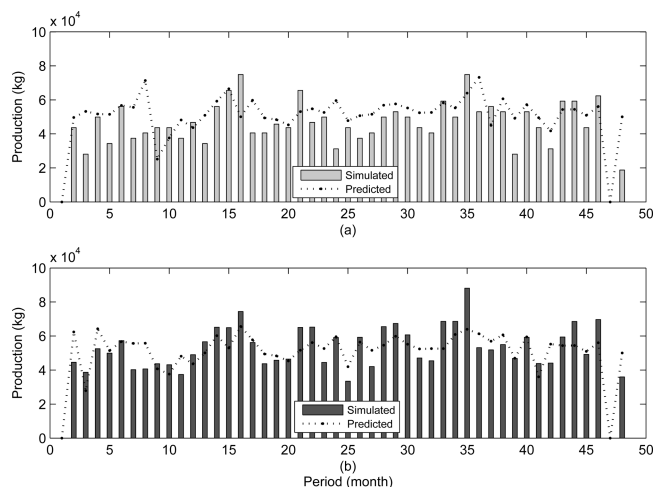
These results show that the integrated approach allows the SC manager to make more appropriate strategic decisions. The deviations observed in the SC performance, when different configurations of the SC are deployed into real scenarios, can be reduced by using the integrated approach. What is more,



**Figure 12.** Production level obtained by the design phase vs production level obtained by simulating scheduling operations for final product *s6* with the (a) sequential approach and (b) integrated approach.



**Figure 13.** Production level obtained by the design phase vs production level obtained by simulating scheduling operations for final product *s7* with the (a) sequential approach and (b) integrated approach.



**Figure 14.** Production level obtained by the design phase vs production level obtained by simulating scheduling operations for final product *s8* with the (a) sequential approach and (b) integrated approach.

the SC configuration proposed by the integrated approach clearly outperforms in terms of CV (15.47%) the one proposed by the

**Table 3.** Production Predicted during Design vs Simulated Production

	production	integrated approach	sequential approach
<i>s5</i>	predicted ( $10^6$ kg)	1.63	1.83
	simulated ( $10^6$ kg)	1.48	1.06
	deviation (%)	-9.35	-41.99
<i>s6</i>	predicted ( $10^6$ kg)	3.87	3.98
	simulated ( $10^6$ kg)	3.91	3.87
	deviation (%)	0.92	-2.60
<i>s7</i>	predicted ( $10^6$ kg)	3.80	3.73
	simulated ( $10^6$ kg)	3.71	2.14
	deviation (%)	-0.02	-0.43
<i>s8</i>	predicted ( $10^6$ kg)	2.44	2.44
	simulated ( $10^6$ kg)	2.47	2.16
	deviation (%)	1.16	-11.38
total	predicted ( $10^6$ kg)	11.74	11.97
	simulated ( $10^6$ kg)	11.56	9.24
	deviation (%)	-1.54	-22.86

**Table 4.** Changes in Production Tasks Due to Equipment Failure

site	material	production level (kg)	
		before failure	after failure
LA	styrene	94437.3	90957.7
	EPS-1	43680.0	26743.9
	EPS-2	18720.0	20056.1
GA	EPS-1	9328.5	15600.0
IA	EPS-1	8489.1	15600.0

sequential approach when they are tested using the production scheduling simulation.

**Failure Consideration.** Regarding the control policy at the low level, a statistical process strategy is considered for the styrene reactor. A multivariate statistical process control strategy based on the latent variable model is implemented for the reactor in order to monitor the polymer production. The monitoring strategy will enable the determination of whether the reactor operation is or is not under control. The monitoring strategy determines whether a reactor breakdown exists or not when the system is found out of control. The system is supposed to be continuously monitoring the reactor operation. Once an abnormal situation is detected and diagnosed, a decision about how to continue the operations has to be made by the SC controller shown in Figure 2. For the sake of simplicity, merely the operation time and the reactor temperature will be considered as control variables and their respective values will be obtained by a simplified reactor simulation model. The monitoring model was developed in Matlab,<sup>21</sup> and the connection with the SC control algorithm was carried out by using matgams (an interface created by Ferris.<sup>22</sup>)

A breakdown in equipment technology *III* in plant site in LA is simulated and detected in the ninth planning period. The main changes in production planning due to this failure are the following:

- Production levels for EPS-I and EPS-II are considerably reduced in LA site. The tasks to produce these two products are performed in equipment technology *III*.

- The styrene production level is slightly reduced in site LA. A tactical decision at the SC level is made by the algorithm to transfer the styrene production that cannot be processed at this site to sites GA and IA.

- The production level of EPS-I is increased at sites GA and IA. Both locations had idle capacity of technology *III* before the failure.

The production and distribution levels before and after the failure for the affected sites are shown in Tables 4 and 5. As shown in this example, styrene has been transferred from LA to GA and IA as a result of this failure. Such movement of

**Table 5. Changes in Distribution Tasks Due to Equipment Failure**

sites		material	distribution level (kg)	
from	to		before failure	after failure
LA	GA	styrene	8862.1	14820.0
LA	IA	styrene	9370.7	14820.0

materials is not usually considered in traditional SC planning models. Furthermore, the capacity of other sites has been contemplated in order to resolve the incident at the LA site. This example demonstrates that when the strategy is applied, namely SC transparency is gained, the decision-making process to resolve incidents is performed at the SC level and not merely at the plant level, thus allowing the selection of the best plan in terms of value creation. It should be mentioned that the planning problem utilized in this work is able to consider movement of raw materials, intermediates, and final products between the different facilities comprising the SC network. Such flexibility increases the number of alternatives available to resolve the incidences since production can be transferred accordingly from one site to another.

### Concluding Remarks

A means of integrating the three standard SC decision levels is presented. Moreover, an SC design-planning model that permits a soft integration with scheduling models is presented. The results show that significant improvements can be gained when all of these decision levels are incorporated into a single model. It is demonstrated that decisions made solely on the SC design and disregarding the production scheduling scenario can lead to aggregated capacity overestimation and consequently to a fictitious better corporate value. Aggregated capacity overestimation can be solved by incorporating scheduling constraints into the design-planning problem. As a result, production rates are adjusted to the real working scenario and expected production profiles can be better sustained during the whole planning horizon. Additionally, the SC operations formulation included in this work is able to transfer any kind of material between any pair of SC components. This is an important feature toward finding the best way of resolving SC incidences.

It is important to highlight that capacity has been utilized as the linking aspect when integrating SC design, planning, and scheduling. Even more, it is demonstrated how capacity is the core factor that integrates supervisory control modules with scheduling formulations. The scheduling formulation requires details about individual equipment capacity which facilitates the exchange of information with the equipment supervisory control module. The integration of process monitoring capability allows us to take into account the effects of equipment failures and out of control operating conditions in the SC activities.

It has been shown that a Lagrangean decomposition can significantly reduce the computational burden associated to the solution of monolithic problems. In this work, optimal condition decomposition is applied. This technique facilitates updating the Lagrangean multipliers. In this way, one of the most common difficulties when applying Lagrangean relaxation methods is overcome.

Even though the computational cost increases with the number of periods, this increase is not significant if it is compared to the computational cost added with the number of scenarios. For that reason, a decomposition scheme is applied to tackle medium size problems. However, the number of scenarios to consider is still limited given the complexity of the integrated model. It

is envisaged that the use of a parallel computing frameworks may help to reduce the time required to the solution of this kind of problems.

### Acknowledgment

Financial support received from the “Generalitat de Catalunya” (FI programs, project I-0898) and European Community (project PRISM-MRTN-CT-2004-512233) is fully appreciated. Also, financial support from AGAUR (I0898) and MEC (DPI 2006-05673) is gratefully acknowledged.

### Nomenclature

#### Indices

$e$  = suppliers  
 $f$  = facility locations  
 $i$  = tasks  
 $j$  = plant equipment  
 $l$  = events in which uncertainty unfolds  
 $s$  = states  
 $t$  = planning periods  
 $t_k$  = scheduling time buckets  
 $h_l$  = combination of events at level  $l$  in the scenario tree

#### Sets

$AH_{h_l}$  = set of combination of events that belong to level  $l$  and are ancestors of  $h_l$   
 $E_s$  = set of suppliers  $e$  that provide raw material  $s$   
 $\hat{E}_{\text{prod}}$  = set of suppliers  $e$  that provide production services  
 $\tilde{E}_{\text{tr}}$  = set of suppliers  $e$  that provide transportation services  
 $FP$  = set of states  $s$  that are final products  
 $\hat{J}_f$  = equipment  $j$  that can be installed at location  $f$   
 $J_i$  = equipment that can perform task  $i$   
 $J_{\text{prod}}$  = equipment that performs production tasks  
 $J_{\text{stor}}$  = storage equipment  
 $M$  = set of market locations  
 $R_f$  = set of raw materials that can be provided from location  $f$   
 $RM$  = set of states  $s$  that are raw materials  
 $RM_e$  = set of raw materials that are offered by supplier  $e$   
 $\text{sup}$  = set of supplier locations  
 $Tr$  = set of distribution tasks  
 $T_s$  = set of tasks producing state  $s$   
 $\bar{T}_s$  = set of tasks consuming state  $s$

#### Parameters

$A_{sft}$  = maximum availability of raw material  $s$  in period  $t$  at location  $f$   
 $\text{cline}^{\text{max}}$  = upper bound of short-term credit line  
 $\text{coef}_{et'}$  = technical discount coefficient for payments to external supplier  $e$  executed in period  $t'$  on accounts incurred in period  $t$   
 $d_e$  = maximum delay on payments of supplier  $e$   
 $\bar{d}_m$  = maximum delay in receivables at market  $m$   
 $\bar{d}_M^{\text{max}}$  = maximum delay in receivables at all markets  
 $D_{it}^{\text{MS}}$  = technical coefficient for investments in marketable securities  
 $\text{dem}_{sft h_l}^{\text{MS}}$  = product  $s$  demand at market  $f$  in period  $t$  for combination of events  $h_l$   
 $E_{it}^{\text{MS}}$  = technical coefficient for sales of marketable securities  
 $E[\text{ROE}]_{h_l}$  = expected return on equity associated with combination of events  $h_l$   
 $\text{FCF}J_{ft}$  = fixed cost per unit of capacity of plant equipment  $j$  at location  $f$  in period  $t$   
 $\text{FCFS}_{ft}$  = fixed cost per unit of distribution center capacity at location  $f$  in period  $t$   
 $ir_t^{\text{LD}}$  = interest rate of long-term debt  
 $ir_t^{\text{SD}}$  = interest rate of short-term debt

$I_{ft}^j$  = investment required to establish a processing facility in location  $f$  in period  $t$   
 $I_{ft}^d$  = investment required to establish a distribution center in location  $f$  in period  $t$   
 $Iu_{st}$  = value of inventory of state  $s$  in period  $t$   
 mincash = lower bound of cash  
 minCSL = lower bound of customer service level  
 other $_t$  = other expected outflows or inflows of cash in period  $t$   
 $pt_i$  = task  $i$  processing time  
 $price_{st}$  = price of product  $s$  at market  $f$  in period  $t$   
 $price_{ft}^{Fj}$  = investment required per unit of capacity of equipment  $j$  increased at facility  $f$  in period  $t$   
 $price_{ft}^{FS}$  = investment required per unit of distribution center capacity increased at facility  $f$  in period  $t$   
 $r_t^{0,j}$  = risk free rate of return in period  $t$   
 $rp$  = risk premium rate  
 $Y_t^{NI}$  = marketable securities of the initial portfolio maturing in period  $t$   
 $trate$  = tax rate  
 $|T|$  = length of planning horizon

#### Binary Variables

$JB_{ft_{i-1}}^j$  = 1 if a processing site at location  $f$  is established in period  $t$ , 0 otherwise  
 $SB_{ft_{i-1}}^j$  = 1 if a distribution center at location  $f$  is established in period  $t$ , 0 otherwise  
 $V_{ft_{i-1}}^j$  = 1 if the equipment  $j$  capacity is increased at location  $f$  in period  $t$ , 0 otherwise  
 $\tilde{V}_{ft_{i-1}}^j$  = duplicated variable for  $V_{ft_{i-1}}^j$   
 $W_{ft_{i-1}}^j$  = 1 if task  $i$  starts at scheduling time bucket  $t_k$  to be performed in equipment  $j$  at site  $s$  for combination of events  $h_i$

#### Continuous Variables

$Apay_{th_i}^j$  = amount of accounts payable in period  $t$  for combination of events  $h_i$   
 $Arec_{th_i}^j$  = amount of accounts receivable in period  $t$  for combination of events  $h_i$   
 $Asales_{tr_{th_i}}^j$  = sales executed in period  $t$  and receivable in period  $t'$  for combination of events  $h_i$   
 $B_{ijft_{i-1}}^j$  = batch size for task  $i$  performed in technology  $j$  at site  $f$  in time bucket  $t_k$  for combination of events  $h_i$   
 $borrow_{th_i}^j$  = total amount borrowed from the short-term credit line in period  $t$  for combination of events  $h_i$   
 $capital_{th_i}^j$  = capital supported by shareholders in period  $t$  for combination of events  $h_i$   
 $cash_{th_i}^j$  = cash in period  $t$  for combination of events  $h_i$   
 $Cline_{th_i}^j$  = short-term debt in period  $t$  for combination of events  $h_i$   
 $CV_{th_L}^j$  = corporate value at the end of the planning horizon for combination of events  $h_L$   
 $dep_{th_i}^j$  = depreciation in period  $t$  for combination of events  $h_i$   
 $DFCF_{th_L}^j$  = sum of discounted free cash flows at the end of the planning horizon for combination of events  $h_L$   
 $Ecash_{th_i}^j$  = exogenous cash in period  $t$  for combination of events  $h_i$   
 $Epur_{th_i}^j$  = economic value of purchases executed in period  $t$  to supplier  $e$  for combination of events  $h_i$   
 $Esales_{th_i}^j$  = economic value of sales carried out in period  $t$  for combination of events  $h_i$   
 $E[CV]$  = expected corporate value  
 $Fasset_{th_i}^j$  = increment in fixed assets in period  $t$  for combination of events  $h_i$   
 $FCF_{th_i}^j$  = free cash flows in period  $t$  for combination of events  $h_i$   
 $Fcost_{th_i}^j$  = fixed cost in period  $t$  for combination of events  $h_i$   
 $FEx_{th_i}^j$  = other financial expenses and incomes in period  $t$  for combination of events  $h_i$

$F_{jft_{i-1}}^j$  = plant equipment  $j$  total capacity during period  $t$  at location  $f$  for combination of events  $h_i$   
 $FE_{jft_{i-1}}^j$  = plant equipment  $j$  capacity increment at location  $f$  during period  $t$  for combination of events  $h_i$   
 $\tilde{FE}_{jft_{i-1}}^j$  = duplicated variable for  $FE_{jft_{i-1}}^j$   
 $Lborrow_{th_i}^j$  = total amount of money borrowed from the long-term credit line in period  $t$  for combination of events  $h_i$   
 $Ldebt_{th_i}^j$  = long-term debt in period  $t$  for combination of events  $h_i$   
 $Lrepay_{th_i}^j$  = total amount repaid to the long-term credit line in period  $t$  for combination of events  $h_i$   
 $net_{th_i}^{Cline,j}$  = total amount of money borrowed or repaid to the short-term credit line for combination of events  $h_i$  in period  $t$   
 $net_{th_i}^{debt,j}$  = total amount of money borrowed or repaid to the long-term credit line in period  $t$  for combination of events  $h_i$   
 $net_{th_i}^{MS,j}$  = total amount received or paid in securities transactions in period  $t$  for combination of events  $h_i$   
 $netdebt_{th_L}^j$  = net total debt at final period  $T$  for combination of events  $h_L$   
 $netinvest_{th_i}^j$  = net investment in period  $t$  for combination of events  $h_i$   
 $P_{ijst_{i-1}}^j$  = production rate of product  $i$  in equipment  $j$  at site  $s$  in period  $t$  for combination of events  $h_i$   
 $pay_{tr_{th_i}}^j$  = payments to external supplier  $e$  executed in period  $t'$  on accounts payable incurred in period  $t$  for combination of events  $h_i$   
 $pled_{tr_{th_i}}^j$  = amount pledged within period  $t'$  on accounts receivable maturing in period  $t$  for combination of events  $h_i$   
 $profit_{th_i}^j$  = profit achieved in period  $t$  for combination of events  $h_i$   
 $pur_{th_i}^{en,j}$  = amount of money payable to supplier  $e$  in period  $t$  for combination of events  $h_i$  associated with consumption of raw materials  
 $pur_{th_i}^{tr,j}$  = amount of money payable to supplier  $e$  in period  $t$  for combination of events  $h_i$  associated with transport services  
 $pur_{th_i}^{ut,j}$  = amount of money payable to supplier  $e$  in period  $t$  associated with consumption of production utilities for combination of events  $h_i$   
 $pur_{th_i}^{rm,j}$  = amount of raw material  $s$  purchased to supplier  $e$  at site  $f$  in period  $t$  for combination of events  $h_i$   
 $repay_{th_i}^j$  = total amount repaid to the short-term credit line in period  $t$  for combination of events  $h_i$   
 $rawM_{st_{i-1}}^j$  = incoming of material  $s$  to facility  $f$  during time bucket  $t_k$  for combination of events  $h_i$   
 $sales_{st_{i-1}}^j$  = amount of product  $s$  sold from location  $f$  in market  $f'$  in period  $t$  for combination of events  $h_i$   
 $Ssched_{st_{i-1}}^j$  = inventory of state  $s$  at location  $f$  in time bucket  $t_k$  for combination of events  $h_i$   
 $St_{st_{i-1}}^j$  = inventory of state  $s$  at location  $f$  in period  $t$  for combination of events  $h_i$   
 $SV_{th_L}^j$  = salvage value of facilities at the end of planning horizon for combination of events  $h_L$   
 $WACC_{th_i}^j$  = weighted average cost of capital in period  $t$  for combination of events  $h_i$   
 $Y_{tr_{th_i}}^{MS}$  = cash invested in period  $t'$  in marketable securities maturing in period  $t$  for combination of events  $h_i$   
 $Z_{tr_{th_i}}^{MS,j}$  = security sold in period  $t'$  maturing in period  $t$  for combination of events  $h_i$   
 $\Delta Apay_{th_i}^j$  = change in amount of accounts payable in period  $t$  for combination of events  $h_i$   
 $\Delta Arec_{th_i}^j$  = change in amount of accounts receivable in period  $t$  for combination of events  $h_i$   
 $\Delta inv_{th_i}^j$  = change in inventory value in period  $t$  for combination of events  $h_i$   
 $\Delta NWC_{th_i}^j$  = change in net working capital in period  $t$



### Greek Symbols

$\alpha_{rij}$  = fixed coefficient for consumption of raw material  $r$  by product  $i$   
 $\beta_{sj}$  = minimum utilization of plant equipment  $j$  capacity allowed at site  $s$   
 $\delta_{stt'}$  = fraction of sales carried out in period  $t$  that are receivable in period  $t'$  in market  $s$   
 $\theta_{ij}$  = capacity consumption of plant equipment  $j$  by product  $i$   
 $\lambda_t$  = proportion of equity over total capital investment in period  $t$   
 $\pi_{f,j,l,h_l,h_{l-1}}^I$  = Lagrange multipliers for eq 28  
 $\pi_{f,j,l,h_l,h_{l-1}}^{II}$  = Lagrange multipliers for eq 29  
 $\rho_{eff}^{tr}$  = unitary transport costs from location  $f$  to location  $f'$  payable to external supplier  $e$   
 $\tau_{ije}^{utl}$  = cost of the utilities associated with task  $i$  manufactured in equipment  $j$  in site  $f$  and payable to external supplier  $e$   
 $\tau_{sfe}^{utl}$  = cost of the utilities associated with handling the inventory of material  $s$  in site  $f$  and payable to external supplier  $e$   
 $\phi_{tt'}$  = face value of accounts maturing in period  $t$  pledged in period  $t'$   
 $\psi_{est}$  = price of raw material  $s$  offered by external supplier  $e$  in period  $t$   
 $v_s$  = specific volume of material  $s$

### Superscripts

L = lower bound  
 U = upper bound

### Appendix: Financial Model

The financial model can be separated into two sections: a cash management formulation and a set of equations for the SC valuation methodology that consist of a stochastic discounted-free-cash-flow approach.

### Cash Management

The cash management formulation also considers the same  $t$  planning periods covering the whole time horizon in the strategic-tactical SC formulation. This assumption allows an easy integration of both sets of constraints into a unique holistic model.

The cash balance for each planning period and combination of events is calculated by means of eq 32. The cash at each period  $t$  ( $\text{cash}_{th_l}^I$ ) is a function of the available cash at period  $t-1$  ( $\text{cash}_{th_l}^{I*}$ ), the exogenous cash from the sales of products ( $\text{Ecash}_{th_l}^I$ ), the amount borrowed or repaid to the short-term credit line ( $\text{net}_{th_l}^{\text{Cline},I}$ ), the raw materials, production, and transport payments on accounts payable incurred in any previous or actual period  $t$  ( $\text{pay}_{et,th_l}^I$ ), the payments of the fixed cost ( $\text{Fcost}_{th_l}^I$ ), the sales and purchases of marketable securities ( $\text{net}_{th_l}^{\text{MS},I}$ ), the amount invested on facilities ( $\text{Fasset}_{th_l}^I$ ), the capital supported by the shareholders of the company ( $\text{capital}_{th_l}^I$ ), the amount borrowed or repaid to the long-term credit line ( $\text{net}_{th_l}^{\text{Ldebt},I}$ ), and, finally, other expected outflows or inflows of cash (other).

$$\text{cash}_{th_l}^I = \text{cash}_{th_l}^{I*} + \text{Ecash}_{th_l}^I + \text{net}_{th_l}^{\text{Cline},I} - \sum_e \sum_{t'=1}^t \text{pay}_{et,th_l}^I - \text{Fcost}_{th_l}^I + \text{net}_{th_l}^{\text{MS},I} - \text{Fasset}_{th_l}^I + \text{capital}_{th_l}^I + \text{net}_{th_l}^{\text{Ldebt},I} + \text{other}_I$$

$$\forall l, h_l, t \in T_l, l^* \in L_{t-1}^*, h_{l^*} \in AH_{l^*h_l} \quad (32)$$

A certain proportion of the accounts receivable may be pledged at the beginning of a period. It can be assumed that a certain proportion of the receivables outstanding at the beginning of a period is received during that period through pledge, as stated by eq 33. In this equation the variable  $\text{pled}_{tt',th_l}^I$  represents

the amount pledged within period  $t'$  on accounts receivable maturing in period  $t$ , while  $\text{Asales}_{tt',th_l}^I$  represents the accounts receivable associated with the sales of products executed in period  $t'$  and maturing in  $t$  (eq 34). Here, the parameter  $\bar{d}_M^{\text{max}}$  denotes the maximum maturing period at markets and  $\delta_{mtt'}$  denotes the fraction of sales carried out in market  $m$  in period  $t$  that will be paid in period  $t'$ .

$$\sum_{t'=t-\bar{d}_M^{\text{max}}}^{t'} \sum_{l^* \in L_{t'}} \sum_{h_{l^*} \in AH_{l^*h_l}} \text{pled}_{tt',th_l}^{I*} \leq \sum_{t'=t-\bar{d}_M^{\text{max}}}^{t'} \sum_{l^* \in L_{t'}} \sum_{h_{l^*} \in AH_{l^*h_l}} \text{Asales}_{tt',th_l}^I \leq \sum_{t'=t-\bar{d}_M^{\text{max}}}^{t'} \sum_{l^* \in L_{t'}} \sum_{h_{l^*} \in AH_{l^*h_l}} \text{Asales}_{tt',th_l}^I \quad \forall l, h_l, t \in T_l, t' \in [t - \bar{d}_M^{\text{max}}, t] \quad (33)$$

$$\text{Asales}_{tt',th_l}^I = \sum_s \sum_{f \in M} \text{sales}_{sft,th_l}^I \delta_{ftr} \text{price}_{sft,th_l}^I \quad \forall l, h_l, t \in T_l, t' > t \quad (34)$$

$$\bar{d}_M^{\text{max}} = \max_{f \in M} \{\bar{d}_f\} \quad (35)$$

The exogenous cash is computed by means of eq 36 as the difference between the amount of accounts receivable maturing in period  $t$  and that incurred in previous periods  $t'$  minus the amount of receivables pledged in previous periods on accounts receivable maturing in period  $t$  plus the amount pledged in the actual period on accounts receivable maturing in future periods. In this expression,  $\phi_{tt'}$  represents the face value of the receivables being pledged.

$$\text{Ecash}_{th_l}^I = \sum_{t'=t-\bar{d}_M^{\text{max}}}^t \sum_{l^* \in L_{t'}} \sum_{h_{l^*} \in AH_{l^*h_l}} \text{Asales}_{tt',th_l}^{I*} - \sum_{t'=t-\bar{d}_M^{\text{max}}}^{t-1} \sum_{l^* \in L_{t'}} \sum_{h_{l^*} \in AH_{l^*h_l}} \text{pled}_{tt',th_l}^{I*} + \sum_{t'=t+1}^{t+\bar{d}_M^{\text{max}}} \phi_{tt'} \cdot \text{pled}_{tt',th_l}^I$$

$$\forall l, h_l, t \in T_l \quad (36)$$

A short-term financing source is represented by an open line of credit with a maximum limit imposed by the bank (eq 37). Equations 38 and 39 make a balance on borrowings, considering for each period the updated debt from the previous periods, the balance between borrows and repayments, and the interest of the credit line. Moreover, the bank regularly requires a repayment greater than or equal to the interests accumulated in previous periods, as it is stated by eq 40.

$$\text{Cline}_{th_l}^I \leq \text{Cline}^{\text{max}} \quad \forall l, h_l, t \in T_l \quad (37)$$

$$\text{Cline}_{th_l}^I = \text{Cline}_{th_l}^{I*} (1 + ir_{th_l}^{\text{SD},I}) + \text{borrow}_{th_l}^I - \text{repay}_{th_l}^I$$

$$\forall l, h_l, t \in T_l, l^* \in L_{t-1}^*, h_{l^*} \in AH_{l^*h_l} \quad (38)$$

$$\text{net}_{th_l}^{\text{Cline},I} = \text{borrow}_{th_l}^I - \text{repay}_{th_l}^I \quad \forall l, h_l, t \in T_l \quad (39)$$

$$\text{repay}_{th_l}^I \geq ir_{th_l}^{\text{SD}} \cdot \text{Cline}_{th_l}^{I*}$$

$$\forall l, h_l, t \in T_l, l^* \in L_{t-1}^*, h_{l^*} \in AH_{l^*h_l} \quad (40)$$

With regard to the accounts payable, eq 41 forces the payments executed in period  $t'$  on accounts payable to supplier  $e$  incurred in period  $t$  to equal the total amount due. The payment constraints belonging to the last periods of time are formulated as inequalities (eq 42), as it is not reasonable to require that total accounts payable be zero at the end of the planning period.

$$\sum_{t'=t-d_e}^t \sum_{l^* \in L_{t'}} \sum_{\tilde{h}_{l^*} \in AH_{l^*} \tilde{h}_l} \text{pay}_{ett^*}^{l^*} \text{coef}_{ett^*} = \sum_{l^* \in L_{t-d_e}} \sum_{\tilde{h}_{l^*} \in AH_{l^*} \tilde{h}_l} \text{Epurch}_{e,t-d_e,\tilde{h}_{l^*}}^{l^*} \quad \forall e, t \in [d_e, |T|], l \in L, \tilde{h}_l \quad (41)$$

$$\sum_{t'=t}^T \sum_{l^* \in L_{t'}} \sum_{\tilde{h}_{l^*} \in AH_{l^*} \tilde{h}_L} \text{pay}_{ett^*}^{l^*} \text{coef}_{ett^*} \leq \sum_{l^* \in L_t} \sum_{\tilde{h}_{l^*} \in AH_{l^*} \tilde{h}_L} \text{Epurch}_{ett^*}^{l^*} \quad \forall e, \tilde{h}_L, t > (|T| - d_e) \quad (42)$$

Equation 43 makes a balance for marketable securities. It is assumed that all marketable securities can be sold prior to maturity at a discount or loss for the firm, as stated by eq 43. Equation 44 is applied to constraint in each period the total amount of marketable securities sold prior to maturity to be lower than the available ones (those belonging to the initial portfolio plus the ones purchased in previous periods minus those sold before).

$$\text{net}_{\tilde{h}_l}^{\text{MS},l} = Y_t^{\text{INI}} - \sum_{t'=t+1}^T Y_{t'\tilde{h}_l}^{\text{MS},l} + \sum_{t'=t+1}^T Z_{t'\tilde{h}_l}^{\text{MS},l} + \sum_{t'=1}^{t-1} \sum_{l^* \in L_{t'}} \sum_{\tilde{h}_{l^*} \in AH_{l^*} \tilde{h}_l} (1 + D_{tt'}^{\text{MS}}) Y_{tt'}^{\text{MS},l^*} - \sum_{t'=1}^{t-1} \sum_{l^* \in L_{t'}} \sum_{\tilde{h}_{l^*} \in AH_{l^*} \tilde{h}_l} (1 + E_{tt'}^{\text{MS}}) Z_{tt'}^{\text{MS},l^*} \quad \forall l, \tilde{h}_l, t \in T_l \quad (43)$$

$$\sum_{t'=1}^{t'} \sum_{l^* \in L_{t'}} \sum_{\tilde{h}_{l^*} \in AH_{l^*} \tilde{h}_l} Z_{tt'}^{\text{MS},l^*} (1 + E_{tt'}^{\text{MS}}) \leq Y_t^{\text{INI}} + \sum_{t'=1}^{t'-1} \sum_{l^* \in L_{t'}} \sum_{\tilde{h}_{l^*} \in AH_{l^*} \tilde{h}_l} (1 + D_{tt'}^{\text{MS}}) Y_{tt'}^{\text{MS},l^*} \quad \forall l, \tilde{h}_l, t \in T_l, t' < t \quad (44)$$

Equation 45 balances the investment with the capital supported by shareholders ( $\text{capital}_{\tilde{h}_l}^l$ ) and the amount borrowed to banks as long-term debt ( $\text{Lborrow}_{\tilde{h}_l}^l$ ) at each time period  $t$  and combination of events  $\tilde{h}_l$ .

$$\text{Fasset}_{\tilde{h}_l}^l = \text{Lborrow}_{\tilde{h}_l}^l + \text{capital}_{\tilde{h}_l}^l \quad \forall l, \tilde{h}_l, t \in T_l \quad (45)$$

Equations 46–48 reflect the payment conditions associated with the long-term debt. These constraints are similar to those associated with the short-term credit line, but the amount repaid in each period of time  $\text{Lrepay}_{\tilde{h}_l}$  remains usually constant in every planning period.

$$\text{Ldebt}_{\tilde{h}_l}^l = \text{Ldebt}_{t-1\tilde{h}_l}^{l^*} (1 + i_{\tilde{h}_l}^{\text{LD},l}) + \text{Lborrow}_{\tilde{h}_l}^l - \text{Lrepay}_{\tilde{h}_l}^l \quad \forall l, \tilde{h}_l, t \in T_l, l^* \in L_{t-1}^*, \tilde{h}_{l^*} \in AH_{l^*} \tilde{h}_l \quad (46)$$

$$\text{net}_{\tilde{h}_l}^{\text{Ldebt},l} = \text{Lborrow}_{\tilde{h}_l}^l - \text{Lrepay}_{\tilde{h}_l}^l \quad \forall l, \tilde{h}_l, t \in T_l \quad (47)$$

$$\text{Lrepay}_{\tilde{h}_l}^l \geq i_{\tilde{h}_l}^{\text{LD},l} \cdot \text{Ldebt}_{t-1\tilde{h}_l}^{l^*} \quad \forall l, \tilde{h}_l, t \in T_l, l^* \in L_{t-1}^*, \tilde{h}_{l^*} \in AH_{l^*} \tilde{h}_l \quad (48)$$

Equation 49 limits the cash in each period ( $\text{cash}_{\tilde{h}_l}^l$ ) to be larger than a minimum value ( $\text{mincash}$ ). A minimum cash is usually required to handle uncertain events.

$$\text{cash}_{\tilde{h}_l}^l \geq \text{mincash} \quad \forall l, \tilde{h}_l, t \in T_l \quad (49)$$

#### Valuation Method: Stochastic DFCF Method

The DFCF method calculates the enterprise value by determining the present value of its future cash flows and discounting

them taking into account the appropriate capital cost during the time horizon for which it is defined.<sup>12</sup>

Equation 50 computes the expected corporate value ( $E[\text{CV}]$ ) as the weighted average of the corporate value calculated for each combination of events at the end of the time horizon. Here,  $P_{\tilde{h}_l}^L$  represents the probability of occurrence of each combination of events.

$$E[\text{CV}] = \sum_{\tilde{h}_L} P_{\tilde{h}_L}^L \text{CV}_{\tilde{h}_L}^L \quad (50)$$

According to financial theory, enterprise market value of a firm is given by the difference between the discounted stream of future cash flows during the planning horizon and the initial net total debt ( $\text{netdebt}_0^L$ ), as it is stated by eq 51. The final total debt includes both the short and the long-term debt and also the cash (eq 52).

$$\text{CV}_{\tilde{h}_L}^L = \text{DFCF}_{\tilde{h}_L}^L - \text{netdebt}_0^L \quad \forall \tilde{h}_L \quad (51)$$

$$\text{netdebt}_0^L = \text{Cline}_0^L + \text{Ldebt}_0^L - \text{cash}_0^L \quad \forall \tilde{h}_L \quad (52)$$

In the calculation of the DFCF, one must discount the free cash flows of each period  $t$  and the salvage value (SV) at a rate equivalent to the capital cost (eq 53). The salvage value could be calculated as a percentage of the total investment or by any other applicable method.

$$\text{DFCF}_{\tilde{h}_L}^L = \left( \sum_t \sum_{l \in L_t} \sum_{\tilde{h}_l \in AD_{\tilde{h}_L}} \frac{\text{FCF}_{t\tilde{h}_l}^l}{(1 + \text{WACC}_{\tilde{h}_l}^l)^t} \right) + \frac{\text{SV}_{\tilde{h}_L}^L}{(1 + \text{WACC}_{\tilde{h}_L}^L)^T} \quad \forall \tilde{h}_L \quad (53)$$

**Capital Cost.** The capital cost can be determined through the weighted average method (eq 54). In this expression,  $\lambda_t$  denotes the proportion of equity over the total capital investment. To compute the expected return on equity, which is denoted by  $E(\text{ROE})$ , eq 55 is applied.

$$\text{WACC}_{\tilde{h}_l}^l = \lambda_t E(\text{ROE})_{\tilde{h}_l}^l + i_{\tilde{h}_l}^l (1 - \lambda_t) (1 - \text{trate}) \quad \forall l, \tilde{h}_l, t \in T_l \quad (54)$$

$$E(\text{ROE})_{\tilde{h}_l}^l = r_{\tilde{h}_l}^{0,l} + \varphi Re \quad (55)$$

**Free Cash Flow.** Free cash flows at every period  $t$  ( $\text{FCF}_{\tilde{h}_l}^l$ ) are given by the profit after taxes, net change in investments, and change in net working capital. Specifically, the free cash flows are the difference between the net operating profit after taxes (NOPAT) and the increase in capital invested. From this definition, it follows that there will be value creation if the incoming value ( $\text{profit}_{\tilde{h}_l}^l (1 - \text{trate})$ ) is greater than the consumed value ( $\Delta \text{NWC}_{\tilde{h}_l}^l$ ) as shown in eq 56.

$$\text{FCF}_{\tilde{h}_l}^l = \text{profit}_{\tilde{h}_l}^l (1 - \text{trate}) - \text{netinvest}_{\tilde{h}_l}^l - \Delta \text{NWC}_{\tilde{h}_l}^l \quad \forall l, \tilde{h}_l, t \in T_l \quad (56)$$

**Net Operating Profit.** Equation 57 is applied to compute the profit at each period  $t$  and combination of events  $\tilde{h}_l$ .

$$\text{profit}_{\tilde{h}_l}^l = \text{Esales}_{\tilde{h}_l}^l - \left( \sum_e \text{Epurch}_{e\tilde{h}_l}^l + \text{Fcost}_{\tilde{h}_l}^l - \Delta \text{inv}_{\tilde{h}_l}^l \right) \quad \forall l, \tilde{h}_l, t \in T_l \quad (57)$$

**Net Fixed Capital.** The net investment at each period  $t$  represents the monetary value of the fixed assets acquired in that period minus the depreciation (eq 58).

$$\text{netinvest}_{t\hat{h}_l}^l = \text{Fasset}_{t\hat{h}_l}^l - \text{dep}_{t\hat{h}_l}^l \quad \forall l, \hat{h}_l, t \in T_l \quad (58)$$

**Net Working Capital.** The change in net working capital associated with period  $t$  and combination of events  $\hat{h}_l$  (NWC <sub>$\hat{h}_l$</sub> ) is computed from the change in accounts receivables, plus the change in inventory, minus the change in accounts payable, plus any other financial expenses or incomes (FEX <sub>$\hat{h}_l$</sub> ), as stated by eq 59.

$$\Delta \text{NWC}_{t\hat{h}_l}^l = (\Delta \text{Arec}_{t\hat{h}_l}^l + \Delta \text{inv}_{t\hat{h}_l}^l - \Delta \text{Apay}_{t\hat{h}_l}^l + \text{FEX}_{t\hat{h}_l}^l) \quad \forall l, \hat{h}_l, t \in T_l \quad (59)$$

Equation 60 computes the accounts receivables corresponding to period  $t$  and combination of events  $\hat{h}_l$ . Equation 61 determines the change in accounts receivable. Equations 62 and 63 express the calculation of inventory value and change and inventory, respectively. The accounts payable are determined by eq 64. The change in accounts payable is represented by eq 65. Finally, eq 66 computes other financial expenses and incomes (FEX <sub>$\hat{h}_l$</sub> ) associated with the SC operation.

$$\text{Arec}_{t\hat{h}_l}^l = \sum_{t'=t-d_M^{\max}+1}^t \sum_{t''=t'+1}^{t'+d_M^{\max}} \sum_{l^* \in L_{t'}} \sum_{\hat{h}_{l^*} \in AH_{l^*}\hat{h}_l} \text{Asales}_{t't''\hat{h}_{l^*}}^{l^*} - \sum_{t'=t+1}^{t+d_M^{\max}} \sum_{t''=t'-d_M^{\max}+1}^t \sum_{l^* \in L_{t''}} \sum_{\hat{h}_{l^*} \in AH_{l^*}\hat{h}_l} \text{pled}_{t't''\hat{h}_{l^*}}^{l^*} \quad \forall l, \hat{h}_l, t \in T_l \quad (60)$$

$$\Delta \text{Arec}_{t\hat{h}_l}^l = \text{Arec}_{t\hat{h}_l}^l - \text{Arec}_{t-1\hat{h}_{l^*}}^{l^*} \quad \forall l, \hat{h}_l, t \in T_l, l^* \in L_{t-1}^*, \hat{h}_{l^*} \in AH_{l^*}\hat{h}_l \quad (61)$$

$$\text{inv}_{t\hat{h}_l}^l = \sum_s \sum_{f \notin (\sup \cup M)} Iu_{st} S_{st\hat{h}_l}^l \quad \forall l, \hat{h}_l, t \in T_l \quad (62)$$

$$\Delta \text{inv}_{t\hat{h}_l}^l = \text{inv}_{t\hat{h}_l}^l - \text{inv}_{t-1\hat{h}_{l^*}}^{l^*} \quad \forall l, \hat{h}_l, t \in T_l, l^* \in L_{t-1}^*, \hat{h}_{l^*} \in AH_{l^*}\hat{h}_l \quad (63)$$

$$\text{Apay}_t = \sum_e \sum_{t'=1}^t \sum_{l^* \in L_{t'}} \sum_{\hat{h}_{l^*} \in AH_{l^*}\hat{h}_l} \text{Epurch}_{et\hat{h}_{l^*}}^{l^*} - \sum_e \sum_{t''=1}^t \sum_{t'=t''}^t \sum_{l^* \in L_{t'}} \sum_{\hat{h}_{l^*} \in AH_{l^*}\hat{h}_l} \text{coef}_{et't\hat{h}_{l^*}} \text{pay}_{et't\hat{h}_{l^*}}^{l^*} \quad \forall l, \hat{h}_l, t \in T_l \quad (64)$$

$$\Delta \text{Apay}_{t\hat{h}_l}^l = \text{Apay}_{t\hat{h}_l}^l - \text{Apay}_{t-1\hat{h}_{l^*}}^{l^*} \quad \forall l, \hat{h}_l, t \in T_l, l^* \in L_{t-1}^*, \hat{h}_{l^*} \in AH_{l^*}\hat{h}_l \quad (65)$$

$$\text{FEX}_{t\hat{h}_l}^l = \sum_{t'=t+1}^{t+d_M^{\max}} (1 - \phi_{t'}) \text{pled}_{t't\hat{h}_l}^l - \sum_e \sum_{t'=1}^t \text{pay}_{et't\hat{h}_l}^l (\text{coef}_{et't\hat{h}_l} - 1) + \sum_{t'=1}^{t-1} \sum_{l^* \in L_{t'}} \sum_{\hat{h}_{l^*} \in AH_{l^*}\hat{h}_l} E_{tt'}^{\text{MS}} Z_{tt\hat{h}_{l^*}}^{\text{MS}, l^*} - \sum_{t'=1}^{t-1} \sum_{l^* \in L_{t'}} \sum_{\hat{h}_{l^*} \in AH_{l^*}\hat{h}_l} D_{tt'}^{\text{MS}} Y_{tt\hat{h}_{l^*}}^{\text{MS}, l^*} \quad \forall l, \hat{h}_l, t \in T_l \quad (66)$$

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Received for review December 21, 2008  
Revised manuscript received June 9, 2009  
Accepted June 30, 2009

IE801973N