

# Direct test of time reversal invariance violation in B mesons

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## Abstract

In this letter we reinterpret and reanalyze the available data of the B meson factories showing the existence of direct experimental evidence of time reversal invariance violation in B mesons. This reinterpretation consists of using the available observables to define a new observable which, in a model independent way and without assuming CPT invariance, compares a transition between a  $B^0$  and a here-defined  $B_\alpha$ -state, with its time reversed transition. The observable then offers a direct way to probe time reversal invariance and it is therefore independent of any conclusion obtained from current experimental information on CP violation and CPT invariance. As far as the authors are concerned, this is the first direct evidence of time reversal invariance violation in B mesons and also the first one obtained from decaying particles whose mean life time difference is negligible.

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In the last fifty years there has been a great theoretical and experimental advance in the study of the discrete symmetries C (charge conjugation), P (parity), T (time reversal) and their relevant combinations CP, T and CPT. This was achieved thanks to the exploration of the Electroweak sector of the Standard Model (SM) where, at the level of energies explored insofar, we understand that the root of CP and T violation lies. On the other hand, CPT symmetry is protected by the general Pauli's CPT theorem and it has not been measured to be violated. During this period we have learned that the study of the discrete symmetries may give us clues to project the behaviour of physics at higher energies. Such is the case of the celebrated prediction of the third generation [1] in answer to the CP violation measured in the Kaon system [2]. It is therefore of great interest the study of the discrete symmetries in the different sectors of the Electroweak Lagrangian.

The violation of CP has been deeply and extensively studied in the Kaon system since its first measurement in 1964. The following experimental and theoretical studies, in the 70's and 80's, pointed out that an important CP violation should be expected in the B sector. In fact, this was measured for the first time by Babar and Belle in 2001 [3, 4], and since then an amazing amount of data and a very high accuracy in the measurements have considerably increased our knowledge of the Electroweak sector.

On the contrary, there is a poor knowledge of T violation, which is a much more difficult symmetry to be tested. In fact, it was not directly measured until 1998 by the CPLEAR collaboration [5]. This measurement was performed in Kaon mesons and, up to now, it is the only direct measurement for T violation that exists. This T-violating observable, however, has been subject of controversy [6, 7, 8] due to its dependence on the Kaons mean life time difference: since T violation is probed in this case through the oscillation probability from  $K^0$  to  $\bar{K}^0$  relative to  $\bar{K}^0$  to  $K^0$ , it might be that a  $\bar{K}^0$  decaying more rapidly than a  $K^0$  mimics a T-violating effect. The search for a similar T-violating observable in the B-sector by the B-factories, which compares the  $\Gamma(B^0 \rightarrow \bar{B}^0, \Delta t)$  and  $\Gamma(\bar{B}^0 \rightarrow B^0, \Delta t)$  transitions, it has thrown negative results [9, 10], as expected due to the negligible mean life time difference ( $\Delta\Gamma$ ) in the neutral B mesons. As a matter of fact, T-invariance is also proposed to be explored in the B system through T-odd observables [11] which show up through the angular analysis of certain  $B$  decays.

The present state of the art indicates that T violation should be expected in the B system. However, owing to its importance, it is of great interest to define and measure a T-violating observable which is, in addition, not null

in the limit of a vanishing  $\Delta\Gamma$ . In fact, in this case the resulting observable will lack of the previously mentioned controversy and, hence, it can be taken as the first time reversal violation effect in the sense of Ref. [6]. Therefore, the goals of this work are the following: *(i)* To show how a proposed T-violating observable for the B-factories exhibits, in a model independent way and without assuming CPT invariance, direct evidence of T violation in a transition between some given neutral B-states; and, therefore, it is independent of any indirect evidence of T violation obtained from our present knowledge on CP violation and CPT invariance in B mesons. And *(ii)* to reanalyze the available experimental data and compute this T-asymmetry to positively find direct evidence of T violation in the B system.

The observable that we propose in this letter has been previously studied in Refs. [12, 13, 14, 15]. However, in those articles, the interpretation of this observable as a T-asymmetry is only valid within the SM, where the formalism of the CP-tag can be applied. Along this work, we go beyond this formalism and we construct the T-violating observable only from quantum mechanics and available experimental information. In addition, we also compute its value from current data.

The T-operation, in contrast with the CP-operation, corresponds to an antiunitary operator and hence it has no conserved quantum numbers. Therefore, we cannot assert that T is either conserved or not in a given process or decay; instead, we should say that the process is *T-invariant* or not. Hence, a way to inquire directly about the T-invariance of a given process is by comparing it with its T-reversed. For instance, if  $\phi_1$  and  $\phi_2$  are two possible quantum states, then the inequality

$$|\langle\phi_2|U(\Delta t)|\phi_1\rangle|^2 \neq |\langle\phi_1|U(\Delta t)|\phi_2\rangle|^2, \quad (1)$$

where  $U(\Delta t)$  stands for the evolution operator, is a clear signal that the process is not T-invariant (usually referred as *T violation*, for short). A direct measurement of an inequality of this type is an explicit measurement of T violation, as it was the case of CPLEAR in the Kaon system [5]. We show here that in the B-factories it is possible to construct an asymmetry which probes, in a model independent way and without assuming CPT-invariance, an inequality of the type in Eq.(1) and, hence, it offers a direct test of T violation in the B system. Moreover, since  $\Delta\Gamma$  is negligible in the B system, this asymmetry is free from any argument that might arise due to a considerable mean life time difference.

For the purposes of this letter, we consider the events in the B-factories in which the  $\Upsilon(4S)$  decays to a pair of correlated neutral B mesons. Once

this correlated pair is created, the first decay of one of these mesons tags the other meson, which evolves during a  $\Delta t$  time until it decays. The process of *tagging* in a correlated couple of particles occurs always no matter which is the first decay. In the B-factories, where the correlated initial state at time  $t$  is written as

$$|i(t)\rangle = \frac{e^{-\Gamma t}}{\sqrt{2}} \left( |B^0(+\vec{k}), \bar{B}^0(-\vec{k})\rangle - |\bar{B}^0(+\vec{k}), B^0(-\vec{k})\rangle \right), \quad (2)$$

a first decay to  $X$  at time  $t_1$  projects the second meson which is still flying to

$$e^{-\Gamma t_1} |B_{\bar{X}}\rangle \equiv \frac{1}{\sqrt{2}} e^{-\Gamma t_1} (\langle X|T|B^0\rangle |\bar{B}^0\rangle - \langle X|T|\bar{B}^0\rangle |B^0\rangle),$$

where  $\Gamma^{-1}$  is the mean life time of the B mesons and  $T$  is the 'scattering' matrix of the direct decay –with no mixing–. Hence, we say that the first decay has acted as a filter in the quantum mechanical sense and has tagged the second meson as a  $|B_{\bar{X}}\rangle$ . The state of the second meson is as physical and determined as it is –for instance– a flavour state  $B^0$ , even if the  $B \rightarrow X$  decay is not theoretically well understood. Therefore, every time that we have an  $X$  decay on one side, we know that a  $B_{\bar{X}}$  must occur on the other side, and this conclusion is achieved beyond any 'a priori' theoretical assumption regarding the decay process.

Suppose that the first decay of the correlated B pair is  $B \rightarrow X = J/\psi K_S$ , then the tagged meson is

$$|B_{\overline{J/\psi K_S}}\rangle = \frac{1}{\sqrt{2}} (\langle K_S|T|B^0\rangle |\bar{B}^0\rangle - \langle K_S|T|\bar{B}^0\rangle |B^0\rangle).$$

(Here and below  $|K_{S,L}\rangle$  stands for  $|J/\psi K_{S,L}\rangle$ ). This motivates us to define a new orthonormal basis in the B-space such that one of its vectors is parallel to  $B_{\overline{J/\psi K_S}}$ ,

$$\begin{aligned} |B_\alpha\rangle &= \frac{1}{\sqrt{N}} (\langle K_S|T|B^0\rangle |\bar{B}^0\rangle - \langle K_S|T|\bar{B}^0\rangle |B^0\rangle) \\ |B_{\alpha_\perp}\rangle &= \frac{1}{\sqrt{N}} (\langle K_S|T|\bar{B}^0\rangle^* |\bar{B}^0\rangle + \langle K_S|T|B^0\rangle^* |B^0\rangle), \end{aligned}$$

where  $N$  is the normalization factor. These two new states are well and unambiguously defined through these equations, and hence they are physical.

It is worth noticing at this point that due to Bose-statistics we can assert a first important feature of this basis, namely the impossibility of  $B_\alpha$  to decay directly to  $J/\psi K_S$ :

$$\langle K_S | T | B_\alpha \rangle = 0. \quad (3)$$

Using this new basis, we can rewrite the initial state of the B-factories at time  $t$ , Eq. (2), as

$$|i(t)\rangle = \frac{e^{-\Gamma t}}{\sqrt{2}} \left( |B_\alpha(+\vec{k}), B_{\alpha_\perp}(-\vec{k})\rangle - |B_{\alpha_\perp}(+\vec{k}), B_\alpha(-\vec{k})\rangle \right).$$

This new expression allows us to understand, with help of Eq. (3), that the  $B_{\alpha_\perp}$  state is the *father* of the  $J/\psi K_S$  decay.

We are now interested in showing that  $B_\alpha$  is the *father* of the  $J/\psi K_L$  decay and that its branching fraction is equal to that corresponding to the decay  $B_{\alpha_\perp} \rightarrow J/\psi K_S$ . Up to here we have used essentially quantum mechanics in our derivation but, in order to achieve this goal, we also need the input of experimental information. In this direction, we first obtain the following relation from measurements of direct CP violation in  $B \rightarrow J/\Psi K_{S,L}$  decays [16]<sup>3</sup>

$$|\langle K_{S,L} | T | B^0 \rangle| = |\langle K_{S,L} | T | \bar{B}^0 \rangle| (1 + \eta_1), \quad (4)$$

with  $\eta_1$  being at most of a few percent ( $\eta_1 \sim 10^{-2}$ ). Next, we express the  $K_{S,L}$  states in the  $\{K^0, \bar{K}^0\}$  flavour basis (a small parameter,  $\eta_2 \sim 10^{-3}$ , accounts for Kaons CP-violation in this change of basis). Then, neglecting  $\eta_{1,2}$  (the  $\eta_{1,2} \neq 0$  case is analyzed below) and combining Eq. (4) –now written in terms of  $K^0$  and  $\bar{K}^0$ – with  $\langle \bar{K}^0 | T | B^0 \rangle = \langle K^0 | T | \bar{B}^0 \rangle = 0$ <sup>4</sup>, we obtain a  $T$ -matrix proportional to the identity in the  $K$ -flavour and  $B$ -flavour basis. Afterwards,

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<sup>3</sup>Although contrived, these measurements may be also interpreted as a fine tuned NP cancellation between CP-violation in the mixing and direct CP-violation in the decay in the  $B \rightarrow J/\psi K_{S,L}$  channels. However, this possibility is unrealistic since it should be also consistent with the measurement of the semileptonic asymmetry (see [9, 10] and footnote (5)).

<sup>4</sup>A small departure from the equality  $\langle \bar{K}^0 | T | B^0 \rangle = \langle K^0 | T | \bar{B}^0 \rangle = 0$  would be still acceptable. In any case, these transitions would involve processes which are highly constrained by, for example,  $\Delta S = 1$  and  $\Delta B = 1$  weak neutral currents modes in  $K^0$  and  $B^0$  decays respectively (see [17]).

we perform two unitary rotations to the  $\{K_S, K_L\}$  and  $\{B_\alpha, B_{\alpha_\perp}\}$  basis and we use Eq. (3) to get

$$\langle K_L | T | B_{\alpha_\perp} \rangle = 0 \quad (5)$$

and

$$|\langle K_S | T | B_{\alpha_\perp} \rangle| = |\langle K_L | T | B_\alpha \rangle|, \quad (6)$$

as we wanted to show.

The result in Eq. (5) is of great usefulness since, together with Eq. (3), it allows us to reduce to a *single transition* between well defined B-states the following two intensities:

$$I(\ell^-, K_L, \Delta t) = c |\langle K_L | T | B_\alpha \rangle|^2 |A_{\ell^-}|^2 |\langle B_\alpha | U(\Delta t) | B^0 \rangle|^2 \quad (7)$$

$$I(K_S, \ell^+, \Delta t) = c |\langle K_S | T | B_{\alpha_\perp} \rangle|^2 |A_{\ell^+}|^2 |\langle B^0 | U(\Delta t) | B_\alpha \rangle|^2, \quad (8)$$

where  $I(X, Y, \Delta t)$  is the probability of having first an  $X$  decay and  $\Delta t$  later an  $Y$  decay,  $\ell^\pm$  refers to flavour specific leptonic decays,  $|A_{\ell^\pm}|^2$  is their corresponding branching ratios and 'c' is a common constant factor to both intensities.

Motivated by the time reversed single B-meson transitions that occur in Eqs. (7) and (8), we propose to measure the following asymmetry,

$$A_T^{exp}(\Delta t) = \frac{I(\ell^-, K_L, \Delta t) - I(K_S, \ell^+, \Delta t)}{I(\ell^-, K_L, \Delta t) + I(K_S, \ell^+, \Delta t)}. \quad (9)$$

As it is easily seen using Eq. (6) and assuming  $|A_{\ell^+}| = |A_{\ell^-}|$ <sup>5</sup>, this asymmetry gets reduced to a T-violating asymmetry in the spirit of Eq. (1),

$$A_T^{exp}|_{\eta_1=\eta_2=0} = A_T(\Delta t) \equiv \frac{|\langle B_\alpha | U(\Delta t) | B^0 \rangle|^2 - |\langle B^0 | U(\Delta t) | B_\alpha \rangle|^2}{|\langle B_\alpha | U(\Delta t) | B^0 \rangle|^2 + |\langle B^0 | U(\Delta t) | B_\alpha \rangle|^2}. \quad (10)$$

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<sup>5</sup>Observe that this equality is a sufficient but not a necessary condition for our argument, and only a deviation of order 10% would spoil our conclusions in Eq. (11). This –widely accepted– equality has not been directly measured, since it is very difficult to separate it from indirect CP-violation in an observable. However, the precise measurement of the semileptonic asymmetry [9, 10] rules out a departure greater than the 1% level from the above equality, unless a fine tuned new physics cancellation between a deviation of this equality and indirect CP-violation occurs. However, in close connection with the discussion given in footnote (3), such fine tuning is highly unrealistic since it should be also consistent with the measurements of the combination of CP violation in mixing and in decay  $-A_f$  in the notation of the paper quoted in [16]– from time-dependent decay rates in non leptonic B decays (see, for instance, the article given in [16]).

This expression represents an asymmetry constructed with a well defined transition between physical states and its T-reversed process without assuming CPT-invariance, therefore it is an asymmetry sensitive to T violation in a direct way and then independent of any deduction which combines measurements of CP violation and CPT invariance.

The case  $\eta_{1,2} \neq 0$  is now easily analyzed perturbatively. In fact, if we think of  $\eta$  as a small parameter whose magnitude is at most as large as the maximum between  $\eta_1$  and  $\eta_2$ , then we have that a term of order  $\eta$  should be added in both RHS of Eqs. (5) and (6). These extra terms end up giving a correction to Eq. (10), which now reads

$$A_T^{exp}(\Delta t) = A_T(\Delta t) + \mathcal{O}(\eta).$$

Therefore, the conclusion is straightforward: *the measurement of*

$$|A_T^{exp}(\Delta t)| \gg \eta \sim 10^{-2} \quad (11)$$

*for some  $\Delta t$ , implies direct evidence of time reversal invariance violation in the mixing of the B mesons in a model independent way.*

The conclusion concerning Eq. (11) is now easily analyzed using the available experimental data of Babar and Belle. We stress at this point that, as far as the authors are concerned, the particular combination of intensities which gives rise to the T-asymmetry  $A_T^{exp}$ , Eq. (9), has not been computed in the literature. The experimental results in References [16] and [18] related to  $I(K_S, \ell^+, \Delta t)$  and  $I(\ell^-, K_L, \Delta t)$  are obtained through a fit to a formula which assumes  $\Delta\Gamma = 0$  (this condition assures that our observable is a true T-violating observable in the spirit of reference [6]). Since current experiments [9, 10] constrain  $\Delta\Gamma/\Gamma \leq 10^{-2}$ , we can safely use this fit: any correction to our results would be at most of this order of magnitude and, hence, it will not modify our conclusion.

The result for the time dependence of the relevant intensities is

$$\begin{aligned} I(\ell^-, K_L, \Delta t) &\propto \frac{e^{-\Gamma\Delta t}}{4\Gamma} \times \begin{cases} (1 + (0.716 \pm 0.080) \sin(\Delta m\Delta t)) & \text{Babar [18]} \\ (1 + (0.641 \pm 0.057) \sin(\Delta m\Delta t)) & \text{Belle [16]} \end{cases} \\ I(K_S, \ell^+, \Delta t) &\propto \frac{e^{-\Gamma\Delta t}}{4\Gamma} \times \begin{cases} (1 - (0.691 \pm 0.040) \sin(\Delta m\Delta t)) & \text{Babar [18]} \\ (1 - (0.643 \pm 0.038) \sin(\Delta m\Delta t)) & \text{Belle [16]} \end{cases} \end{aligned}$$

and therefore, using Eq. (9), we obtain the result for the model independent T-violating observable,

$$A_T^{exp}(\Delta t) = \begin{cases} (0.703 \pm 0.044) \sin(\Delta m\Delta t) & \text{Babar} \\ (0.642 \pm 0.034) \sin(\Delta m\Delta t) & \text{Belle.} \end{cases} \quad (12)$$

As it is easily seen, Eq. (12) clearly satisfies the requirement in Eq. (11) and, therefore, we can positively assert that a reinterpretation and a reanalysis of the B-factories available data provides a direct evidence of T violation in the B system.

Even if T violation should be expected in this sector, the relevance of the result resides in defining properly –i.e., without assuming CPT-invariance and model independently– a suitable observable to probe it and, finally, in obtaining a direct experimental evidence of T violation. It is worth to stress that the T-violating observable is, in consequence, independent of any result arising from constrains imposed by measurements of CP violation and CPT invariance.

In summary, we have reanalyzed the observables measured by the B-factories and we have shown that they imply direct evidence of T violation in the B system, where the negligible mean life time difference reinforces this result. We have proposed an asymmetry which, in a model independent way and without assuming CPT invariance, consists of the comparison of the transition between a  $B^0$  and a  $B_\alpha$ -state, and its T-reversed:  $\Gamma(B^0 \rightarrow B_\alpha, \Delta t)$  versus  $\Gamma(B_\alpha \rightarrow B^0, \Delta t)$ . We have shown how the precise definition of the  $B_\alpha$ -state arises from the concept of tagging, which filters the information of a given decay into the structure of the tagged meson. The argument in this letter states that if the measurement of the proposed T-asymmetry,  $A_T^{exp}(\Delta t)$  (Eq. (9)), is greater than the quoted  $\eta \sim 10^{-2}$  for some  $\Delta t$  then there is a direct signal of T violation. The reanalysis of the available experimental data fully overshoots this condition by more than ten standard deviations (Eq. (12)) and hence it provides clear direct evidence of T violation in the B system.

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