Reply to Comment on "alpha decay in the complex-energy shell model" by R. G. Lovas

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We reply to the Comment by Lovas, which concerns the shell model calculations of alpha-decay width. In the Comment, the author claims, that the alpha-formation amplitudes obtained in our work "look unusual." Here we reaffirm the results and conclusions of our original article.

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In his Comment Lovas claims that the amount of clustering $\mathcal{S} = 0.011$ obtained in our work [1] is inconsistent with the "classical result" [2, 3] of $\mathcal{S} \approx 0.3$, and that the shapes of the formation amplitude $g(R)$ and the modified formation amplitude $G(R)$ are defective.

The hybrid model of Ref. [2] and many cluster models, including those mentioned in the Comment, introduce alpha particle as a separate degree of freedom. While such models can often provide good fits to experimental data, they do not offer a microscopic description of the alpha decay phenomenon that is rooted in the nucleonic picture. While the calculation of the absolute width using the amount of clustering is a valid procedure, the actual amount of clustering strongly depends on the model assumtion used. In particular, for $\mathcal{S} = 0.011$ [1] one obtains $\Gamma = 0.137 \times 10^{-14}$ MeV while for $\mathcal{S} = 0.3$ [2] one gets $\Gamma = 3.74 \times 10^{-14}$ MeV.

There are significant differences between the assumptions of Ref. [2] and our work [1], which makes it difficult to compare these models directly: (i) our four-body wave function does not contain a phenomenological alpha-cluster component whose amplitude is governed by an ad-hoc phenomenological Hamiltonian; (ii) our calculations are carried out in a shell model space that is vastly larger as compared to that of [2] (one major shell, including the unusual-parity intruder); (iii) the particle continuum is neglected in [2] while our extended space M4 consists of all resonant states with a width less than 1 MeV; the asymptotic behavior of the unbound resonant states is very different from that of the bound wave functions employed in [2].

In Fig. 1 we show the norm eigenvalues according to the convention of Ref. [2]. It can be seen that this plot resembles Fig. 3 of Ref. [2] and Fig. 3 of Ref. [4]. In particular, we see a stabilization of results with respect to ΔR for $\nu \geq 12$, and this is consistent with the previous works. We confirm the point made by Lovas that the norm eigenvalues are generally ordered in the increasing node number order.

We agree with Lovas that understanding of the eigenvalue problem of the norm operator is essential. As far as the small eigenvalues of the norm kernel ($n_\nu < 10^{-3}$) are concerned, one can notice that they are strictly zero when equal oscillator lengths are used for the alpha-particle wave functions and the shell-model basis, see Refs. [5, 6]. As stated in our paper, below Fig. 1, the omission of the small norm eigenvalues does impact the inner region. This is again consistent with Lovas's comment. However, as stated in Ref. [4], 'States with those small eigenvalues are partially forbidden'. In particular, they found that the allowed states must have at least 11 nodes, and

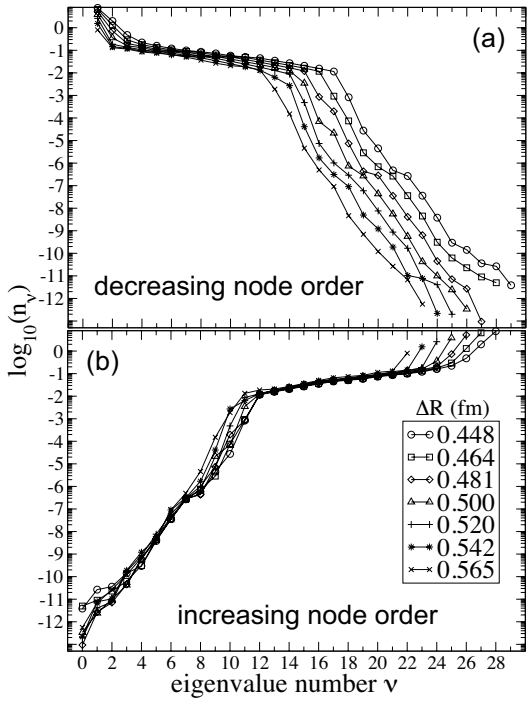

FIG. 1. Eigenvalues of the norm kernel for ^{212}Po for $R_{\max} = 13$ fm for different values of ΔR ordered in (a) decreasing node number as in Ref. [1] and (b) increasing node number order as in Ref. [2].

that the forbidden states aline in a straihg line. This observation is consisten with the finding in ref. [1]: in pag. 6, second column reads 'To eliminate these spurious eigenvectors, we define the cutoff at the value where the eigenvalue distribution changes slope'. But – in addition to physical arguments – one also needs to consider numerical aspects of the problem. Indeed, as pointed out in our paper (cf. discussion around Figs. 5-7 and Eq. (49) therein), the spectroscopic factor $\mathcal{S}$ does depend on both ΔR and $R_{\max}$. In particular, Fig. 5 of our paper shows the sensitivity of $\mathcal{S}$ as a function of $R_{\max}$ for various values of ΔR for $n_{\min} = 0.001$. The new Fig. 2 illustrates the situation for $n_{\min} = 0.00016$. Here, it is impossible to obtain a result that is independent on the parameters ΔR and $R_{\max}$ of the shifted Gaussian basis. So we are not talking about "slight numerical inaccuracies" here.

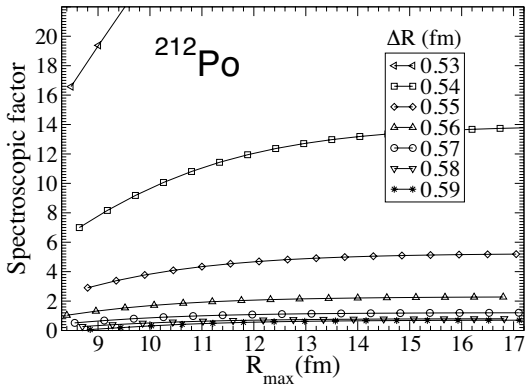

FIG. 2. Similar to Fig. 5 of Ref. [1] but for $n_{\min} = 0.00016$

The source of the problem is captured by Eq. (49) of [1] for the spectroscopic factor in terms of the spectral representation of the norm kernel [7]:

$$\mathcal{S} = \sum_{\nu} \frac{g_\nu^2}{n_\nu}. \qquad (1)$$

While Eq. (1) is mathematically correct, it is numerically extremely unstable since the eigenvalues n_ν rapidly drecease to zero. To this end, in Ref. [1] we have proposed a systematic procedure to determine the norm eigenvalue cutoff $n_{\min}$ to be sure that a local plateau in ΔR and $R_{\max}$ is achieved. We wish to emphasize that the basis of the norm kernel eigenstates is orthonormal with an accuracy of 10^{-10} for all eigenvalues so there is no question about the precision of the expansion of $g(R)$. The lowest n_ν-values obtained are around and below 10^{-11}; hence, they are practically impossible to control numerically. It is worth noting that an important consistency check is provided by the expansion of $g(R)$ in the eigenfunctions of the norm kernel (see Fig. 4 in our paper) and by the asymptotic behavior of the overlap integral as it is determined by the Q_α-value of ^{212}Po. Moreover, the R-matrix result is not that far from the alpha spectroscopic factor approach.

As demonstrated in our work (see, e.g., Fig. 8 of [1]), the shape of the formation amplitudes strongly depends on the configuration space employed. In this respect, we do not see the relevance of simple harmonic oscillator arguments to the discussion at hand. We have difficulty to accept Lovas's statement that the function g^{sh} ($g(R)$ in Ref. [1]) is bound to have at least twelve nodes: in general, the nodal structure of the expanded function is not simply related to the nodal structure of the underlying basis.

In his comment, Lovas has neither demonstrated flaws in the shell-model formalism employed, nor has he presented a solid evidence against our results. He has merely pointed out that some of our findings are not consistent with his previous papers and schematic arguments. As discussed in the conclusions of our paper, much work still needs to be done on both the modeling and algorithmic side. But we believe that our results, and related discussion, represent a step toward microscopic understanding of alpha decay, without invoking an ad-hoc cluster component.

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