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# Instantaneous frequency measurement by in-fiber 0.5th order fractional differentiation

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ABSTRACT

We experimentally demonstrate the possibility to retrieve the instantaneous frequency profile of a given temporal light pulse by in-fiber fractional order differentiation of 0.5th-order. The signal's temporal instantaneous frequency profile is obtained by simple dividing two temporal intensity profiles, namely the intensities of the input and output pulses of a spectrally-shifted fractional order differentiation. The results are supported by the experimental measurement of the instantaneous frequency profile of a mode-locked laser.



## 1. Introduction

Non-integer or fractional order differentiation dates back to the birth of the theory of differential calculus of integer order [1]. In the photonic domain, as far as we know, the first device capable of calculating the fractional order differentiation on the complex envelope of an incoming optical waveform was proposed in 2008 [2]. Since then, different proposals were presented to perform this task including: an asymmetrical $\pi$ phase-shifted fiber Bragg grating [3], long-period grating [4,5], tilted fiber Bragg grating [6], silicon-on-isolator micro-ring resonators [7,8], and electrically assisted Mach–Zehnder interferometer [9]. The possibility to tune the fractional order of differentiation within some range is present in some of these devices [6–9]. Although is clear that a noticeable effort was done in the development of new photonic fractional order differentiator devices with better capabilities (such as fractional order tuning), little progress was achieved demonstrating some advantage in the use of these devices for a specific task.

On the other hand, due to its importance in the performance of fiber-optic communication systems today, new solutions are demanded for the instantaneous frequency monitoring of optical waveforms. There are renowned techniques able to perform this task, such as the frequency-resolved optical gating (FROG) [10,11], the spectral phase interferometry for direct electric field reconstruction (SPIDER) [12,13], and the multi-photon intra-pulse interference phase scan (MIIPS) [14]. However, they are typically best suited for short high intensity pulses well in the femtosecond regime, being of more limited application for broader optical pulses, i.e. from a few ps to well into the ns regime. Thus, new techniques have been proposed to retrieve the instantaneous frequency profile for longer optical temporal waveforms. In addition, the in-fiber solutions in which we are especially interested might be more practical for optical fiber systems. In References [15,16], a direct phase recovery technique was proposed from temporal intensity measurements at the input and output of a linear optical filter. However, precise knowledge of the filter's impulse response is necessary in amplitude and phase. More recently, a direct method for phase recovery based on the use of the transport of intensity equation was introduced, where two temporal intensity profiles at the input and output of a linear dispersive device are required [17]. On the other hand, in Ref. [18] it was shown that a spectrally shifted differentiator can be used to retrieve the phase profile of a given temporal optical waveform. However, the proposed algorithm also needs the numerical calculation of the first-order derivative of the modulus of the input signal. As expected, this numerical procedure is very sensitive to the presence of noise.

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Finally, in Ref. [19] it was demonstrated, through fractional calculus tools, that a very simple expression relates the instantaneous frequency, with the temporal intensities of the temporal waveform under test and either its 0.5th order fractional integration or differentiation. The developed theory was supported by numerical simulations; however no experimental realization was provided.

In this work we provide experimental evidence of a photonic 0.5th order fractional differentiator measuring the instantaneous frequency of a light pulse in the ten of ps regime. Next, we compare this measurement with another technique able to retrieve the instantaneous frequency profile [17]. To the best of our knowledge, this is the first work that experimentally demonstrates the convenience of using a fractional order differentiator for a specific task in the photonic domain, i.e. the instantaneous frequency measurement. This works opens the door for the use of fractional calculus operators solving specific problems in the photonic signal processing.

## 2. Theory

Let us suppose a given optical pulse, whose complex temporal envelope is given by $g(t) = |g(t)|\exp[j\varphi(t)]$, with $j=\sqrt{-1}$. Now, if we perform on this pulse not a standard, but a spectrally shifted 0.5th order fractional differentiation (with angular frequency shifting given by $\omega_s$), the signal processed by the photonic fractional order differentiator could be written as $g(t)f(t)$, with $f(t) = \exp(j\omega_s t)$. Next, let us use the generalized Leibnitz rule for the differentiation of the product of two arbitrary functions:

$$\frac{d^r fg}{dt^r} = \sum_{q=0}^{\infty} \frac{\Gamma(r+1)}{\Gamma(r-q+1)q!}\frac{d^{r-q}f}{dt^{r-q}}\frac{d^q g}{dt^q}, \tag{1}$$

where $\Gamma(.)$ is the gamma function.

Now, by replacing $r=1/2$, and taking into account that $d^a[\exp(j\omega_s t)]/dt^a = (j\omega_s)^a\exp(j\omega_s t)$, with $a\in\mathbf{R}$; it can be demonstrated that the instantaneous angular frequency profile of the input pulse is related to the intensities of the original plus the spectrally shifted 0.5th order differentiation $d^{0.5}/dt^{0.5}$ by the following approximation:

$$\frac{d\varphi(t)}{dt} \approx \frac{\left|\frac{d^{0.5}}{dt^{0.5}}[g(t)\exp(j\omega_s t)]\right|^2}{|g(t)|^2} - \omega_s, \tag{2}$$

The mathematical details of this derivation can be followed in Appendix A of Ref. [19]. Eq. (2) shows that the instantaneous frequency profile can be obtained by simply dividing the temporal intensity profiles of the light pulse under test $|g(t)|^2$, and that of its corresponding spectrally-shifted 0.5th order fractional differentiation $\left|d^{0.5}[g(t)\exp(j\omega_s t)]/dt^{0.5}\right|^2$. The spectral shift $\omega_s$ should be high enough that the spectral content of the input pulse is mainly located at one side of the 0.5th order fractional differentiator resonance frequency. If required, the pulse's temporal phase profile can be obtained by numerical integration of Eq. (2), except by an undetermined numerical constant. It is worth noting the non-iterative nature of the proposed procedure; as opposed to other well-known techniques such as the Gerchberg-Saxton algorithm, which precludes real-time applications. On the contrary, the technique proposed here is potentially well-suited for real-time applications and non-repetitive events.

We deliberative postponed until now the characteristics required to a photonic fractional order differentiator, whose basic operation principle will be explained in the following. To this end, it is very useful to remind one property of the Fourier transform, namely:

$$\Im[g(t)] = G(\omega) \Rightarrow \Im\left[\frac{d^n g(t)}{dt^n}\right] = (j\omega)^n G(\omega), \tag{3}$$

i.e. the Fourier transform of the $n$th time derivative of a given function is $(j\omega)^n$ times the original Fourier transform; where the $\Im$ symbol stands for the Fourier transform, and $n$ is the order of differentiation, which is not necessarily restricted to be an integer. Therefore, and from a strictly spectral point of view, a 0.5th order fractional differentiator is essentially a high-pass filtering device with a transfer function given by $(j\omega)^{0.5}$ [3], where $\omega$ is the baseband angular frequency i.e. the difference between the optical angular frequency $\omega_{opt}$ and the central optical angular frequency of the signal $\omega_0$.

## 3. Experimental

The photonic fractional order differentiation was performed in this work by using a long period fiber grating (LPG). A detailed characterization of this device working as a fractional order differentiator is out of the scope of this work; the interested reader can follow the fabrication details and performance characterization through Ref. [5]. Only for completeness, its main features will be summarized in the following. The LPG was inscribed in a boron doped photosensitive fiber (PS980 by Fibercore, numerical aperture of 0.13 and a cut-off wavelength of 980 nm) by using the point-by-point technique. The selected periodicity was of 187.6 μm, with a final LPG length of 146.5 mm. This LPG was specially fabricated to behave as a 0.5th order fractional differentiator around the resonance wavelength $\lambda_0 = 1035.5$ nm, with a −14 dB transmittance dip and a 3 dB bandwidth of 1.14 nm. The experimental measurement of the LPG transmission can be observed in Fig. 1 (in amplitude). In the same figure, it is compared with the theoretical amplitude response of an ideal 0.5th order fractional differentiator, i.e.. There is a good degree of resemblance between both within the whole operative optical bandwidth (determined by the first transmission maxima at both sides of the resonance dip), except at the resonance frequency, where the transmission should decay to −∞. However, it should be emphasized that a slight deviation in the magnitude has lower consequences than a deviation of the phase

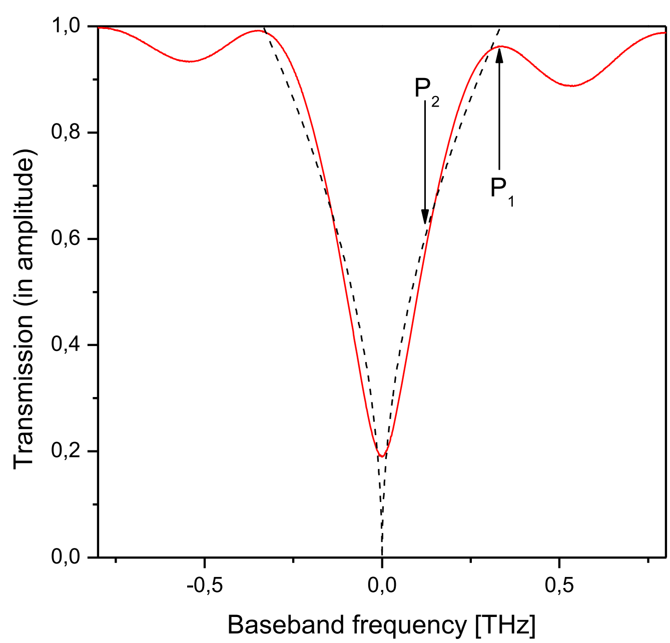


**Fig. 1.** Measured optical spectrum of the LPG (solid curve) and theoretical response (dashed curve) of a 0.5th order fractional differentiator, both in amplitude.

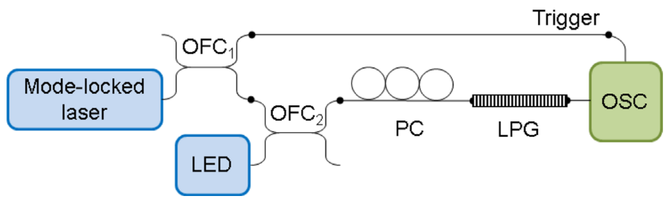


**Fig. 2.** Experimental setup used to measure the instantaneous angular frequency; where OFC, OSC, and PC stand for optical fiber coupler, oscilloscope, and polarization controller, respectively. The LPG is used to perform the 0.5th order fractional differentiation.

response for the implementation of a photonic fractional order differentiator [5,20]. Finally, this LPG was fixed to a linear micrometer translation stage in order to select its operation wavelength properly.

The experimental setup is shown in Fig. 2. The light pulses under test whose instantaneous frequency will be measured, were provided by a passively mode-locked ytterbium fiber laser, emitting at a fixed wavelength $\lambda_0 = 1038.5$ nm. The repetition rate of the mode-locked laser was of 23.15 MHz, and the output light pulses can be approximately fitted with an hyperbolic secant profile $g(t) = \text{sech}(t/T_0)$, with $T_0 = 13$ ps, i.e., a FWHM of 23 ps. This signal was split by a first optical fiber coupler ($OFC_1$, 80/20); and one of its outputs (20%) was sent to the trigger input of the oscilloscope through a 1.2 GHz bandwidth photodetector (not shown). The second output port of the $OFC_1$ was split by a second fiber optic coupler ($OFC_2$, 50/50) and propagated towards the LPG, where the state of polarization of the signal at the input of the LPG was previously adjusted through a polarization controller (PC). The temporal intensity waveform at the output of the LPG was detected with a $>63$ GHz sampling oscilloscope (OSC) provided with a fast built-in photodetector (53 GHz). Finally, a LED source in conjunction with an optical spectrum analyzer (replacing the OSC in Fig. 2) was used in those cases where the knowledge of the spectral position of the resonance transmission dip of the LPG was necessary.

Now, we will describe the experimental procedure used to measure the instantaneous frequency. The first transmission maxima at one side of the transmission dip of the LPG, see position 1 ($P_1$) in Fig. 1, was tuned into the emission wavelength of the mode-locked laser, by using the micrometer translation stage. In this condition, the oscilloscope measured the temporal intensity profile of the pulse under test; see Fig. 3. Next, we measured the intensity profile of its spectrally shifted 0.5th order fractional differentiator, by translating the transmission dip of the LPG to one slope of the 0.5th order fractional differentiator by using the micrometer translation stage, see position 2 ($P_2$) in Fig. 1, a wavelength shift below 1 nm was enough. In this case, the oscilloscope registered the temporal intensity profile of the spectrally shifted 0.5th order fractional differentiation, see Fig. 3. The differences between both intensity profiles are due to the spectrally shifted fractional order differentiation performed on the signal, since the trigger signal used for the oscilloscope was the same in both cases; see the experimental setup in Fig. 2. On the other hand, it can be estimated that the delay introduced by the slight stretching in the LPG for the wavelength shift is below 1 ps; which has no consequences according to Ref. [19]. The instantaneous angular frequency experimentally obtained is shown in Fig. 4, and it was obtained by dividing the temporal intensity profiles shown in Fig. 3, according to Eq. (2). It is worth to mention, that we should focus our attention where the pulse energy is located; the unshaded central area in Fig. 4 concentrates 95% of the input pulse energy. The measured instantaneous frequency profile of the input pulse $g(t)$ can be fitted linearly within the area of interest. This linearity for the instantaneous frequency necessarily implies a

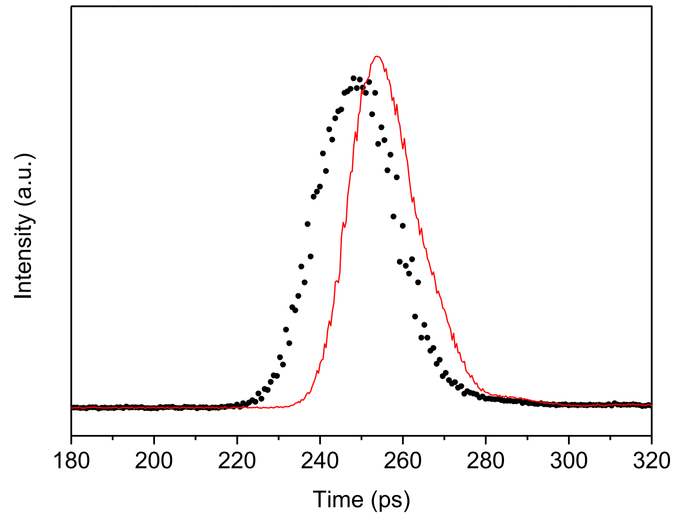


**Fig. 3.** Measured temporal intensity profiles of the optical pulse under test; and spectrally shifted 0.5th order fractional differentiation, scatter points and solid curve, respectively.

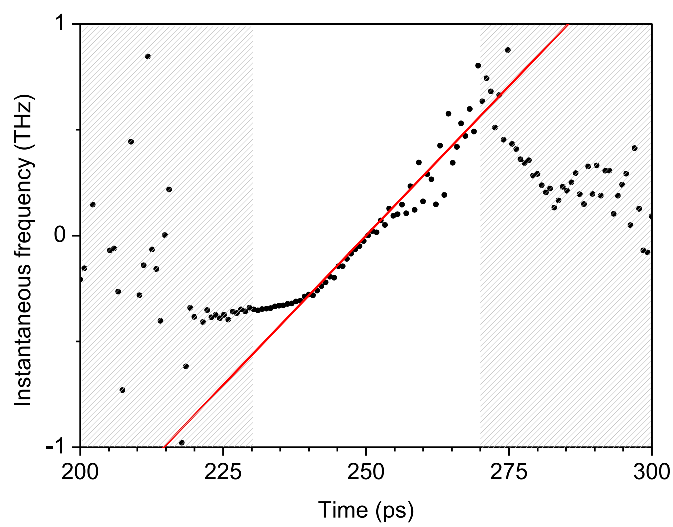


**Fig. 4.** Instantaneous frequency profile of the light pulse under test measured through in-fiber fractional order differentiation, and linear instantaneous frequency profile corresponding to a chirp parameter $C = -30$, scatter points and solid curve, respectively.

parabolic phase profile for the output pulse of the mode-locked laser $g(t)$, which could be described by $\exp(-jCt^2/2T_0^2)$, where $C$ is the chirp parameter, being $C = -30$ according to our fitting. Regarding the accuracy, one of the mains sources of errors in proposed fractional order differentiators is located at the central frequency of operation, where there is a gradual phase transition instead of the required phase discontinuity of $n \times \pi$. Fortunately, in this proposed technique, this spectral region is avoided, since the required fractional differentiation is fully spectral shifted.

It is useful to compare the results obtained with another technique which also relies in temporal intensity measurements, namely the instantaneous frequency measurement by using optical fiber dispersion [17]. In this technique it is necessary to perform two temporal intensity measurements, namely at the input and output of an optical fiber of known dispersion. With both temporal intensity profiles plus the optical fiber dispersion, the instantaneous frequency profile is obtained; via the temporal transport-of-intensity equation -further details can be followed in Ref. [17]. Therefore, we propagate the light pulse under test provided by our mode-locked laser through a dispersion line (low

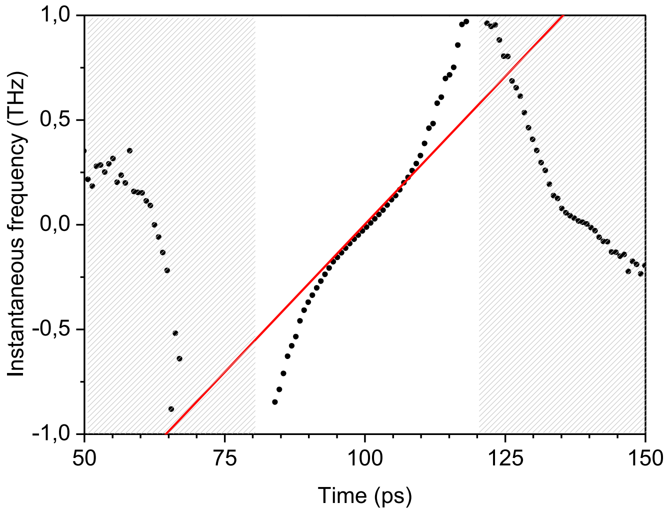


**Fig. 5.** Instantaneous frequency profile of the light pulse under test measured through optical fiber dispersion, and linear instantaneous frequency profile corresponding to a chirp parameter $C = -30$, scatter points and solid curve, respectively.

numerical aperture optical fiber SM980 by Fibercore, length 102 m, measured first order dispersion $D = -44$ ps/nm km at $\lambda_0$). Fig. 5 shows the instantaneous frequency profile obtained; in the same figure, a linear fitting for the instantaneous frequency is shown, corresponding again to a chirp parameter $C = -30$, in correspondence with our previous measurement. Again, as in Fig. 4, we should focus our attention where the pulse energy is concentrated; the unshaded central area in Fig. 5 concentrates 95% of the input pulse energy.

## 4. Conclusion

In this work we measured the instantaneous frequency profile of a light pulse provided by a mode-locked laser by using a LPG based in-fiber 0.5th order fractional differentiator. This work is the first experimental application, to our knowledge, of a photonic device performing a fractional calculus operation in a specific problem, namely the instantaneous frequency profile measurement. We believe this result opens the door for the use of photonic devices performing fractional calculus operation on light pulses for a specific task; which could be expected given the rapid growth in the applications of fractional calculus tools in an increasing number of fields, such as electromagnetism, control engineering, and signal processing. Finally, since the measurement relies on time-domain intensity detection, the technique is limited by the bandwidth of the oscilloscopes and detectors (100 GHz in real time electronic oscilloscopes, $>500$ GHz in sampling optical oscilloscopes).

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