Internal - External stimulus competition in a moving particle interacting system: persuasion vs propaganda

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We propose a general non linear analytical framework to study the effect of an external stimulus in the internal state of a population of moving particles. This novel scheme allows us to study a broad range of excitation transport phenomena. In particular, considering social systems, it gives insight of the spatial dynamics influence in the competition between propaganda (mass media) and convincement. By extending the framework presented by Terranova et al. (Europhys. Lett. **105**, 30007 (2014)), we now allow changes in individual's opinions due to a reflection induced by mass media. The equations of the model could be solved numerically, and, for some special cases, it is possible to derive analytical solutions for the steady states. We implement computational simulations for different social and dynamical systems to check the accuracy of our scheme and to study a broaden variety of scenarios. In particular, we compare the numerical outcome with the analytical results for two possible real cases, finding a good agreement. From the results, we observe that mass media dominates the opinion state in slow dynamics communities; whereas, for higher agent active speeds, the rate of interactions increases and the opinion state is determined by a competition between propaganda and persuasion. This difference suggests that kinetics can not be neglected in the study of transport of any excitation over a particle system.

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I. INTRODUCTION

The concept that many laws of nature arises from statistical origin is firmly grounded in most fields of modern physics. In the last years, given its success and general conceptual framework, statistical physics has become a discipline on its own. This is evidenced in a trend toward applications of statistical physics to interdisciplinary fields as diverse as biology, medicine, information technology, computer science; including problems from behavioral, social, and political sciences [1–4].

A crucial topic for many of these applications is to understand how information propagates through a system of mobile particles. It is possible to find examples of this problem in diverse areas, ranging from chemical reactions to epidemic diseases[5]. In general, the transmission dynamics occurs in a main direction; i.e. the stimulus or excitation spreads from an excited particle to another one that is not excited, while the reverse process is not possible (the non-exited agent can not "unstimulate" the exited agent). It is such the case of an epidemic spreading where the disease goes from an ill agent to a healthy one [6]. But, what happens when an excitation becomes a bidirectional (or multidirectional) mechanism, where the quiescent agent could deactivate an excited agent? These phenomena are typical in opinion dynamics, where agents share the information in both directions; and where the spread of a given excitation occurs not only through the usual agent-agent interaction, but by many different ways. Then, we can test possible answers to these questions by studying the flow of opinion among the inhabitants of a community. Actually, opinion dynamics is the subject of sociophysics that has become one of its main streams, attracting the attention of many physicists [7]. Its modeling represents a challenging field to study the evolution of opposite ideas and the emergence of a collective consensus in communities. Moreover, simulations of agent-based models have provided frameworks and tools to gain insight about the collective behavior of a social system [8–12].

The essential idea behind opinion dynamics models is that individuals (or agents) have opinions that can change under the influence of other individuals or by external stimuli. Indeed, interactions among agents or agent-external influences, for example propaganda, give rise to collective behaviors (political movements or religions) from individual perceptions (believes or thoughts). There are many opinion formation models in the literature that pursue to describe community behaviors or decision-making. However, only a few of them take into account free moving agents[13, 14]. Commonly, opinion formation models are based on network theory [15–17], i.e considering agents that occupy fixed positions into a lattice or space. The dynamics of social contacts due to agent mobility represents an important element to model the spreading of ideas or opinions. It was already shown their relevance in the outcome of an epidemic disease[6], and also in the transmission of opinions[18]. Indeed, we have found that ideas spreading through social contacts are strongly influenced by mobility. This last novel scheme allowed us to tackle a broad range of social phenomena, such as the influence of agent interaction dynamics in the opinion formation, the inclusion of different individual's idiosyncrasies or the different characteristics of the community (mobility, density, etc).

In this work we extend the framework presented previously by Terranova et al.[18], to study the opinion evolution of a community, when the opinion state is not only ruled by agent interactions but also by external stimuli. Carletti et al.[19] introduced the concept of propaganda as an opinion attractor to the agents close in the opinion space. They found conditions required for an efficient spread of the "message". Moreover, Vaz Martins et al.[20] extended that model to incorporate possible repulsive interactions. Interestingly, they showed that the receptiveness of the external information is optimal for an intermediate fraction of repulsive links. In both works there are no space dynamics of the agents and the opinion interaction is only possible if the interacting agents are close in the opinion space. Our scheme allows us to analyze deeply these aspects, considering moving agents and all possible opinion interactions. The introduction of mobility is of great interest because it let us model realistic situations. Social contacts due to mobility are important factors for the spreading of ideas, in contrast with previous models where they spread across individuals fixed in their positions. In particular, it allows us to analyze new different scenarios, ranging from slow dynamics with scarce interactions, to the extreme of a fast changing agent vicinity. Moreover, our framework could be solved exactly in certain situations, obtaining analytical expressions for the steady state populations. The system studied is composed by agents having a discrete set of different viewpoints. They can change their opinion states through two possible process: by "persuasion", while interacting with other agents, or by "reflection", due to a change of mind when the agent does not interact. The reflection process could be originated as a mass media effect or due to the meditation of the agent about the subject in social debate. Summing up, the opinion evolution of each agent depends on its interaction status.

In the following sections we first present the model, and then, by considering only binary collisions, we deduce an analytical expression to compute the opinion evolution of the system. The model allows us to analyze many influence-reflection scenarios including the effect of rejection of the opinions of the interacting agents. We present analytical results for the steady states of the system for some particular cases. In order to check the theoretical results for different social conventions, we show in the last sections the outcomes obtained with a computational model of self propelled agents, moving in a 2D area and interacting through a soft core potential.

II. ANALYTICAL MODEL

In the present work we extend the model presented in [18] in order to introduce an external stimulus or a spontaneous change ("reflection") in the opinion status. We consider a system composed by N agents (individuals), each of them having an inner state representing its excitation situation. For the sake of simplicity we will work on the sociophysics problem, assuming two opposite possible opinions or beliefs (A and B) regarding a specific topic. They could be, for example, left and right parties/idiologies in politics, Windows vs Linux in computer science, or "Barcelona" vs "Real Madrid" in sports. All agents support one of these positions, but with m different degrees of confidence. Then, there are $2m$ possible agent opinion states, that can be represented in a one-dimensional lattice or opinion space as in Fig.1. This was just an arbitrary choice for the number of possible intermediate states. There could be an odd number of them, representing the middle one a neutral opinion. Nevertheless, the model and its analytical expressions are independent of the number of moderate states. Of course, increasing this number will imply a smoother transition among the extreme populations. Each opinion state i can be occupied by any number of agents, (populations P_i, $(i = 1, ..., 2m)$) indicating a specific degree of adhesion of the community to a given position. In the scheme, $i = 1$ and $i = 2m$ are the radical convictions for opinion A and B respectively (see Fig. 1), while the intermediate sites represent a smooth transition (moderate opinions) between the two extreme positions. Henceforth, we will refer to the opinion lattice when we talk about first "neighboring" states or opinions; or opinion changes to the "right" or to the "left". We also assume that the number of members of the community is constant, i.e. $\sum_i P_i = N$ at all times. In fact this not only means a constant population, but also that all N individuals of the system have always an opinion.

Following Terranova's work[18], we will consider that agents can move freely in a 2D finite space with periodic boundary conditions, interacting with each other by direct contact. This interaction can not only change their movement dynamics but also their opinion states; we define this mechanism as *persuasion*. When agents are not interacting they can still change their opinion; we define this new mechanism as *reflection*. We will assume that people may change their opinion smoothly, considering improbable an abrupt change of ideas. So the state of mind of an agent after a reflection or persuasion process can not be radically different from the original one. As a consequence, we will allow only first-neighbor jumps in the opinion lattice. Nevertheless, it is easy to generalize our scheme by letting radical changes of opinion (jumps to second neighbors or still larger ones). Finally, we consider that changes of opinion can be in either direction, nearing or separating the opinion state of the interacting agents.

At each time step t, agents will modify their opinion states depending on the interaction status:

- Interaction process (*persuasion*). In a society, people usually debates about public matters, and this interaction involves, in general, an exchange of opinions among individuals, influencing their position about the topic in debate. In order to model persuasion processes, we define two $2m \times 2m$ matrices, $\mathbf{\Phi}$ ($\mathbf{\Omega}$). Their coefficients ϕ_{ij} (ω_{ij}) represent the frequency of opinion change to the right (left)

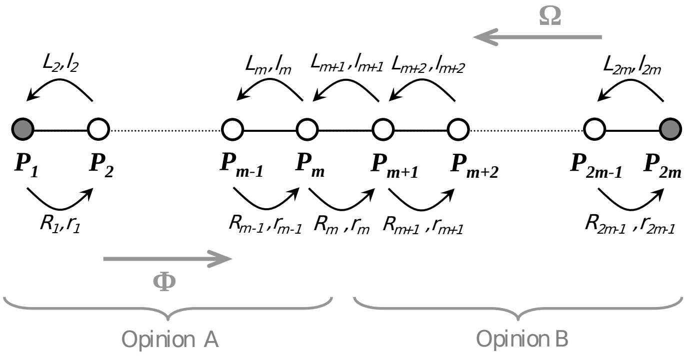

Figure 1. One-dimensional representation of the populations P_i with opinion states i. The arrows R_i, r_i (L_i, l_i) represent the fraction of agents changing its opinion to the right (left) due to textitpersuasion (R, L) or *reflection* (r, l).

in an encounter; i.e., the probability per unit time that an agent with opinion j changes to $j+1$ ($j-1$), while interacting with another one in state i. Accordingly, as it is impossible for the agents at the "end" populations, P_1 and P_{2m}, to have a more radical belief in opinion A or B, respectively, they can only change their opinions in just one direction (see Fig. 1), weakening their confidence. Therefore, the coefficients $\phi_{i,2m} = \omega_{i,1} = 0$, $\forall i$. As a first approximation to obtain some analytical results we consider only binary interactions between agents and a mean field approach, where the probability of an encounter is proportional to the agent populations. Hence, we can represent with $R_i = \sum_k \phi_{ki} P_k$ and $L_i = \sum_k \omega_{ki} P_k$ the rates of agent fraction belonging to opinion i changing its opinion to the right and to the left, respectively, in an encounter with any other agent. In matrix notation $\mathbf{R} = \mathbf{P}^T \mathbf{\Phi}$ and $\mathbf{L} = \mathbf{P}^T \mathbf{\Omega}$, where $\mathbf{R} = (R_1, ..., R_{2m-1}, 0)$, $\mathbf{L} = (0, L_2, ..., L_{2m})$, $\mathbf{P} = (P_1, ..., P_{2m})$ and $\mathbf{P}^T$ is the transposed vector of $\mathbf{P}$.

- Free movement process (*reflection*). Nowadays, it is possible for people to access, through mass media or internet, to articles or direct advertising about the issues that are in public debate. They can also analyze deeply the same issue, influenced by its cultural background. Both mechanisms may lead to modifications in people's confidence in their opinion, causing the phenomenon which we call reflection. In other words, reflection is the process in which an external mechanism, such as mass media or the previous history, influences an agent opinion.

  In the present model, the reflection mechanism implies that agents can still change their opinion states when they do not interact. The reflection is a linear process because the change in the opinion state does not depend on other agents. Then, we can define the coefficients r_i and l_i ($i = 1, ..., 2m$) as the probability per unit time that an agent with opinion i changes to $i+1$ or to $i-1$, respectively. In other words, r_i and l_i represent the rate of change of opinion to the right or to the left in the scheme presented in 1). As we already explained for the persuasion process, agents who have radical positions can only change their opinion in one direction, weakening their confidence, and thus $r_{2m} = l_1 = 0$.

  Following the same ideas of the persuasion process, it is possible to define a 3-diagonal square matrix, $\mathbf{S}$, with coefficients $S_{i,j}$ that represent the agent fraction that changes opinion from state j to i by reflection per unit of time. $\mathbf{S}$ has dimension $2m$, and coefficients $S_{i,i} = -(r_i + l_i)$ (main diagonal), $S_{i,i+1} = l_{i+1}$ (upper diagonal), $S_{i+1,i} = r_i$ (lower diagonal) and $S_{ij} = 0$ elsewhere.

Considering simultaneously both processes described, it is possible to construct the master equation for P_i as:

$$\dot{P}_i = R_{i-1}P_{i-1} + L_{i+1}P_{i+1} - (R_i + L_i)\, P_i + r_{i-1}P_{i-1} + l_{i+1}P_{i+1} - (r_i + l_i)\, P_i, \tag{1}$$

or in a matrix form as:

$$\dot{\mathbf{P}} = [\mathbf{T}(\mathbf{P}) + \mathbf{S}] \times \mathbf{P}, \tag{2}$$

where $\mathbf{T}$ is also a 3-diagonal square matrix of dimension $2m$ with coefficients $T_{i,i} = -(R_i + L_i)$ (main diagonal), $T_{i,i+1} = L_{i+1}$ (upper diagonal), $T_{i+1,i} = R_i$ (lower diagonal) and $T_{ij} = 0$ elsewhere. Note that this is a quadratic equation in $\mathbf{P}$ as $\mathbf{T}$ is a linear function of P, which is a consequence of dealing with just binary interactions. However, the present scheme could be extended to include interactions among many agents, considering a more complex expression for $\mathbf{T}$. Moreover, as we have only allowed changes of opinions to neighboring sites, the matrices $\mathbf{T}$ and $\mathbf{S}$ have just one lower and one upper diagonal. If we let the agents perform more radical changes of ideas, at a "distance" d in the opinion lattice (to second neighbors, etc), then $\mathbf{T}$ and $\mathbf{S}$ will have d lower and upper non-zero diagonals.

III. ANALYTICAL RESULTS

If it was possible to solve exactly Eq.2 then the time evolution of each opinion population would be known. However, the former master equation is nonlinear due to the population dependence of the persuasion $\mathbf{T}$ matrix. In other words people encounters are directly related to the degree of the $\mathbf{P}$ polynomial in this matrix. If we consider gatherings of n agents, then $\mathbf{T}$ will have elements of degree $n-1$, making difficult to solve the equation. Nevertheless, for the approximation addressed in this work where we deal with binary interactions, it is possible to obtain analytical expressions for the asymptotic opinion populations in some particular cases of the matrix evolution equation (Eq. 2). Independently from the existence of an exact solution, it will be always possible to find the asymptotic populations by performing numerical approximations.

The most simple case is to consider constant persuasion probability rates, $\phi_{ij} = \phi$ and $\omega_{ij} = \omega \; \forall \; i, j$. We denote homogeneous persuasion system when $\phi = \omega = C_p$, and biased persuasion system in the opposite. In these cases $\mathbf{T}$ matrix is linear and then it is possible a diagonalization of the system in order to find stationary states. For the sake of simplicity we will consider a system with only two degrees of confidence in each opinion (m=2). The agents could then be found in one of the four different opinion states: two of them belong to moderate positions (P_{A^-} and P_{B^-}) and the other two ones represent radical opinions (P_{A^+} and P_{B^+}).

Solving the Eq. 2 in the stationary regime ($\dot{\mathbf{P}} = 0$) for the homogeneous ($\omega = \phi$) and biased persuasion ($\omega \neq \phi$) system we found:

$$
\begin{aligned}
P_{A^+} &= \frac{1}{\epsilon}(\omega + l_2)(\omega + l_3)(\omega + l_4) \\
P_{A^-} &= \frac{1}{\epsilon}(\phi + r_1)(\omega + l_3)(\omega + l_4) \\
P_{B^-} &= \frac{1}{\epsilon}(\phi + r_1)(\phi + r_2)(\omega + l_4) \\
P_{B^+} &= \frac{1}{\epsilon}(\phi + r_1)(\phi + r_2)(\phi + r_3)
\end{aligned}
\tag{3}
$$

where ϵ is:

$$
\begin{aligned}
\epsilon = {} & (\omega + l_2)(\omega + l_3)(\omega + l_4) + (\phi + r_1)(\omega + l_3)(\omega + l_4) \\
& + (\phi + r_1)(\phi + r_2)(\omega + l_4) + (\phi + r_1)(\phi + r_2)(\phi + r_3)
\end{aligned}
$$

Starting from this solution it is possible to analyze some particular cases:

- If reflection and persuasion are both homogeneous, i.e. $\phi = \omega = l_i = r_i \; \forall \; i$ then all the states converge to 1/4. The stationary states do not depend on neither the persuasion rate nor the reflection rate. It is a trivial and expected result, where there is not a preferred state.

- Considering a system where the persuasion mechanism is not present then the effect of reflection is apparent. We have found in a previous work [18] that in an homogeneous persuasion system with no reflection, the opinion populations suffer a shift to both extreme positions due to the presence of the borders in the opinion space. Now, for no persuasion and considering all reflection rates distinct to zero, the stationary populations are:

$$
\begin{aligned}
P_{A^+} &= \frac{1}{\epsilon_1}(l_2)(l_3)(l_4) \\
P_{A^-} &= \frac{1}{\epsilon_1}(r_1)(l_3)(l_4) \\
P_{B^-} &= \frac{1}{\epsilon_1}(r_1)(r_2)(l_4) \\
P_{B^+} &= \frac{1}{\epsilon_1}(r_1)(r_2)(r_3)
\end{aligned}
\tag{4}
$$

where ϵ_1 is:

$$
\begin{aligned}
\epsilon_1 = {} & (l_2)(l_3)(l_4) + (r_1)(l_3)(l_4) \\
& + (r_1)(r_2)(l_4) + (r_1)(r_2)(r_3)
\end{aligned}
$$

Is interesting to remark the dependence of the A (B) populations with the l_i (r_i) parameters. If all reflection rates to the left and to the right are equal each other ($l_i = l$ and $r_i = r$) then all populations will depend on the relation r/l. A bias to the right or to the left arise depending on $r/l > 1$ or $r/l < 1$.

- A more interesting situation occurs when competition between persuasion and reflection exists. In order to do it we consider a biased persuasion, $\phi_{ij} = \phi$, $\omega_{ij} = \omega$, and biased reflection, $l_i = l$, $r_i = r \; \forall, \; i, j$, system. Then, by setting the parameters $\omega > \phi$ and $l < r$, we can construct a system where reflection drives the individual opinions towards left (opinion B) while persuasion drives it to the opposite (opinion A). From Eq.3 we obtain the stationary opinion populations as:

$$
\begin{aligned}
P_{A^+} &= \frac{1}{\Delta}(\omega + l)^3 \\
P_{A^-} &= \frac{1}{\Delta}(\phi + r)(\omega + l)^2 \\
P_{B^-} &= \frac{1}{\Delta}(\phi + r)^2(\omega + l) \\
P_{B^+} &= \frac{1}{\Delta}(\phi + r)^3
\end{aligned}
\tag{5}
$$

where Δ is:

$$
\Delta = (\omega + \phi + l + r)((\omega + l)^2 + (\phi + r)^2)
$$

From the former equation it is possible to see that the different populations depend only on a competition rate, $C_c = (\omega + l)/(\phi + r)$, in a power law behavior: $P_{A^+} = {C_c}^3/\Delta^*$, $P_{A^-} = {C_c}^2/\Delta^*$, $P_{B^-} = C_c/\Delta^*$ and $P_{B^+} = 1/\Delta^*$, with $\Delta^* =$

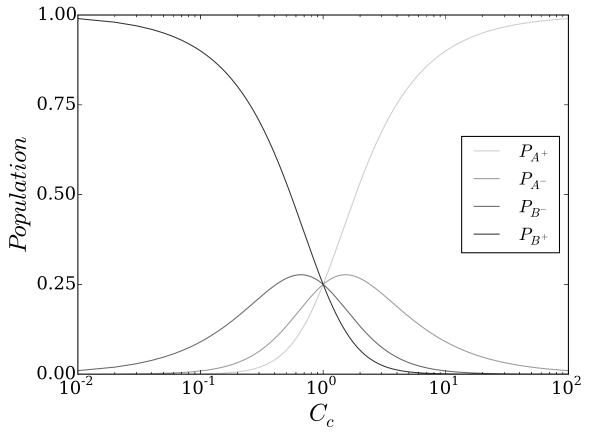

Figure 2. Stationary states as a function of the overall drift rate C_c. From the figure is apparent that when persuasion and reflection dynamics are strongly biased, fundamentalist states predominate over the moderate ones. Slight biased dynamic permits the coexistence among the different degrees of opinions.

$(C_c + 1)(C_c{}^2 + 1)$. Then the stationary behavior depends on the sum of the rates toward an opinion, A or B, contributing independently of the mechanism of opinion change. In these systems persuasion and reflection influence the opinion state in the same way; which is a consequence of having a homogeneous persuasion system that linearize the equations. This behavior will be lost for non homogeneous systems, where persuasion strongly depends on the opinion populations. From the figure 2 we observe that for very high or low C_c values the entire population belong to a radical opinion. Moreover, they sum up to $P_{A^+} + P_{B^+} = 1 - (C_c + 1/C_c)^{-1} \geq 0.5 \; \forall C_c$. Therefore the radical opinion populations dominate the system in the entire range of C_c, except for $C_c = 1$ where the system is completely homogeneous and all opinion populations reach out the 0.25 average value.

IV. COMPUTATIONAL MODEL

We develop a computational model to perform agent-based simulations for the evolution of the opinion dynamics in a moving population. It will allow us to check the accuracy of our analytical results considering just binary interactions and a spatial mean field. In order to simulate the kinetic dynamics of an excitable mobile agent system, we follow the scheme used in the previous work [18] adding the reflection mechanism, i.e. when the agents do not interact, they can still modify its inner state (opinion) with a certain probability.

We assume agents are self-propelled disks that, in absence of interactions, move at a constant speed in a 2D space with periodic boundary conditions. Agents change their direction of motion at Poissonian distributed times, i.e. the agent changes the active direction of motion with a rate α while keeping constant the active speed. Meanwhile, the agent-agent interaction is model by a repulsive soft-core two-body potential[6]. In this way, the collision between agents is a relatively slow process in which agents keep physical contact for a non-vanishing time. The excitation transmission could occur only while in contact, and then the mean collision time, λ, arise as a crucial variable for the persuasion process. We have found previously[21] that at low enough densities λ depends with the mean active speed, v, as $\lambda(v) = K(\rho/v)^{-\xi}$, where ξ and K are positive constants, and ρ is the characteristic size of the agents.

To simulate real world situations of individual interactions, where only one person "wins" a discussion by influencing the other with his speech, we allow just one change of opinion state per encounter. Besides, for reflection processes, we allow an unlimited amount of changes in the opinion state of an agent, following the idea that a person may change his thoughts many times by reflection or influenced by propaganda. Accordingly, the mean free time, τ, is the key variable in the reflection process. It is possible to demonstrate[6] that $\tau = (v\sigma_0\delta)^{-1}$ where $v\sigma_0\delta$ is the agent-agent collision rate. Here, $\sigma_0 = 4\rho$ is the scattering cross section of agents, and $\delta = N/V$ is the agent density, considering $V = L \times L$ the system area of characteristic size L.

In order to compare numerical and analytical results we have to include the agent dynamics in our theoretical approach, redefining the opinion change coefficients. Thereby, we have to include the probability to be interacting $(\frac{\lambda}{\lambda+\tau})$ or non interacting $(\frac{\tau}{\lambda+\tau})$; and the probability to perform a transition while it is in that state. It is worth remarking that, for the persuasion transitions, i.e. while interacting, we have to consider the probability to perform an specific transition among all possible. In this case, interacting agents i and j can change to four different states (less if i or j are in the borders): i, $j \pm 1$, and $i \pm 1$, j. Moreover, the possibility to perform only one change of state by persuasion implies that there had not been any previous transitions during that interaction: $(1 - e^{-\Sigma_{ij}\lambda})$ [6], where $\Sigma_{ij} = \phi_{ij} + \omega_{ij} + \phi_{ji} + \omega_{ji}$. Then, it is possible to write ϕ'_{ij} and ω'_{ij}, the re-scaled opinion change coefficients, as a function of v as:

$$\phi'_{ij} = \frac{1}{\lambda + \tau}\left[\frac{\phi_{ij}}{\Sigma_{ij}}\left(1 - e^{-\Sigma_{ij}\lambda}\right)\right]$$

$$\omega'_{ij} = \frac{1}{\lambda + \tau}\left[\frac{\omega_{ij}}{\Sigma_{ij}}\left(1 - e^{-\Sigma_{ij}\lambda}\right)\right] \quad (6)$$

For r'_i and l'_i, the re-scaled reflection transition rates, we have to include the probability for an agent to be not interacting, obtaining the following expressions:

$$r'_i = \frac{\tau}{\lambda + \tau} r_i, \qquad l'_i = \frac{\tau}{\lambda + \tau} l_i \quad (7)$$

It is interesting to study the parameter behavior in extreme mobility situations, i.e $v >> max(\Sigma_{ij})$ and $v << min(\Sigma_{ij})$. As it was already mentioned $\lambda \sim \rho/v$, revealing a strong dependence of the coefficients ϕ'_{ij} and ω'_{ij} with speed v. For a very energetic agent regime ($v >> max(\Sigma_{ij})$), the interacting time tends to zero, while the number of interactions increases. Here, the matrix coefficients tend to $\phi'_{ij} \sim \sigma_0 \delta\phi_{ij}$, $\omega'_{ij} \sim \sigma_0 \delta\omega_{ij}$, $r'_i = r_i$ and $l'_i = l_i$, from where we can observe that none of the coefficients depends on the velocity. In the opposite v limit, for $v << min(\Sigma_{ij})$, the interactions are scarce, but each of them lasts for a long time. Now the persuasion coefficients tend to zero with the active speed as $\phi'_{ij} = v\sigma_0\delta\phi_{ij}/\Sigma_{ij}$ and $\omega'_{ij} = v\sigma_0\delta\omega_{ij}/\Sigma_{ij}$, while the reflection parameters remain constant. These results strongly suggest that propaganda will dominate in a society with slow kinetics, while the persuasion process will increase its influence for frenzy or excited communities where the interaction dominates the opinion state.

Besides, due to a large rate of collisions, in one limit, or to very long collisions, in the other, there will be an increasing number of agents involved in the interaction. It is expected then, the appearance of clustering in these limits for high-density communities, giving rise to a complete different behavior, where interaction, and then persuasion, dominates.

V. NUMERICAL RESULTS

We perform numerical simulations to check first the accuracy of our analytical approximations by studying the opinion behavior in different situations. According to the parameter analysis carried out in the previous chapter, and due to the absence of an analytical result depending on the active speed, the solutions found in section III can only be compared with the simulations in the extreme mobility situations, i.e $v >> max(\Sigma_{ij})$ and $v << min(\Sigma_{ij})$.

Fig.3 shows simulation results for the steady state populations with opinion A, P_{A^+} and P_{A^-}, as a function of the active speed for different homogeneous reflection strengths in a homogeneous persuasion system. The reflection constant is $C_r = r_i = l_i \; \forall_i$ and the rest of the system parameters are given in [22]. The outcome is symmetrical for the B opinion populations, $P_{B^+} = P_{A^+}$ and $P_{B^-} = P_{A^-}$. We compare the simulations with numerical solutions of Eq.2 observing a good general agreement for the entire range of active speeds. Furthermore the analytical results agrees with the corresponding analytical solutions found at the high and slow active speed limits. For $v >> max(\Sigma_{ij})$ the parameters coefficient became all the same, and then the population divides equally among all opinions. For the case where there is no reflection process involved ($C_r = 0$), we reproduce the behavior obtained previously in [18]. If reflection processes are included, the behavior changes in the low speed limit $v << min(\Sigma_{ij})$; overruling the shift toward

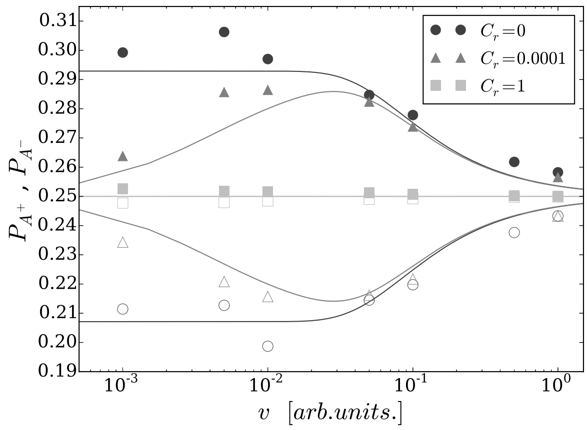

Figure 3. Steady state agent populations P_{A^+} (open symbols) and P_{A^-} (closed symbols) as a function of the agents active speed. Lines correspond to steady state analytical results obtained from solving Eq. (2).

moderate opinions introduced by persuasion. These results confirm the speed dependence of the parameters discussed in the previous section: the reflection processes dominates in the slow dynamics situation. Stronger the propaganda, more homogeneous the population distribution among the opinions. This system revels the different mechanisms of influence of both opinion process. Persuasion depends on the interaction among the different populations, while reflection depends only on the own change of opinion rate value. At slow mobility, the agents surroundings change slowly, creating zones with majority of one or other opinion. Any region with majority of a radical opinion must be surrounded by moderate majority regions, leading to an increase of the moderate populations. A behavior that is perturbed by the presence of the reflection process.

To analyze further the binary collision approximation and the mean field approach, we performed simulations of the previous system[22] for different agent densities, at constant active speed $v = 0.05$ $a.u.$ Figure 4 depicts the steady state population P_{A^+} as a function of the size of the box L for both the simulations and numerical results for different reflection constants C_r. It is evident that Eq.2 is valid only in the low density limit ($L > 200$). At high densities (small L) the number of collisions increases forming clusters of interacting agent, making invalid our assumptions.

Nevertheless, it is interesting to observe the effect of density on the opinion dynamics. At high density agent mobility is strongly reduced due to the formation of clusters, and agents remain just a short time not interacting, and then, nor having much time to perform a reflection. However its effects can be observed for the entire range of system sizes. In the maximum packing fraction limit, the system behaves similar to a fixed network: the agents do not mix and there is no diffusion process, existing a permanent interaction among them. Then the interaction

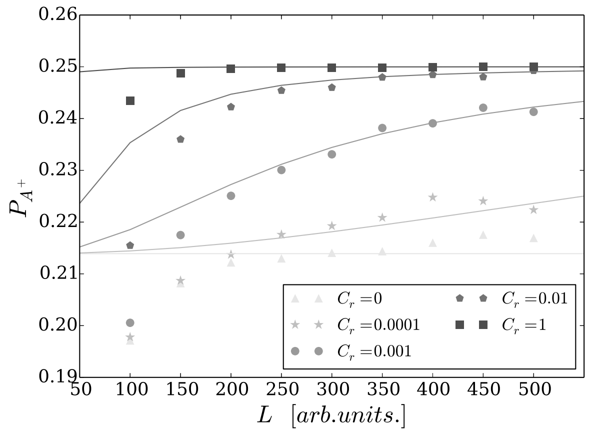

Figure 4. Steady state agent population as a function of the box length L for P_{A+}. Simulation data (symbols) and analytical stationary solutions of Eq. (2) for different values of L (lines). The active speed is $v = 0.05$ *a.u.* and the others parameters are given in [22].

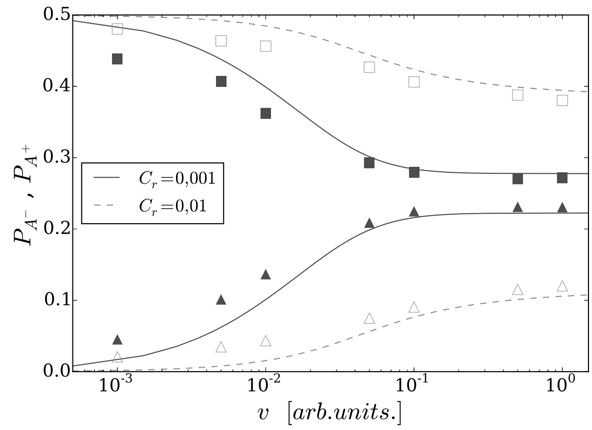

Figure 5. Simulation data (symbols) and analytical stationary solutions of Eq. (2) (lines) for case 1 (fundamentalist do not reflect). The squares correspond to P_{A+} and the triangles correspond to P_{A-}. For P_{B+} and P_{B-} the behavior is analogous

process dominates, reducing even more the radical opinion populations than in the low speed limit. In general we can observe that, for a given system size L, the radical opinion population increases due to reflection, reaching, for reflection parameters C_r large enough, the value of 1/4, i.e. it generates an homogenization of the opinions.

VI. "REAL" SITUATIONS

We can set the parameters of the model in order to make predictions on the opinion behavior of different communities or situations, in an attempt to represent real cases. We present here two possible situations:

Case 1: Fundamentalists do not reflect.

It is a common situation in a discussion, to find people that have a strong formed opinion and are not able to put their points of view into question. To model this behavior we propose that those agents who belong to radical opinion populations, P_{A+} and P_{B+}, do not reflect. Then we set the parameters $r_1 = l_4 = 0$, and the others at a constant rate $r_i = l_i = C_r$ the rest. Since we want to analyze the behavior of reflection, we also set all the persuasion parameters with the same value ($C_p = 0.05$ a.u.); thus, all populations have the same probability to convince each other. The defined system is symmetric in the opinion space, and therefore it is expected to have the same population for both extreme opinions, and another value for both moderate opinions. In other words, $P_{B+} = P_{A+}$ and $P_{B-} = P_{A-}$.

Fig.5 shows simulations and the corresponding analytical results for the opinion A populations as a function of the active speed. The introduction of individuals who do not reflect in a system with equal persuasion probabilities during interaction, results in an increase in the population of these stubborn groups. This is because the moderates (P_{A-} and P_{B-}) are more prone to change their opinion due to reflection, but once the agents are in a fundamentalist opinion state they can only change due to persuasion. It can be seen in the figure that this behavior is enhanced as the reflection parameter increases. As it was mentioned in section III, the reflection process dominates at slow kinetics. Therefore, almost all the population ends up in a radical opinion state due to the reflection asymmetry. In the opposite v limit, persuasion soften this tendency. Nevertheless, stationary states do not reach the homogenous situation observed in other cases. It is a consequence of the mentioned asymmetry in the transition rates among moderate and radical opinions.

In order to analyze further the effects of agent kinetics in this process, we study the problem taking into account the analytical results previously obtained. In this case the minimum value of Σ_{ij} is 0.1 a.u.; which defines the threshold point for the active speed at which reflection dominates. In figure 5 it can be observed that effectively this is the threshold value for two different behaviors. In the energetic regime, $v >> max(\Sigma_{ij})$, the probabilities rates are independent of velocity, and the populations correspond to those values obtained in Eq.3. The asymmetry introduced in the reflection by the stubborn agents produces a shift in the populations; the gap between the populations increases with C_r, favoring the inflexible populations.

On the other hand, in the limit $v << min(\Sigma_{ij})$, the persuasion rates depend linearly on the active speed v, and the reflection process dominates. At very low active speeds the persuasion process, that allows the agents to escape the radical opinion states, losses strength. Then, the population is polarized between the radical opinions A^+ and B^+, tending them to the limiting value of 1/2 when $v \to 0$.

From the preceding discussion, we can conclude that when every opinion state has the same probability to persuade, the existence of a stubborn fundamentalist group implies a bias favoring this opinion.

Case 2: Persuasion vs Propaganda.

Generally, engaging in discussions implies rethinking our own opinions. When we discuss about a topic we can change our minds because we were persuaded. However, the new adopted idea could be against our personal beliefs, or be opposite to what we observe in propaganda. Therefore, when we reflect about that topic, we could reverse the opinion transition. In section III we analyze this situation by choosing the parameters for the model in order to create a convincement flow in favor of one of the opinions (A), opposed to a reflection flow which favors the opposite opinion (B). This could be the case in which propaganda pushes the opinions in one direction, while, the ordinary ideas exchange of daily experiences shows a different shared reality, pushing the opinions in the opposite direction. But, what happens in this case if we change the society kinetics? We performed numerical simulations varying the active speed, setting the homogeneous biased persuasion parameters as $\omega = 0.1\, a.u.$ and $\phi = 0.05\, a.u.$ for different values of r and l. The simulations and the corresponding theoretical results are shown in Fig.6.

In all the plots of the figure we can observe that at low speeds the reflection flow determines the opinion population steady state, as was previously shown. The asymptotic low v regimen corresponds with the analytical results found for a system with no persuasion (see eq.4). It is interesting to note that asymptotic stationary populations values are the same in the three plots depicted, as a consequence of its dependence with r/l, already established analytically in section III. As active speed increases, the interaction process begins to gain influence, diminishing the difference between the populations introduced by reflection. At high active speeds the steady state populations corresponds to those found in eq.5 for an homogenous biased persuasion and reflection system. As was previously mentioned, for $C_c \geq 1$ the persuasion flow overcome the reflection, reversing opinion majority (see fig.6.a). In this v limit, if $C_c \leq 1$ the persuasion flux is not high enough to overcome the reflection drift. In this case the opinion B, supported by a propaganda bias, is a majority independently of the society kinetics (see fig.6.b).

It is interesting to analyze the transition in the opinion behavior. At first sight there is a threshold occurring at the crossing point among the populations. As C_c is reduced to 1, the threshold tends to infinity. For $C_c \leq 1$ the curves do not cross anymore. But, this crossover could confuse the active speed point at which the persuasion starts to affect the opinion. It can be noted in all the plots that the $max(\Sigma_{ij})$ threshold always exists. The persuasion is constant in the three cases depicted, and starts to influence at $v \sim \Sigma_{ij} \sim 0.3\ a.u.$, regardless the reflection value.

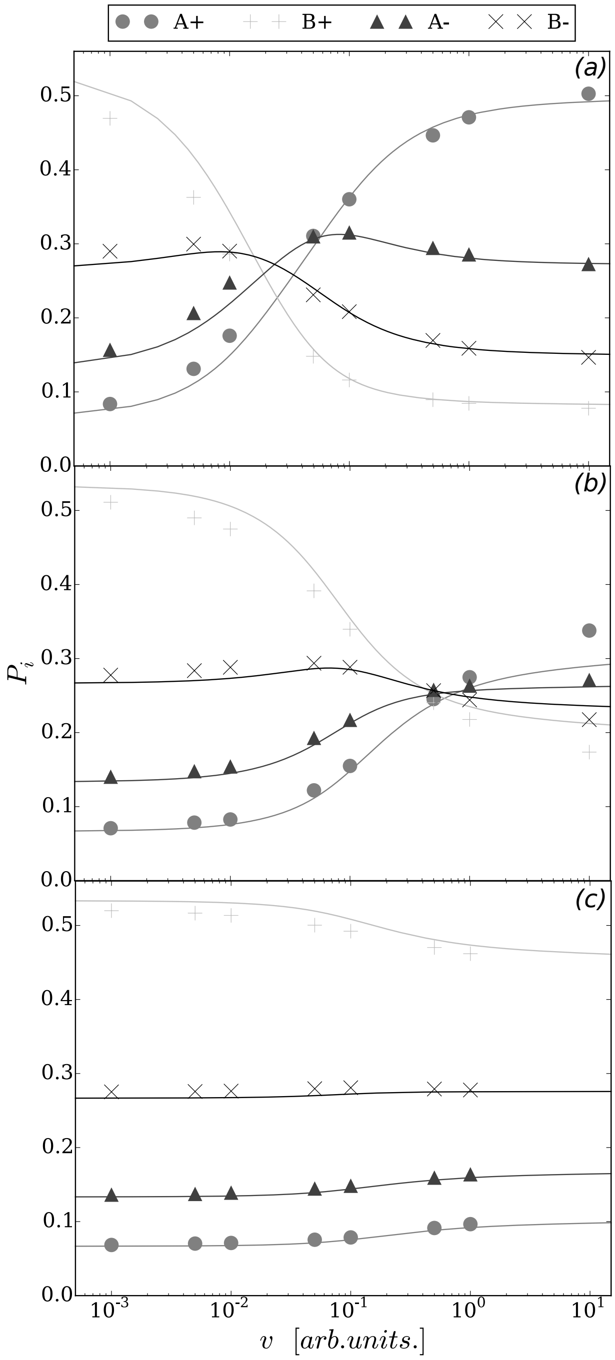

Figure 6. Simulation data (symbols) and analytical steady state solutions of Eq. (2) (lines) for the populations in case 2 (persuasion vs reflection). In case (a) the parameters are $r_i = 6\text{x}10^{-4}$ and $l_i = 3\text{x}10^{-4}$; in case (b) are $r_i = 6\text{x}10^{-3}$ and $l_i = 3\text{x}10^{-3}$; and in case (c) are $r_i = 6\text{x}10^{-2}$ and $l_i = 3\text{x}10^{-2}$.

To conclude the analysis of this situation, we observe that no matter the intensity of persuasion, or interaction, the opinion state of the community could be modified by an adequate external stimulus. Then, a strong propaganda campaign could generate a status-quo in the opinion of a group.

VII. CONCLUDING REMARKS

In the present work we have addressed, by studying a simple but complete opinion formation model, the problem of characterizing the excitation dynamics over a system of moving particles, when they are subjected to both internal and external stimuli. Besides, our simple scheme allows us to obtain valuable information about the competition among different factors in realistic systems where the transitions depend on the agent state. As a first approximation we have considered particles moving at an average speed, but it is straightforward to extend the analysis to consider different types of speed distributions. In particular we have studied the effect of a linear external force (mass media) in an opinion formation model analyzing the interplay between this external stimulus and a process dependent on the interaction among the moving agents (persuasion). It is noticeable (see Fig.3 and Fig.5) that agent velocity plays an important role in the dynamic of this system. Our results suggest that kinetics can not be neglected in the study of transport of any excitation over a particle system. Moreover, it is interesting to highlight that our analysis clearly shows that external stimulus is determinant in a slow moving system, while the interplay among both stimuli determines the outcome in an high kinetic situation.

We also observe that when the number of encounters/ideas exchanged among the individuals increases in a community, external excitation starts to lose effectiveness. In a broad frame, low dynamics systems tend to be more stimulated by internal conditions than by their own debated ideas. This could explain some social behavior under conflict: when individuals start to interact more frequently, communication among them dominates the public opinion, diminishing the mass media impact. Indeed, in a more participative society, where ideas can be debated among the members of the community, internal stimuli are more important than the external ones. This behavior drives the society to a moderate position, diminishing the influence of fundamentalist parties or movements.

It is worth remarking that the obtained analytical results are valid in the low density limit, i.e. for small or scattered communities, as a consequence of considering only binary interactions. This effect is emphasized by the 2D spatial dynamic of the agents. Nevertheless, at high speeds, the low density induce a well mixed system, and then, mean field is valid. The kinetic range of validity of our mean field approach will increase if we were considering movement in a 3D space, as for example, in the transport of information in a flock of birds or fish[23]. The numerical simulations performed here reproduce complex situations in 2D, just like many-agent encounters. If a better analytical description was necessary, our model could be easily extended to tackle more interactions. However, in this case, it will be more difficult to obtain exact expressions.

The analytical and numerical results obtained for the competition between propaganda and persuasion show that the extreme opinions always dominates in the stationary regimes. Nevertheless, the typical real situation in many societies is the opposite, the majority of the population is in a moderate opinion state. The reason of this discrepancy is quite obvious, we are considering a oversimplified situation of external-internal stimuli competition, where the transition rates among the opinion states do not depend on the state. The defined transition matrices could be more complex than the analyzed case. We have found situations in which the majority belongs to the moderate opinions. Therefore, social experiments must be performed to determine which are the transition matrices of a given society.

What happens if the parameters of our model are time dependent? For example, propaganda or advertising campaigns are carried out during a period of time, and they do not have the same effect at the end of it, as it was at the beginning. In this way the agents considered in our model could be pushed by reflection in one direction or other, in the opinion space, at different times, moderating the opinions.

We want to mention that we are considering a conservative population, i.e. without birth or death of individuals, that can modify the composition of each opinion state. In this aspect the population dynamics of agents on each opinion state could be very different. It could be possible that individuals with radical opinion have a lower reproduction rate than in moderate states. Then there would be a stronger incorporation of agents in moderate opinion states than in the radical ones, compensating the flux observed in our work.

A possible extension to our work is to consider more complex parameters. In the analyzed model each transition rate represents a mean value of the probabilities to change opinion for all agents in a given opinion state. In real cases each individual has its own personal education and previous experience that makes it to react in a very different way than others in the same situation and under the same stimulus. This problem could be analyzed with our framework considering stochastic transition rates. Recently, two works proposed situations in which the heterogeneity of the population is modeled through other parameters such as the stubbornness [24] or convincing power [25]. In both works authors reported that one extreme opinion reaches out the majority, as was observed in the present work. Then, we consider that a possible low variability among the agent properties will not introduce a strong difference with the results observed here.

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