Simultaneous correlation architecture for multilevel complementary sequences

M.N. Hadad, M.A. Funes, P.G. Donato and D.O. Carrica

Complementary sequences are being widely used in applications ranging from communication to radar and sonar systems. The properties of these sequences render them suitable for applications requiring signal detection in noisy environments and/or multi-user systems. The use of complementary sequences in practical applications with reduced hardware resources has promoted the development of efficient architectures that reduce the computational requirements for the generation and correlation of the sequences. An optimised architecture for the simultaneous correlation of pairs of multilevel complementary sequences is presented, which achieves that reduction by removing redundant operations. The proposal has an impact also on the correlation of binary complementary pairs of sequences of lengths of a power of 10 and 26 that use the multilevel architecture to generate the kernels.

Introduction: A review of some signal processing topics reveals that there is ongoing interest in the development of algorithms to process coded signals. Among them, there is sustained interest in complementary sequences algorithms. The need for coding systems to provide higher signal-to-noise ratios and/or to support more simultaneous users has been moving from binary encodings to other types of multilevel and/or polyphase codes. To the extent that the encodings are more complex, the processing architectures are also more complex. In this context, it is necessary to develop new signal processing architectures which allow optimisation of the calculations.

Multilevel complementary sequences are attractive not only due to their properties, but also due to the use of their architectures to process binary complementary sequences of kernels 2, 10 and 26 [1]. These sequences were first mentioned in [2] and several studies were dedicated to them [3–5]. In [6], an iterative algorithm for the generation of multilevel complementary sequences was defined as

$$\begin{aligned} S_{1,0}(z) &= 1 \\ S_{2,0}(z) &= a_0 \\ S_{1,n}(z) &= S_{1,n-1} + a_n w_n S_{2,n-1} z^{-d_n} \\ S_{2,n}(z) &= a_n S_{1,n-1} - w_n S_{2,n-1} z^{-d_n} \end{aligned} \tag{1}$$

where z represents the Z-domain variable; $S_{r,n}$, with $r \in \{1, 2\}$, is the sequence r at the iteration $n(n \in \{1, \ldots, N\})$; $N \in \mathbb{N}$ is the total number of iterations of the algorithm; a_n is an arbitrary real number; d_n is a delay defined as $d_n = 2^{P_n}$, where P_n is any permutation of the numbers $[0, 1, \ldots, N]$ and $w_n \in \{+1, -1\}$ is a seed coefficient.

From (1), the architecture of a stage (iteration) of the generator algorithm can be obtained (Fig. 1).

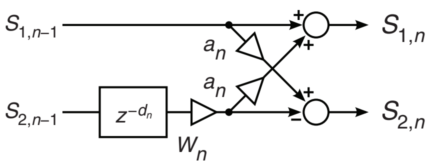

Fig. 1 *Architecture of stage of multilevel complementary sequences generator*

The final sequences are obtained after concatenation of the N stages, which can be represented in the matrix form as shown in (2)

$$\boldsymbol{S}(z) = \prod_{n=1}^{N} \boldsymbol{B}_{a_n, w_n}\left(z^{-d_n}\right) \begin{bmatrix} 1 \\ 1 \end{bmatrix} \tag{2}$$

where

$$\boldsymbol{S}(z) = \begin{bmatrix} S_1 & S_2 \end{bmatrix}^{\mathrm{T}} \tag{3}$$

$$\boldsymbol{B}_{a_n, w_n}\left(z^{-d_n}\right) = \begin{bmatrix} 1 & a_n w_n z^{-d_n} \\ a_n & -w_n z^{-d_n} \end{bmatrix} \tag{4}$$

The main property of these sequences is that their sum of autocorrelation functions results in a Kronecker delta

$$C_{S_{0,N}, S_{0,N}}[k] + C_{S_{1,N}, S_{1,N}}[k] = \prod_{n=0}^{N} \left(1 + a_n^2\right) \delta[k] \tag{5}$$

where $C_{S_{r,N}, S_{r,N}}[k]$ is the autocorrelation function of $S_{r,N}[k]$

$$C_{S_{r,N}, S_{r,N}}[k] = \sum_{m=1}^{L} S_{r,N}[m] S_{r,N}[m+k] \tag{6}$$

From (5) it can be inferred that to exploit the properties of these sequences in signal coding applications, it is necessary to compute the sum of correlations between two inputs and both sequences of the complementary pair. The objective of this Letter is to propose a correlation architecture that allows simultaneous correlation with respect to a pair of multilevel complementary sequences, using a reduced amount of calculations.

Simultaneous correlation architecture: The equation generic of the sum of correlations with two input signals ($R_1(z)$, $R_2(z)$), in the Z-domain, is

$$Y(z) = \overline{S_1}(1/z) R_1(z) + \overline{S_2}(1/z) R_2(z) \tag{7}$$

where $\overline{S_1}(1/z)$ and $\overline{S_2}(1/z)$ are the complex conjugates of $S_1(1/z)$ and $S_2(1/z)$, respectively. Writing it in a matrix form

$$\begin{aligned} \boldsymbol{Y}(z) &= \begin{bmatrix} \overline{S_1}(1/z) & \overline{S_2}(1/z) \end{bmatrix} \begin{bmatrix} R_1(z) \\ R_2(z) \end{bmatrix} \\ &= \boldsymbol{S}(1/z)^{\mathrm{H}} \begin{bmatrix} R_1(z) \\ R_2(z) \end{bmatrix} \end{aligned} \tag{8}$$

where H is the Hermitian operation. Working with $\boldsymbol{S}(1/z)^{\mathrm{H}}$

$$\begin{aligned} \boldsymbol{S}(1/z)^{\mathrm{H}} &= \left(\prod_{n=1}^{N} \boldsymbol{B}_{a_n, w_n}\left(z^{d_n}\right) \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right)^{\mathrm{H}} \\ &= \begin{bmatrix} 1 \\ 1 \end{bmatrix}^{\mathrm{H}} \left(\prod_{n=1}^{N} \boldsymbol{B}_{a_n, w_n}\left(z^{d_n}\right) \right)^{\mathrm{H}} \\ &= \begin{bmatrix} 1 & 1 \end{bmatrix} \prod_{n=N}^{1} \boldsymbol{B}_{a_n, w_n}\left(z^{d_n}\right)^{\mathrm{H}} \end{aligned} \tag{9}$$

Analysing $\boldsymbol{B}_{a_n, w_n}\left(z^{d_n}\right)^{\mathrm{H}}$

$$\begin{aligned} \boldsymbol{B}_{a_n, w_n}\left(z^{d_n}\right)^{\mathrm{H}} &= \begin{bmatrix} 1 & a_n w_n z^{d_n} \\ a_n & -w_n z^{d_n} \end{bmatrix}^{\mathrm{H}} \\ &= \begin{bmatrix} 1 & a_n \\ a_n \bar{w}_n z^{d_n} & -\bar{w}_n z^{d_n} \end{bmatrix} \end{aligned} \tag{10}$$

To make a causal and practically feasible system, $\boldsymbol{B}'_{a_n, w_n}\left(z^{-d_n}\right)$ is defined

$$\begin{aligned} \boldsymbol{B}'_{a_n, w_n}\left(z^{-d_n}\right) &= z^{-d_n} \boldsymbol{B}_{a_n, w_n}\left(z^{d_n}\right)^{\mathrm{H}} \\ &= \begin{bmatrix} z^{-d_n} & a_n z^{-d_n} \\ a_n \bar{w}_n & -\bar{w}_n \end{bmatrix} \end{aligned} \tag{11}$$

rendering (8) as

$$Y(z) = \begin{bmatrix} 1 & 1 \end{bmatrix} \prod_{n=N}^{1} \boldsymbol{B}'_{a_n, w_n}\left(z^{-d_n}\right) \begin{bmatrix} R_1(z) \\ R_2(z) \end{bmatrix} \tag{12}$$

Defining $C_{r,n}$ as the intermediate correlation results so as to make it an iterative process, a correlation stage is defined as

$$\begin{bmatrix} C_{1,n-1}(z) \\ C_{2,n-1}(z) \end{bmatrix} = \boldsymbol{B}'_{a_n, w_n}\left(z^{-d_n}\right) \begin{bmatrix} C_{1,n}(z) \\ C_{2,n}(z) \end{bmatrix} \tag{13}$$

with

$$\begin{bmatrix} C_{1,N}(z) \\ C_{2,N}(z) \end{bmatrix} = \begin{bmatrix} R_1(z) \\ R_2(z) \end{bmatrix} \tag{14}$$

$$Y(z) = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} C_{1,0}(z) \\ C_{2,0}(z) \end{bmatrix} \tag{15}$$

Splitting $\boldsymbol{B}'_{a_n, w_n}\left(z^{-d_n}\right)$ into three matrices in order to find an efficient correlation architecture for (12) using the same approach of [7]

$$\boldsymbol{B}'_{a_n, w_n}\left(z^{-d_n}\right) = \boldsymbol{D}_n(z) \boldsymbol{W}_n \boldsymbol{A}_n \tag{16}$$

where

$$\boldsymbol{D}(z) = \begin{bmatrix} z^{-d_n} & 0 \\ 0 & 1 \end{bmatrix} \quad (17)$$

$$\boldsymbol{W}_n = \begin{bmatrix} 1 & 0 \\ 0 & \bar{w}_n \end{bmatrix} \quad (18)$$

$$\boldsymbol{A}_n = \begin{bmatrix} 1 & a_n \\ a_n & -1 \end{bmatrix} \quad (19)$$

the architecture of a stage of the simultaneous correlator of multilevel complementary sequences is developed (Fig. 2).

The complete correlation process is completed by a concatenation of stages and a final addition, as shown in Fig. 3.

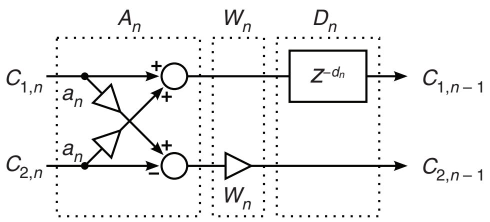

Fig. 2 *Stage of simultaneous correlator of multilevel complementary sequences*

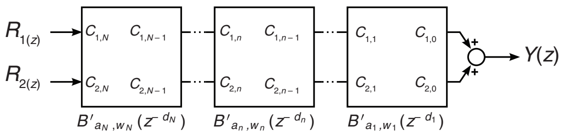

Fig. 3 *Simultaneous correlator of multilevel complementary sequences*

Results: To evaluate the efficiency of the proposed algorithm, it was compared with the efficient correlator for complementary pairs of sequences of [8] based on multilevel sequences. In that work it was stated that for a pair of sequences of length $L = 2^N 10^M 26^P$, $Q = N + 4M + 12P$ stages are needed. Table 1 shows the operations required to compute the sum of correlation of a pair of complementary sequences with the generic length.

Table 1: Operations required for correlation of complementary sequences

	Additions	Multiplications	Delays
Straightforward	$2(2^N 10^M 26^P) - 1$	$2 \times (2^N 10^M 26^P)$	$2 \times (2^N 10^M 26^P - 1)$
Efficient [8]	$4(N + 4M + 12P) - 1$	$6(N + 4M + 12P)$	$2 \times (2^N 10^M 26^P - 1)$
Proposed	$2(N + 4M + 12P) + 1$	$3(N + 4M + 12P)$	$2^N 10^M 26^P - 1$

The Table shows that both the efficient and proposed correlation architectures require less additions and multiplications than the straightforward correlator, as these two correlation architectures use two additions and three multiplications per stage, whereas the straightforward correlator needs an addition and a multiplication per sequence element. In addition, it can be concluded that the proposed architecture needs half as many operations as the efficient architecture. The obtained reduction in the amount of operations is given by the fact that the architecture of [8] needs two correlation architectures to perform the sum of correlations, whereas the proposed architecture uses a single correlation architecture. Additionally, it is important to note that the proposed architecture reduces the amount of required delays to compute the correlation with half of the other architectures, which has a great impact when they need to be implemented in the hardware.

Conclusions: The existence of ongoing interest in new coding schemes and the optimisation of existing ones have brought multilevel complementary sequences into focus. This Letter present, as a proposal, an optimised correlation architecture for multilevel complementary sequences that performs the correlation of two input signals with respect to a pair of sequences with a reduced amount of calculations. The proposal is theoretically demonstrated and compared with the straightforward correlation and the architecture presented by García *et al.* [8]. The comparison showed the reduction in the amount of required calculations.

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