

## Accepted Manuscript

Title: Oversizing Analysis in Plant-wide Control Design for Industrial Processes

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PII: S0098-1354(13)00086-0

DOI: <http://dx.doi.org/doi:10.1016/j.compchemeng.2013.03.021>

Reference: CACE 4678

To appear in: *Computers and Chemical Engineering*

Received date: 31-7-2012

Revised date: 19-2-2013

Accepted date: 6-3-2013



Please cite this article as: David A.R. Zumoffen, Oversizing Analysis in Plant-wide Control Design for Industrial Processes, *Computers and Chemical Engineering* (2013), <http://dx.doi.org/10.1016/j.compchemeng.2013.03.021>

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# Oversizing Analysis in Plant-wide Control Design for Industrial Processes

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## Abstract

In this work, an alternative plant-wide control design approach based on oversizing analysis is presented. The overall strategy can be divided in two main sequential tasks: 1- defining the optimal decentralized control structure, and 2- setting the controller interaction degree and its implementation. Both problems represent combinatorial optimizations based on multi-objective functional costs and were solved efficiently by genetic algorithms. The first task defines the optimal selection of controlled and manipulated variables simultaneously, the input-output pairing, and the overall controller dimension in a sum of square deviations context. The second task analyzes the potential improvements by defining the controller interaction degree via the net load evaluation approach. In addition, some insights are given about the feasibility (implementation load) of these control structures for a decentralized or centralized framework. The well-known Tennessee Eastman (TE) process is selected here for sake of comparison with other multivariable control designs.

**Keywords:** Plant-wide control, Oversizing analysis, Servo-regulator trade-off, Sparse controller, Controller interaction degree

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## 1. Introduction

Plant-wide control (PWC) design is a very important topic in industrial process control. In general it involves the selection of controlled and manipulated variables (CVs and MVs), input-output pairing, the definition of the controller structure, tuning, etc. The solution to these problems will define (restrict) the future operability degree for the plant under study. In fact, both investment and operating costs can be seriously affected if the plant-wide control problem is not solved properly (Downs and Skogestad, 2011; Yuan et al., 2011b; Sharifzadeh and Thornhill, 2012). The best PWC design must be able to meet all the outlined process objectives and use the minimum number of control loops, i.e. a parsimonious CVs and MVs selection.

It is clear that some systematic and generalized approach is required for quantifying the optimal solutions to topics stated previously. Note that, the problem size quickly becomes intractable when the process dimension increases, i.e. exhaustive search is unpractical. On the other hand, any holistic approach to solve the PWC problems requires several knowledge bases with different insights which shows the complexity of an unified (all in one) methodology (Downs and Skogestad, 2011).

There are some approaches for addressing these problems almost systematically and covering the broad spectrum from strategies based on purely heuristic/engineering judgment (Buckley, 1964; Luyben et al., 1998) to optimization routines. General topics involved in these proposals include stability and/or controllability assessments (Yuan et al., 2011a,b), input-output pairing problems (Bristol, 1966; Chang and Yu, 1990; McAvoy et al., 2003; He et al., 2009; Assali and McAvoy, 2010), operating cost and self-optimizing (Skogestad, 2000; Alstad and Skogestad, 2007; Downs and Skogestad, 2011), performance and/or robustness indicators (Grosdidier et al., 1985; Skogestad and Morari, 1987; Skogestad and Postlethwaite, 2005), and deviation-based indexes or some combination of these into a multi-objective criteria (Downs and Skogestad, 2011; Sharifzadeh and Thornhill, 2012). Usually, the suggested design framework considers all possible degrees of freedom in a classical control structure (centralized/full or decentralized/diagonal). A good review of some relevant techniques and comparisons can be found in previous works of the author: Molina et al. (2011); Zumoffen et al. (2010, 2011); Zumoffen and Basualdo (2012); and two excellent books as Skogestad and Postlethwaite (2005) and Khaki-Sedigh and Moaveni (2009).

In this article, an alternative methodology for PWC design based on oversizing analysis is presented. In fact, this methodology, called extended minimum square deviations (extended MSD), complements significantly the approaches suggested in Molina et al. (2011) and Nieto Degliuomini et al. (2012) via a simultaneous CVs and MVs parametrization. On the other hand, the work recently appeared (succinct) in Zumoffen and Basualdo (2012) is extended here by adding a complete controller interaction degree analysis, a genetic algorithms (GA)-based representation, and several controller synthesis evaluations. The overall procedure is based on a multi-objective optimization (combinatorial) framework by accounting the sum of square de-

viations (SSD) index in the presence of setpoint changes and disturbances. Initially, the simultaneous CVs and MVs selection is performed based on the SSD evaluation for uncontrolled variables (UVs). The original control requisites from process engineering, as well as the degrees of freedom, are parameterized suitably into the combinatorial problem to perform an oversizing analysis along several PWC designs with different dimensions. The second part of the proposed sequential algorithm relies on the so-called controller interaction design via the net load evaluation (NLE) index. This approach also represents a multi-objective combinatorial problem and gives valuable information about the controller interaction degree (diagonal, sparse, or full) for a servo/regulator trade-off solution as well as the implementation load for internal model control (IMC) or model predictive control (MPC) contexts. All the combinatorial problem formulations are solved via GA due to the following two reasons: 1- they provide the optimal and suboptimal set of solutions (Chipperfield et al., 1994; Molina et al., 2011) and 2- they are less prone to getting trapped in local optima (Sharifzadeh and Thornhill, 2012). Although the overall strategy is not a holistic approach, it has some systematic and generalization degree by minimizing the heuristic considerations. The suggested methodology is tested on the well-known Tennessee Eastman (TE) process giving a complete set of dynamic simulations, performance indexes and required hardware resources for sake of comparison with other multivariable control designs (McAvoy and Ye, 1995; Ricker, 1996; Larsson et al., 2001; Banerjee and Arkun, 1995; Molina et al., 2011).

The paper is organized as follows: Section 2 presents the proposed extended MSD methodology. Subsection 2.1 analyzes the optimal CVs and MVs selection based on SSD criterion. Subsection 2.2 complements the above procedure addressing the controller interaction degree analysis via NLE. These sections define various combinatorial problems which need to be solved efficiently. In this context, Section 3 summarizes some backgrounds about the GA procedure and the problem representation used in this case. Section 4 shows the case study suggested to check the performance of the extended MSD approach. The main results are displayed in this section. Conclusions of the work are stated in Section 4 and some additional information about modeling and tuning are presented in Appendix.

## 2. Extended MSD methodology

Let's consider a stable industrial process,  $\mathbf{P}$ , with  $m$  potential controlled variables (CVs),  $n$  available manipulated variables (MVs) and  $p$  disturbance variables (DVs). Considering a plant model based on transfer functions matrix (TFM),  $\mathbf{G}(s)$  and  $\mathbf{D}(s)$ , with dimension  $(m \times n)$  and  $(m \times p)$  respectively, the process can be partitioned as shown in eq. (1). Here,  $q \leq \min(m, n)$  represents the number of variables which should be controlled (a subset of potential CVs).

$$\mathbf{y}(s) = \mathbf{G}(s)\mathbf{u}(s) + \mathbf{D}(s)\mathbf{d}(s) = \begin{bmatrix} \mathbf{y}_s(s) \\ \mathbf{y}_r(s) \end{bmatrix} = \begin{bmatrix} \mathbf{G}_s(s) & \mathbf{G}_s^*(s) \\ \mathbf{G}_r(s) & \mathbf{G}_r^*(s) \end{bmatrix} \begin{bmatrix} \mathbf{u}_s(s) \\ \mathbf{u}_r(s) \end{bmatrix} + \begin{bmatrix} \mathbf{D}_s(s) \\ \mathbf{D}_r(s) \end{bmatrix} \mathbf{d}(s) \quad (1)$$

The subsystems and signals involved in eq. (1) have the following description:  $\mathbf{G}_s(s)$  is the square  $q \times q$

Figure 1: Generalized IMC structure

subprocess to be controlled,  $\mathbf{G}_s^*(s)$ ,  $\mathbf{G}_r(s)$ ,  $\mathbf{G}_r^*(s)$ ,  $\mathbf{D}_s(s)$  and  $\mathbf{D}_r(s)$  are remaining matrices with dimension  $q \times (n - q)$ ,  $(m - q) \times q$ ,  $(m - q) \times (n - q)$ ,  $q \times p$  and  $(m - q) \times p$  respectively. On the other hand,  $\mathbf{u}(s)$  and  $\mathbf{d}(s)$  represent the input and disturbance vectors respectively. Note that,  $\mathbf{u}_s(s)$  ( $q \times 1$ ) are the selected MVs subset for controlling the output variables subset  $\mathbf{y}_s(s)$ . In this work the remaining input variables called  $\mathbf{u}_r(s)$  ( $(n - q) \times 1$ ) are not used for control purposes, i.e. they are fixed. Vector  $\mathbf{y}_r(s)$  groups together the so called uncontrolled variables (UVs). Thus, the CVs and UVs subsets are represented via  $\mathbf{P}_s$  and  $\mathbf{P}_r$  subprocesses respectively as shown in Fig. 1.

Henceforth, the steady-state operation is represented without the Laplace variable  $s$ , i.e  $\mathbf{G}_s(s = 0) = \mathbf{G}_s$ . Considering the internal model control (IMC) theory and steady-state perfect control ( $\mathbf{y}_s = \mathbf{y}_s^{sp}$ ), the following relationships can be stated,

$$\begin{aligned} \mathbf{u}_s &= \mathbf{G}_s^{-1} \mathbf{y}_s^{sp} - \mathbf{G}_s^{-1} \mathbf{D}_s \mathbf{d} - \mathbf{G}_s^{-1} \mathbf{G}_s^* \mathbf{u}_r \\ \mathbf{y}_r &= \mathbf{G}_r \mathbf{G}_s^{-1} \mathbf{y}_s^{sp} + (\mathbf{D}_r - \mathbf{G}_r \mathbf{G}_s^{-1} \mathbf{D}_s) \mathbf{d} + (\mathbf{G}_r^* - \mathbf{G}_r \mathbf{G}_s^{-1} \mathbf{G}_s^*) \mathbf{u}_r \end{aligned} \quad (2)$$

where  $\mathbf{y}_r$  consider the UVs deviations from their nominal working points when set points and disturbances changes were considered (Fig. 1). It is important to note that partitioning in eq. (1) and eq. (2) are function of “ $q$ ”, i.e. the number of variables to be controlled which also defines the “*controller size*”.

In this context, some questions appear: What is the minimum  $q$  to fulfill the control requirements?, Is it necessary to use all the available MVs?, What are the additional CVs?, and MVs?. The answers are non trivial and require a previous analysis about the potential plant-wide control scenarios and the problem formulation in each case.

Figure 2: Plant-wide control alternatives

### 2.1. Optimal CVs and MVs selection

The potential multivariable control alternatives are defined by the process dimension,  $m$ ,  $n$ ,  $p$ , and the control requisites,  $q = q_o + q_a$ . It is worth to perform here the following explanations: the original process is stable or stabilized (inventory control is not addressed by the extended MSD approach) and  $q_o$  is the number of output variables which must be controlled “*indefectibly*”. The latter represents the process engineering requirements, i.e. production rate, product quality, etc. On the other hand,  $q_a$  is the number of additional output variables which “*could/should*” be controlled in order to complete the multivariable controller configuration. In this context, the potential plant-wide control alternatives are divided into four cases (I to IV) as shown in Fig. 2. Each scenario requires a particular approach for solving the original problem.

Let’s consider the binary decision variables,  $\mathbf{c}^c$  and  $\mathbf{c}^m$ , which parameterize the CVs and MVs subset selection respectively, then the PWC problem can be defined via a combinatorial one. Henceforth,  $\mathbf{A}(\mathbf{c}^i)$  represents a particular selection of the steady-state matrix  $\mathbf{A}$  with the parametrization variable  $\mathbf{c}^i$  and  $\|\cdot\|_F$  is the Frobenius norm for matrices. Section 3 summarizes some details about the parametrization and solution based on GA. Before presenting the “*extended MSD methodology*”, which improves and generalizes

the basic approach stated in Molina et al. (2011), it is important to analyze the PWC alternatives shown in Fig. 2 (remember  $q = q_o + q_a$ ):

**Case I:** Optimal MVs Selection. Here, all potential outputs need to be controlled ( $q = m$  and  $m < n$ ), so  $\mathbf{G}_r = \mathbf{G}_r^* = 0$  and all entries in  $\mathbf{c}^c$  are one. In this case, the combinatorial problem dimension results  $n!/(q!(n-q)!)$ . The functional cost used here is the sum of square deviations (SSD) on MVs from eq. (2):

$$\text{SSD}_{\mathbf{u}_s}(\mathbf{c}^m) = \|\mathbf{G}_s(\mathbf{c}^m)^{-1}\|_F^2 + \|\mathbf{G}_s(\mathbf{c}^m)^{-1}\mathbf{D}_s(\mathbf{c}^m)\|_F^2 \quad (3)$$

**Case II:** Optimal MVs and CVs Selection. In this case,  $q < \min(m, n)$  and the problem dimension results  $[(m - q_o)!/(q_a!(m - q)!)] [n!/(q!(n - q)!)]$ . Here, the SSD index on UVs is used to drive the search,

$$\text{SSD}_{\mathbf{y}_r}(\mathbf{c}^c, \mathbf{c}^m) = \|\mathbf{G}_r(\mathbf{c}^c, \mathbf{c}^m)\mathbf{G}_s(\mathbf{c}^c, \mathbf{c}^m)^{-1}\|_F^2 + \|\mathbf{D}_r(\mathbf{c}^c, \mathbf{c}^m) - \mathbf{G}_r(\mathbf{c}^c, \mathbf{c}^m)\mathbf{G}_s(\mathbf{c}^c, \mathbf{c}^m)^{-1}\mathbf{D}_s(\mathbf{c}^c, \mathbf{c}^m)\|_F^2 \quad (4)$$

**Case III:** Optimal CVs Selection. Here, all potential MVs are used for control purposes ( $q = n$  and  $m > n$ ), so  $\mathbf{G}_s^* = \mathbf{G}_r^* = 0$  and all entries in  $\mathbf{c}^m$  are one. In this case, the combinatorial problem dimension results  $(m - q_o)!/(q_a!(m - q)!)$ . Again here, the functional cost is the SSD index on UVs but only parameterized with  $\mathbf{c}_c$ :

$$\text{SSD}_{\mathbf{y}_r}(\mathbf{c}^c) = \|\mathbf{G}_r(\mathbf{c}^c)\mathbf{G}_s(\mathbf{c}^c)^{-1}\|_F^2 + \|\mathbf{D}_r(\mathbf{c}^c) - \mathbf{G}_r(\mathbf{c}^c)\mathbf{G}_s(\mathbf{c}^c)^{-1}\mathbf{D}_s(\mathbf{c}^c)\|_F^2 \quad (5)$$

**Case IV:** There are no possible combinations. Direct pairing based on relative gain array (RGA) or similar approaches is used.

In fact, case III represents the design scenario used by the classical MSD approach presented opportunely in Zumoffen et al. (2010, 2011) and Molina et al. (2011). In these works some useful properties about the minimization of  $\text{SSD}_{\mathbf{y}_r}(\mathbf{c}^c)$  were analyzed.

The extended MSD methodology is displayed in Algorithm 1 and it groups together all cases, from I to IV, in a single layout. Note that, for  $0 \leq q_a \leq \min(m, n) - q_o$ , Algorithm 1 gives a complete overview of the potential plant-wide decentralized control structures. In fact, for each  $q_a$ , selected by the designer, the extended methodology provides an optimal decentralized control structure by selecting the CVs and MVs subsets and the corresponding input-output pairing.

Note that, eqs. (3), (4), and (5) can be augmented with diagonal weighting matrices,  $\Lambda_1$  and  $\Theta_1$ , for including the process control objectives such as set point/disturbance magnitudes (useful when the process models is not normalized or scaled), and  $\Lambda_2$  and  $\Theta_2$  weighting the relative degree of importance among the overall outputs. For example, the weighted version of eq. (5) results

$$\text{SSD}_{\mathbf{y}_r}(\mathbf{c}^c) = \|\Lambda_2 \mathbf{G}_r(\mathbf{c}^c) \mathbf{G}_s(\mathbf{c}^c)^{-1} \Lambda_1\|_F^2 + \|\Theta_2 (\mathbf{D}_r(\mathbf{c}^c) - \mathbf{G}_r(\mathbf{c}^c) \mathbf{G}_s(\mathbf{c}^c)^{-1} \mathbf{D}_s(\mathbf{c}^c)) \Theta_1\|_F^2.$$

## 2.2. Sparse controllers: improvements via NLE

If the problem stated in Section 2.1 is solved efficiently, then the subprocess  $\mathbf{G}_s(s)$  and the corresponding decentralized input-output pairing are already defined. The problem to be addressed now is the controller

**Algorithm 1:** Extended MSD methodology

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**Data:**  $\mathbf{G}$ ,  $\mathbf{D}$ ,  $m$ ,  $n$ , original control requisites  $q_o$   
**Result:**  $\mathbf{G}_s$ ,  $RGA$ , decentralized pairing, model parametrization  $\Gamma$

```

1 for  $q_a = 0$  to  $\min(m, n) - q_o$  do
2    $q = q_o + q_a$ ;
3    $case = f(m, n, q)$ ;
4   switch  $case$  do
5     case I “Optimal MVs Selection”
6        $\min_{\mathbf{c}^m} [\text{SSD}_{\mathbf{u}_s}(\mathbf{c}^m)]$ ;
7     endsw
8     case II “Optimal MVs and CVs Selection”
9        $\min_{(\mathbf{c}^c, \mathbf{c}^m)} [\text{SSD}_{\mathbf{y}_r}(\mathbf{c}^c, \mathbf{c}^m)]$ ;
10    endsw
11    case III “Optimal CVs Selection”
12       $\min_{\mathbf{c}^c} [\text{SSD}_{\mathbf{y}_r}(\mathbf{c}^c)]$ ;
13    endsw
14  endsw
15  Save:  $\mathbf{c}^c$  and  $\mathbf{c}^m$  for each  $q_a$ ;
16   $RGA = \mathbf{G}_s(\mathbf{c}^c, \mathbf{c}^m) \otimes (\mathbf{G}_s(\mathbf{c}^c, \mathbf{c}^m)^{-1})^T$ ;
17 end
18 Analyze “ $\min(m, n) - q_o + 1$ ” optimal decentralized control structures;
19 Evaluate potential improvements via NLE for selected ( $q_a$ ) control policy;
20  $\min_{\Gamma} [\text{NLE}(\Gamma)]$ ;

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structure design (lines 19 and 20 in Algorithm 1). In fact, note that the controller structure may be diagonal (decentralized/without interaction), full (centralized/full interaction) or sparse (partial interaction).

Considering again Fig. 1, the controlled outputs can be represented as

$$\mathbf{y}_s(s) = \tilde{\mathbf{G}}_s(s) \mathbf{G}_c(s) \mathbf{y}_s^{sp}(s) + (\mathbf{I} - \tilde{\mathbf{G}}_s(s) \mathbf{G}_c(s)) \mathbf{y}_s^{net}(s), \quad (6)$$

with

$$\mathbf{y}_s^{net}(s) = \mathbf{A}(s) \mathbf{y}_s^{sp}(s) + \mathbf{B}(s) \mathbf{d}_*(s) \quad (7)$$

where  $\mathbf{A}(s) = f(\mathbf{G}_s(s), \tilde{\mathbf{G}}_s(s), \mathbf{G}_c(s))$  and  $\mathbf{B}(s) = g(\mathbf{G}_s(s), \tilde{\mathbf{G}}_s(s), \mathbf{G}_c(s))$  are the net load matrices with specific structure.  $\mathbf{B}(s) \mathbf{d}_*(s)$  is the so called net load effect (Chang and Yu, 1992) and  $\mathbf{y}_s^{net}(s)$  the augmented form considering references and disturbances changes (Nieto Degliuomini et al., 2012). In the latter work only a “sparse suboptimal control policy” was evaluated in a decentralized context.

There are two ways to avoid the adverse effects of  $\mathbf{y}_s^{net}(s)$  on  $\mathbf{y}_s(s)$ : 1- to adjust the controller tuning for fast responses ( $(\mathbf{I} - \tilde{\mathbf{G}}_s(s) \mathbf{G}_c(s)) \rightarrow 0$  quickly) or 2- to minimize the multivariable gain of  $\mathbf{y}_s^{net}(s)$  at steady state. The former is limited by stability and robustness issues. The second option is selected here and depends on the plant-model mismatch adopted. In fact, at steady state, the net load matrices reduce to  $\mathbf{A} = \mathbf{I} - \tilde{\mathbf{G}}_s \mathbf{G}_s^{-1}$  and  $\mathbf{B} = \tilde{\mathbf{G}}_s \mathbf{G}_s^{-1} \mathbf{D}_s$ , respectively. From eq. (7) it is clear that if  $\tilde{\mathbf{G}}_s = \mathbf{G}_s$  (if full IMC controller is used) then  $\mathbf{A} = \mathbf{0}$  and  $\mathbf{B} = \mathbf{D}_s$ , so changes in the references do not affect  $\mathbf{y}_s^{net}(s)$  (and therefore the CVs) but, in contrast, disturbances are not attenuated.



A specific process model selection,  $\tilde{\mathbf{G}}_s(\Gamma)$ , could generate a trade-off solution between servo and regulator behaviors by minimizing a new scalar index called net load evaluation,  $\text{NLE}(\Gamma) = \|\mathbf{y}_s^{\text{net}}(\Gamma)\|_F^2$  in a SSD sense. In this context, the combinatorial problem can be stated as follows,

$$\min_{\Gamma} \text{NLE}(\Gamma) = \min_{\Gamma} \left[ \|\Delta_2(\mathbf{I} - \tilde{\mathbf{G}}_s(\Gamma)\mathbf{G}_s^{-1})\Delta_1\|_F^2 + \|\Xi_2\tilde{\mathbf{G}}_s(\Gamma)\mathbf{G}_s^{-1}\mathbf{D}_s\Xi_1\|_F^2 \right] \quad (8)$$

subject to

$$\begin{aligned} \tilde{\mathbf{G}}_s(\Gamma) &= \mathbf{G}_s \otimes \Gamma \\ \text{Re} \left[ \lambda_i \left( \mathbf{G}_s \tilde{\mathbf{G}}_s(\Gamma)^{-1} \right) \right] &> 0, \quad \text{with } i = 1, \dots, q \end{aligned} \quad (9)$$

where  $\Gamma$  is a binary parametrization matrix for selecting specific parts of  $\mathbf{G}_s$ , “ $\otimes$ ” is the element-by-element product, and the inequality in eq. (9) is the stability/robustness criterion developed by Garcia and Morari (1985) for multivariable control structures based on IMC theory.  $\text{Re}[\cdot]$  is the real part function and  $\lambda_i(\cdot)$  is the  $i$ -th eigenvalue. Again here,  $\Delta_1$ ,  $\Delta_2$ ,  $\Xi_1$  and  $\Xi_2$  are diagonal weighting matrices which allow to define the process control objectives according to their relative importance in the system, in particular when the process model used is not normalized. The optimization defined in eq. (8) has  $2^{(q \times q)}$  potential solutions. According to the problem size, this minimization can be done by exhaustive search or implementing some mixed-integer optimization routine (deterministic or stochastic). Additional details about the approach used to solve the combinatorial problem are given in the following section.

### 3. Solution via genetic algorithms

The number of potential solutions in the problems defined in Sections 2.1 and 2.2 increase suddenly with the size of the system. Indeed, a purely heuristic approach quickly becomes impractical (Yuan et al., 2011b; Sharifzadeh and Thornhill, 2012). A methodology based on genetic algorithms (GA) is selected here to solve these problems for the following two reasons: 1- it provides an optimal and suboptimal set of solutions (Chipperfield et al., 1994; Molina et al., 2011) and 2- it is less prone to getting trapped in local optima (Sharifzadeh and Thornhill, 2012). Genetic algorithms are defined as stochastic global search methods which mimic natural biological evolution. Thus, the individuals are merged, mate, and mutate along the generations in order to find the best population according to some particular fitness function (environment). Specific details about how these algorithms can be used to solve combinatorial problems (parametrization and tuning) can be found in Zumoffen and Basualdo (2010) and Molina et al. (2011).

Each individual,  $\mathbf{c}_i = [c_i^1, c_i^2, \dots, c_i^{n_c}]$ , is represented with a particular alphabet which parameterizes the decision variables. In this case,  $c_i^j$  belongs to the binary alphabet (0 or 1) indicating the absence or presence of the signal  $j$ , being  $n_c$  the individual length. The following parametrization is particularly useful to solve the optimization problem stated in Section 2.1,

$$\mathbf{c}_i = [\mathbf{c}_i^c, \mathbf{c}_i^o, \mathbf{c}_i^m] \quad (10)$$

with

$$\mathbf{c}_i^c = [c_i^{c1}, c_i^{c2}, \dots, c_i^{c(m-q_0)}], \quad \mathbf{c}_i^o = [c_i^{o1}, c_i^{o2}, \dots, c_i^{oq_0}], \quad \mathbf{c}_i^m = [c_i^{m1}, c_i^{m2}, \dots, c_i^{cn}] \quad (11)$$

where  $\mathbf{c}_i^c$  and  $\mathbf{c}_i^m$  are the decision variables parametrization for selecting the CVs and MVs subsets respectively. On the other hand, the original control requisites are included in the vector  $\mathbf{c}_i^o$ , which is not used as decision variable, but has a direct influence on the functional cost. Vectors in eq. (11) have the following lengths:  $m - q_0$ ,  $q_0$ , and  $n$ , respectively. Hence  $n_c = m - q_0 + n$ .

Without loss of generality, let us consider **case II**, which is the most complex of the scenarios considered. For each  $q_a \in [0, \min(m, n) - q_0]$  the optimization problem stated in eq. (12) must be solved subject to the constraints in eq. (13). Note that  $\mathbf{G}_s$ ,  $\mathbf{G}_r$ ,  $\mathbf{D}_s$ , and  $\mathbf{D}_r$  are functions of the parametrization  $\mathbf{c}_i = [\mathbf{c}_i^c, \mathbf{c}_i^o, \mathbf{c}_i^m]$ .

$$\min_{(\mathbf{c}_i^c, \mathbf{c}_i^m)} SSD_{\mathbf{y}_r}(\mathbf{c}_i^c, \mathbf{c}_i^o, \mathbf{c}_i^m) \quad (12)$$

subject to

$$\|\mathbf{c}_i^c\|_1 = q_a, \quad \|\mathbf{c}_i^m\|_1 = q_a + q_0, \quad \det(\mathbf{G}_s) \neq 0 \quad (13)$$

Restrictions in eq. (13) guarantee the invertible square subprocess selection of  $q \times q$ , called  $\mathbf{G}_s$ . Note that  $\|\cdot\|_1$  is the 1-norm for vectors, i.e. the sum of the absolute values.

For addressing the problem stated in Section 2.2 it is necessary the following considerations: 1- perform a decentralized pairing for the selected  $\mathbf{G}_s$ , 2- reorder  $\mathbf{G}_s$  for diagonal pairing, and 3- use this diagonal control structure as a starting point for the NLE approach. Then, the parametrization shown in eq. (14) is useful for solving the optimization problem stated in eqs. (8) and (9),

$$\tilde{\mathbf{G}}_s(\Gamma_i) = \mathbf{G}_s \otimes \Gamma_i = \mathbf{G}_s \otimes \begin{bmatrix} 1 & c_i^1 & \dots & c_i^{q-1} \\ c_i^q & 1 & \dots & c_i^{2(q-1)} \\ \vdots & \vdots & \ddots & \vdots \\ c_i^{(q-1)(q-1)+1} & \dots & c_i^{q(q-1)} & 1 \end{bmatrix} \quad (14)$$

where  $\mathbf{c}_i^{NLE} = [c_i^1, c_i^2, \dots, c_i^{n_c}]$  is the individual representation in this case with  $n_c = 2^{(q \times q) - q}$ . Thus, the NLE approach defines the best controller interaction level by selecting (or not) specific off-diagonal elements in the process model, considering the decentralized structure as a base case (Zumoffen et al., 2011; Nieto Degliuomini et al., 2012).

In this context, it is also useful to know how the NLE index evolves/degrades when the individual parametrization is constrained to take fewer controller components with respect to the optimal solution. Thus, if the best solution ( $\mathbf{c}_{op}^{NLE}$ ) to the problem stated in eqs. (8), (9), and (14) presents  $n_{op}$  additional

Table 1: TE process variables

no.	Inputs	var.	no.	Outputs	var.
$u_1$	D Flow [kg/h]	$XMV(1)$	$y_1$	Recycle flow [kscmh]	$XME(5)$
$u_2$	A Flow [kg/h]	$XMV(3)$	$y_2$	Reactor flow [kscmh]	$XME(6)$
$u_3$	A/C Flow [kscmh]	$XMV(4)$	$y_3$	Reactor temp. [°C]	$XME(9)$
$u_4$	Compressor rec. valve [%]	$XMV(5)$	$y_4$	Separator temp. [°C]	$XME(11)$
$u_5$	Purge valve [%]	$XMV(6)$	$y_5$	Separator pressure [kPa]	$XME(13)$
$u_6$	Stripper steam valve [%]	$XMV(9)$	$y_6$	Stripper pressure [kPa]	$XME(16)$
$u_7$	RCWO temp. set point [°C]	$XME(21)_{sp}$	$y_7$	Stripper temp. [°C]	$XME(18)$
$u_8$	CCW Flow [m3/h]	$XMV(11)$	$y_8$	Compressor work [kW]	$XME(20)$
no.	Disturbances	var.			
$d_1$	Composition stream 4 (A/C)	$IDV(1)$			
$d_2$	Composition stream 4 (B)	$IDV(2)$			

off-diagonal elements, now the problem can be redefined as

for  $k = 1$  to  $(n_{op} - 1)$

$$\min_{\Gamma_i} \text{NLE}(\Gamma_i)$$

subject to

$$\tilde{\mathbf{G}}_s(\Gamma_i) \text{ (eq. 14)} \quad (15)$$

$$\text{Re} \left[ \lambda_j \left( \mathbf{G}_s \tilde{\mathbf{G}}_s(\Gamma_i)^{-1} \right) \right] > 0, \quad \text{with } j = 1, \dots, q$$

$$\|\mathbf{c}_i^{NLE}\|_1 \leq k$$

end

and a complete NLE profile from decentralized to full control structures is obtained.

#### 4. Case study: Tennessee Eastman process

The Tennessee Eastman (TE) process is a well-known benchmark simulation case from the process control community for testing new developments. In this section, only basic details are given about the process (see Downs and Vogel (1992) and Molina et al. (2011)). The TE process is open-loop unstable, so a stabilizing control structure is required before applying the extended MSD methodology. In this paper, the stabilizing control policy opportunely suggested by McAvoy and Ye (1995) is adopted, which consists of flow (inner) and level (cascade) controllers for the reactor, the separator, and the stripper units. Table 1 summarizes the remaining CVs and MVs. In this case,  $m = 12$  outputs and  $n = 8$  inputs (case II, Fig. 2). In addition,  $IDV(1)$  and  $IDV(2)$  disturbance scenarios are considered for the extended MSD approach, i.e.  $p = 2$ . These perturbations represent composition changes in the fresh feed entering to the stripper unit. The normalized steady-state model used here is shown in Table 2.

Considering the original control requisites, stated by Downs and Vogel (1992), it is required to control the following variables:  $y_9, y_{10}, y_{11}$  y  $y_{12}$  (gray background in Table 1), which means that  $q_o = 4$  and  $0 \leq q_a \leq \min(m, n) - q_o = 4$ . In this context, there are “ $\max(q_a) + 1 = 5$ ” optimal multivariable servo/regulator

Table 2: Normalized steady-state process model

	G								D	
	$u_1$	$u_2$	$u_3$	$u_4$	$u_5$	$u_6$	$u_7$	$u_8$	$d_1$	$d_2$
$y_1$	-0.39	-0.03	-0.12	0.07	-0.13	-0.03	0.78	-0.08	0.59	0.55
$y_2$	-0.37	-0.01	0.10	0.10	-0.13	-0.03	0.70	-0.09	0.65	0.50
$y_3$	0.27	0.03	0.60	-0.09	0.04	0.06	-0.78	0.01	-1.02	-1.05
$y_4$	0.38	0.07	0.22	-0.11	0.08	0.04	-0.80	-0.04	-0.95	-1.09
$y_5$	-0.44	-0.13	0.43	0.10	-0.21	0.01	0.56	-0.05	0.98	1.09
$y_6$	-0.44	-0.13	0.44	0.08	-0.21	0.01	0.55	-0.05	0.98	1.09
$y_7$	0.38	0.06	0.23	-0.10	0.08	0.10	-0.79	-0.08	-0.97	-1.08
$y_8$	-0.38	0.04	-0.01	0.20	-0.11	-0.04	0.78	-0.14	-1.29	0.97
$y_9$	-0.44	-0.13	0.43	0.10	-0.21	0.01	0.56	-0.05	0.99	1.09
$y_{10}$	-0.02	0.03	0.70	-0.02	0.02	0.02	-0.11	-0.00	-0.20	-0.06
$y_{11}$	0.44	0.01	0.26	-0.13	-0.19	0.01	-0.84	0.06	-1.04	1.07
$y_{12}$	0.60	-0.03	-0.53	0.03	-0.01	-0.01	-0.00	-0.01	0.56	0.09

control structures with dimensions:  $(4 \times 4)$ ,  $(5 \times 5)$ ,  $(6 \times 6)$ ,  $(7 \times 7)$ , and  $(8 \times 8)$ . In this case, the overall combinatorial problem dimension is  $2^{16} = 65536$  with only 1820 feasible solutions. For avoiding the heuristic evaluation of all these solutions, the extended MSD approach (GA parameterized with eqs. (12) and (13)) is applied here for obtaining the optimal selection of CVs and MVs. The GA setting is shown in Table 3, where the following parameters were defined: initial population ( $n_i$ ), number of generations ( $n_g$ ), crossover probability ( $p_c$ ), mutation probability ( $p_m$ ), individual length ( $n_c$ ), selection and crossover methodologies, weighting matrices, and the number of additional control loops ( $q_a$ ).

Table 4 displays the optimal solutions,  $[\mathbf{c}_{op}^c, \mathbf{c}_o^o, \mathbf{c}_{op}^m]$ , to the problem stated previously for each  $q_a$ . Note that,  $\mathbf{c}_i^o = [1, 1, 1, 1]$  is fixed for variables  $y_9$ ,  $y_{10}$ ,  $y_{11}$ , and  $y_{12}$  (Table 1) according to the original control requisites. Extreme cases called  $q_a = 0$  and  $q_a = 4$  represent the optimal selection of MVs and CVs alone respectively. Intermediate cases  $q_a = 1, 2, 3$  constitute a simultaneous MVs and CVs combinatorial problem. The functional cost profiles,  $SSD_{yr}(\mathbf{c}_i^c, \mathbf{c}_i^m)$ , are shown in Fig. 3. For improving the visualization a logarithmic scale is used. Note that, when  $q_a$  increases the achievable SSD index value decreases. In fact, when the dimension ( $q = q_o + q_a$ ) of the control structure increases, the number of UVs decreases ( $m - q$ ). On the other hand, the RGA-based decentralized input-output pairings for each optimal solution  $q_a$  are shown in Table 5.

It is worth mentioning, that the dimension of the final control policy is  $(v + q) \times (v + q)$  with  $v$  being the number of stabilizing control loops. In this case,  $v = 7$ ,  $q_o = 4$ , and  $q_a = 0, 1, 2, 3, 4$ , which generate five optimal control structures with size  $(11 \times 11)$ ,  $(12 \times 12)$ ,  $(13 \times 13)$ ,  $(14 \times 14)$ , and  $(15 \times 15)$  respectively. Note that, the overall PWC problem was reduced from 1820 feasible solutions to testing only 5 optimal decentralized policies. These latter alternatives are dynamically evaluated (servo and regulator) under the most challenging scenarios suggested by Downs and Vogel (1992): A- Set point changes for  $XME(7)$ ,  $XME(17)$ ,  $XME(30)$ , and  $XME_{G/H}$  (called here sp1, sp2, sp3, and sp4), and B- Disturbances:  $IDV1$ ,  $IDV2$ ,  $IDV4$ ,  $IDV8$ , and  $IDV12/IDV15$  simultaneously (called here d1, d2, d3, d4, and d5).

Figure 4 summarizes the normalized integral absolute tracking error (IAE) for the main process variables

Table 3: GA parameter settings for CVs and MVs optimal selection

$n_i$	$n_g$	$p_c$	$p_m$	$n_c$	Selection	Crossover	$\Lambda_1, \Lambda_2, \Theta_1, \Theta_2$	$q_a$
500	60	0.7	$0.7/n_c$	16	roulette-wheel	double-point	equally-weighted	$[0, 4]$

Table 4: Optimal CVs and MVs selection

$q_a$	$\mathbf{c}_{op}^c$								$\mathbf{c}^o$				$\mathbf{c}_{op}^m$							
	$y_1$	$y_2$	$y_3$	$y_4$	$y_5$	$y_6$	$y_7$	$y_8$	$y_9$	$y_{10}$	$y_{11}$	$y_{12}$	$u_1$	$u_2$	$u_3$	$u_4$	$u_5$	$u_6$	$u_7$	$u_8$
0	0	0	0	0	0	0	0	0	1	1	1	1	1	0	1	0	1	0	1	0
1	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	0	1	0	1	0
2	1	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	1	0
3	1	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	0	1	1
4	1	0	0	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1

(original control requisites) and the process operating cost for each control structure under the nine scenarios mentioned previously. From Figs. 4(a) to 4(e) the following observations can be made:

- A- The smallest control structure,  $(11 \times 11)$ , does not have an appropriate behavior because it violates the high pressure limit for d1. The remaining structures,  $(12 \times 12)$  to  $(15 \times 15)$ , fulfill all the specifications.
- B- The IAE decreases when the controller size increases, only for d5. Figures 4(a) to 4(d) show that IAE for the structures  $(12 \times 12)$  to  $(15 \times 15)$ , under scenarios sp1 to d4, remain quite similar.
- C- Figure 4(e) shows that the operating cost increases with the control structure dimension for cases sp1, sp2 and sp4. For scenarios sp3, d1, d2, d3, and d4 the operating costs have practically the same IAE.
- D- Hence, the best solution to avoid a plant-wide control oversizing is the optimal control structure  $(12 \times 12)$ , which presents a suitable trade-off between dynamic performance and operating cost as shown in Fig. 5.

Figure 5 shows the pareto profile for the mean normalized IAEs from Figs. 4(a) to 4(d) versus the mean normalized cost from Fig. 4(e) for each feasible control structure and considering all the simulation scenarios. The  $(12 \times 12)$  control structure improves the operating cost about  $\approx 10.2\%$  and resigns  $\approx 16\%$  in the overall mean performance.

On the other hand, Table 6 shows a comparison of the hardware requirements among different control structures proposed in the literature. While all these control policies fulfill the main objectives, there are significant differences related to the number of measured variables, control loops and composition measurements needed. Therefore, the configuration  $(12 \times 12)$  (or  $q_a = 1$  in Table 4) is proposed here as the optimal decentralized plant-wide control structure for the TE process.

The next step in the extended MSD approach (lines 19 and 20 in Algorithm 1) is the analysis of potential improvements via the NLE methodology. In fact, the controller interaction degree (decentralized, sparse or full) for servo-regulatory control loops can be evaluated and defined. Considering the previously selected control policy with dimension  $(12 \times 12)$  (5 servo/regulatory + 7 stabilizing control loops) the procedure stated in eqs. (8), (9), and (15) is applied here. Note that, the combinatorial problem dimension is

Figure 3: Functional cost profiles -  $SSD_{yr}(\mathbf{c}_i^c, \mathbf{c}_i^m)$ 

Table 5: Decentralized pairings for optimal solutions

Structures - $q_a$				
0	1	2	3	4
$u_1 - y_{12}$	$u_1 - y_{12}$	$u_1 - y_{12}$	$u_1 - y_{12}$	$u_1 - y_{12}$
$u_3 - y_{10}$	$u_2 - y_9$	$u_2 - y_9$	$u_2 - y_9$	$u_2 - y_9$
$u_5 - y_{11}$	$u_3 - y_{10}$	$u_3 - y_{10}$	$u_3 - y_{10}$	$u_3 - y_{10}$
$u_7 - y_9$	$u_5 - y_{11}$	$u_4 - y_8$	$u_4 - y_8$	$u_4 - y_8$
	$u_7 - y_8$	$u_5 - y_{11}$	$u_5 - y_{11}$	$u_5 - y_{11}$
		$u_7 - y_1$	$u_7 - y_1$	$u_6 - y_7$
			$u_8 - y_7$	$u_7 - y_1$
				$u_8 - y_4$

$2^{(5 \times 5 - 1)} = 2^{20} = 1048576$ . Initially, the optimal solution  $\mathbf{c}_{op}^{NLE}$  is found via GA with  $n_i = 2000$ ,  $n_g = 50$ ,  $n_c = 20$ ,  $\Delta_2 = \text{diag}([1, 1, 1, 1, 1])$ ,  $\Delta_1 = \text{diag}([0, 1, 1, 1, 1])$ ,  $\Xi_2 = \text{diag}([1, 1, 1, 1, 1])$ , and  $\Xi_1 = \text{diag}([1, 1])$ . The remaining GA parameters are the same as those shown in Table 3. The next procedure, stated in eq. (15), is the NLE index profile evaluation when individual parametrization is constrained to take fewer model/controller components with respect to the optimal solution,  $\mathbf{c}_{op}^{NLE}$ . The results of these optimization problems applied to the TE process are shown in Fig. 6. All the NLE profiles are displayed in Fig. 6(a). The optimal case (■) provides  $\mathbf{c}_{op}^{NLE}$  which selects only  $n_{op} = 14$  specific model components with the structure  $\Gamma_{op}$  shown in eq. 16. The remaining profiles represent suboptimal cases with constraints from  $\|\mathbf{c}_i^{NLE}\|_1 = 1$  to  $\|\mathbf{c}_i^{NLE}\|_1 = n_{op} - 1 = 13$ . Figure 6(b) provides a comparison of the NLE index for several different model components selections. In fact, any suboptimal-constrained solution ( $\Gamma_i$ ) with fewer/more components than the optimal one ( $\Gamma_{op}$ ) degrades its performance until reaching the NLE value corresponding to the decentralized/full case ( $\Gamma_d/\Gamma_f$ ). Solutions with more components than  $\Gamma_{op}$  are not considered here

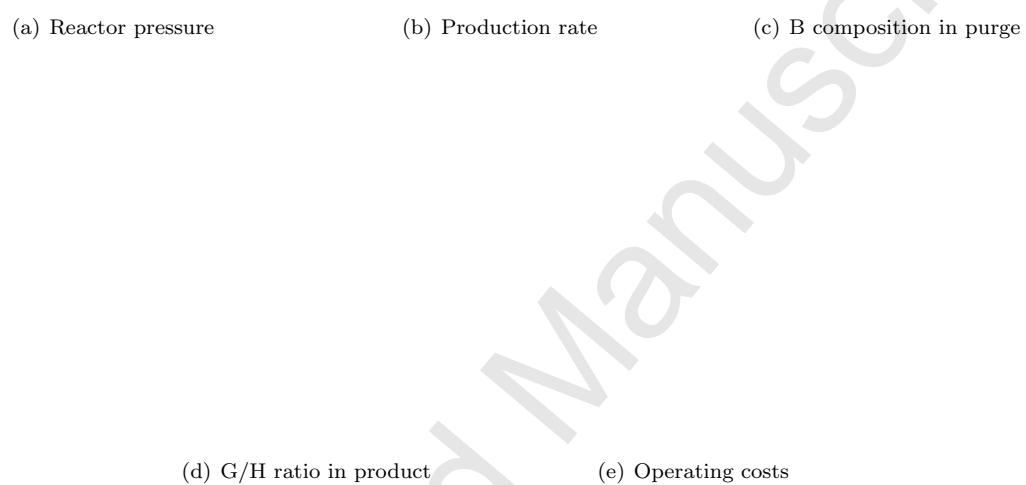


Figure 4: Normalized IAEs - Decentralized control structures

Figure 5: Pareto: Performance / Cost / Control structure size

Table 6: Hardware requirements

Approaches	Total Measurements	Control loops	Composition Measurements
McAvoy and Ye (1995)	22	22	4
Larsson et al. (2001)	22	17	3
Ricker (1996)	16	19	3
Banerjee and Arkun (1995)	15	16	5
Molina et al. (2011)	15	15	3

because they increase the controller's complexity without improving the performance. In this case, it is important to evaluate this complexity when several model parametrizations ( $\Gamma_i$ ) are used in the context of IMC controller design. Figure 6(c) shows the relationship between the optimal model components selection and the final number of controller elements. This evaluation allows to define some complexity degree in the controller design and decide about the implementation policy, i.e. decentralized or centralized. It is clear that any model parametrization above  $\Gamma_3$  introduces several elements in the controller design which leads to a complex and tedious design if a decentralized context is considered. For these control structures an implementation based on MPC policies is more convenient and feasible.

Indeed, eq. 16 summarizes four different model parametrizations which define four multivariable control structures for the TE process. The decentralized control policy is represented with  $\Gamma_d$  and it does not consider additional model components, i.e. a controller without interaction. Selection  $\Gamma_3$  is the sparse suboptimal control structure by adding only 3 model components, this provides a partially interacting controller with 4 additional off-diagonal elements and designed via IMC concepts. The third plant-wide control policy considers the optimal model parametrization  $\Gamma_{op}$  which generates a strongly interacting sparse controller with 20 additional off-diagonal components. Finally, the classic full control structure is considered where  $\Gamma_f$  gives a complete interaction degree. In the last two cases the controller is implemented via the MPC philosophy without constraints and the former ones are designed based on a decentralized PID approach.

$$\Gamma_d = \begin{bmatrix} 10000 \\ 01000 \\ 00100 \\ 00010 \\ 00001 \end{bmatrix}, \quad \Gamma_3 = \begin{bmatrix} 10001 \\ 01010 \\ 00100 \\ 00010 \\ 00101 \end{bmatrix}, \quad \Gamma_{op} = \begin{bmatrix} 10101 \\ 11111 \\ 11110 \\ 10111 \\ 10101 \end{bmatrix}, \quad \Gamma_f = \begin{bmatrix} 11111 \\ 11111 \\ 11111 \\ 11111 \\ 11111 \end{bmatrix} \quad (16)$$

Figure 7 shows the dynamic behavior of the closed-loop TE process (five main variables) when servo-regulatory control loops are implemented according to eq. 16. Similarly to Fig. 4, the normalized IAE index was selected here for simplifying the visualization. In fact, several simulation scenarios were suggested by accounting 4 reference changes ( $sp_1$  to  $sp_4$ ), 5 disturbances ( $d_1$  to  $d_5$ ) and 4 control structures (eq. 16), which summarize 36 dynamic responses.



(a) Optimization profiles

(b) NLE index comparison

(c) Model vs controller components

Figure 6: Controller interaction degree - NLE approach

All control structures have similar servo performances.  $\Gamma_{op}$  and  $\Gamma_f$ -based control structures improve the set point tracking for  $sp_2$  and  $sp_4$  (Figs. 7(b) and 7(d)), and  $\Gamma_d$  and  $\Gamma_3$ -based ones improve the  $sp_1$  and  $sp_3$  (Figs. 7(a) and 7(c)) cases. Servo interaction of sparse and full controllers can be observed clearly in Fig. 7(a) for the reactor pressure. Although the performance is affected, this degree of interaction is useful to give a suitable trade-off solution between servo-regulator behaviors. In fact, disturbance  $d_1$  is critical because it produces harmful excursions near the upper operation limit for the reactor pressure. It is clear that decentralized control ( $\Gamma_d$ ) and sparse suboptimal one ( $\Gamma_3$ ) are not the best structures for rejecting this effect (Fig. 7(a)). The optimal sparse multivariable control based on  $\Gamma_{op}$  has the best performance for the disturbance  $d_1$ , and it improves the behavior for the disturbances  $d_2$ ,  $d_4$ , and  $d_5$ . The production rate in Fig. 7(b) has virtually the same performance for all control structures. Similarly to Fig. 7(a), the composition of B in purge at Fig. 7(c) shows that the  $\Gamma_{op}$ -based control structure has significant improvements under

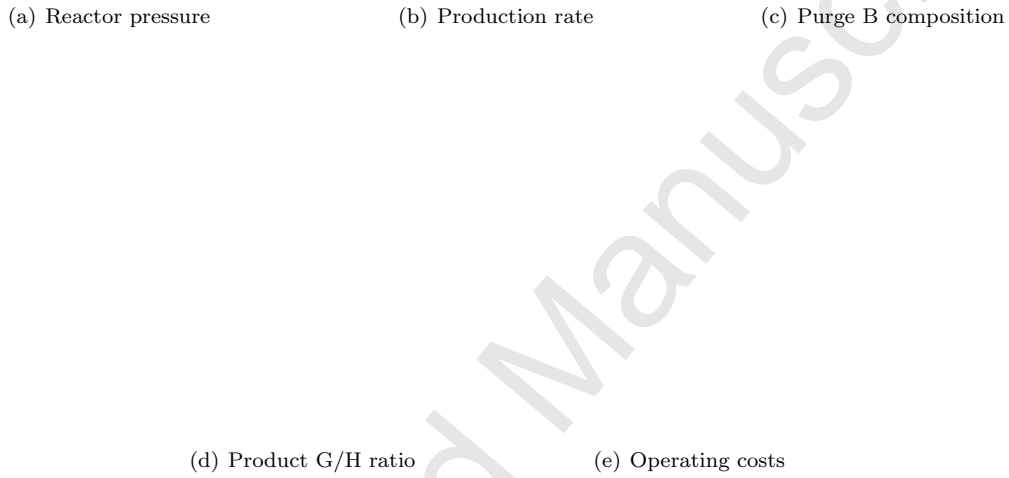


Figure 7: Normalized IAEs - Control structures based on  $\Gamma_d$ ,  $\Gamma_3$ ,  $\Gamma_{op}$ , and  $\Gamma_f$

disturbances  $d_1$ ,  $d_2$ ,  $d_4$ , and  $d_5$ . The G/H ratio in the product (Fig. 7(d)) displays similar normalized IAE values for all scenarios. In fact, optimal sparse control improves the  $sp_2$ ,  $sp_4$ ,  $d_2$ , and  $d_4$  cases only. On the other hand, Fig. 7(e) summarizes the operating costs for all the simulation instances evaluated here. It is clear that the  $\Gamma_{op}$ -based multivariable control structure presents good performance for scenarios  $sp_1$ ,  $sp_2$ ,  $d_1$ ,  $d_2$ , and  $d_5$ , maintaining similar values of the IAE index for the remaining cases.

Summarizing, sparse optimal plant-wide control based on  $\Gamma_{op}$  parametrization is selected here because it provides the best rejection performance for the challenging  $d_1$  scenario ( $\Gamma_d$  and  $\Gamma_3$  have the worst indices). Moreover, this control structure represents a good trade-off solution for the remaining simulation instances without degrading the operating costs. Figure 8 shows the TE process layout and both, the decentralized and centralized control policies. The  $\Gamma_d$ -based and  $\Gamma_{op}$ -based MPC control approaches are displayed in Fig. 8(a) and 8(b) respectively.

Finally, all the simulations and evaluations were performed in a PC with the following characteristics: Intel® Core™ i5 3.1 GHz, 3 GB RAM, Matlab® 6.5, and the Genetic Algorithms Toolbox for Matlab (Chipperfield et al., 1994). The main oversizing analysis (lines 1 to 17) in the Algorithm 1 applied to the

(a) Decentralized

(b) Centralized

Figure 8: TE process and control layouts

TE process takes  $\approx 11.1720$  seconds to give all the solutions displayed in Table 4.

## Conclusions

The extended MSD approach provides a complete evaluation of several multivariable control sizes to select the optimal CVs and MVs simultaneously, i.e. an overzising analysis. Thus, for example, the PWC problem suggested by the TE process with  $2^{16}$  potential solutions is reduced to testing the performance of 5 optimal decentralized control policies only. The optimal control structure from this procedure gives a more consistent framework for applying the NLE approach. In addition, a feasibility (implementation) analysis is given based on the number of controller components required to be tuned when this parametrization moves from diagonal to full selection passing through the optimal one. This information supports the decision about the sparse controller synthesis in the context of decentralized policies (PID) or centralized advanced structures (MPC). The overall procedure suggested here provides a systematic and generalized methodology for PWC design, minimizing the heuristic considerations. Additionally, the hypothesis “specific interaction via  $\Gamma$ -based model parameterizations could improve dynamic performance” was tested also in the MPC context. Future work will be focused on deepening this last topic.

Note that, for the case study addressed here, the extended MSD provides a  $(12 \times 12)$  (7 stabilizing + 5 servo/regulatory control loops) decentralized plant-wide control structure against the  $(15 \times 15)$  (7 stabilizing + 8 servo/regulatory control loops) one suggested in Molina et al. (2011). Furthermore, sparse optimal plant-wide control based on  $\Gamma_{op}$  parametrization and MPC is selected here because it provides the best rejection performance for the challenging  $d_1$  scenario and a good trade-off solution for the remaining simulation instances, without degrading the operating costs.

Table A.7: PI controller tuning based on  $\Gamma_d$  and  $\Gamma_3$  parametrizations

	$e_8$	$e_9$	$e_{10}$	$e_{11}$	$e_{12}$
$u_7$	$K_i^{11} = 0.2$ $\tau_i^{11} = 2$				
$u_2$		$K_i^{22} = -0.2$ $\tau_i^{22} = 10$			
$u_3$			$K_i^{33} = 1.5$ $\tau_i^{33} = 1$		
$u_5$				$K_i^{44} = -2.7$ $\tau_i^{44} = 2$	
$u_1$					$K_i^{55} = 9.5$ $\tau_i^{55} = 1$

Table A.8: Reduced and Normalized TE process model -  $\mathbf{G}_s(s)$  for  $q_a = 1$ 

	$u_7$	$u_2$	$u_3$	$u_5$	$u_1$
$y_8$	$\frac{-0.94}{(0.5s+1)}$	$\frac{0.13}{(s+1)}$	$\frac{0.36}{(0.5s+1)}$	$\frac{-0.12}{(s+1)}$	$\frac{-0.18}{(0.5s+1)}$
$y_9$	$\frac{-0.68}{(0.5s+1)}$	$\frac{(-10s+1)(-0.12)}{(10s+1)}$	$\frac{0.71}{(5s+1)}$	$\frac{-0.18}{(s+1)}$	$\frac{-0.31}{(0.5s+1)}$
$y_{10}$	$\frac{0.1}{(0.1s+1)}$	$\frac{8 \times 10^{-3}}{(s+1)}$	$\frac{0.6}{(0.5s+1)}$	$\frac{0.01}{(5s+1)}$	$\frac{-0.04}{(0.5s+1)}$
$y_{11}$	$\frac{0.96}{(0.5s+1)}$	$\frac{-0.05}{(0.5s+1)}$	$\frac{-0.10}{(0.5s+1)}$	$\frac{(-5s+1)(-0.21)}{(15s+1)}$	$\frac{0.24}{(s+1)}$
$y_{12}$	$\frac{-0.02}{(0.1s+1)}$	$\frac{-9 \times 10^{-3}}{(0.1s+1)}$	$\frac{-0.56}{(2s+1)}$	$\frac{(-20s+1)(-0.02)}{(15s+1)}$	$\frac{0.61}{(2s+1)}$

## Acknowledgements

The author thanks the financial support of CONICET (Consejo Nacional de Investigaciones Científicas y Técnicas) and ANPCYT (Agencia Nacional de Promoción Científica y Técnica) from Argentina. The author also acknowledges the support of the UTN-FRRo, CAPEG-CIFASIS and the invaluable collaboration of Dr. Alejandro Marchetti in the review process.

## Appendix A. Modeling and tuning

Table A.7 shows the tuning parameters used for implementing the decentralized ( $\Gamma_d$ ) and suboptimal sparse ( $\Gamma_3$ ) PI-based control structures. These settings are computed based on the IMC theory. The decentralized control uses the parameters placed diagonally in Table A.7. On the other hand, the  $\Gamma_3$ -based control structure adds four off-diagonal controller components, displayed with gray background, to the original decentralized one. Note that,  $e_i$  represents the tracking error between the CV  $y_i$  and its corresponding reference. The time and gain units are given according to the Table 1. Table A.8 shows the reduced and normalized TE process model used here for implementing the MPC structures based on  $\Gamma_{op}$  and  $\Gamma_f$ . For these cases the tuning parameters are: prediction horizon  $[h_w, h_p] = [5, 50]$  samples, control horizon  $h_u = 4$  samples, prediction error weights  $[q_1, q_2, q_3, q_4, q_5] = [1, 1, 1, 1, 1]$ ,  $\Delta u$  penalties  $[r_1, r_2, r_3, r_4, r_5] = [0.15, 0.1, 3, 0.05, 1.7]$ , and sampling time  $T_s = 5$  minutes.

## AppendixB. Comments on reliability

The extended MSD methodology presented here uses scalar functional costs based on a linearized model of the process under study. In this case, no dynamic implications are considered and only a steady-state model is required. Obviously, like any model-based control approach (RGA, IMC, MPC, etc), the performance and confidence of the extended MSD methodology are bounded by the validity zone of the process model itself. The model may not be valid due to multiple factors and basically if the process model is wrong the decisions obtained by any model-based methodology also will be unreliable.

The extended MSD methodology can deal with unscaling and unnormalized processes by augmenting the functional cost with diagonal weighting matrices as commented at the end of Sections 2.1 and 2.2. Eventually, the scaling procedure suggested by (Skogestad and Postlethwaite, 2005, Chap. 1) is very useful in this case. On the other hand, if the plant changes its operating point it is likely that the process model will be no longer valid due to changes in the inner relationships among variables. These changes will be severe or not depending on the nonlinearity of the process under study. A clear example of these effects are displayed in an earlier work of the author Molina et al. (2011) (preliminaries of the MSD approach) where control structures for the TE process were designed at two operating points, i.e. base and optimal cases. The final control policies are different, mainly, due to the severe changes in some steady-state gains of the process model which leads to different solutions from the SSD functional cost and RGA points of view. Recent analysis about the properties of the SSD-based optimization can be found in Zumoffen and Basualdo (2013).

Summarizing, modifications in the linearized steady-state process model can generate different solutions from the MSD point of view as well as input-output pairing problems. If these changes are severe, not necessarily in magnitude (Grosdidier et al., 1985; Skogetad and Morari, 1987), it is recommended a new steady-state model identification for reliable conclusions.

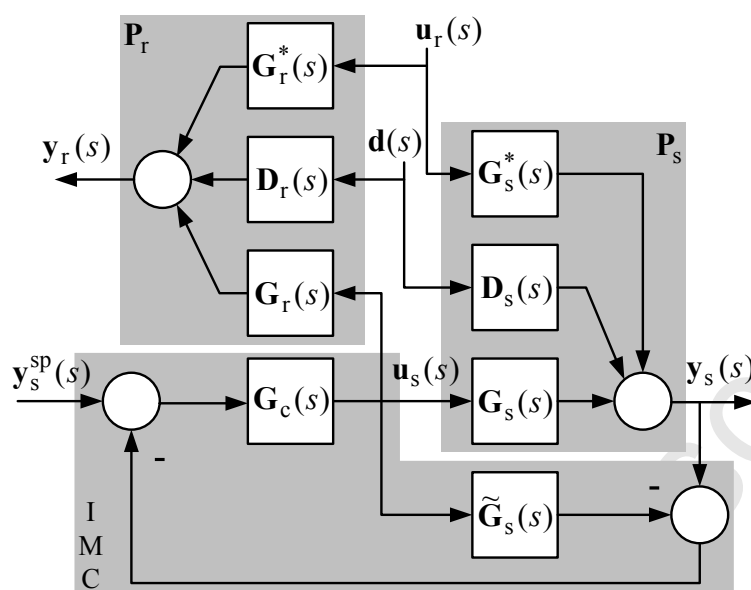
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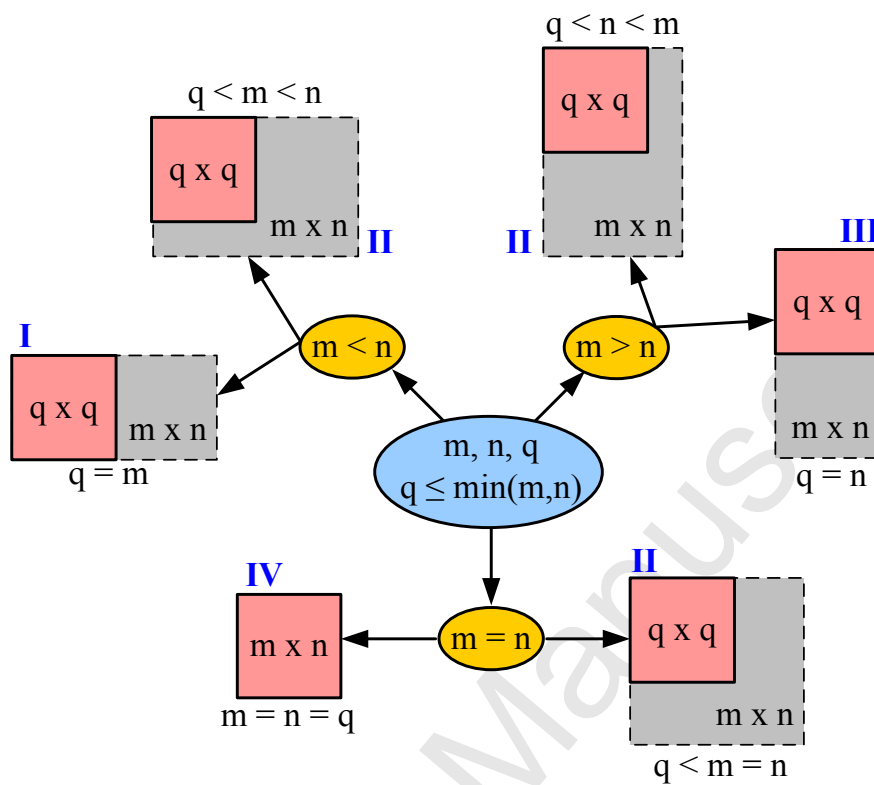
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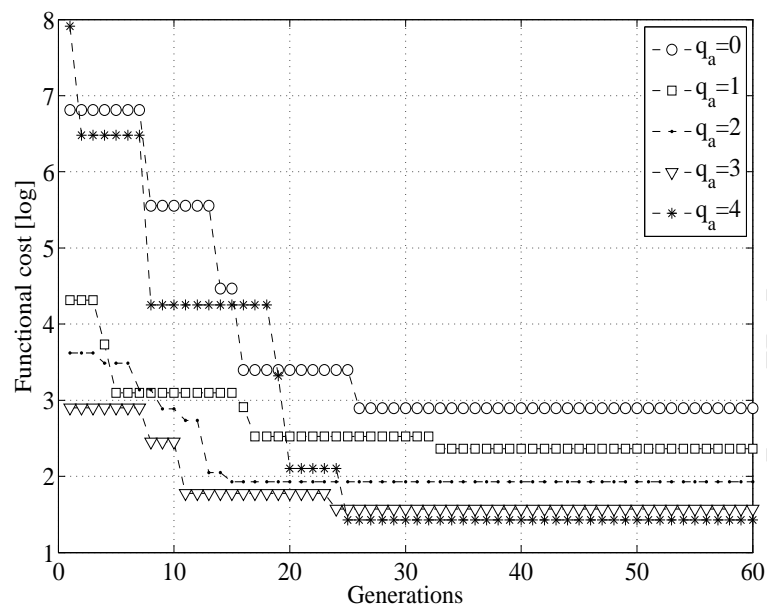
### Highlights

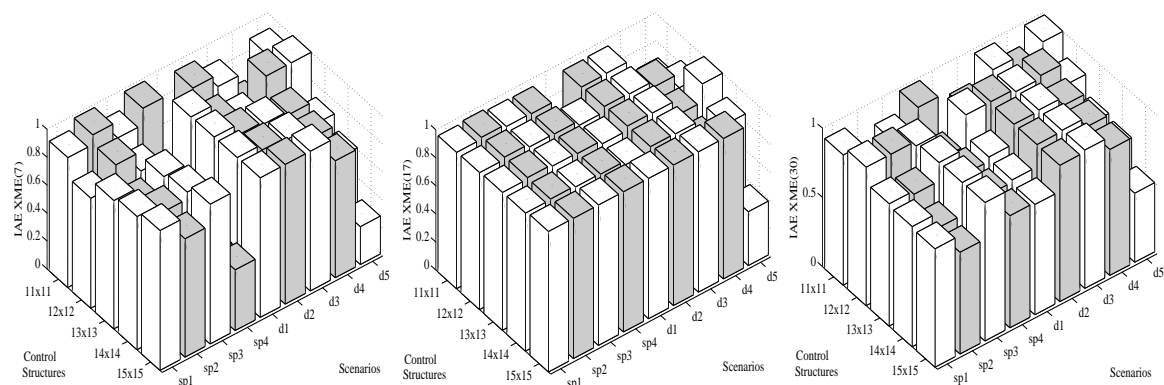
- Oversizing analysis for decentralized plant-wide control design.
- Problem reduction. It is required to test few optimal solutions only.
- Improvements based on sparse multivariable control structures.
- Implementation load analysis based on IMC theory.
- Sparse plant-wide control implementation based on MPC.
- Hardware requirements comparison with other multivariable control structures.







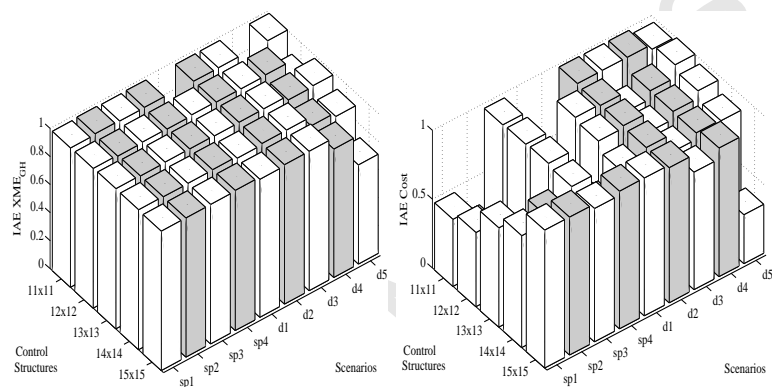


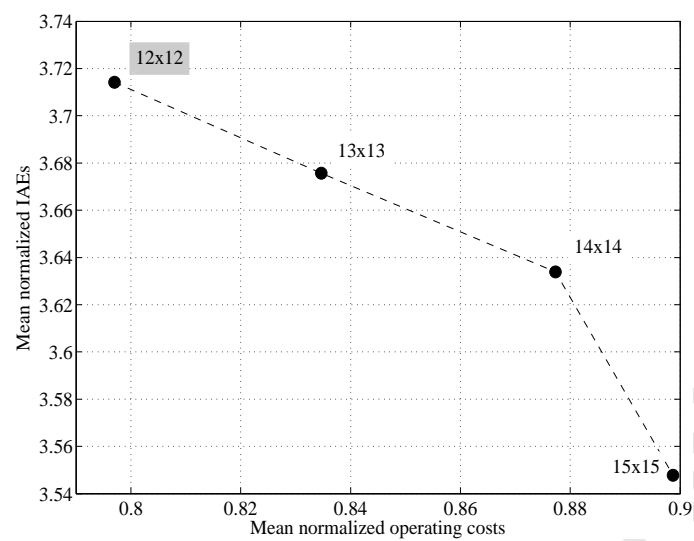


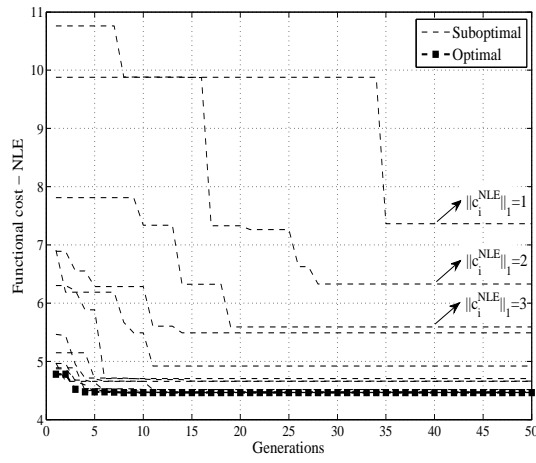
(a) Reactor pressure

(b) Production rate

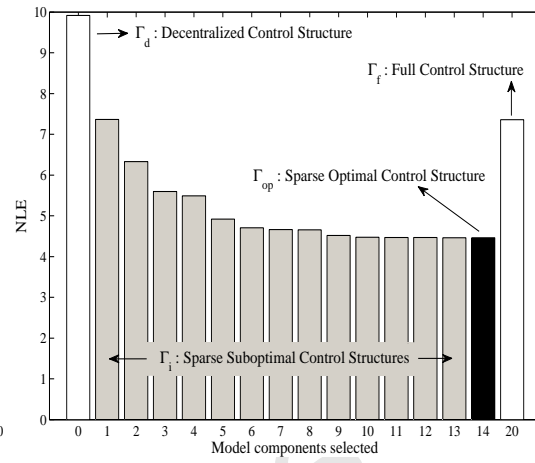
(c) B composition in purge



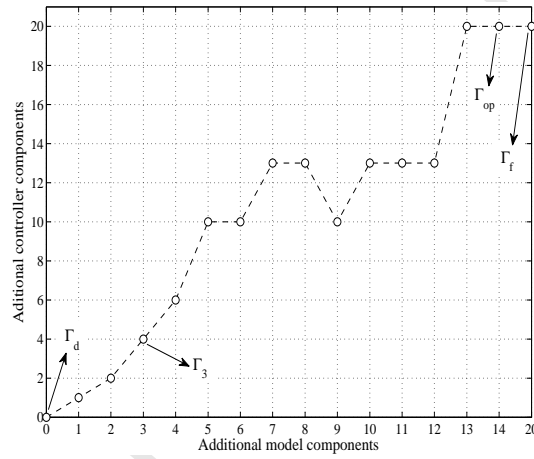


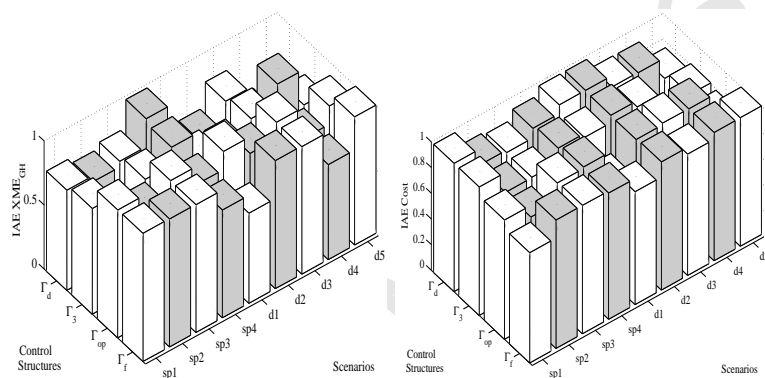
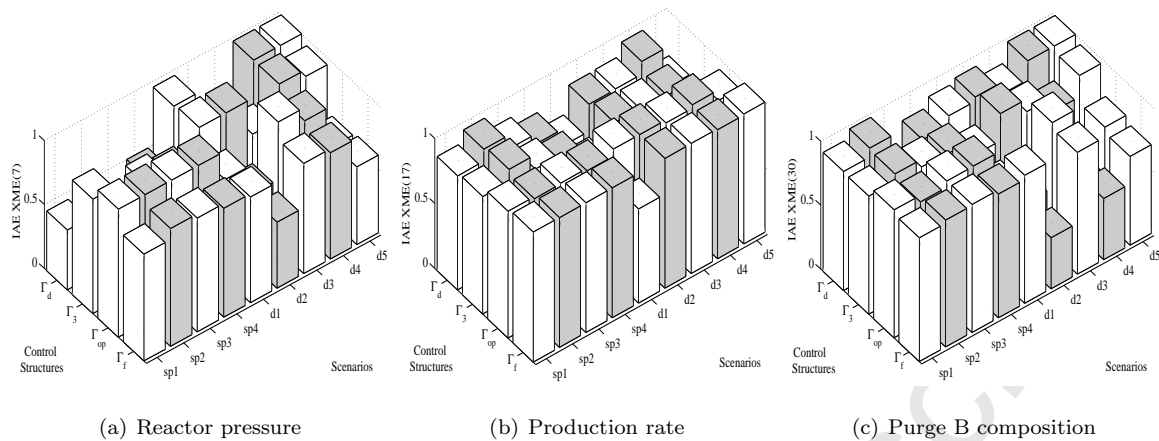


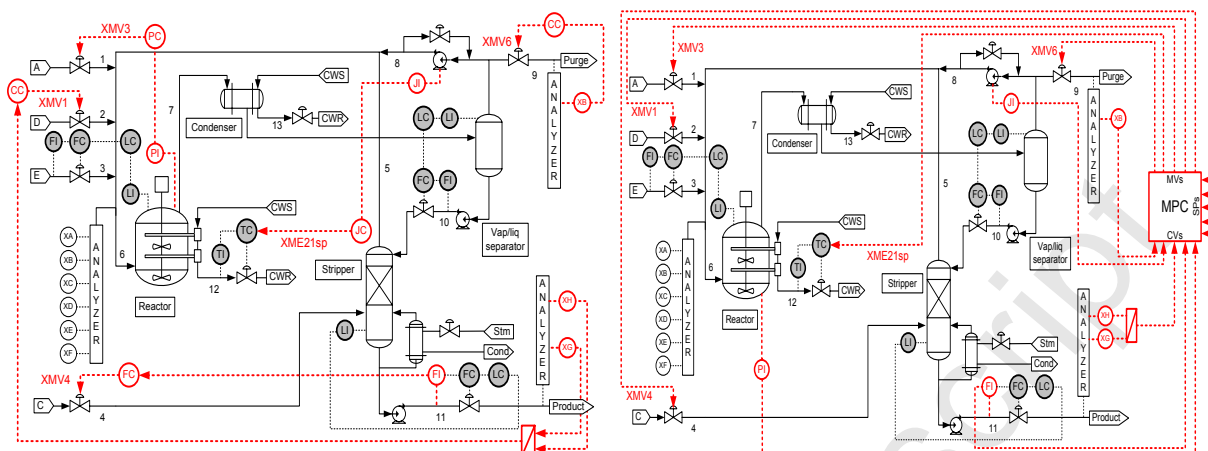
(a) Optimization profiles



(b) NLE index comparison







(a) Decentralized