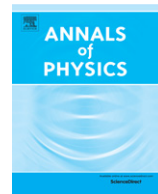




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Quantum histories without contrary inferences



Marcelo Losada^a, Roberto Laura^{b,*}

^a Instituto de Física Rosario, Pellegrini 250, 2000 Rosario, Argentina

^b Facultad de Ciencias Exactas, Ingeniería y Agrimensura and Instituto de Física Rosario, Pellegrini 250, 2000 Rosario, Argentina

HIGHLIGHTS

- We prove ordinary quantum mechanics has no contrary properties.
- Contrary properties in consistent histories are reviewed.
- We prove generalized contexts for quantum histories have no contrary properties.

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ABSTRACT

In the consistent histories formulation of quantum theory it was shown that it is possible to retrodict contrary properties. We show that this problem do not appear in our formalism of generalized contexts for quantum histories.

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1. Introduction

In the consistent histories formulation of quantum theory [1–5], the probabilistic predictions and retrodictions depend on the choice of a consistent set. It was shown that this freedom allows the formalism to retrodict two contrary properties [6]. This is not a problem for the defenders of the theory, because each retrodiction is obtained in a different consistent sets of histories, i.e. in different descriptions of the physical system not to be considered simultaneously [7,8]. However, this fact is considered by some authors as a serious failure of the theory of consistent histories [6,9,10].

* Corresponding author.

E-mail addresses: marcelolosada@yahoo.com (M. Losada), rlaura@fceia.unr.edu.ar (R. Laura).

We are going to analyze this problem with our formalism of generalized contexts [11,12], developed to deal with expressions involving properties at different times. This formalism is an alternative to the theory of consistent histories, which has proved to be useful for the time dependent description of the logic of quantum measurements [13], the decay processes [14] and the double slit experiment with and without measurement instruments [12]. More recently, we have discussed the relation of our formalism with the theory of consistent histories [15].

In Section 2 we show that there is no possibility for contrary inferences in ordinary quantum mechanics. In Section 3 we discuss the retrodiction of contrary properties in the theory of consistent histories. In Section 4 we show that there are no retrodiction of contrary properties in the formalism of generalized contexts. Some applications of this formalism are discussed in Section 5 and the main conclusions are given in Section 6.

2. Contrary properties in an ordinary quantum context

In quantum mechanics, a property p is represented by a projector Π_p in the Hilbert space \mathcal{H} , or alternatively by the corresponding Hilbert subspace $V_p = \Pi_p \mathcal{H}$. By definition [6], two quantum properties p and q are said to be *contrary* if they satisfy the order relation $p \leq \bar{q}$, which can also be expressed in terms of the inclusion of the corresponding Hilbert subspaces in the form

$$\Pi_p \mathcal{H} \subseteq (I - \Pi_q) \mathcal{H}. \quad (1)$$

The inclusion of subspaces is equivalent to the following relation between the corresponding projectors (see [16, Section 1.3])

$$\Pi_p(I - \Pi_q) = (I - \Pi_q)\Pi_p = \Pi_p,$$

from which we easily deduce that $\Pi_p \Pi_q = \Pi_q \Pi_p = 0$, that means the projectors Π_p and Π_q are orthogonal.

As they also commute, p and q are compatible properties. The projectors Π_p , Π_q and $\Pi_{\bar{p} \vee \bar{q}} = I - \Pi_p - \Pi_q$ form a projective decomposition of the Hilbert space, i.e. they are orthogonal and their sum is the identity operator. Therefore, the properties p , q and $\bar{p} \vee \bar{q}$ can be considered the atomic properties generating a *context* of quantum properties with well defined probabilities [12].

For any state of the system represented by a state operator ρ , the probability of any property p' in the context is obtained with the Born rule, i.e. $\text{Pr}_\rho(p') = \text{Tr}(\rho \Pi_{p'})$. For the atomic properties p , q and $\bar{p} \vee \bar{q}$ we obtain

$$\text{Pr}_\rho(p) + \text{Pr}_\rho(q) + \text{Pr}_\rho(\bar{p} \vee \bar{q}) = 1. \quad (2)$$

From this equation we easily deduce that if $\text{Pr}_\rho(p) = 1$ then $\text{Pr}_\rho(q) = 0$ and if $\text{Pr}_\rho(q) = 1$, $\text{Pr}_\rho(p) = 0$.

We conclude that in ordinary quantum mechanics it is impossible for any state ρ that two contrary properties p and q have probability equal to one. These results are the stochastic version of contrary proposition in ordinary logic. This can be interpreted by saying that whenever the property p (q) is true, the property q (p) is false. By the way, this result also justifies to have given the name *contrary* to quantum properties p and q satisfying Eq. (1).

More generally, it is easy to see that if p and q are contrary properties, it is not possible to have a state ρ and another property r for which

$$\text{Pr}_\rho(p|r) = 1, \quad \text{Pr}_\rho(q|r) = 1. \quad (3)$$

Taking into account that p , q and r should be represented by commuting projectors, so that the conditional probabilities be well defined, we would have

$$\text{Pr}_\rho(p|r) = \frac{\text{Tr}(\rho \Pi_p \Pi_r)}{\text{Tr}(\rho \Pi_r)} = \text{Tr}(\rho^* \Pi_p) = \text{Pr}_{\rho^*}(p), \quad \text{Pr}_\rho(q|r) = \text{Tr}(\rho^* \Pi_q) = \text{Pr}_{\rho^*}(q),$$

where $\rho^* \equiv \frac{\Pi_r \rho \Pi_r}{\text{Tr}(\Pi_r \rho \Pi_r)}$. Taking into account Eq. (2) with $\rho = \rho^*$ we conclude that there are no state ρ and property r for which Eqs. (3) can be both valid.

3. Contrary properties in the theory of consistent histories

In the theory of consistent histories n different contexts of properties at each time t_j ($j = 1, \dots, n$), satisfying a state dependent consistency condition, can be used to define a *family of consistent histories*, i.e. a set of n times sequences of properties with well defined probabilities [1–5]. According to this theory, each possible family of consistent histories is an equally valid description of the quantum system. In general it is not possible to include two different families in a single larger one. Different families of this kind are complementary descriptions of the system, which the theory excludes to be considered simultaneously.

Adrian Kent [6] proved that it is possible the retrodiction of contrary properties in different families of consistent histories.

J.B. Hartle [7] developed an example of contrary inferences, which is also a special case of the example 1 given by Adrian Kent [6]. A quantum system is in a state represented by the vector $|\Psi\rangle = \frac{1}{\sqrt{3}}(|A\rangle + |B\rangle + |C\rangle)$ at time t_0 , where $|A\rangle$, $|B\rangle$ and $|C\rangle$ are three orthogonal and normalized vectors of a three-dimensional Hilbert space. For simplicity, the Hamiltonian is chosen to be zero. The property Φ , represented by the projector $P_\Phi = |\Phi\rangle\langle\Phi|$, where $|\Phi\rangle = \frac{1}{\sqrt{3}}(|A\rangle + |B\rangle - |C\rangle)$, is considered at the time $t_2 > t_0$.

For the system in the state $|\Psi\rangle$ at the initial time t_0 , and having the property Φ at the later time t_2 , it is asked whether the system has the property A represented by $P_A = |A\rangle\langle A|$ at an intermediate time t_1 ($t_0 < t_1 < t_2$).

A suitable family of two-times consistent histories (i.e. a family of histories verifying consistency conditions) is given by considering the properties generated by the projectors P_A and $P_{\bar{A}} = I - P_A$ at time t_1 and by the projectors P_Φ and $P_{\bar{\Phi}} = I - P_\Phi$ at time t_2 . Within this family of consistent histories the following results are obtained

$$\Pr_\Psi(A, t_1|\Phi, t_2) = 1, \quad \Pr_\Psi(\bar{A}, t_1|\Phi, t_2) = 0, \quad (4)$$

stating that the property Φ at time t_2 implies the property A at the previous time $t_1 < t_2$.

Using a different family of consistent histories, including the properties generated by the projectors $P_B = |B\rangle\langle B|$ and $P_{\bar{B}} = I - P_B$ at time t_1 and by the projectors P_Φ and $P_{\bar{\Phi}} = I - P_\Phi$ at time t_2 , the following results are obtained

$$\Pr_\Psi(B, t_1|\Phi, t_2) = 1, \quad \Pr_\Psi(\bar{B}, t_1|\Phi, t_2) = 0, \quad (5)$$

giving the retrodiction of property B at time t_1 conditional to property Φ at time $t_2 > t_1$.

From the point of view of the theory of consistent histories Eqs. (4) and (5) cannot be interpreted as the retrodiction of two contrary properties A and B , because they were obtained from two different and complementary descriptions, which cannot be included in a single consistent family [7,8]. However, some authors have considered the results given in Eqs. (4) and (5) as a serious objection for the internal consistency of the theory of consistent histories [6,9,10].

In the following section we will analyze this example from the point of view of our generalized contexts formalism. We will also present a general proof of the absence of contrary inferences in this formalism.

4. Contrary properties in the formalism of generalized contexts

In this section contrary quantum properties will be considered from the point of view of our formalism of generalized contexts. We start with a brief description of the formalism, which was presented in full details in our previous papers [11,12].

Quantum mechanics do not give a meaning to the joint probability distribution of observables whose operators do not commute. It can only deal with a set of properties belonging to a context.

A context of properties \mathcal{C}_i at time t_i is obtained starting from a set of atomic properties $p_i^{k_i}$ ($k_i \in \sigma_i$) represented by projectors $\Pi_i^{k_i}$ corresponding to a projective decomposition of the Hilbert space \mathcal{H} ,

i.e. verifying

$$\sum_{k_i \in \sigma_i} \Pi_i^{k_i} = I, \quad \Pi_i^{k_i} \Pi_i^{k'_i} = \delta_{k_i k'_i} \Pi_i^{k_i}. \quad (6)$$

Any property p of the context \mathcal{C}_i is represented by a sum of the projectors of the projective decomposition,

$$\Pi_p = \sum_{k_i \in \sigma_p} \Pi_i^{k_i}, \quad \sigma_p \subset \sigma_i. \quad (7)$$

The context \mathcal{C}_i is an orthocomplemented distributive lattice, with the complement \bar{p} of a property p defined by $\Pi_{\bar{p}} \equiv I - \Pi_p$ and the order relation $p \leq p'$ defined by $\Pi_p \mathcal{H} \subseteq \Pi_{p'} \mathcal{H}$.

A well defined probability (i.e. additive, non negative and normalized) is defined by the Born rule $\text{Pr}_{t_i}(p) \equiv \text{Tr}(\rho_{t_i} \Pi_p)$ on the context \mathcal{C}_i . In Heisenberg representation, the probability of a property p at time t_i can be written in terms of the state at a reference time t_0 , i.e.

$$\text{Pr}_{t_i}(p) = \text{Tr}(\rho_{t_0} \Pi_{p,0}), \quad \Pi_{p,0} \equiv U(t_0, t_i) \Pi_p U(t_i, t_0), \quad U(t_i, t_0) = e^{-\frac{i}{\hbar} H(t_i - t_0)}. \quad (8)$$

Taking into account Eqs. (7) and (8), the Heisenberg representation of the property p of the context \mathcal{C}_i at time t_i is given by

$$\Pi_{p,0} = \sum_{k_i \in \sigma_p} \Pi_{i,0}^{k_i}, \quad (9)$$

where the projectors $\Pi_{i,0}^{k_i} = U(t_0, t_i) \Pi_i^{k_i} U(t_i, t_0)$ represent the time translation of the atomic properties $p_i^{k_i}$ from time t_i to the time t_0 . The projectors $\Pi_{i,0}^{k_i}$ also satisfy Eqs. (6).

The Heisenberg representation of the context \mathcal{C}_i at time t_i suggests a generalization of quantum mechanics for including the joint probability of properties belonging to different contexts $\mathcal{C}_1, \dots, \mathcal{C}_i, \dots, \mathcal{C}_n$ corresponding to n different times $t_1 < \dots < t_i < \dots < t_n$.

By extending what is a common assumption in ordinary quantum mechanics, we proposed to give a meaning to the joint probability of properties at different times if they correspond to commuting projectors in Heisenberg representation. This will be the case if the atomic properties generating each of the n contexts are represented by projectors satisfying

$$[\Pi_{i,0}^{k_i}, \Pi_{j,0}^{k_j}] = 0, \quad i, j = 1, \dots, n, \quad k_i \in \sigma_i, \quad k_j \in \sigma_j.$$

If these projectors commute the projectors $\Pi_0^{\mathbf{k}} \equiv \Pi_{1,0}^{k_1} \dots \Pi_{i,0}^{k_i} \dots \Pi_{n,0}^{k_n}$, with $\mathbf{k} = (k_1, \dots, k_n)$ and $k_i \in \sigma_i$, form a projective decomposition of the Hilbert space \mathcal{H} , as they satisfy

$$\sum_{\mathbf{k}} \Pi_0^{\mathbf{k}} = I, \quad \Pi_0^{\mathbf{k}} \Pi_0^{\mathbf{k}'} = \delta_{\mathbf{k}\mathbf{k}'} \Pi_0^{\mathbf{k}}, \quad \mathbf{k}, \mathbf{k}' \in \sigma_1 \times \dots \times \sigma_n.$$

In our formalism we postulate that an expression of the form “property $p_1^{k_1}$ at time t_1 and ... and $p_n^{k_n}$ at time t_n ” is an atomic generalized property $\mathbf{p}^{\mathbf{k}}$ with the Heisenberg representation given by the projector $\Pi_0^{\mathbf{k}}$. A generalized context is defined by all the generalized properties \mathbf{p} having a Heisenberg representation given by an arbitrary sum of the projectors $\Pi_0^{\mathbf{k}}$, i.e.

$$\Pi_{\mathbf{p}} = \sum_{\mathbf{k} \in \sigma_{\mathbf{p}}} \Pi_0^{\mathbf{k}},$$

where $\sigma_{\mathbf{p}}$ is a subset of $\sigma_1 \times \dots \times \sigma_n$. The generalized context is an orthocomplemented distributive lattice, with the complement $\bar{\mathbf{p}}$ of \mathbf{p} defined by $\Pi_{\bar{\mathbf{p}}} \equiv I - \Pi_{\mathbf{p}}$, and the order relation $\mathbf{p} \leq \mathbf{p}'$ defined by the inclusion of the corresponding Hilbert subspaces ($\Pi_{\mathbf{p}} \mathcal{H} \subseteq \Pi_{\mathbf{p}'} \mathcal{H}$).

An extension of the Born rule provides a definition of an additive, non negative and normalized probability on the generalized context, given by

$$\text{Pr}(\mathbf{p}) \equiv \text{Tr}(\rho_{t_0} \Pi_{\mathbf{p}}). \quad (10)$$

We can now show that the contrary inferences of the example developed by J.B. Hartle [7] and presented in the previous section do not appear with our formalism. The projectors $P_A = |A\rangle\langle A|$ and $P_\Phi = |\Phi\rangle\langle\Phi|$ do not commute, there is no description of the quantum system involving properties A at time t_1 and Φ at time t_2 , and therefore the conditional probabilities appearing in Eqs. (4) are not even defined. The conditional probabilities appearing in Eqs. (5) are also not defined in our formalism, because the projectors $P_B = |B\rangle\langle B|$ and P_Φ do not commute. The negative results of this example can be generalized. In what follows, we prove for an arbitrary quantum system that our formalism of generalized contexts do not allow for the retrodiction of contrary properties.

For the general case, we consider a state ρ_{t_0} at time t_0 , two contrary properties p and q at time $t_1 > t_0$ and another property r at time $t_2 > t_1$, and we search for the possibility to obtain for both conditional probabilities the results $\text{Pr}_{\rho_{t_0}}(p, t_1|r, t_2) = 1$ and $\text{Pr}_{\rho_{t_0}}(q, t_1|r, t_2) = 1$.

The projectors Π_p and $\Pi_{p,0} = U(t_0, t_1)\Pi_p U(t_1, t_0)$ are respectively Schrödinger and Heisenberg representations of the property p at time t_1 . Analogously, Π_q and $\Pi_{q,0} = U(t_0, t_1)\Pi_q U(t_1, t_0)$ are representations of the property q at time t_1 . Moreover, Π_r and $\Pi_{r,0} = U(t_0, t_2)\Pi_r U(t_2, t_0)$ are representations of the property r at time t_2 .

The conditional probabilities are meaningful in our formalism if the following compatibility conditions are satisfied

$$[\Pi_{p,0}, \Pi_{r,0}] = 0, \quad [\Pi_{q,0}, \Pi_{r,0}] = 0, \quad (11)$$

while the contrary properties p and q are represented by orthogonal projectors, and therefore

$$[\Pi_{p,0}, \Pi_{q,0}] = 0. \quad (12)$$

The commutation relations given in Eqs. (11) and (12) are the compatibility conditions required to consider a two times generalized context including the contrary properties p and q at time t_1 and property r at time t_2 , in which both conditional probabilities $\text{Pr}_{\rho_{t_0}}(p, t_1|r, t_2)$ and $\text{Pr}_{\rho_{t_0}}(q, t_1|r, t_2)$ are meaningful.

In our formalism, the required retrodictions would have the explicit forms

$$\text{Pr}_{\rho_{t_0}}(p, t_1|r, t_2) = \frac{\text{Tr}(\rho_{t_0}\Pi_{p,0}\Pi_{r,0})}{\text{Tr}(\rho_{t_0}\Pi_{r,0})} = 1, \quad \text{Pr}_{\rho_{t_0}}(q, t_1|r, t_2) = \frac{\text{Tr}(\rho_{t_0}\Pi_{q,0}\Pi_{r,0})}{\text{Tr}(\rho_{t_0}\Pi_{r,0})} = 1.$$

Taking into account the commutation relations given in Eqs. (11), the previous equations are equivalent to

$$\text{Tr}(\rho_{t_0}^* \Pi_{p,0}) = 1, \quad \text{Tr}(\rho_{t_0}^* \Pi_{q,0}) = 1, \quad \rho_{t_0}^* \equiv \frac{\Pi_{r,0}\rho_{t_0}\Pi_{r,0}}{\text{Tr}(\Pi_{r,0}\rho_{t_0}\Pi_{r,0})}. \quad (13)$$

As $\Pi_{p,0}$ and $\Pi_{q,0}$ represent contrary properties at the same time t_0 , we can follow the arguments given at the end of Section 2 to show that there is no $\rho_{t_0}^*$ for which both equations given in Eqs. (13) can be valid. Therefore we conclude that the problem of retrodiction of contrary properties do not arise in our formalism of generalized contexts for quantum histories.

5. On the usefulness of the formalism of generalized contexts

In axiomatic quantum theories the states are considered as functionals acting on the space of observables and therefore appearing after the observables in a somehow subordinate position [17,18]. Quantum histories play the role of the observables of ordinary quantum theory and it seems reasonable that the allowed sets of histories satisfy state independent conditions. The consistency conditions of the theory of consistent histories produce state dependent families of consistent histories. On the contrary, the compatibility conditions of the generalized context formalism are state independent.

The state independent compatibility conditions of the generalized context formalism produce an important difference with respect to the theory of consistent histories. Each quantum history has a Heisenberg representation given by a projection operator and each valid set of quantum histories is generated by a projective decomposition of the Hilbert space. As a consequence, a generalized context of quantum histories has the logical structure of a distributive orthocomplemented lattice of subspaces of the Hilbert space, i.e. the same logical structure of the quantum properties of an ordinary

context. It is because of this logical structure that in our formalism there is no place for the retrodiction of contrary properties, as we already found in the example analyzed in the previous section.

In the opinion of some authors the theory of consistent histories allows too many histories and some of them are difficult to interpret [19,20]. We have recently proved that our compatibility conditions, given by the commutation of the projectors representing the properties translated to a common time, are equivalent to the consistency conditions imposed on all possible states of the system [15]. Therefore, the formalism of generalized contexts imposes more restrictions than the theory of consistent histories for the valid families of quantum histories and it allows less families of histories. This can be an advantage of our formalism.

However, although the absence of contrary inferences may be considered as an advantage of the generalized context formalism, we should consider the possibility that some physically relevant families of consistent histories may be eliminated by it. We do not have yet a definite answer to this question, but we present in what follows a brief description of two physically relevant applications of our formalism, presented in our previous works [12,13].

5.1. Quantum measurements

A non ideal measurement of an observable Q of a system S is an interaction during the time interval (t_1, t_2) of the measured system S with the instrument A . It is represented in the Hilbert space $\mathcal{H}_S \otimes \mathcal{H}_A$ by a unitary transformation $U = U(t_2, t_1)$ satisfying

$$|q_i\rangle|a_0\rangle \xrightarrow{U} |\phi_i\rangle|a_i\rangle,$$

where $|q_i\rangle$ is an eigenvector of the observable Q with eigenvalue q_i , $|a_0\rangle$ is the initial reference state of the instrument and $|a_i\rangle$ is a state of the instrument with the value a_i of the pointer variable. The state of the composed system at time t_1 is $|\Psi_1\rangle = |\varphi_1\rangle|a_0\rangle$, where $|\varphi_1\rangle = \sum_i c_i |q_i\rangle$.

The formalism of generalized contexts can provide a description of the process involving the possible values q_i of the observable Q of system S at time t_1 and the possible pointer values a_j of the instrument A at time t_2 . These properties are represented by the projectors

$$\Pi_{q_i} = |q_i\rangle\langle q_i| \otimes I_A, \quad \Pi_{a_j} = I_S \otimes |a_j\rangle\langle a_j|,$$

and satisfy the compatibility conditions when translated to the common time t_1 , i.e. $[U^{-1}\Pi_{a_j}U; \Pi_{q_i}] = 0$. Therefore, the generalized context formalism allows to compute the conditional probability

$$\Pr(q_i, t_1 | a_j, t_2) = \frac{\Pr((q_i, t_1) \wedge (a_j, t_2))}{\Pr(a_j, t_2)} = \frac{\langle \Psi_1 | U^{-1} \Pi_{a_j} U \Pi_{q_i} | \Psi_1 \rangle}{\langle \Psi_1 | U^{-1} \Pi_{a_j} U | \Psi_1 \rangle} = \delta_{ij}.$$

For the composed system prepared in the state $|\Psi_1\rangle = |\varphi_1\rangle|a_0\rangle$, this result can be interpreted by saying that if the instrument pointer variable has the value a_j after the measurement, the system S had the property $Q = q_j$ before the measurement. More details of the application of this formalism to the logic of quantum measurements can be found in Ref. [13].

5.2. The double-slit experiment

The generalized context formalism was used to describe the double-slit experiment [12]. A particle in a state represented by a wave packet, coming from left to right, passes through a double-slit at time t_1 . The particle reaches a vertical zone located to the right of the double slit at a later time t_2 .

We can attempt to give a description of the process involving through which slit has passed the particle at time t_1 that appear to be in some region of the vertical zone at a later time t_2 . As we assume no measurement instruments, the Hilbert space to describe this process is the Hilbert space of the particle ($\mathcal{H} = \mathcal{H}_{\text{particle}}$).

The projectors representing the particle located in each slit at time t_1 are

$$\Pi_{t_1}^u \equiv \int_{V^u} d^3r |\vec{r}\rangle\langle\vec{r}|, \quad \Pi_{t_1}^d \equiv \int_{V^d} d^3r |\vec{r}\rangle\langle\vec{r}|, \quad (14)$$

where V^u (V^d) is the volume of the upper (lower) slit, and $|\bar{r}\rangle$ is a generalized eigenvector of the position operator of the particle with generalized eigenvalue \bar{r} .

For the later time t_2 , the projectors corresponding to the particle in small regions of the vertical zone to the right of the double slit are

$$\Pi_{t_2}^n \equiv \int_{V^n} d^3r |\bar{r}\rangle \langle \bar{r}|, \quad (15)$$

where V^n is the volume of the small region of the vertical zone labeled by the index n .

We proved in Ref. [12] that the properties represented by the projectors (14) and (15), translated to the common time t_1 , are represented by non commuting projectors, i.e.

$$[\Pi_{t_1}^u; U^{-1} \Pi_{t_2}^n U] \neq 0, \quad [\Pi_{t_1}^d; U^{-1} \Pi_{t_2}^n U] \neq 0,$$

where $U = U(t_2, t_1) = e^{-iH_0(t_2-t_1)/\hbar}$ is the unitary evolution generated by the free particle Hamiltonian, $H_0 = p^2/2m$.

Therefore, our formalism shows the well known fact that it is not possible a description of the quantum process suitable to talk about which slit passed the particle before reaching a region of the vertical zone.

We also considered a modified double-slit experiment with an ideal measurement instrument A located in the slits zone, interacting with the particle during the short time interval $[t_1, t_1 + \Delta_1]$, and with its pointer variable indicating a_u (a_d) if the particle is detected passing through the upper (lower) slit. A second ideal measurement instrument B is located in the vertical zone to the right of the double-slit, interacting with the particle in the short time interval $[t_2, t_2 + \Delta_2]$, and with a pointer variable indicating b_n if the particle is detected in the small zone labeled by the index n of the vertical zone. The Hilbert space for the description of this process is the tensor product of the Hilbert space of the particle and the two Hilbert spaces of the detectors, i.e. $\mathcal{H} = \mathcal{H}_{particle} \otimes \mathcal{H}_A \otimes \mathcal{H}_B$. The unitary time evolution is assumed to be dominated by the interaction between the particle and instrument A in the short time interval $[t_1, t_1 + \Delta_1]$, by the free evolution in the time interval $[t_1 + \Delta_1, t_2]$ and by the interaction of the particle and instrument B in the time interval $[t_2, t_2 + \Delta_2]$.

The possible pointer indications of the instrument A at time $t_1 + \Delta_1$ are represented by the projectors

$$\Pi_{t_1+\Delta_1}^{a_u} \equiv I_{particle} \otimes |a_u\rangle \langle a_u| \otimes I_B, \quad \Pi_{t_1+\Delta_1}^{a_d} \equiv I_{particle} \otimes |a_d\rangle \langle a_d| \otimes I_B, \quad (16)$$

and the possible indications of instrument B at time $t_2 + \Delta_2$ are represented by the projectors

$$\Pi_{t_2+\Delta_2}^{b_n} \equiv I_{particle} \otimes I_A \otimes |b_n\rangle \langle b_n|. \quad (17)$$

We proved in Ref. [12] that the properties corresponding to the projectors (16) and (17), translated to a common time, are represented by commuting projectors. Therefore, the generalized context formalism allows a history of the composed system involving through which slit the particle is measured to pass and in which region of the vertical plane it is measured to be at a later time. The corresponding conditional probabilities give the expected non interference pattern.

6. Conclusions

In ordinary quantum mechanics, contrary properties are represented by orthogonal subspaces of the Hilbert space associated with the physical system. In Section 2, we proved that given two contrary properties p and q , there is no state ρ and property r for which the probability of p conditional to r and the probability of q conditional to r can be both equal to one. Therefore, there is no possibility of contrary inferences in ordinary quantum mechanics. This result corresponds to a state and properties considered at a single time.

As we discussed in Section 3, this is not the case for the theory of consistent histories, where a state at time t_0 , two contrary properties p and q at time $t_1 > t_0$ and another property r at time $t_2 > t_1$ can be found in such a way that the probability of p conditional to r and the probability of q conditional to r are both equal to one. Although these conditional probabilities are defined in different sets of

consistent histories [7,8], some authors have considered this fact as a serious problem for the logical consistency of the theory [6,9,10].

The main purpose of this paper was to analyze the problem of contrary inferences in the framework of our formalism of generalized contexts. In this formalism, as it was explained in Section 4, ordinary contexts of properties at different times can be used to obtain a valid set of quantum histories if they satisfy a compatibility condition. This condition is given by the commutation of the projectors corresponding to the time translation of the properties to a single common time. These compatibility conditions are state independent, an important difference with respect to the state dependent consistency conditions of the theory of consistent histories. Each quantum history has a Heisenberg representation given by a projection operator and each valid set of quantum histories is generated by a projective decomposition of the Hilbert space. As a consequence, a generalized context of quantum histories has the logical structure of a distributive orthocomplemented lattice of subspaces of the Hilbert space, i.e. the same logical structure of the quantum properties of an ordinary context. It is because of this logical structure that in our formalism there is no place for the retrodiction of contrary properties.

The formalism of generalized contexts imposes more restrictions than the theory of consistent histories for the valid families of quantum histories and it allows less families of histories. Although the absence of contrary inferences can be considered an advantage of this formalism, we should also consider the possibility that some physically relevant families of consistent histories may be eliminated by it. We do not have yet a definite answer to this question, but we can mention the results we obtained with physically relevant applications [14,13,12], some of which have been briefly discussed in Section 5. These partial results encourages us to continue our future research considering more applications of the formalism of generalized contexts.

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