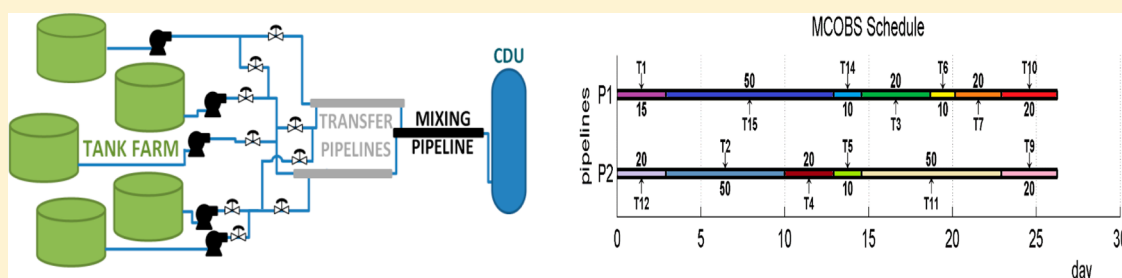


Scheduling Multipipeline Blending Systems Supplying Feedstocks to Crude Oil Distillation Columns

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^S Supporting Information



ABSTRACT: Different types of crude oils, including heavy sour crudes, are usually processed in oil refineries. They are suitably mixed in multipipeline crude oil blending systems (MCOBS) to get qualified feedstocks for the crude distillation units (CDUs) that separate them into various fractions or cuts. Current research on the scheduling of such refinery operations seeks to minimize the total operating cost while limiting the concentration of some impurities in the feedstock. However, an essential property like the feedstock composition usually approximated by the true boiling point (TBP) distribution curve is often ignored. In fact, one of the major goals of the blending process is to supply feedstocks to the CDUs that consistently produce the desired proportions of final cuts to match refined product demands. This work introduces a novel slot-based mixed-integer nonlinear programming (MINLP) formulation for the MCOBS scheduling problem that explicitly accounts for the TBP curves of the crude oils to get a series of suitable and stable feedstocks for the CDUs. An MILP-NLP solution strategy is proposed to find near-optimal schedules in reasonable CPU times.

1. INTRODUCTION

The purpose of the oil refining industry is to transform crude oil into a number of final products. Leading petroleum refiners have revamped their assets to process a wide variety of crude oils, including heavy sour crudes, to maximize margins while meeting final product demands. The value of a particular crude oil not only depends on its quality but also on whether by mixing with other crude oils the refiner can economically obtain a refined product pattern that matches the market demand. Crude distillation units (CDUs) are the first processing units in any refinery separating the crude oil into several fractions or final cuts, namely, naphtha, kerosene, jet fuel, and light and heavy diesel. Because different types of crude oils are processed, blending them in tanks or mixing pipelines is necessary to meet the quality specifications of the feedstock for the CDUs established at the planning level. A variety of properties like the yields of premium products, the true boiling point distillation curve, and the concentrations of trace elements such as sulfur, nitrogen, and heavy metals is used in practice to characterize the mixed feedstock (Reddy et al.,¹ Li et al.,² Bai et al.^{3,4}). For operational reasons, the proportion of lighter components in the feedstock should belong to some specific interval to get a suitable reflux flow and avoid pressure control difficulties in the CDUs. Moreover, the concentration of impurities should be kept very low.

In coastal and inland oil refineries, a substantial number of charging tanks supply feedstock to crude distillation units. The blending process can occur either in the charging tanks receiving different crude oils from very large crude carriers (VLCC) or storage tanks located at the port terminal or by mixing flows of crude blends coming from two or more charging tanks in the mixing pipeline delivering the feedstock to some particular CDU. Often, oil refiners use dedicated charging tanks storing a single type of crude oil. In such cases, no blending of crudes can occur, and the crude composition in each tank remains almost constant over the planning horizon. Other refiners handle nondedicated charging tanks receiving different crude oils, and consequently, the composition of the crude mix in the tank varies with time. In fact, depending on the refiner, alternative operational rules are adopted as next described: (i) Charging tanks are devoted to a single type of crude oil or instead receive and blend different crude oils. (ii) Any charging tank can at most feed a single CDU or it simultaneously provides feedstock to different CDUs over the scheduling horizon. (iii) The feedstock for a crude distillation unit should always be supplied by only one charging tank that

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may change with time or it is the result of mixing flows of crudes coming from several charging tanks. (iv) When the mixing option is allowed to get a qualified feedstock for the CDU, the number of charging tanks simultaneously acting as sources are always the same or it may change along the scheduling horizon. (v) The transfer of crude oil from a charging tank to some CDU can be interrupted at any time or it should be continued until the tank is empty. (vi) Just the maximum concentration of some trace elements like sulfur are used to define the qualified feedstock or, additionally, some other properties like the yields of pseudocomponents are also considered. In all cases, the CDUs are continuously operated without any interruption over the scheduling horizon. As a result, the mathematical formulation for the scheduling of crude oil refinery operations varies according to the operational rules adopted by the refiner.

Current research on the scheduling of crude oil operations mainly intends to minimize the total operating cost while limiting the concentration of some impurities in the feedstock by blending crude oils in tanks and/or mixing pipelines. However, such trace elements have no impact on some important issues such as the yields of final cuts. As stated by Bai et al.,^{3,4} other crude properties ignored in previous contributions are essential for the crude oil blending problem. Such critical data are obtained by making a crude oil assay analysis that uniquely characterizes the feedstock. They allow the refiner to make good decisions in the areas of crude selection and blending operations. The crude oil assay uses a batch distillation process to separate the crude oil into a number of pseudocomponents or microcuts, each one featuring a predefined true boiling point (TBP) interval of approximately 10 °C. As the batch distillation proceeds, the TBP of the crude sample increases from the boiling point of methane to approximately 850 °C. By measuring the mass and specific gravity of the crude distilled during every TBP interval, the volume or the weight yield of each microcut is determined. In addition, a set of relevant properties including the concentration of sulfur and other trace elements are also measured. In this way, the characteristic boiling temperature vs volumetric yield data for the crude oil is obtained. The TBP distribution curve represents the variation of the true boiling point of the crude oil as a function of the cumulative volumetric distilled fraction. Pseudocomponents of a crude oil can be classified into two groups: heart and swing microcuts. Heart-cuts refer to material that must always be allocated to certain final cut. In turn, swing-cuts can be distributed between the two adjacent heart cuts. Swing-cut models have been proposed to make a correct distribution of light and heavy swing-cuts (Menezes et al.⁵). Therefore, the yields and other properties of the final cuts can be determined by mixing a predefined set of microcuts. Reciprocally, the TBP distribution data of the feedstock required to produce the desired proportions of the final cuts can approximately be established using swing-cut models. Suitable feedstocks with trace element concentrations below the upper limit and TBP distribution data close to the desired values can consistently be obtained through a proper selection of the crude oils to blend and their individual feed flow rates (Bai et al.^{3,4}).

In previous contributions (Lee et al.,⁶ Jia et al.,⁷ Furman et al.,⁸ Karuppiyah et al.,⁹ Saharidis et al.,¹⁰ Mouret et al.,^{11–13} Yadav and Shaik,¹⁴ and Castro and Grossmann¹⁵), the problem goal is to minimize the operating costs mostly related to the unloading of crude parcels from VLCCs (crude vessel harboring, VLCC sea-waiting cost), crude inventories in storage/charging tanks, and feedstock changeovers. Similarly to Bai et al.,^{3,4} this work is focused on the efficient operation of the multipipeline crude oil

blending system (MCOBS) delivering a sequence of feedstocks to the CDU. But instead of tackling the problem using hybrid metaheuristic algorithms,^{3,4} it has developed a novel mixed-integer nonlinear (MINLP) formulation for the scheduling of the MCOBS operations that accounts for the TBP distribution data of the crude oils available in charging tanks. The selected goal is to feed the CDUs with suitable and stable feedstocks that consistently produce the desired proportions of final cuts, with low variation in composition with time and trace element concentrations below the maximum allowed value. Downstream refinery operations, even the primary separation process of crude oil in the CDU units, are out of the scope of this work. An MILP-NLP solution strategy permits us to find good MINLP feasible solutions in reasonable CPU times.

2. PREVIOUS CONTRIBUTIONS

Most papers dealing with the scheduling of front-end refinery operations are devoted to inland oil refineries with separate storage and charging tanks (Lee et al.,⁶ Jia et al.,⁷ Furman et al.,⁸ Karuppiyah et al.,⁹ Saharidis et al.,¹⁰ Mouret et al.,^{11–13} Yadav and Shaik,¹⁴ and Castro and Grossmann¹⁵). They all assume the following: (a) Nondedicated charging tanks receive different kinds of crude oils from storage tanks. (b) The crude blending process just occurs in the charging tanks. (c) Only one tank provides the feedstock to each CDU at any time. (d) Each charging tank can sequentially feed different CDUs over the time horizon. (e) The transfer of crude oil from a charging tank to a CDU can be interrupted before the tank is empty. (f) Limiting values for the concentration of one or two trace elements are specified to define a qualified feedstock. Then, the charging schedule is determined by choosing the sequence of charging tanks connected to each CDU, the connection period, and the flow rate of the crude blend supplied by each one. The selected problem goal usually seeks to minimize the total operating cost comprising the inventory holding cost, the crude vessel harboring, the sea-waiting cost, and the feedstock changeover cost. For simplicity, some works ignore the inventory holding cost.^{11–13} By assuming the operational rules listed before, discrete and continuous-time formulations for the scheduling of upstream operations in inland refineries have been proposed. Examples involving up to four charging tanks and three CDUs were solved.

Some other contributions have been focused on the crude oil scheduling problem for coastal refineries with a set of charging tanks directly receiving crude oils from very large crude carriers (VLCCs) and supplying feedstock to CDUs (Reddy et al.^{1,16} Li et al.,² Li et al.,¹⁷ Cerdá et al.¹⁸). All of these works assume the following: (i) Nondedicated charging tanks can store different kinds of crude oils. (ii) A particular CDU can receive a mix of crude blends simultaneously supplied by several charging tanks, normally at most two. (iii) Any charging tank may sequentially feed multiple CDUs, normally at most two. (iv) The number of charging tanks providing the feedstock to a particular CDU can change with time, i.e., from one to two or vice versa. (v) To have control on the feedstock composition, the streams of crude blends coming from two or more charging tanks to the same CDU must start and end at the same time, and their flow rates will remain constant while they concurrently feed the CDU. (vi) The feedstock quality is defined by specifying the acceptable ranges for a number of properties including concentrations of trace elements (sulfur, nitrogen, carbon residue), Reid vapor pressure, specific gravity, viscosity, etc. By adopting these operational rules, the quality of the feedstock not only depends

on the composition of the crude blends concurrently supplied by the charging tanks to the CDU but also on their feed flow-rates. Reddy et al.^{1,16} developed discrete and continuous-time MILP formulations for the problem by including linear bounding constraints on trace element concentrations instead of the typical bilinear mixing equations. To eliminate the composition discrepancy of the MILP without solving an NLP model, the authors proposed a solution algorithm that consists of iteratively solving new MILPs. But the algorithm may fail to find a feasible solution. Based on the discrete MILP formulation, Li et al.² later introduced a simple way to recover the approach from a failure by identifying and removing the infeasible combinations of binary variables. The selected objective function is aimed at maximizing the total netback obtained by making the difference between the estimated revenues and the operational costs. Examples involving up to eight charging tanks and three CDUs were solved.

Li et al.¹⁷ developed a novel unit-specific continuous-time MINLP formulation that exploit recent advances in piecewise-linear underestimation of bilinear terms to find a global MINLP optimum. Several important crude property indices were considered to ensure the quality of the feedstock. In turn, Cerdá et al.¹⁸ proposed an MINLP continuous-time approach based on (a) global-precedence sequencing variables to establish the ordering of loading and unloading operations in the storage tanks and (b) an efficient mode of tracking the composition and the inventory of crude mix in the charging tanks. Moreover, synchronized time slots of variable length are used to model the sequence of feedstock supplied to each CDU. The MINLP formulation is solved using an MILP-NLP strategy that provides very good solutions at low computational cost.

More recently, Bai et al.^{3,4} consider another refinery configuration where each CDU has a blending system composed of (1) a set of dedicated charging tanks, (2) a limited number of transfer pipelines that continuously receive crude oil from the charging tanks, all converging into a single mixing pipeline, and (3) a mixing pipeline supplying the mixed feedstock to the CDU (Figure 1). Different types of crude oils are available, but only

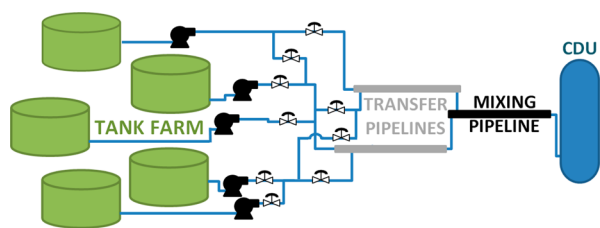


Figure 1. Schematic of a multipipeline crude oil blending system (MCOBS).

one is stored in every charging tank all over the scheduling horizon. Lots of crude oil are unloaded from the tanks into a number of transfer pipelines and subsequently blended in the mixing pipeline connected to the associated CDU. In contrast to previous contributions, some attention is paid on the relative proportions of the final cuts leaving the CDUs. The following operation rules are considered: (i) Each tank can at most feed one transfer pipeline all over the scheduling horizon. (ii) There is only one tank connected to the transfer line at any time, i.e., charging tanks should feed a transfer pipeline one by one in a certain sequence. (iii) Transfer pipelines should continuously receive crude oil from charging tanks. (iv) The unloading of crude from the currently connected charging tank should be

continued until that tank is empty. (v) There is an emptying due date for every tank as long as new parcels of crude oils are received from trunk pipelines or VLCCs during the planning horizon. (vi) The required quality of the feedstock is defined by specifying the allowed maximum trace element concentrations, the acceptable range of light components and, especially, the desired true boiling point (TBP) distribution data matching the product demand pattern. In this case, the scheduling task seeks to allocate tanks to transfer pipelines in a proper sequence so as to feed the CDU with a feedstock of qualified and stable properties (Bai et al.^{3,4}). The objective function intends to make the TBP distillation curve of the mixed feedstock as close as possible to the target TBP curve and, at the same time, keep mostly stable the composition of the feedstock along the planning horizon. The authors model the scheduling of a multipipeline crude oil blending system as a two-level problem, with the outer level allocating and sequencing storage tanks to transfer pipelines and the inner level optimizing the crude flow rates coming from every tank. The two-level problem is solved by applying a pair of metaheuristic optimization algorithms. Tabu search (TS) and differential evolution (DE) were used by Bai et al.,³ while Bai et al.⁴ proposed a two-level optimization framework based on constrained ordinal optimization (COO) and evolutionary algorithms (EA).

In turn, Fu et al.¹⁹ focused on the operation of the distillation units. They presented a small-sized model of a crude distillation unit (preflash, atmospheric, and vacuum towers) suitable for all these applications. Combined with volumetric and energy balances, partial least-squares models allow us to predict the product true boiling point (TBP) curves from the feed TBP curve and the unit operating conditions. The results are very close to those obtained with rigorous tray-to-tray distillation model. Associated properties (e.g., gravity, sulfur) are computed for each product based on its distillation curve and the corresponding property distribution in the feed.

In this work, we develop an MINLP formulation for the operations scheduling of the multipipeline crude oil blending system (MCOBS) by assuming a refinery configuration and operational rules similar to the ones considered by Bai et al.⁴ Instead of applying metaheuristic algorithms to find a feasible solution for the problem, better schedules are found by developing an MINLP model and using an MILP-NLP solution strategy to solve it.

3. PROBLEM DESCRIPTION

To reduce the complexity of the crude oil blend scheduling problem, charging tanks are usually dedicated to a single type of crude oil and gathered into as many groups as the number of CDUs. In the short term planning, each group provides crude oil to a single crude distillation unit. Moreover, the tanks belonging to the same group should contain different types of crude oils so as to obtain the desired composition of the feedstock by mixing the flows coming from two or more tanks. To carry out the blending process, there is a multipipeline crude oil blending system (MCOBS) associated with each CDU composed by a limited number of transfer pipelines receiving crude oil from the assigned group of charging tanks (Figure 1). Transfer pipelines converge into a single mixing pipeline that provides the mixed feedstock to the related CDU. The discharge of batches of different crude oils into the transfer pipelines should be synchronized in both quality and flow-rates to get a qualified feedstock. A flow control system is available to keep the blending flow rates always stable. In this way, the scheduling of upstream

refinery operations simply reduces to solving a series of scheduling subproblems each one involving a single CDU.

When a charging tank is connected to a transfer pipeline, the unloading operation must continue until it becomes empty. Just after that, the unloading of the next tank can be started. Moreover, a charging tank should discharge crude oil into at most a single transfer pipeline, and tanks allocated to the same transfer pipeline are unloaded one at a time. As a result, the scheduling of crude oil operations mostly implies to optimally allocate charging tanks to a limited number of transfer pipelines (normally 2 or 3) and choose the order in which they discharge crude into the assigned pipeline. Synchronizing the unloading of crude oils into the transfer lines to get a series of qualified and stable feedstocks can be modeled by introducing the notion of time slots. At each slot, a single tank can at most be connected to each pipeline. Another key decision is the selection of tanks concurrently feeding the transfer pipelines. As the CDUs must be continuously operated, the transfer pipelines should receive crude oil from the charging tanks over the scheduling horizon without any interruption. To plan the MCOBS operations for each CDU, just the amounts of crude oils currently available in the charging tanks are first considered. As new parcels of crude oils will be unloaded from the arriving vessels over the time horizon, due dates are specified for emptying some charging tanks so that they can receive the new parcels during the planning horizon. In short, the following operational rules are considered:

- Each charging tank is dedicated to a single type of crude oil and linked to a single transfer pipeline.
- Crude oil should be continuously supplied to each transfer pipeline without interruption over the whole time horizon.
- At any time, each transfer pipeline should receive crude oil from a single charging tank.
- When a charging tank is connected to the assigned transfer pipeline, it should supply crude oil until it becomes empty.
- Each charging tank should be emptied before the specified due date.
- The concentration of trace elements in the feedstock produced in the mixing pipeline should remain below the specified maximum value.
- The flow-rate at which crude oil is discharged from a charging tank into a transfer pipeline must be within the specified allowable range.
- At any time, the mixed feedstock must be supplied to the CDU at the specified fixed flow-rate.
- The amount of crude oil initially contained in the charging tanks should be discharged into the transfer pipelines. The new arriving parcels will be partially or entirely processed by the CDU if there is still time remaining until the end of the planning horizon.

The goal of the MCOBS operations scheduling problem is to consistently feed the CDU with a series of qualified and stable feedstocks featuring TBP distribution data as close to the specified TBP target as possible and, at the same time, minimum changes in composition between consecutive time slots.

4. MATHEMATICAL MODEL

The key decisions when scheduling MCOBS operations in an oil refinery are the assignment of charging tanks to transfer pipelines and the selection of the emptying sequence of the tanks allocated to each pipeline. Hence, the mixed-integer nonlinear (MINLP) formulation that is proposed will be based on the use of (a) direct precedence binary variables to choose the tank emptying

sequence for each transfer pipeline and (b) time slots to synchronize the unloading of crude oil from different tanks into the IPI transfer lines to get a series of qualified and stable feedstocks. In contrast to Bai et al.,⁴ the proposed formulation can generate additional feedstock by mixing arriving parcels of crude oil received from pipelines or vessels along the planning horizon and stored in preassigned charging tanks, if there is still time available for their discharge in the transfer pipelines. The MINLP model has been linearized to get an approximate MILP representation that provides a good feasible solution for the problem. By using the MILP solution as the starting point and fixing the integer variables in the original MINLP formulation, the resulting NLP is subsequently solved. In other words, an MILP-NLP sequential strategy has been adopted to find a near-optimal solution for the problem. Before presenting the MINLP formulation, we introduce the major model elements and the model assumptions.

4.1. Model Elements. To solve the MCOBS scheduling problem, the following data arising as model parameters are assumed to be available: (a) the types of crude oils $cr \in CR$ initially stored in the charging tanks $t \in T$ and the subset of tanks storing the crude type cr (T_{cr}), (b) the amount of crude initially contained in each tank (inv_t), (c) the set of transfer pipelines $p \in P$, (d) the limiting discharge rates of crude oil from the charging tanks into the transfer pipelines ($rmin_p, rmax_p$), (e) the emptying due date for every tank (dd_t), (f) the fixed flow-rate in the mixing pipeline (f_{rate}), (g) the trace elements $k \in K$ to be controlled and their concentrations in the available crude oils ($c_{k,cr}$), (h) the maximum allowed concentration of trace elements in the mixed feedstock (c_k^{max}); (i) the true boiling point (TBP) distribution data for the crude oils available in the charging tanks, i.e., the volumetric yield of each microcut $mc \in MC$ ($yld_{mc,cr}$), (j) the desired TBP distribution data for the mixed feedstock ($yld_{ref,mc}$), (k) the minimum/maximum yield of light components in the mixed feedstock ($lyld_{min}/lyld_{max}$), and (l) the normalized coefficient for each microcut $mc \in MC$ (wf_{mc}) weighting the quadratic deviation of the microcut yield in the feedstock from both the desired value and from the one featured by the corresponding microcut of the preceding feed stream. Because it is more important to keep the total yield of light pseudocomponents stable and closer to the desired values, larger wf_{mc} are assigned to lighter microcuts.

On the other hand, the 0–1 key model variables are related to the assignment of charging tanks to transfer pipelines (YT_{tp}) and the emptying sequence of tanks allocated to the same pipeline. The direct precedence variables (XF_{tp} , $X_{t,t',p}$, XL_{tp}) are used to determine the tank emptying sequence for the transfer pipeline p . If $XF_{tp} = 1$, then the tank t is allocated to pipeline p (i.e., $YT_{tp} = 1$) and the first discharging crude oil into that pipeline. Otherwise, $XF_{tp} = 0$. The binary variable $X_{t,t',p}$ denotes that tank t' is discharged just after emptying tank t into pipeline p only if both tanks have been assigned to that pipeline (i.e., $YT_{tp} = YT_{t'p} = 1$) and $X_{t,t',p} = 1$. In turn, XL_{tp} is equal to 1 when tank t is the last unloading crude into pipeline p .

As already stated, the feedstock supplied to the CDU is obtained by mixing crude oils concurrently discharged into the IPI transfer pipelines from a similar number of charging tanks. As usually $|T| > |P|$ and, in addition, a charging tank has to be paired with other ($|P| - 1$) tanks, a sequence of mixed feedstocks is supplied to the CDU over the planning horizon. Then, a set of time slots $s \in S$ must be defined with a different feedstock being processed by the CDU during each slot. It must be emphasized that a charging tank delivering crude oil to pipeline p during the

slot s is not necessarily empty at the end of that slot. It may be discharging batches of crude oil of different sizes into pipeline p over two or more successive slots s , $(s+1)$, $(s+2)$ To efficiently deal with the assignment of a charging tank to several time slots, it is required that the problem model also handles an ordered set of batches $b \in B$. Note that the cardinality of the S should be an integer greater than or equal to $|T|/|P|$. If $|P| = 2$ and $|T| = 15$, then $|S| \geq 8$. In turn, $|B| = |P| \times |S|$ as long as $|S|$ batches are pumped into each pipeline over the time horizon. The set B is said to be an ordered set because the first $|S|$ batches are preassigned to the first element of the set P , the following $|S|$ lots to the second pipeline and so on. Other important 0–1 decision variables are denoted by $YB_{b,t,p}$ and $YS_{b,p,s}$, respectively. The binary variable $YB_{b,t,p}$ indicates that the batch b is unloaded from tank t and discharged into pipeline p whenever $YT_{t,p} = 1$ and $YB_{b,t,p} = 1$. In turn, batch b is delivered to pipeline p during the time slot s only if $YB_{b,t,p} = 1$ for some $t \in T$, and $YS_{b,p,s} = 1$.

Continuous variables are mostly related to batches and time slots. Batch-related variables ($QB_{b,t,p}$, $CB_{b,t,p}$, $LB_{b,t,p}$) denote the size, the completion time, and the length of the unloading of batch b from tank t into pipeline p , respectively. In turn, the slot-based variables (SS_s , CS_s) stand for the initial and end times of slot s . If $YS_{b,p,s} = 1$, then the unloading operation of batch b into pipeline p should start at time SS_s and end at time CS_s . On the other hand, K is the set of trace elements whose concentration should be below some maximum value. To meet that constraint, introduced is the variable $QK_{k,b,p,s}$ denoting the amount of the trace element $k \in K$ in batch b delivered from some tank t to pipeline p during the time slot s , only in case $YT_{t,p} = 1$. Other important continuous variables include $QMB_{mc,b,p,s}$ representing the amount of microcut $mc \in MC$ in batch b of the crude oil discharged into pipeline p during the slot s and $YMF_{mc,s}$ denoting the yield of microcut mc in the mixed feedstock supplied to the CDU over the time slot s .

4.2. Model Assumptions. To develop the proposed mathematical formulation, the following assumptions have been made:

- (1) The TBP distribution data is the major property used to evaluate the feedstock quality.
- (2) The changeover time for switching the charging tank connected to a transfer pipeline is neglected.
- (3) A perfect mixing of the batches coming from the charging tanks is obtained in the mixing pipeline.
- (4) The properties of the flows concurrently discharged into the transfer pipelines, including their TBP distribution data, blend linearly in the mixing line.
- (5) Initial inventories of crude oil available in the charging tanks should be fully discharged before the end of the planning horizon.
- (6) The mixed feedstocks should be supplied to the CDU at a constant feed rate over the scheduling horizon.
- (7) The desired TBP distribution data for the mixed feedstock ($yd_{ref,mc}$) is given.

4.3. Model Constraints. The proposed MINLP model includes different groups of constraints for (a) allocating charging tanks to transfer pipelines, (b) sequencing charging tanks assigned to the same pipeline, (c) obtaining acyclic solutions, (d) sizing batches discharged from the tanks and determining the flow-rate and the initial/end times of each unloading operation, (e) assigning discharged batches to time slots, and (f) determining the quadratic deviation from the

desired TBP distribution curve and the change of feedstock composition in two consecutive slots.

4.3.1. Assigning Charging Tanks to Transfer Pipelines. Each charging tank should be assigned to at most one transfer pipeline. In eq 1, the binary variable $YT_{t,p}$ denotes the assignment of tank t to pipeline p whenever $YT_{t,p} = 1$.

$$\sum_{p \in P} YT_{t,p} \leq 1 \quad t \in T \quad (1)$$

4.3.2. Sequencing Unloading Operations from Charging Tanks Assigned to the Same Pipeline. A charging tank t does not belong to the queue of the transfer pipeline p if it is not assigned to that pipeline. The binary variable $X_{t',t,p}$ is introduced to indicate that tank t immediately precedes tank t' in the queue of pipeline p only if both tanks have been assigned to that pipeline (i.e., $YT_{t,p} = YT_{t',p} = 1$) and $X_{t',t,p} = 1$. Otherwise, $X_{t',t,p} = 0$.

$$\sum_{\substack{t' \in T \\ t' \neq t}} X_{t',t,p} \leq YT_{t,p} \quad t \in T, p \in P \quad (2)$$

$$\sum_{\substack{t' \in T \\ t' \neq t}} X_{t,t',p} \leq YT_{t,p} \quad t \in T, p \in P \quad (3)$$

Each charging tank assigned to the transfer pipeline p is either the first unloaded or it has a direct predecessor in the tank queue of pipeline p . In eq 4, the binary variable $XF_{t,p}$ denotes that tank t among those allocated to pipeline p is the first unloaded.

$$XF_{t,p} + \sum_{\substack{t' \in T \\ t' \neq t}} X_{t',t,p} = YT_{t,p} \quad t \in T, p \in P \quad (4)$$

Similarly, a charging tank connected to pipeline p is either the last unloaded or it has a successor in the tank queue of pipeline p . In eq 5, the binary variable $XL_{t,p}$ denotes that tank t among those assigned to pipeline p is the last unloaded.

$$XL_{t,p} + \sum_{\substack{t' \in T \\ t' \neq t}} X_{t,t',p} = YT_{t,p} \quad t \in T, p \in P \quad (5)$$

Only one charging tank is first unloaded into each transfer pipeline. In the same way, just one charging tank among those assigned to pipeline p is the last unloaded. These constraints are given by eqs 6 and 7.

$$\sum_{t \in T} XF_{t,p} = 1 \quad p \in P \quad (6)$$

$$\sum_{t \in T} XL_{t,p} = 1 \quad p \in P \quad (7)$$

4.3.3. Cycle-Breaking Constraints. Cycle-breaking constraints are incorporated into the mathematical model to avoid cycles in the sequence of tanks discharging crude oil to the same transfer pipeline. $RT_{t,p}$ is an integer variable denoting the position of the tank t in the queue of pipeline p . The value of $RT_{t,p}$ is always equal or greater than one and never larger than the number of tanks discharging crude oil into pipeline p . Moreover, $RT_{t',p}$ is larger than $RT_{t,p}$ by one if tank t' immediately succeeds tank t in the queue of pipeline p , i.e., $X_{t',t,p} = 1$. In addition, the value of $RT_{t,p}$ for the charging tank last emptied is equal to the number of tanks assigned to pipeline p given by $|T|$. These cycle-breaking constraints are given by eqs 8–11.

$$RT_{t,p} \geq XF_{t,p} \quad t \in T, p \in P \quad (8)$$

$$RT_{t',p} \geq RT_{t,p} + 1 - |T| \times (1 - X_{t,t',p}) \\ t, t' \in T (t' \neq t), p \in P \quad (9)$$

$$RT_{t,p} \leq \sum_{t' \in T} YT_{t',p} \quad t \in T, p \in P \quad (10)$$

$$RT_{t,p} \geq \sum_{t' \in T} YT_{t',p} - |T| \times (1 - XL_{t,p}) \quad t \in T, p \in P \quad (11)$$

4.3.4. Allocating Batches of Crude Oil to Charging Tanks.

Let us introduce the ordered set of batches B whose first $|S|$ elements are assigned to tanks allocated to the first pipeline in $|P|$, the next $|S|$ batches to the second element of P , and so on. In this way, we define the subsets of batches B_p for each pipeline. As stated by eq 12, every batch of the subset $B_p \subset B$ can at most be assigned to a single charging tank allocated to pipeline p . From eq 13, the binary variable $YB_{b,t,p}$ denoting the assignment of batch $b \in B_p$ to tank t is equal to 1 only if tank t discharges crude oil into pipeline p (i.e., $YT_{t,p} = 1$). Otherwise, $YB_{b,t,p} = 0$. Equation 14 reserves the last elements of B_p for fictitious batches featuring $\sum_{t \in T} YB_{b,t,p} = 0$.

$$\sum_{t \in T} YB_{b,t,p} \leq 1 \quad p \in P, b \in B_p \quad (12)$$

$$YB_{b,t,p} \leq YT_{t,p} \quad p \in P, b \in B_p, t \in T \quad (13)$$

$$\sum_{t \in T} YB_{b,t,p} \leq \sum_{t \in T} YB_{b',t,p} \\ p \in P, (b, b') \in B_p (b > 1, b' < b) \quad (14)$$

Equations 15a–15c remove equivalent solutions from the feasible region by assigning the elements of B_p to charging tanks in the same order that they are discharged into pipeline p . To meet that condition, it is defined the new binary variable $YBN_{b,t,p}$. From eq 15a, $YBN_{b,t,p} = 1$ whenever some batch $b' < b$ is unloaded from tank t' into pipeline p . Otherwise, eq 15b makes $YBN_{b,t,p} = 0$. If tank t' directly succeeds tank t (i.e., $X_{t,t',p} = 1$) and batch $b' < b$ has been assigned to tank t' , then $YBN_{b,t,p} = 1$ and eq 15c drives $YB_{b,t,p}$ to zero. In case several batches are unloaded from the same charging tank, they should be consecutive elements of the set B_p as ruled out by eq 16.

$$YBN_{b,t,p} \geq YB_{b',t',p} \quad p \in P, b, b' \in B_p (b' < b), t' \in T \quad (15a)$$

$$YBN_{b,t,p} \leq \sum_{\substack{b' \in B_p \\ b' < b}} YB_{b',t',p} \quad p \in P, b \in B_p, t' \in T \quad (15b)$$

$$YB_{b,t,p} \leq 1 - YBN_{b,t',p} + |B| \times (1 - X_{t,t',p}) \\ b \in B_p, t, t' \in T (t \neq t'), p \in P \quad (15c)$$

$$YB_{b,t,p} + YB_{b'',t,p} \leq 1 + YB_{b',t,p} \\ t \in T, p \in P, (b, b', b'') \in B_p (b' = b + 1, b'' = b' + 1) \quad (16)$$

The continuous variable $WP_{b,p}$ restricted to the interval $[0,1]$ is introduced to denote the existence of batch $b \in B_p$ whenever the value of $WP_{b,p}$ resulting from eq 17 is equal to 1. Otherwise, such an element is a fictitious lot. By eq 18, the number of batches injected into pipeline p should be equal to that injected into any other pipeline $p' (\neq p)$. Note that the sequence of feed streams supplied to the CDU results from successively mixing batches concurrently coming from the $|P|$ transfer pipelines.

$$WB_{b,p} = \sum_{t \in T} YB_{b,t,p} \quad p \in P, b \in B_p \quad (17)$$

$$\sum_{b \in B_p} WB_{b,p} = \sum_{b' \in B_{p'}} WB_{b',p'} \quad p, p' \in P (p' \neq p) \quad (18)$$

4.3.5. Batch Sizing Constraints. The continuous variable $QB_{b,t,p}$ represents the size of batch $b \in B_p$ discharged from the charging tank t into pipeline p . If batch b is not discharged from tank t , then $YB_{b,t,p} = 0$, and consequently, $QB_{b,t,p} = 0$. When $YB_{b,t,p} = 1$, the size of batch b should never be lower than a specified minimum amount $qmin$. As the full content of tank t must be transferred to the assigned pipeline, then the total amount of crude oil contained in the batches discharged from tank t featuring $YB_{b,t,p} = 1$ should be equal to the initial content of tank t (inv_t). Such constraints are given by eqs 19–21. The parameter M is a large number.

$$QB_{b,t,p} \leq M \times YB_{b,t,p} \quad p \in P, b \in B_p, t \in T \quad (19)$$

$$QB_{b,t,p} \geq qmin \times YB_{b,t,p} \quad p \in P, b \in B_p, t \in T \quad (20)$$

$$\sum_{p \in P} \sum_{b \in B_p} QB_{b,t,p} = inv_t \quad t \in T \quad (21)$$

4.3.6. Start and End Times of Tank Unloading Operations.

The unloading rate of nonfictitious batches of crude oil from charging tanks should belong to the interval $[rmin, rmax]$. If the continuous variable $LB_{b,t,p}$ denotes the length of the unloading operation of batch $b \in B_p$ from tank t , then its value is determined by eqs 22 and 23. When $QB_{b,t,p} = 0$, then $LB_{b,t,p}$ is also equal to 0.

$$QB_{b,t,p} \leq rmax \times LB_{b,t,p} \quad p \in P, b \in B_p, t \in T \quad (22)$$

$$QB_{b,t,p} \geq rmin \times LB_{b,t,p} \quad p \in P, b \in B_p, t \in T \quad (23)$$

The unloading of batch $b' \in B_p$ from the charging tank t' should start after completing the discharge of batch $b \in B_p$ from tank t only if this tank is the direct predecessor of tank t' (i.e., $X_{t,t',p} = 1$) in the queue of pipeline p , and $YB_{b,t,p} = YB_{b',t',p} = 1$. If batches $b, b' \in B_p$ come from the same tank t and $b' > b$, then the unloading of batch b' should start after completing the discharge of batch b . Both constraints are given by eqs 24 and 25. In such equations, the continuous variables $SB_{b,t,p}$ and $CB_{b,t,p}$ stand for the times at which the unloading of batch b begins and ends, respectively.

$$CB_{b,t,p} \leq SB_{b',t',p} + H \times (2 - YB_{b,t,p} - YB_{b',t',p}) \\ + H \times (1 - X_{t,t',p}) \\ p \in P, (b, b') \in B_p (b' > b), t, t' \in T (t \neq t') \quad (24)$$

$$CB_{b,t,p} \leq SB_{b',t',p} + H \times (2 - YB_{b,t,p} - YB_{b',t',p}) \\ p \in P, (b, b') \in B_p (b' = b + 1), t \in T \quad (25)$$

From eq 26, $CB_{b,t,p}$ can be obtained from $SB_{b,t,p}$ by adding the length of the unloading operation. Moreover, eq 27 states that the value of $CB_{b,t,p}$ for any batch $b \in B_p$ featuring $YB_{b,t,p} = 1$ must never exceed the due date dd_t for emptying the tank t . From eqs 28 and 29, the unloading operations from charging tanks consecutively discharged into the same pipeline take place one after another without delay.

$$CB_{b,t,p} = SB_{b,t,p} + LB_{b,t,p} \quad b \in B_p, t \in T, p \in P \quad (26)$$

$$CB_{b,t,p} \leq dd_t \times YB_{b,t,p} \quad b \in B_p, t \in T, p \in P \quad (27)$$

$$\sum_{t \in T} CB_{b,t,p} \leq \sum_{t \in T} \sum_{\substack{b' \in B_p \\ b' \leq b}} LB_{b',t,p} \quad p \in P, b \in B_p \quad (28)$$

$$\sum_{t \in T} CB_{b,t,p} \geq \sum_{t \in T} LB_{b,t,p} + \sum_{t \in T} CB_{b-1,t,p} \quad p \in P, (b-1), b \in B_p \quad (29)$$

4.3.7. Using Time Slots To Determine the Series of Feedstocks Supplied to the CDU. The series of feed streams supplied to the CDU over the planning horizon results from sequentially mixing batches concurrently coming from the $|P|$ transfer pipelines. The unloading of such batches into the pipelines should begin and end at the same time. To comply with that constraint, a common set of time slots $s \in S$ for the $|P|$ pipelines and the assignment variable $YS_{b,p,s}$ are defined. The binary variable $YS_{b,p,s}$ denotes that the unloading of batch $b \in B_p$ into pipeline p is performed during the time slot s whenever $YS_{b,p,s} = 1$. If batch $b \in B_p$ does exist, then its unloading operation must be assigned to only one time slot. Reciprocally, a single batch $b \in B_p$ can at most be discharged into the transfer pipeline p during the time slot s . These constraints are given by eqs 30 and 31.

$$\sum_{s \in S} YS_{b,p,s} = WB_{b,p} \quad b \in B_p, p \in P \quad (30)$$

$$\sum_{b \in B_p} YS_{b,p,s} \leq 1 \quad p \in P, s \in S \quad (31)$$

Accounting for the fact that batches are allocated to tanks in the same order that they are discharged into the assigned pipeline, eq 32 states that batch $b \in B_p$ cannot be assigned to slot s if another batch $b' \in B_p$ with $b' < b$ is discharged in a later slot $s' > s$. Moreover, the number of unloading operations into any pair of pipelines p and p' (with $p' \neq p$) during any time slot s must be equal, and the last elements of the set S are reserved for fictitious slots by eqs 33 and 34, respectively.

$$YS_{b,p,s} + YS_{b',p',s'} \leq 1 \quad p \in P, b, b' \in B_p (b' < b), s, s' \in S (s < s') \quad (32)$$

$$\sum_{b \in B_p} YS_{b,p,s} = \sum_{b \in B_{p'}} YS_{b,p',s} \quad p, p' \in P (p \neq p'), s \in S \quad (33)$$

$$\sum_{b \in B_p} YS_{b,p,s} \leq \sum_{b \in B_p} YS_{b,p,s-1} \quad p \in P, s \in S \quad (34)$$

4.3.8. Length of Time Slot s and Total Amount of Crude Oil Allocated to Each one. The length of the unloading of batch $b \in B_p$ discharged into pipeline p and allocated to slot s is

represented by the continuous variable $LS_{b,p,s}$. Its value is calculated from eqs 35 and 36. Note that $LS_{b,p,s} = 0$ in case the discharge of batch b does not occur during the time slot s and/or it is not injected into pipeline p . As stated by eq 37, $LS_{b,p,s} = LS_{b',p',s}$ if the unloading of batches $b \in B_p$ and $b' \in B_{p'}$ into the pipelines p and p' ($p' \neq p$) occur during the same time slot s (i.e., $YS_{b,p,s} = YS_{b',p',s} = 1$). Then, the length of slot s (L_s) is given by eq 37.

$$LS_{b,p,s} \leq H \times YS_{b,p,s} \quad p \in P, b \in B_p, s \in S \quad (35)$$

$$\sum_{s \in S} LS_{b,p,s} = \sum_{t \in T} LB_{b,t,p} \quad p \in P, b \in B_p \quad (36)$$

$$L_s = \sum_{b \in B_p} LS_{b,p,s} \quad p \in P, s \in S \quad (37)$$

Let us introduce the continuous variable $QS_{b,p,s}$ representing the amount of crude oil in batch $b \in B_p$ allocated to time slot s . Its value is determined through eqs 38 and 39. As stated in Section 3, the feed stream to the CDU should be supplied at a constant rate given by the parameter *frate*. Then, the ratio between the total amount of crude oil allocated to any slot s and the length of that slot L_s must be equal to the fixed flow rate *frate*. This constraint is enforced by eq 40.

$$QS_{b,p,s} \leq M \times YS_{b,p,s} \quad p \in P, b \in B_p, s \in S \quad (38)$$

$$\sum_{s \in S} QS_{b,p,s} = \sum_{t \in T} QB_{b,t,p} \quad p \in P, b \in B_p \quad (39)$$

$$\sum_{p \in P} \sum_{b \in B_p} QS_{b,p,s} = \text{frate} \times L_s \quad s \in S \quad (40)$$

4.3.9. Limiting the Concentration of Trace Elements. The concentration of trace elements $k \in K$ contained in the crude oil, like sulfur and some other contaminants, should be kept below some upper limit c_k^{\max} . To this end, we introduce the continuous variable $QK_{k,b,p,s}$ representing the amount of trace element k in batch $b \in B_p$ allocated to the time slot s , whose value is determined by eqs 41 and 42. In turn, eq 43 states that the limiting concentration for the trace element k must not be exceeded. The parameter M in eq 41 is a relatively large number equal to $\max_i(\text{inv}_i)$.

$$QK_{k,b,p,s} \leq M \times c_k^{\max} \times YS_{b,p,s} \quad p \in P, s \in S, k \in K, b \in B_p \quad (41)$$

$$\sum_{s \in S} QK_{k,b,p,s} = \sum_{t \in T} \sum_{cr \in CR_t} c_{k,cr} \times QB_{b,t,p} \quad p \in P, k \in K, b \in B_p \quad (42)$$

$$\sum_{p \in P} \sum_{b \in B_p} QK_{k,b,p,s} \leq c_k^{\max} \times \left(\sum_{p \in P} \sum_{b \in B_p} QS_{b,p,s} \right) \quad k \in K, s \in S \quad (43)$$

4.3.10. Volumetric Yield of Microcuts in the CDU Feedstock during Slot s . We introduce the continuous variable $QMB_{mc,b,p,s}$ to denote the amount of microcut mc contained in batch b injected into pipeline p during the time slot s . From eq 44, its value is driven to zero if $YS_{b,p,s} = 0$. In eq 44, the parameter $yl_{mc,cr}$ represents the percentage yield of microcut mc in a crude oil of type cr , and the set MC includes all the microcuts used to define its true boiling point distribution curve. In turn, $yl^{\max} =$

$\max_{mc,cr}\{yld_{mc,cr}\}$ is the maximum predicted percentage yield of a microcut in the crude oil cr . Equations 44 and 45 provide the value of $QMB_{mc,b,p,s}$. The amount of microcut mc in the mixed feedstock during the time slot s is given by the continuous variable $QMF_{mc,s}$ whose value is computed using eq 46. In turn, the percentage yield of microcut mc in the feedstock supplied to the CDU during the time slot s represented by the variable $YMF_{mc,s}$ is calculated using the nonlinear eq 47.

$$\sum_{mc \in MC} QMB_{mc,b,p,s} \leq M \times yld_{mc,cr}^{max} \times YS_{b,p,s} \quad p \in P, b \in B_p, s \in S \quad (44)$$

$$\sum_{s \in S} QMB_{mc,b,p,s} = \sum_{cr \in CR} \sum_{t \in T_{cr}} yld_{mc,cr} \times QB_{b,t,p} \quad mc \in MC, p \in P, b \in B_p \quad (45)$$

$$QMF_{mc,s} = \sum_{p \in P} \sum_{b \in B_p} QM_{mc,b,p,s} \quad mc \in MC, s \in S \quad (46)$$

$$YMF_{mc,s} = QMF_{mc,s} / \left(\sum_{p \in P} \sum_{b \in B_p} QS_{b,p,s} \right) \quad mc \in MC, s \in S \quad (47)$$

4.3.11. Limiting the Yield of Light Components in the Feedstock. As stated by Bai et al.,⁴ the feedstock should be neither too light nor too heavy. If it is too light, difficulties in pressure control will arise. In turn, insufficient reflux flow rate is produced by a feedstock that is too heavy. According to eq 48, the total percentage yield of light components in the feedstock should belong to the interval $[yld_{min}, yld_{max}]$. In eq 48, the subset MC_L includes all light microcuts with TBP ≤ 250 °C.

$$yld_{min} \leq \sum_{mc \in MC_L} YMF_{mc,s} \leq yld_{max} \quad s \in S \quad (48)$$

4.3.12. Considering Parcels of Crude Oil Arriving during the Planning Horizon. Let us assume that new parcels of crude oil are received from trunk pipelines or vessels and stored in some preassigned charging tanks $t \in T' \subset T$ over the planning horizon. Such elements of the set T' are assumed to be initially empty and the parcels of crude oil they will be available for discharging into the transfer pipelines at some predefined release times $rt_t > 0$ for $t \in T'$. Instead, rt_t is equal to 0 for the other tanks with finite initial inventories. The release time constraint is given by eq 49.

$$SB_{b,t,p} \geq rt_t \times YB_{b,t,p} \quad p \in P, b \in B_p, t \in T \quad (49)$$

Moreover, no due dates are specified for emptying the charging tanks $t \in T'$. In addition, eq 21 that forces to completely unload a charging tank is substituted by the weakened constraint (21') just for tanks $t \in T'$.

$$\sum_{p \in P} \sum_{b \in B_p} QB_{b,t,p} \leq inv_t \quad t \in T \quad (21')$$

4.4. Objective Function. To achieve the desired feedstocks, the batches of crude oils to mix and their flow rates are to be selected in such a way that the accumulate deviation of the property $YMF_{mc,s}$ with regards to the desired value $yldref_{mc}$ for all microcuts $mc \in MC$ is minimized. In addition, the cumulative change of that property in two consecutive time slots should be reduced as much as possible. The quadratic property deviation $(YMF_{mc,s} - yldref_{mc})^2$ and the quadratic property change $(YMF_{mc,s} - YMF_{mc,s-1})^2$ in two consecutive slots for each microcut mc is weighted by the coefficient wf_{mc} . Larger coefficients are assigned to lighter microcuts. Then, the problem objective function is given by eq 50.

– $YMF_{mc,s-1})^2$ in two consecutive slots for each microcut mc is weighted by the coefficient wf_{mc} . Larger coefficients are assigned to lighter microcuts. Then, the problem objective function is given by eq 50.

$$Z = \sum_{mc \in MC} wf_{mc} \times \sum_{s \in S} (YMF_{mc,s} - yldref_{mc})^2 + \sum_{mc \in MC} wf_{mc} \times \sum_{s \in S, s > 1} (YMF_{mc,s} - YMF_{mc,s-1})^2 \quad (50)$$

4.5. Approximate MILP Model and MILP-NLP Solution Strategy. In the proposed problem formulation, nonlinearities arise in eq 47 and the objective function 50. In order to get a good initial point for the local MINLP solution algorithm, an approximate MILP representation has been developed. Let the continuous variable $QREF_{mc,s}$ be the desired amount of microcut mc in the mixed feedstock supplied during the time slot s . Its value is given by eq 51. To linearize the problem formulation, the property deviation $(YMF_{mc,s} - yldref_{mc})$ for the microcut mc at the time slot s is now expressed in terms of the linear difference $DEV_{mc,s}$ between the related amount of mc in the feedstock $QMF_{mc,s}$ and the reference amount $QREF_{mc,s}$ during slot s , i.e., $(QMF_{mc,s} - QREF_{mc,s})$. The absolute value of the property deviation for the pair (mc,s) is provided by eqs 52 and 53.

$$QREF_{mc,s} = yldref_{mc} \times \sum_{p \in P} \sum_{b \in B_p} QS_{b,p,s} \quad mc \in MC, s \in S \quad (51)$$

$$DEV_{mc,s} \geq QMF_{mc,s} - QREF_{mc,s} \quad mc \in MC, s \in S \quad (52)$$

$$DEV_{mc,s} \geq QREF_{mc,s} - QMF_{mc,s} \quad mc \in MC, s \in S \quad (53)$$

In turn, the property change of the feedstock in two consecutive slots is approximated by the positive variable $DIF_{mc,s}$. This variable is defined as the difference between the property deviation from the target value in two consecutive slots s and $(s + 1)$, and its value is given by eq 54. The absolute value of $DIF_{mc,s}$ called $ADIF_{mc,s}$ is determined by eqs 55 and 56. In this way, the objective function for the approximate MILP formulation is given by eq 57.

$$DIF_{mc,s} \geq (QMF_{mc,s} - QREF_{mc,s}) - (QMF_{mc,s-1} - QREF_{mc,s-1}) \quad mc \in MC, s \in S (s > 1) \quad (54)$$

$$ADIF_{mc,s} \geq DIF_{mc,s} \quad mc \in MC, s \in S (s > 1) \quad (55)$$

$$ADIF_{mc,s} \geq -DIF_{mc,s} \quad mc \in MC, s \in S (s > 1) \quad (56)$$

$$Z = \sum_{mc \in MC} (wf_{mc} \times \sum_{s \in S} DEV_{mc,s}) + \sum_{mc \in MC} (wf_{mc} \times \sum_{s \in S, s > 1} ADIF_{mc,s}) \quad (57)$$

The approximate MILP usually provides a good feasible solution for the MINLP. By fixing the integer variables in the MINLP to their MILP-values, the original MINLP is converted into an NLP model. Using the MILP solution as the starting point, the resulting NLP can be solved very fast. In this way, a near-optimal solution for the MINLP can be found using a simple two-step MILP-NLP procedure. When the best MILP-solution discovered within the CPU time limit has a large optimality gap, the current values of the sequencing variables ($XF_{t,p}$, $X_{t',p}$, $XL_{t,p}$) or the

assignment variables $YT_{i,p}$ are fixed, and the MILP is solved again up to optimality in a short CPU time before solving the NLP model.

5. RESULTS AND DISCUSSION

The MINLP formulation for the scheduling of multipipeline crude oil blending systems (MCOBS) and the proposed MILP-NLP solution strategy have been tested by solving a pair of examples first introduced by Bai et al.^{3,4} In addition, two variants of the example presented by Bai et al.⁴ have also been generated and solved to study the effect of using more transfer pipelines and handling nonzero release times for some charging tanks. Problem data and detailed computational results for the four case studies are given as [Supporting Information](#). The models are implemented in GAMS 24.8.3 and solved using the solvers CPLEX 12.7 for the MILPs, and CONOPT 3.17C for the NLPs. All computations were performed on an Intel Core i7-6820HQ quad-core CPU, with 16 GB RAM. The relative optimality gap tolerance has been fixed to 0.01 for the MILP models and 0.0001 for the NLPs in all examples. Besides, a maximum CPU time of 3600 s was allowed.

5.1. Example 1. Example 1 first proposed by Bai et al.³ deals with the scheduling of a multipipeline crude oil blending system (MCOBS) composed by two transfer pipelines and one mixing line providing feedstocks to a single CDU (see [Figure 2](#)). Five

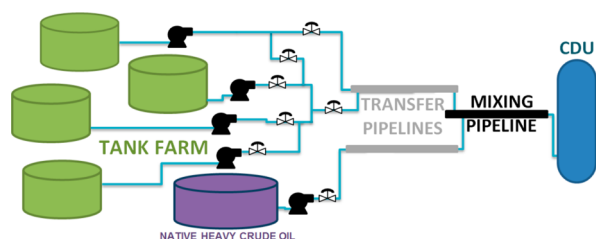


Figure 2. Schematic of the MCOBS configuration for Example 1.

different types of imported crude oils stored in 13 charging tanks are blended with only one type of native crude that is continuously supplied to one of the transfer pipelines ($Pipe_{nat}$) from a nearby oil field. In turn, the charging tanks sequentially deliver their imported crudes one-by-one into the other transfer pipeline ($Pipe_{imp}$).

In this case, assignment variables allocating tanks to pipelines are not needed. A tank connected to $Pipe_{imp}$ remains delivering crude oil until it becomes empty. Moreover, due dates for emptying charging tanks have been specified. Sulfur is the only trace element whose concentration should be monitored. Data for Example 1 include information about (a) the type and the amount of imported crude initially available in every tank and their emptying due dates, (b) the sulfur concentration in native and imported crudes and the maximum sulfur concentration

allowed in the feedstock, (c) the TBP distribution data for the available crude oils and the normalized weight coefficient for each microcut wf_{mc} arising in the objective function, (d) the desired TBP distribution curve for the feedstock and the maximum percentage yield of light components that it should contain, (e) the min/max delivery flow-rates of crude oils from charging tanks to transfer pipelines, and (f) the fixed streamflow-rate in the mixing pipeline over the entire planning horizon. All these data for Example 1 are given as [Supporting Information](#).

Since $|T|/|P| = 13$ for pipeline $Pipe_{imp}$, a minimum of 13 time slots should be considered to solve the MCOBS scheduling problem. If 13 time slots are adopted, then the whole content of each imported crude tank will be discharged into the pipeline $Pipe_{imp}$ during only one slot. Similarly to Bai et al.,³ we choose 13 time slots to obtain by blending an equal number of feedstocks. Then, we assume that suitable flow-rates for imported and native crude oils can be selected at each time slot to get a qualified feedstock. To this end, the discharge rate of the native crude into $Pipe_{nat}$ is also allowed to vary with the time slot. The selected number of batches $|B|$ is given by $|P| \times |S| = 2 \times 13 = 26$. The resulting sizes of the approximate MILP and the NLP models are shown in [Table 1](#).

The best solution for the approximate MILP has been found in 852 s of CPU time, but it needed 1428 s to also reach the specified optimality gap of 0.01 ([Table 2](#)). Since the approximate MILP provides a good feasible solution for the MINLP formulation, its overall quadratic yield deviation amounting to 7.87 is reported as the MILP optimal value. Adopting the MILP solution as the starting point and additionally fixing the integer variables, the resulting NLP model that now includes the nonlinear constraint 47 and the nonlinear objective function 50 is solved in merely 4.2 s. The use of exact expressions for computing both the microcut percentage yields in the mixed feedstock and the objective function produces a significant reduction of the overall quadratic yield deviation from 7.87 to 6.94, i.e., 11.8% reduction. Additional results shown as [Supporting Information](#) indicate that both terms of the objective function 50 diminish after solving the NLP model. The first term representing the deviation from the yield target drops from 5.66 to 5.16, and the second one standing for the change of the feedstock in two consecutive slots decreases from 2.22 to 1.78.

[Table 3](#) reports some features of the best solution for Example 1 such as the emptying sequence of charging tanks connected to $Pipe_{imp}$, the types and flow-rates of the crude oils supplied to the transfer pipelines and blended in the mixing line during each time slot, the slot initial and end times, and the sulfur concentration in the feedstock. Sulfur concentration never exceeds the specified upper limit of 2500 ppm, and unloading operations exactly finish at the end of the planning horizon (30 days). The Gantt chart of the best schedule found for Example 1 is shown in [Figure 3](#), while [Figure 4](#) displays the TBP distillation curves for the 13 feedstocks

Table 1. Selected Number of Time Slots and Model Sizes for Examples 1–4

Example	# transf. pipelines	# slots	Approximate MILP model			NLP model	
			Equations	Binary variables	Continuous variables	Equations	Continuous variables
Example 1	2	13	28011	1136	9783	26895	10253
Example 2	2	8	32003	1118	4395	31337	5117
Example 2	2	9	56664	1242	5323	55908	6115
Example 3	3	6	35214	1953	3934	34768	5572
Example 4	2	9	45864	1482	5323	45108	6355

Table 2. Best Solutions Found and CPU Time Requirements for Examples 1–4

Example	# transf. pipelines	# slots	MILP best solution	CPU time	Gap (%)	Time to find it	NLP best solution	CPU time
Example 1	2	13	7.87	1428	—	852	6.94	4.2
Example 2	2	8	14.36	2602	—	1415	13.50	1.3
Example 2	2	9	10.89	3600 ^a	20.4	3156	9.74	3.5
Example 3	3	6	8.81	3600 ^a	4.4	3314	8.21	1.5
Example 4	2	9	14.92	3600 ^a	5.5	3010	14.33	3.2

^aCPU time limit.

Table 3. Best MCOBS Schedule Found for Example 1 Using 13 Time Slots

		Time slot												
		s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}	s_{11}	s_{12}	s_{13}
start time (day)		0	6.96	8.69	10.10	11.63	15.10	19.12	20.48	21.84	22.56	23.59	25.48	27.77
end time (day)		6.96	8.69	10.10	11.63	15.10	19.12	20.48	21.84	22.56	23.59	25.48	27.77	30.00
tank queue in	p_1	t_{12}	t_1	t_7	t_8	t_{13}	t_9	t_2	t_3	t_4	t_{11}	t_{10}	t_6	t_5
crude oil in	p_1	cr_5	cr_1	cr_3	cr_3	cr_5	cr_4	cr_1	cr_1	cr_1	cr_4	cr_4	cr_2	cr_2
crude oil in	p_2	cr_{nat}	cr_{nat}	cr_{nat}	cr_{nat}	cr_{nat}	cr_{nat}	cr_{nat}	cr_{nat}	cr_{nat}	cr_{nat}	cr_{nat}	cr_{nat}	cr_{nat}
flow rate in (m ³ /h)	p_1	119.8	240.8	294.2	273.7	119.8	207.4	306.0	306.9	290.4	202.5	220.1	182.0	186.8
flow rate in (m ³ /h)	p_2	380.2	259.2	205.8	226.3	380.2	292.6	194.0	193.1	209.6	297.5	279.9	318.0	313.2
batch size in (10 ³ m ³)	p_1	20.00	10.00	10.00	10.00	10.00	20.00	10.00	10.00	5.00	5.00	10.00	10.00	10.00
batch size in (10 ³ m ³)	p_2	63.48	10.77	7.00	8.27	31.74	28.21	6.34	6.29	3.61	7.35	12.71	17.48	16.76
sulfur (ppm)		2500	1265	1307	1358	2500	2369	1054	1052	1105	2361	2389	1850	1847

supplied to the CDU. The TBP curve with the label REF corresponds to the desired feedstock.

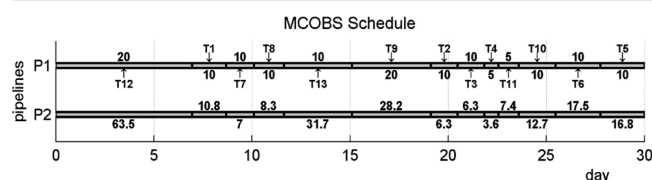


Figure 3. Best MCOBS operations schedule for Example 1.

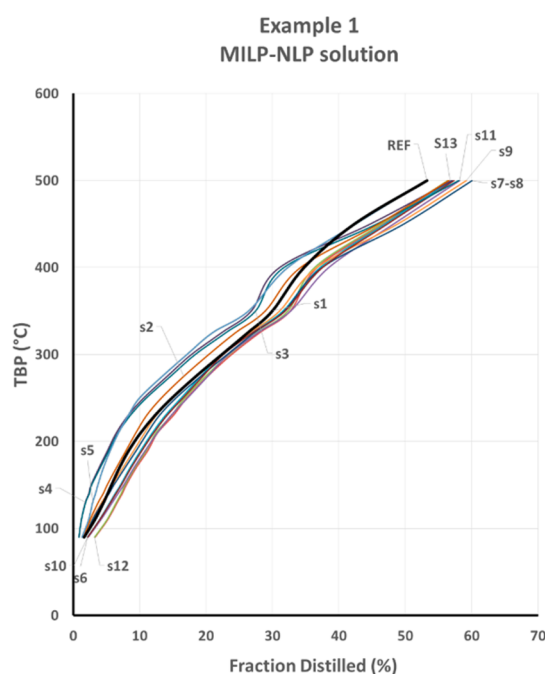


Figure 4. TBP curves of the feedstocks for the CDU at the best solution of Example 1.

A comparison with the solution discovered by Bai et al.³ is made in Table 4. Using the results reported in that contribution, given as Supporting Information, we determine the overall quadratic yield deviation for the solution of Bai et al.³ It was equal to 7.52 (>6.94), i.e., 8.4% larger than the one presented by our best solution. Moreover, Bai et al.³ also reported the mean computer time, equal to 79.2 s, needed per major iteration of the proposed metaheuristic algorithm to solve Example 1.

Part of our improvement comes from supplying a lower number of feedstocks to the CDU, i.e., fewer terms are accounted for in the objective function. In addition, quadratic yield deviations in our solution are mostly associated with heavy microcuts with lower weight coefficient $w_{f_{mc}}$.

5.2. Example 2. Example 2 is a real-world MCOBS scheduling problem introduced by Bai et al.⁴ involving 15 charging tanks that store six types of crude oil, two transfer pipelines (P_1 , P_2), one mixing pipeline, and a single CDU. In this case, charging tanks can be allocated to either transfer pipeline, and those assigned to the same pipeline must be unloaded one at a time. Then, the configuration of the MCOBS is similar to the one shown in Figure 1. Assignment variables allocating tanks to transfer pipelines and sequencing variables controlling the tank emptying sequence for each pipeline are needed. Emptying due dates are also specified for every tank. Data for Example 2 are mostly similar to those considered for Example 1 (Supporting Information). There is only one additional datum consisting of the minimum percentage yield of light components in the feedstock. Moreover, tank t_1 began discharging crude oil into pipeline P_1 during the previous scheduling horizon and still contain some amount of crude oil cr_1 at the start of the current horizon. Therefore, tank t_1 was already allocated to P_1 and it will be the first tank delivering crude oil to P_1 during the current horizon.

For Example 2, the number of time slots should not be lower than $|T|/|P| = 15/2 = 7.5$, i.e., $|S| \geq 8$. As the number of feedstock changeovers increases with $|S|$, it has been sequentially adopted $|S| = 8$ and $|S| = 9$. Using more time slots than nine, no further improvement on the objective function is achieved. If $|S| = 8$, only

Table 4. Comparing Results Found for Examples 1 and 2 with Those Reported in Previous Contributions

Examples	Approximate MILP	MILP-NLP approach	Bai et al. ³	Bai et al. ⁴
Example 1	Number of transfer pipelines	2	2	—
	Number of time slots	13	13	—
	Total quadratic deviation	7.87	6.94	7.52
	CPU time (s)	—	1433 ^a	79.2 ^b
Example 2	Number of transfer pipelines	2	—	2
	Number of time slots	8	—	15
	Number of time slots	9	—	—
	Total quadratic deviation	14.42	13.50	15.53
	Total quadratic deviation	10.89	9.74	—
	CPU time (s)	—	3604 ^a	350 ^b

^aTotal CPU time. ^bMean CPU time per major iteration or run.

Table 5. Best MCOBS Operations Schedule Found for Example 2 Using Eight Time Slots

		Time slot							
		s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8
start time (day)		0	2.08	9.24	12.08	17.92	21.25	24.58	26.25
end time (day)		2.08	9.24	12.08	17.92	21.25	24.58	26.25	27.92
tank queue in	p_1	t_1	t_{15}	t_{15}	t_4	t_{10}	t_{12}	t_8	t_6
tank queue in	p_2	t_{14}	t_2	t_3	t_{11}	t_9	t_7	t_{13}	t_5
crude oil in	p_1	cr_1	cr_6	cr_6	cr_2	cr_4	cr_5	cr_3	cr_3
crude oil in	p_2	cr_6	cr_1	cr_2	cr_4	cr_3	cr_3	cr_5	cr_2
flow rate in	p_1	300.0	208.8	207.2	142.9	250.0	250.0	250.0	250.0
flow rate in	p_2	200.0	291.2	292.8	357.1	250.0	250.0	250.0	250.0
batch size (m ³)	p_1	15,000	35,850	14,150	20,000	20,000	20,000	10,000	10,000
batch size (m ³)	p_2	10,000	50,000	20,000	50,000	20,000	20,000	10,000	10,000
sulfur (ppm)		1468	1485	1342	637	1693	1814	1814	1841

one charging tank should deliver crude oil into a transfer pipeline over two consecutive time slots. In turn, $|B| = |P| \times |S| = 2 \times 8 = 16$. Sizes of the approximate MILP and the NLP models for $|S| = 8$ and $|S| = 9$ can be found in Table 1. When $|S| = 8$, the best solution of the approximate MILP has been discovered in 1415 s of CPU time, and the optimality gap of 0.01 is found in 2602 s. The MILP solution features a quadratic yield deviation of 14.36 that drops to 13.50 by subsequently solving the NLP (Table 2). Since the MILP provides a very good solution for the MINLP, it only takes 1.3 s of CPU time to solve the NLP using the MILP solution as the starting point. Both terms of the objective function decrease when the model nonlinearities are considered through the NLP. The first term is reduced from 9.71 to 9.42 and the second one from 4.65 to 4.08. Solution features such as the sequence of tanks allocated to each transfer pipeline, the types and flow-rates of crude oils blended in each time slot, the slot initial and end times, and the sulfur concentration in the feedstock along the planning horizon are all given in Table 5. Tank t_{15} is the only one discharging its initial content into pipeline P_1 in two consecutive time slots. The other ones are connected to the assigned pipeline during a single slot. The TBP curves for the series of eight feedstocks are given in Figure 5.

The best schedule for Example 2 reported by Bai et al.⁴ was found by using 15 slots and one virtual tank (t_{16}), adopting a maximum computing time of 350 s per run, and executing 35 independent runs. Table 4 compares the best solution provided by our approach with the one discovered by Bai et al.⁴ Based on the results published by Bai et al.⁴ (Supporting Information), we compute its total quadratic yield deviation and obtain a value of $15.53 > 13.50$. Then, our approach with eight time slots discovered a MCOBS operational schedule having a better

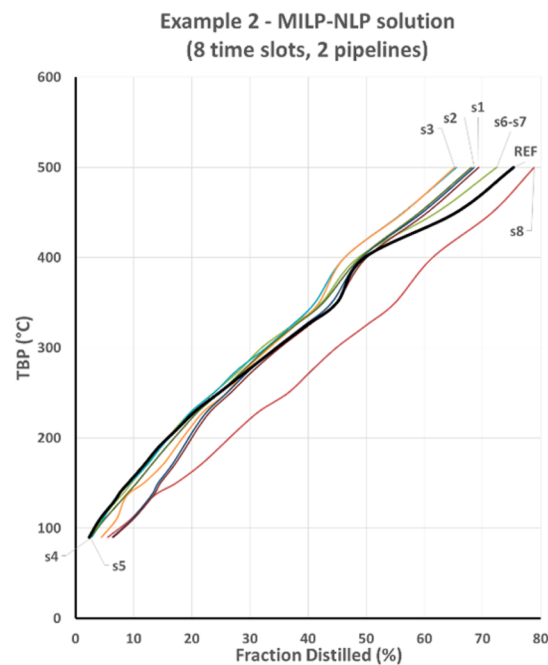


Figure 5. TBP distribution curves of the feedstocks for Example 2 using eight time slots.

quality. Part of the improvement comes from supplying fewer feedstocks to the CDU.

When $|S| = 9$ is adopted, then $|B| = 2 \times 9 = 18$. By increasing the number of time slots by one, the approximate MILP cannot be solved to optimality within the CPU time limit. Nonetheless, a

Table 6. Best MCOBS Operations Schedule for Example 2 Using Nine Time Slots

		Time slot								
		s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9
start time (day)		0	2.92	10.00	12.92	14.58	18.68	20.17	22.92	26.25
end time (day)		2.92	10.00	12.92	14.58	18.68	20.17	22.92	26.25	27.92
tank queue in	p_1	t_1	t_{15}	t_{15}	t_{14}	t_3	t_6	t_7	t_{10}	t_8
tank queue in	p_2	t_{12}	t_2	t_4	t_5	t_{11}	t_{11}	t_{11}	t_9	t_{13}
crude oil in	p_1	cr_1	cr_6	cr_6	cr_6	cr_2	cr_3	cr_3	cr_4	cr_3
crude oil in	p_2	cr_5	cr_1	cr_2	cr_2	cr_4	cr_4	cr_4	cr_3	cr_5
flow rate in (m^3/h)	p_1	214.3	205.9	214.3	250.0	203.3	280.6	303.0	250.0	250.0
flow rate in (m^3/h)	p_2	285.7	294.1	285.7	250.0	296.7	219.4	197.0	250.0	250.0
batch size (m^3)	p_1	15,000	35,000	15,000	10,000	20,000	10,000	20,000	20,000	10,000
batch size (m^3)	p_2	20,000	50,000	20,000	10,000	29,182	7,818	13,000	20,000	10,000
sulfur (ppm)		920	1479	1359	1444	672	1832	1934	1692	1813

solution much better than the one obtained with $|S| = 8$ was discovered. Model sizes and CPU time requirements are given in Tables 1 and 2, respectively. As the best MILP-solution found within the CPU time limit presents a rather large optimality gap, the approximate MILP was solved again after fixing the sequencing variables ($XF_{t,p}$, $X_{t,t',p}$, $XL_{t,p}$) to their current values. The new feasible solution for the MILP found in only 1.2 s of CPU time presents a total quadratic yield deviation equal to 10.89 that drops to 9.74 when the MILP-NLP approach is applied (Table 2). Both terms of the objective function present lower values at the NLP solution. The first term is reduced from 7.67 to 7.31, while the second falls from 3.22 to 2.43. Compared with the results reported by Bai et al.,⁴ the total quadratic yield deviation has been decreased from 15.30 to 9.74, i.e., a significant 57.1% reduction (Table 4). The best MCOBS operations schedule using nine time slots is shown in Table 6 and Figure 6,

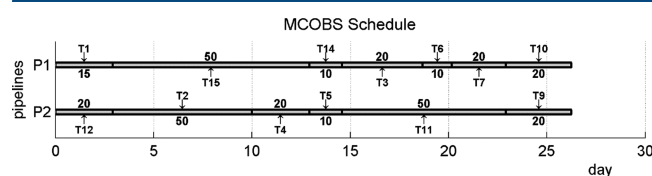


Figure 6. Best MCOBS operations schedule for Example 2 using nine time slots.

while Figure 7 displays the TBP distribution curves for the nine feedstocks supplied to the CDU along the planning horizon. Again, the TBP curve labeled REF corresponds to the desired feedstock. It is observed that the yield deviations mostly occur for heavy microcuts with low wf_{mc} . Using 10 time slots, the quadratic yield deviation rises to 11.33.

The time at which all the MCOBS operations finish can be determined by dividing the total amount of crude initially available in the charging tanks and the fixed flow-rate of the blend obtained in the mixing line, i.e., $[335,000 \text{ m}^3 / (500 \text{ m}^3/\text{h} \times 24 \text{ h/day})] = 27.92$ days.

5.3. A Pair of New Variants of Example 2. Two variants of Example 2, called Examples 3 and 4, have been generated and solved to show some additional advantages of the proposed MINLP formulation: the use of an arbitrary number of transfer pipelines and the handling of nonzero release times for the discharge of crude oils from charging tanks. The latter case arises when new parcels of crude oils are received from pipes or vessels and stored in some preassigned charging tanks during the current planning horizon. The delivery of crude oil can start after loading the whole crude parcel in the assigned charging tank. We assume

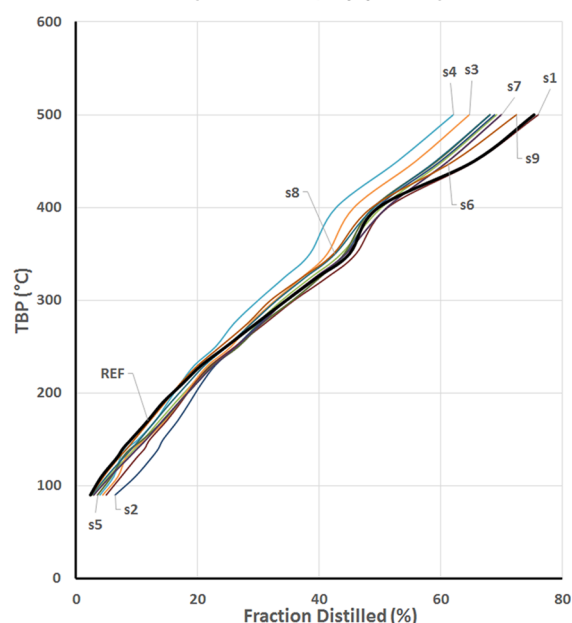
Example 2 - MILP-NLP solution
(9 time slots, 2 pipelines)

Figure 7. TBP distribution curves of the feedstocks for Example 2 using 9 time slots.

that such release times (rt_i) are problem data. Tanks delivering their initial crude inventory to transfer pipelines have $rt_i = 0$.

5.3.1. Example 3. In Example 3, an additional transfer pipeline is available, i.e., $|P| = 3$. Then, flows from three charging tanks are blended in the mixing pipeline at every time slot. With regards to Example 2, a single datum has been changed. To produce feedstocks with acceptable sulfur concentration and TBP distribution data closer to the desired values, the minimum discharge flow-rate of crude oils from charging tanks has been reduced from 100 to 50 m^3/h . Despite $|P| = 3$, a total of 27.92 days is still needed to complete the MCOBS operations because the total amount of crude oil to be discharged from the charging tanks and the fixed flow-rate in the mixing line both remain unchanged. The minimum number of slots for $|P| = 3$ must not be lower than $|T|/|P| = 15/3 = 5$. We choose $|S| = 6$ slots and $|B| = 3 \times 6 = 18$ batches. Model sizes and computational results are given in Tables 1 and 2, respectively. The best MCOBS operations schedule found by the proposed MILP-NLP solution strategy is shown in Table 7. The quadratic yield deviation drops from 9.74 to 8.21 when $|P|$ is increased from 2 to 3 mostly

Table 7. Best Schedule of Unloading Operations for Example 3 Using Six Slots and Three Transfer Pipelines

		Time slot					
		s_1	s_2	s_3	s_4	s_5	s_6
start time (day)		0	2.31	10.00	14.58	18.10	24.58
end time (day)		2.31	10.00	14.58	18.10	24.58	27.92
tank queue in	p_1	t_1	t_{15}	t_{15}	t_4	t_4	t_8
tank queue in	p_2	t_{14}	t_5	t_9	t_{13}	t_{11}	t_{12}
tank queue in	p_3	t_2	t_2	t_3	t_{10}	t_7	t_6
crude oil in	p_1	cr_1	cr_6	cr_6	cr_2	cr_2	cr_3
crude oil in	p_2	cr_6	cr_2	cr_3	cr_5	cr_4	cr_5
crude oil in	p_3	cr_1	cr_1	cr_2	cr_4	cr_3	cr_3
flow rate in (m ³ /h)	p_1	270.0	189.8	136.4	144.8	50.0	125.0
flow rate in (m ³ /h)	p_2	180.0	54.2	181.8	118.4	321.4	250.0
flow rate in (m ³ /h)	p_3	50.0	256.0	181.8	236.8	128.6	125.0
batch size in (m ³)	p_1	15,000	35,000	15,000	12,222	7,778	10,000
batch size in (m ³)	p_2	10,000	10,000	20,000	10,000	50,000	20,000
batch size in (m ³)	p_3	2,778	47,222	20,000	20,000	20,000	10,000
sulfur (ppm)		1430	1423	1895	695	1168	1814

because of the reduction in the number of feedstocks supplied to the CDU. The TBP curves for the feedstocks are shown in Figure 8.

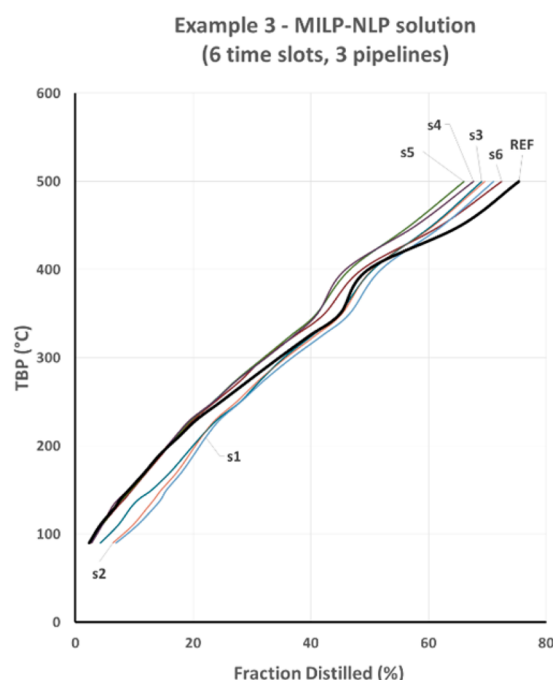


Figure 8. TBP curves of the feedstocks at the best solution found for Example 3.

5.3.2. Example 4. Example 4 is derived from Example 2 by assuming that parcels of crude oils in charging tanks (t_3 , t_4 , t_{12} and t_{13}) will be received during the current planning horizon. The release times specified for those tanks are the start of days 12, 19, 16 and 23, respectively. To solve the example, it has been adopted: $|S| = 9$ and $|B| = 2 \times 9 = 18$. Computational results are given in Tables 1, 2, and 8.

It is observed that the best solution for Example 4 satisfies all release-time constraints, but the quadratic yield deviation grows from 9.74 to 14.33 when compared with the nonconstrained solution for Example 2 with $|S| = 9$. The TBP distribution curves for the resulting feedstocks are shown in Figure 9.

6. CONCLUSIONS

This work has presented a novel MINLP formulation for the scheduling of crude oil blending operations in oil refineries. The proposed model assumes a refinery configuration previously studied by Bai et al.^{3,4} involving a multipipeline crude oil blending system (MCOBS) composed of IPI transfer pipelines continuously receiving crude oils from the charging tanks and converging into a single mixing pipeline. The resulting blend is supplied to the crude distillation unit. Different types of crude oils are available, but only one of them is stored in every tank all over the short-term scheduling horizon. Besides, charging tanks sequentially discharge crude oils into the assigned transfer pipeline, and the unloading operation from each one continues until the tank is empty. Direct precedence variables are used to choose the emptying sequence of the tanks assigned to the same pipeline. In addition, the MINLP model includes a set of time slots to synchronize the simultaneous unloading of crude oils from IPI different tanks into the same number of transfer lines. Some tanks may be unloading crude oil during two or more time slots. A simple rule is given to choose a proper number of time slots. The desired true boiling point (TBP) distribution curve for the feedstock matching the final product demands is the property selected as the problem target. Then, a series of qualified and stable feedstocks are obtained by using an objective function that minimizes the weighted sum of both the cumulative quadratic deviation of that property from the target value and the cumulative quadratic change of the feedstock TBP curve in two consecutive time slots.

An MILP-NLP sequential strategy has been adopted to solve the MINLP formulation. To this end, the MINLP model has been linearized to get an approximate MILP that provides good feasible solutions for the original problem. By using the MILP solution as the starting point and fixing the integer variables in the original MINLP formulation, the resulting NLP is subsequently solved. The proposed formulation and the MILP-NLP solution strategy have been tested by solving a pair of examples first introduced by Bai et al.^{3,4} involving a multipipeline blending system composed of two transfer pipelines that receive six different types of crude oils from up to 15 charging tanks. For both examples, largely improved schedules compared with those reported in previous contributions have been obtained. In one of the examples, the best solution found with our approach presents a decrease in the objective function as large as 57.1%. Moreover,

Table 8. Best MCOBS Operations Schedule for Example 4 Using Nine Slots and the MILP-NLP Approach

		Time slot								
		s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9
start time (day)		0	2.33	8.98	10.42	13.75	16.25	19.58	24.52	26.25
end time (day)		2.33	8.98	10.42	13.75	16.25	19.58	24.52	26.25	27.92
tank queue in	p_1	t_1	t_2	t_5	t_{10}	t_3	t_7	t_{11}	t_{11}	t_{13}
tank queue in	p_2	t_{15}	t_{15}	t_{15}	t_9	t_{14}	t_{12}	t_4	t_6	t_8
crude oil in	p_1	cr_1	cr_1	cr_2	cr_4	cr_2	cr_3	cr_4	cr_4	cr_5
crude oil in	p_2	cr_6	cr_6	cr_6	cr_3	cr_6	cr_5	cr_2	cr_3	cr_3
flow rate in (m ³ /h)	p_1	268.5	313.3	289.5	250.0	333.3	250.0	331.3	258.7	250.0
flow rate in (m ³ /h)	p_2	231.5	186.7	210.5	250.0	166.7	250.0	168.7	241.3	250.0
batch size in (m ³)	p_1	15,000	50,000	10,000	20,000	20,000	20,000	39,278	10,722	10,000
batch size in (m ³)	p_2	12,931	29,800	7269	20,000	10,000	20,000	20,000	10,000	10,000
sulfur (ppm)		1528	1443	1350	1693	1245	1814	652	1653	1814

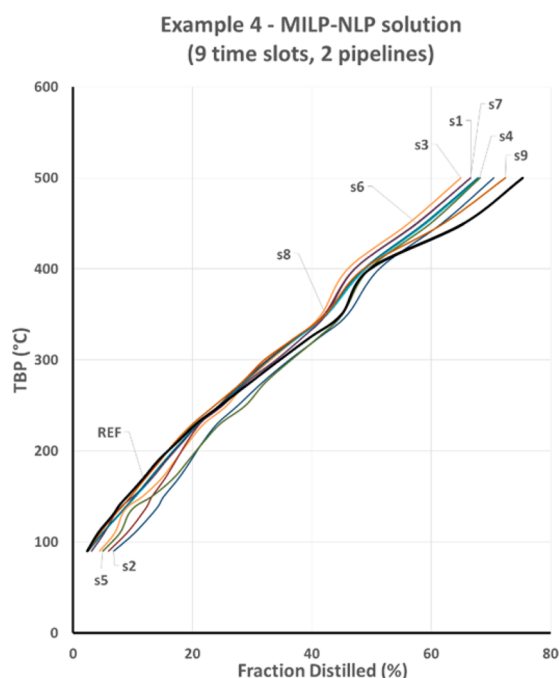


Figure 9. TBP curves of the feedstocks at the best solution for Example 4.

two variants of the example presented by Bai et al.⁴ have also been generated and successfully solved to study the effect of using more transfer pipelines and handling nonzero release times for the discharge of crude oils from some charging tanks.

We are currently working on an algorithmic approach for the optimal assignment of charging tanks to CDUs in oil refineries with a very large number of tanks. After allocating charging tanks to CDUs, the schedule of the MCOBS operations for each CDU will be sequentially determined using the MINLP approach proposed in this work.

■ ASSOCIATED CONTENT

Supporting Information

The Supporting Information is available free of charge on the ACS Publications website at DOI: 10.1021/acs.iecr.7b02449.

Tables with all the data for Examples 1–4, model sizes and computational results obtained for all of them, and the schedule of unloading operations. Figures depicting TBP curves for feedstocks at best solutions for Examples 1–4 as

well as those reported in previous contributions for Examples 1 and 2. (PDF)

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Notes

The authors declare no competing financial interest.

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■ NOTATION

Sets

- B = batches
- CR = crude oils
- K = trace elements
- MC = pseudocomponents or microcuts of a crude oil
- P = transfer pipelines
- S = time slots
- T = charging tanks

Parameters

- $c_{k,cr}$ = concentration of trace element k in crude oil cr
- c_k^{max} = maximum allowed concentration of trace element k in the feedstock
- $cydref_{mc}$ = desired cumulative yield up to microcut mc in the mixed feedstock
- dd_t = emptying due date for tank t
- inv_t = crude inventory in charging tank t
- $frate$ = fixed feedstock flow-rate in the mixing line
- $lyld_{min}, lyld_{max}$ = limiting values for the total yield of light components in the feedstock
- $qmin$ = minimum batch size
- $rmin, rmax$ = limiting discharge flow-rates of crude oil from charging tanks
- wf_{mc} = normalized weight coefficient for microcut mc
- $ydref_{mc}$ = desired volumetric yield of microcut mc in the mixed feedstock
- $ylld_{mc,cr}$ = volumetric yield of microcut mc in the crude oil cr

Binary Variables

- XF_{tp} = denotes that tank t is the first delivering crude oil to pipeline p if $XF_{tp} = 1$
 XL_{tp} = denotes that tank t is the last delivering crude oil to pipeline p if $XL_{tp} = 1$
 $X_{t,t',p}$ = denotes that tank t immediately precedes tank t' in the tank emptying sequence of pipe p
 $YB_{b,tp}$ = denotes that batch b is discharged from tank t into pipeline p
 $YS_{b,p,s}$ = denotes that batch b is discharged into pipeline p during the time slot s
 YT_{tp} = assign charging tanks to transfer pipelines

Integer Variables

- RT_{tp} = indicates the position of tank t in the line of tanks of pipeline p

Continuous Variables

- $ADIF_{mc,s}$ = absolute change of the amount of microcut mc in two consecutive time slots
 $CB_{b,tp}$ = completion time for the discharge of batch b from tank t into pipeline p
 CS_s = end time of slot s
 $CYMF_{mc,s}$ = cumulative yield up to microcut mc in the feedstock supplied during slot s
 $DEV_{mc,s}$ = absolute deviation between the size of mc and the reference one over slot s
 $DIF_{mc,s}$ = change of the amount of microcut mc in consecutive time slots ($s-1$) and s
 $LB_{b,tp}$ = length of the unloading operation of batch b from tank t into pipeline p
 $QB_{b,tp}$ = size of batch b discharged from tank t into pipeline p
 $QK_{k,b,p,s}$ = amount of trace element k in batch b discharged into pipe p during slot s
 $QMB_{mc,b,p,s}$ = amount of microcut mc in batch b discharged into pipe p during slot s
 $QMF_{mc,s}$ = amount of microcut mc in the feedstock during the slot s
 $QREF_{mc,s}$ = desired amount of microcut mc in the feedstock supplied during slot s
 $SB_{b,tp}$ = starting time for the discharge of batch b from tank t into pipeline p
 SS_s = starting time of slot s
 $WP_{b,p}$ = denotes the existence of batch b discharged into pipeline p
 $YMF_{mc,s}$ = volumetric yield of microcut mc in the feedstock supplied during the slot s

REFERENCES

- (1) Reddy, P. C.; Karimi, I. A.; Srinivasan, R. A new continuous-time formulation for scheduling crude oil operations. *Chem. Eng. Sci.* **2004**, *59*, 1325–1341.
- (2) Li, J.; Li, W.; Karimi, I. A.; Srinivasan, R. Improving the robustness and efficiency of crude scheduling algorithms. *AIChE J.* **2007**, *53*, 2659–2680.
- (3) Bai, L.; Jiang, Y. H.; Huang, D. X.; Liu, X. G. A novel scheduling strategy for crude oil blending. *Chin. J. Chem. Eng.* **2010**, *18* (5), 777–786.
- (4) Bai, L.; Jiang, Y. H.; Huang, D. X. A novel two-level optimization framework based on constrained ordinal optimization and evolutionary algorithms for scheduling of multipipeline crude oil blending. *Ind. Eng. Chem. Res.* **2012**, *51*, 9078–9093.
- (5) Menezes, B. C.; Kelly, J. D.; Grossmann, I. E. Improved swing-cut modeling for planning and scheduling of oil-refinery distillation units. *Ind. Eng. Chem. Res.* **2013**, *52*, 18324–18333.

- (6) Lee, H.; Pinto, J. M.; Grossmann, I. E.; Park, S. Mixed-integer linear programming model for refinery short-term scheduling of crude oil unloading with inventory management. *Ind. Eng. Chem. Res.* **1996**, *35*, 1630–1641.
- (7) Jia, Z.; Ierapetritou, M.; Kelly, J. D. Refinery short-term scheduling using continuous time formulation: Crude oil operations. *Ind. Eng. Chem. Res.* **2003**, *42*, 3085–3097.
- (8) Furman, K. C.; Jia, Z.; Ierapetritou, M. A robust event-based continuous time formulation for tank transfer scheduling. *Ind. Eng. Chem. Res.* **2007**, *46*, 9126–9136.
- (9) Karuppiyah, R.; Furman, K. C.; Grossmann, I. E. Global optimization for scheduling refinery crude oil operations. *Comput. Chem. Eng.* **2008**, *32*, 2745–2766.
- (10) Saharidis, G.; Minoux, M.; Dallery, Y. Scheduling of loading and unloading of crude oil in a refinery using event-based discrete time formulation. *Comput. Chem. Eng.* **2009**, *33*, 1413–1426.
- (11) Mouret, S.; Grossmann, I. E.; Pestiaux, P. A novel priority-slot based continuous-time formulation for crude-oil scheduling problems. *Ind. Eng. Chem. Res.* **2009**, *48*, 8515–8528.
- (12) Mouret, S.; Grossmann, I. E.; Pestiaux, P. Time representations and mathematical models for process scheduling problems. *Comput. Chem. Eng.* **2011**, *35*, 1038–1063.
- (13) Mouret, S.; Grossmann, I. E.; Pestiaux, P. A new Lagrangian decomposition approach applied to the integration of refinery planning and crude-oil scheduling. *Comput. Chem. Eng.* **2011**, *35*, 2750–2766.
- (14) Yadav, S.; Shaik, M. A. Short-term scheduling of refinery crude oil operations. *Ind. Eng. Chem. Res.* **2012**, *51*, 9287–9299.
- (15) Castro, P.; Grossmann, I. E. Global optimal scheduling of crude oil blending operations with RTN continuous-time and multiparametric disaggregation. *Ind. Eng. Chem. Res.* **2014**, *53*, 15127–15145.
- (16) Reddy, P. C.; Karimi, I. A.; Srinivasan, R. Novel solution approach for optimizing crude oil operations. *AIChE J.* **2004**, *50*, 1177–1197.
- (17) Li, J.; Misener, R.; Floudas, C. A. Continuous-time modeling and global optimization approach for scheduling crude oil operations. *AIChE J.* **2012**, *58*, 205–226.
- (18) Cerdá, J.; Pautasso, P. C.; Cafaro, D. C. An efficient approach for scheduling crude oil operations in marine-access refineries. *Ind. Eng. Chem. Res.* **2015**, *54*, 8219–8238.
- (19) Fu, G.; Sanchez, Y.; Mahalec, V. Hybrid model for optimization of crude oil distillation units. *AIChE J.* **2016**, *62* (4), 1065–1078.