Research Paper

INTERACTION BETWEEN HYDRODYNAMICS AND CHEMICAL REACTION RATE IN VERTICAL GAS-SLURRY COCURRENT TURBULENT TUBE REACTOR.

I. PERFORMANCE INDICATORS

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Abstract

To analyze the performance of a vertical tube reactor involving a gas-slurry reacting system in dispersed bubble flow and upflow, downflow, and upflow-downflow (alternating) configurations, several performance indicators are defined and expressed in terms of integral equations. A local effectiveness factor associated with the efficiency for gas-slurry mass transfer, conversion of the liquid reactant, the local unconverted mass fraction of the gas phase, and the pressure drop are taken as performance indicators. Bounding criteria for diffusion-controlled and chemical reaction-controlled regimes are established, and their transitions along the reactor are also discussed. A generic heterogeneous catalytic hydrogenation, exemplified by the single reaction $A(l) + H_2(g) \rightarrow B(l)$, is taken as a suitable reference system.

Introduction

The study of multi-phase chemical reactors operating at low superficial velocities (*e.g.,* bubble or packed columns) has received considerable attention. However, much less is still known about continuous slurry reactors operated at high flow velocities, such as the compact devices derived from the simple gas-liquid tubular contactor. Reactors belonging to this class are appropriate to carry out gas-liquid reactions in fast or instantaneous reaction regimes, as well as heterogeneous catalytic reactions (gas-liquid-solid systems) in a moderately fast reaction regime. The turbulent tube reactor (TTR) can be classified within the cocurrent flow category as dispersed gas flows cocurrently upwards or downwards together with the slurry phase in which chemical reaction takes place.

A distinctive feature of TTRs stems from the fact that the gas-liquid interfacial area depends entirely upon the turbulence level of the multiphase mixture flowing in forced flow. Thus, high flow velocities are required to create intimate gas-liquid-solid mixing, at the expense of important friction losses. As a consequence, high pressure drops and pronounced changes of the gas holdup are often established along the tubular device. Gravitational effects can also play an important role in vertical configurations. Therefore, the mass, momentum and energy balances must be simultaneously solved to account for the coupled effects of the chemical reaction, compressible flow, pressure drop and variable holdups on the reactor performance.

Scant information is available about the complex interaction between hydrodynamics and chemical reaction rate for systems involving heterogeneous catalytic reactions taking place in TTRs operated in the moderately fast reaction regime. We used a one-dimensional mathematical model to describe the overall behavior of a TTR operating in the dispersed bubble flow pattern (DBP), focusing on its application to the hydrogenation of soybean oil (Cantero, 1993; Cantero *et al.,* 1994, 1996). However, detailed analyses of the constraints imposed by the hydrodynamics on the reactor performance were not made then. This work now extends the analysis, focusing on the complex intertwining of hydrodynamics and chemical reaction rate, at least for simple situations, such as isothermal operation in upflow (U-configuration), downflow (D-configuration) or alternating upflow-downflow (A-configuration), as outlined in **Fig. 1**. To this end, a single heterogeneous catalytic hydrogenation reaction, such as the following,

$$\mathbf{A}(l) + \mathbf{H_2}(g) \rightarrow \mathbf{B}(l)$$

will be considered to avoid additional complexities owing to complex reaction kinetics.

Since the TTR is an integral reactor, this first part of the present work deals with the definition of performance indicators using integral equations as the basis from which more appropriate mathematical expressions of those indicators are obtained. A local effectiveness factor, the local conversion of the liquid reactant, the local unconverted mass fraction of the gas phase and the pressure drop make up a set of performance indicators which provides a satisfactory description of the efficiency of the TTR. Basic aspects of the DBP that complete the physical model are also briefly presented.

In the second part, the mathematical expressions of the performance indicators are applied to the analysis of a continuous industrial process: mild hydrogenation (‘brushing’) of soybean oil.

1. Flow Patterns

Three flow regimes are possible in a vertical TTR: (i) dispersed bubble, (ii) slow bubble, and (iii) intermittent flow. The steady-state cocurrent flow patterns map for the hydrogen-soybean oil system are shown in **Fig. 2**. The dispersed bubble pattern (DBP) is the most adequate, since it maximizes the gas-slurry interfacial area, and it allows slurry holdups in the range $0.5 < \varepsilon_L < 1$ (Cantero *et al.*, 1996). Hence, the attention is restricted here to conditions under which the reacting mixture flows in the desirable DBP.

In order to attain the DBP throughout the TTR, a minimum value of the local energy dissipation rate per unit mass of slurry (E_{Hmin}) is needed to sustain the high dispersion of bubbles (Barnea *et al.*, 1985):

$$E_{\mathrm{Hmin}} = 0.556\, g\, (g\, D)^{0.5} B_0^{-0.25} F_{\mathrm{C}}^{2.5} \tag{1}$$

where

$$B_0 = \rho_{\mathrm{L}} g\ D^2 \sigma^{-1} \tag{2}$$

$$F_{\mathrm{C}} = 0.725 + 4.15\ \varepsilon_{\mathrm{G}}^{0.5} \tag{3}$$

Equations (1)-(3) show that the higher the interfacial tension or the gas holdup, the larger the E_{Hmin}. The effect of each of the former variables can be independently evaluated by the Bond number (B_0) and the coalescence factor (F_{C}), respectively.

The dynamic equilibrium between coalescence and break-up mechanisms is established whenever the local power dissipation (E_{H}) is greater than E_{Hmin}, or, in mathematical terms whenever (Barnea *et al.*, 1985):

$$E_{\mathrm{H}} = 2\, f_{\mathrm{L}} Fr\ \phi_{\mathrm{L}}^2 (V_{\mathrm{m}} / \rho_{\mathrm{m}}) \geq E_{\mathrm{Hmin}} \tag{4}$$

where the right hand side expresses the local power dissipation. The friction factor (f_{L}) in DBP and the Lockhart-Martinelli correction factor (ϕ_{L}^2) in near-homogeneous bubble flow are given by (Wallis, 1969):

$$f_{\mathrm{L}} = 4.6 \times 10^{-2}\ Re_{\mathrm{L}}\ ^{-0.2} \tag{5}$$

$$\phi_{\mathrm{L}}^2 = (1 - \varepsilon_{\mathrm{G}})^2 \tag{6}$$

and the mass mean velocity (V_{m}) and the mean density (ρ_{m}) of the multiphase mixture are:

$$V_{\mathrm{m}} = (G_{\mathrm{L}} + G_{\mathrm{G}}) / \rho_{\mathrm{m}} \tag{7}$$

$$\rho_{\mathrm{m}} = \rho_{\mathrm{L}} (1 - \varepsilon_{\mathrm{G}}) + \rho_{\mathrm{G}}\ \varepsilon_{\mathrm{G}} \tag{8}$$

In most cases, for both upflow and downflow conditions, a stable DBP can be achieved whenever the liquid superficial velocity is higher than 1 or 2 m/s and the gas holdup (ε_G) is kept under 0.5. If $\varepsilon_G \leq 0.2$, E_{Hmin} increases more than E_H as the gas holdup is larger, whereas the opposite occurs if $\varepsilon_G \geq 0.6$. Hence, the changes in holdup resulting from the gas consumption by chemical reaction, combined with simultaneous frictional losses or hydrostatic pressure changes may lead to significant modification of the local values of the E_H / E_{Hmin} ratio along the TTR, eventually impairing the achievement of the DBP.

2. Mathematical Model of the TTR

Table 1 summarizes the coupled system of non-linear ordinary differential equations (ODEs) which describes the mass and momentum balances, i.e., Eqs. (9)-(17). The main hypotheses leading to this one-dimensional gas-slurry reactor model are: (i) Continuous and isothermal operation; (ii) Steady-state regime; (iii) Pure ideal-gas phase; (iv) Physical properties of the slurry phase identical to those of the liquid phase; (v) Non-volatile liquid phase; (vi) Negligible backmixing in both slurry and gas phases; (vii) Negligible intraparticle and extraparticle resistances; (viii) Negligible enhancement of the gas absorption rate if an eventual presence of small catalyst particles inside the gas-liquid film takes place; and (ix) First-order reaction rate with respect to both A and H_2 reactants. Additional details on the whole set of model hypotheses can be found elsewhere (Cantero *et al.*, 1996).

Analysis of the TTR using integral equations rather than ODEs leads to particularly advantageous results, owing to the integral nature of this tubular reactor. The advantages rest on the following facts: (i) it is possible to associate the terms of the integral equations with contributions due to the gas-slurry mass transfer rate, the chemical reaction rate and the feed conditions, and (ii) the integral equations can be numerically solved by successive approximations using simple iterative schemes. The system of coupled integral equations which corresponds to the original set of

ODEs is shown in Table 1 as well, i.e., Eqs. (18)-(25). Detailed mathematical and numerical features of this system of equations can be found elsewhere (Cantero *et al.*, 1994).

3. Performance Indicators and their Functional Interrelationships

3.1 Gas-slurry mass transfer effectiveness factor

In the absence of selectivity requirements, the highest efficiency to convert the liquid reactant **A** into **B** is obtained when the absorption rate of $\mathbf{H_2}$ is highest. That is, when either the catalytic activity or the catalyst loading (or both) are high and the TTR operates as an efficient gas-slurry contactor. In order to evaluate the local gas-slurry mass transfer efficiency, it is convenient to introduce the following definition of a local effectiveness factor:

$$\eta(\xi) = \frac{R_H(\xi)}{R_H^*(\xi)} \tag{26}$$

where R_H is the actual rate of reaction per unit volume of the reactor

$$R_H(\xi) = \varepsilon_L(\xi)\, k_C\, w_C\, w_A(\xi)\, w_H(\xi) \tag{27}$$

and, R_H^* is the reaction rate obtained by neglecting the gas-slurry transport resistance

$$R_H^*(\xi) = \varepsilon_L(\xi)\, k_C\, w_C\, w_A(\xi)\, w_H^*(\xi) \tag{28}$$

where w_H^* is the local saturation solubility of $\mathbf{H_2}$.

The combination of Eqs. (26)-(28) yields

$$\eta(\xi) = \frac{w_H(\xi)}{w_H^*(\xi)} \tag{29}$$

It can be readily noticed that $\eta \to 1$ whenever the TTR operates in the absence of gas-slurry mass transfer resistance, whereas $\eta < 1$ if the transport resistance is not negligible.

The value of the local effectiveness factor is a function of upstream conditions because the TTR is a continuous integral reactor. Consequently, it seems appropriate to cast the gas-slurry mass transfer effectiveness factor in terms of an integral equation. So, by substituting Eq. (20) into Eq. (29), this performance indicator takes the following form:

$$\eta(\xi) \;=\; w_{\mathrm{H}}^{*\mathrm{o}} \Big/ w_{\mathrm{H}}^{*}(\xi) + \int_{0}^{1} d\bar{\xi}\, G_{\mathrm{H}}(\xi / \bar{\xi})\; \varepsilon_{\mathrm{G}}(\bar{\xi})\; St\;\; w_{\mathrm{H}}^{*}(\bar{\xi}) \Big/ w_{\mathrm{H}}^{*}(\xi) \tag{30}$$

which in the absence of dissolved gaseous reactant in the feed stream simplifies directly to:

$$\eta(\xi) \;=\; \int_{0}^{1} d\bar{\xi}\, G_{\mathrm{H}}(\xi / \bar{\xi})\; \varepsilon_{\mathrm{G}}(\bar{\xi})\, St\;\; w_{\mathrm{H}}^{*}(\bar{\xi}) \Big/ w_{\mathrm{H}}^{*}(\xi) \tag{31}$$

where the Green function, defined by Eq. (24), can be conveniently rewritten as:

$$G_{\mathrm{H}}(\xi / \bar{\xi}) = \exp\left\{ -\int_{\bar{\xi}}^{\xi} d\hat{\xi}\, \varepsilon_{\mathrm{G}}(\hat{\xi})\, St \left[1 + \frac{\varepsilon_{\mathrm{L}}(\hat{\xi})\, Da_{\mathrm{H}}\; w_{\mathrm{A}}(\hat{\xi})}{\varepsilon_{\mathrm{G}}(\hat{\xi})\, St} \right] \right\} \tag{32}$$

The expression in brackets on the right-hand side of Eq. (32), in turn, is the inverse of the effectiveness factor when the accumulation term in the mass balance equation of **H_2** is negligible; more exactly, if

$$\frac{d}{d\xi} w_{\mathrm{H}} = 0 \tag{33}$$

then

$$\eta(\xi) = \frac{1}{1 + \dfrac{\varepsilon_{\mathrm{L}}(\xi)\, Da_{\mathrm{H}}\; w_{\mathrm{A}}(\xi)}{\varepsilon_{\mathrm{G}}(\xi)\, St}} \tag{34}$$

The leading term on the right-hand side of Eq. (34) is clearly the ratio between the local contributions of the chemical reaction and the gas-slurry mass transfer rates. Thus, it directly defines an appropriate criterion to know the local controlling regime in the reaction device. In fact, under a chemical reaction rate-controlled regime:

$$\frac{Da_{\mathrm{H}}\;\, \varepsilon_{\mathrm{L}}(\xi)\; w_{\mathrm{A}}(\xi)}{St\;\, \varepsilon_{\mathrm{G}}(\xi)} \le 0.1 \tag{35}$$

and, consequently

$$0.91 \le \eta(\xi) \le 1.00 \tag{36}$$

whereas, under a gas-slurry mass transfer rate-controlled regime:

$$10 \leq \frac{Da_{\mathrm{H}}\ \varepsilon_{\mathrm{L}}(\xi)\ w_{\mathrm{A}}(\xi)}{St\ \varepsilon_{\mathrm{G}}(\xi)} \tag{37}$$

and, then

$$0.00 \leq \eta(\xi) \leq 0.09 \tag{38}$$

By inspecting Eqs. (35)-(38), it can be seen that the local effectiveness factor tends to approach its highest value whenever the gas-slurry mass transfer resistance becomes negligible with respect to the chemical reaction resistance. In practice, when operating near the depletion of the gaseous reactant ($\varepsilon_{\mathrm{G}} \to 0$), the condition given by Eq. (35) may not be fulfilled even though the local values of the Da_{H}/St ratio were small. Also, from a theoretical standpoint, operating a TTR with very low slurry holdup ($\varepsilon_{\mathrm{L}} \to 0$) might not grant the fulfillment of the condition established by Eq. (37) even for high values of the Da_{H}/St ratio (certainly this last scenario is just hypothetical, since for $\varepsilon_{\mathrm{L}} < 0.5$ the DBP becomes unstable). Likewise, it is obvious that under conditions of high conversion of the liquid reactant ($w_{\mathrm{A}} \to 0$) the chemical reaction-controlled regime can be eventually reached. Therefore, even if the Da_{H}/St ratio were kept constant along the reactor, changes in the controlling regime would be possible as a direct consequence of changes in the local holdups and/or concentration of the liquid reactant.

Equations (36) and (38) suggest the definition of a transition regime from the chemical reaction rate-controlled regime to the gas-slurry mass transfer rate-controlled regime whenever

$$0.10 < \eta(\xi) < 0.90 \tag{39}$$

Asymptotic behaviors able to give unity effectiveness factor are possible when: (i) the gas-slurry mass transfer rate is much higher than the chemical reaction rate, (ii) the catalyst loading and/or its activity are very low, and (iii) the liquid reactant becomes depleted. If the reaction is not fast, condition $\eta \to 1$ is most likely to occur. But this condition only indicates that the process rate is entirely controlled by the chemical reaction rate and does not necessarily imply that the tubular device operates as the more efficient TTR to convert the liquid reactant **A**, as discussed in the following.

3.2 Local conversion of the liquid reactant

By substituting Eqs. (19) and (34) into the classical definition of conversion, the interrelationship between conversion and local effectiveness factor is readily obtained. Thus, the conversion of the liquid reactant can be written, in a somewhat more convenient integral form, as

$$x_{\mathrm{A}}(\xi) = \int_0^1 d\bar{\xi}\, G_{\mathrm{A}}(\xi/\bar{\xi})\, Da_{\mathrm{H}}\ \varepsilon_{\mathrm{L}}(\bar{\xi})\, w_{\mathrm{H}}^{*}(\bar{\xi})\ \eta(\bar{\xi}) \left[1 - x_{\mathrm{A}}(\bar{\xi})\right] \tag{40}$$

Then, if the local conversion of the liquid reactant is used rather than the corresponding mass fraction in the slurry phase, Eq. (19) of the original set of integral equations may be substituted by Eq. (40).

Notice that, for any *Da* number, the higher the local effectiveness factor, the higher the local conversion of the liquid reactant. However, if the *Da* number is very low (i.e., the reaction involved is extremely slow) the local conversion of the liquid reactant will be low, too, even though $\eta \rightarrow 1$. This point will be discussed in some detail in Part II of this paper.

3.3 Local unconverted mass fraction of gas phase

The local unconverted mass fraction of the gaseous reactant indicates the extent of gas phase removal in the TTR. From Eq. (18), this performance indicator can be related to the local effectiveness factor by the following dimensionless expression

$$Y(\xi) = Y(0) - \int_0^1 d\bar{\xi}\, G_{\mathrm{Y}}(\xi/\bar{\xi})\ \varepsilon_{\mathrm{L}}(\bar{\xi})\, Da_{\mathrm{A}}\ w_{\mathrm{A}}(\bar{\xi})\, w_{\mathrm{H}}^{*}(\bar{\xi})\ \eta(\bar{\xi}) \tag{41}$$

The higher the local effectiveness factor, the higher the local unconverted mass of the pure gaseous reactant, for any given Da number. Other variables, such as slurry holdup and mass fraction of liquid reactant, have similar effects.

3.4 Pressure drop

The pressure drop is an important parameter in designing the TTR because the gas-slurry interfacial area depends strongly on the pumping energy supplied to the device which, in turn, influences the former. From Eq. (21), the pressure drop can be expressed as

$$\Delta P(\xi) = \Delta P^{\mathrm{T}} - \int_{0}^{1} d\bar{\xi}\, G_{\mathrm{P}}(\xi / \bar{\xi})\, Ge\, [2 f_{\mathrm{L}} Fr\, \phi_{\mathrm{L}}(\bar{\xi}) - \varepsilon_{\mathrm{L}}(\bar{\xi}) \cos\theta(\bar{\xi})] \tag{42}$$

where ΔP^{T} indicates the total pressure drop (the pressure at the reactor exit is normally prefixed). The pressure drop is directly proportional to the geometric factor (*Ge*), as shown in the leading term of the expression. The net contribution due to frictional losses and hydrostatic pressure change is given by the bracketed difference inside the integral of Eq. (42). In upflow ($\cos\theta < 0$), the hydrostatic pressure and friction losses are additive and, then, the local pressure always decreases upwards. As a consequence, gas holdups will increase by the compounded decompression effects, and the DBP may become unstabilized if ε_{G} reaches the limit value of 0.5. Conversely, in downflow the hydrostatic pressure increase ($\cos\theta > 0$) opposes friction losses. This attenuates the pressure drop and increases the possibilities of keeping the operation within the DBP.

Conclusion

A single criterion, useful to analyze the controlling regimes of the process rate in a TTR, has been established, namely: chemical reaction-rate controlled, intermediate and mass transfer rate-controlled. Even though the Da_{H}/St ratio is essentially constant along the tube, changes in the controlling regime along the tubular device are possible owing to the interrelationship of the chemical reaction rate and the energy dissipation (needed to generate interfacial area) with phase holdups and mass fraction of the liquid reactant, both of which strongly change along the reactor.

Four performance indicators have been defined and conveniently expressed in terms of integral equations: the gas-slurry mass transfer effectiveness factor, the local conversion of the liquid reactant, the local unconverted mass fraction of the gas phase and the pressure drop. Their strong interrelation, shown in mathematical terms by Eqs. (31), (40) and (41), fully describes the intimate, complex interaction between hydrodynamics and chemical reaction rate involved in a vertical cocurrent TTR.

Some illustrative results will be discussed in Part II, where the continuous process of the mild hydrogenation of soybean oil is chosen as a case study.

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Nomenclature

a = gas-liquid interfacial area referred to gas phase [$m^2 \bullet m^{-3}$]

B_o = Bond number, Eq.(2) [—]

D = tube diameter [m]

Da = Damköhler number, Eqs. (14) and (15) [—]

E_H = turbulent energy dissipation rate per unit mass of slurry phase [$W \bullet kg^{-1}$]

f_L = liquid-phase friction factor, Eq. (5) [—]

F_C = coalescence factor, Eq. (3) [—]

Fr = Froude number, Eq. (17) [—]

Fr_{min} = minimum Froude number needed to achieve dispersed bubble pattern [—]

g = acceleration of gravity [$m \bullet s^{-2}$]

$G(\xi / \bar{\xi})$ = Green´s function, Eqs. (22)-(25) [—]

Ge = geometrical parameter, Eq. (16) [—]

G_G = gas superficial mass velocity [$kg \bullet s^{-1} \bullet m^{-2}$]

G_L = slurry superficial mass velocity [$kg \bullet s^{-1} \bullet m^{-2}$]

k_C = second-order kinetic constant based on mass fractions [s^{-1}]

k_L = mass transfer coefficient [$m \bullet s^{-1}$]

L = reactor length [m]

M_H = molecular weight of reactant $\mathbf{H_2}$ [$kg \bullet kmol^{-1}$]

M_A = molecular weight of reactant **A** [$kg \bullet kmol^{-1}$]

P = local pressure [Pa]

R_H = reaction rate per unit volume of the TTR, Eq. (27) [$kmol \bullet m^{-3} \bullet s^{-1}$]

Re_{L} = Reynolds number [—]

St = Stanton number, Eq. (13) [—]

U_{D} = characteristic velocity ($g^{1/2}D^{1/2}$) [m•s^{-1}]

U_{L} = mean superficial velocity of slurry phase [m•s^{-1}]

V_{m} = mass mean velocity, Eq. (7) [m•s^{-1}]

w_{A} = mass fraction of **A** [—]

$w_{\mathrm{A}}^{\mathrm{o}}$ = reference mass fraction of **A** [—]

w_{C} = mass fraction of catalyst into slurry [—]

w_{H} = mass fraction of dissolved gas $\mathbf{H_2}$ [—]

w_{H}^{*} = local equilibrium mass fraction of $\mathbf{H_2}$ [—]

$w_{\mathrm{H}}^{*\mathrm{o}}$ = reference mass fraction of $\mathbf{H_2}$ [—]

x_{A} = conversion of **A** [—]

Y = unconverted mass fraction of gas phase [—]

Greeks

ε_{G} = local gas holdup [m^3•m^{-3}]

ε_{L} = local slurry holdup [m^3•m^{-3}]

η = local effectiveness factor for gas-liquid mass transfer, Eqs. (30) and (34) [—]

ξ = dimensionless axial coordinate [—]

ρ_{G} , ρ_{L} = density of gas and slurry phases [kg•m^{-3}]

ρ_{m} = mean density of the multiphase mixture $\left(\rho_{\mathrm{L}}(1-\varepsilon_{\mathrm{G}})+\rho_{\mathrm{G}}\ \varepsilon_{\mathrm{G}}\right)$ [kg•m^{-3}]

ϕ_{L} = multiphase flow correction for friction factor [—]

σ = interfacial tension [N•m^{-1}]

Θ = step function [—]

θ = flow inclination angle [rad]

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Table 1: One-dimensional model of the TTR

In terms of differential equations			
Mass balances		Dimensionless parameters	
Gas-phase: $\frac{d}{d\xi}Y = -\varepsilon_{\mathrm{G}}\,St\left[w_{\mathrm{H}}^{*} - w_{\mathrm{H}}\right]$	(9)	$St = (k_{\mathrm{L}} a\, w_{\mathrm{H}}^{*\mathrm{o}} D / U_{\mathrm{D}})\, Ge\, Fr^{-0.5}$	(13)
Liquid-phase: $\frac{d}{d\xi}w_{\mathrm{A}} = -\,\varepsilon_{\mathrm{L}}\, Da_{\mathrm{A}}\, w_{\mathrm{A}}\, w_{\mathrm{H}}$	(10)	$Da_{\mathrm{A}} = (k_{\mathrm{C}} w_{\mathrm{C}} w_{\mathrm{H}}^{*\mathrm{o}} D / U_{\mathrm{D}})\, Ge\, Fr^{-0.5}$	(14)
$\frac{d}{d\xi}w_{\mathrm{H}} = \varepsilon_{\mathrm{G}}\, St\left[w_{\mathrm{H}}^{*} - w_{\mathrm{H}}\right] - \varepsilon_{\mathrm{L}}\, Da_{\mathrm{H}}\, w_{\mathrm{A}}\, w_{\mathrm{H}}$	(11)	$Da_{\mathrm{H}} = (k_{\mathrm{C}} w_{\mathrm{C}} w_{\mathrm{A}}^{\mathrm{o}} D / U_{\mathrm{D}})\,(M_{\mathrm{H}} / M_{\mathrm{A}})\, Ge\, Fr^{-0.5}$	(15)
Linear momentum balance		$Ge = D / L$	(16)
$\frac{d}{d\xi}P = -Ge\,(L/D)\, Fr\left\{2 f_{\mathrm{L}} \varPhi_{\mathrm{L}} + \varepsilon_{\mathrm{L}} \cos\theta(\xi)\right\}$	(12)	$Fr = U_{\mathrm{L}}^{2} / g D$	(17)
In terms of integral equations (Cantero *et al.*, 1994)			
Mass balances		Green´s function	
$Y(\xi) = Y(0) - \int_{0}^{1} d\bar{\xi}\; G_{\mathrm{Y}}(\xi/\bar{\xi})\, \varepsilon_{\mathrm{G}}(\bar{\xi})\, St\left[w_{\mathrm{H}}^{*}(\bar{\xi}) - w_{\mathrm{H}}(\bar{\xi})\right]$	(18)	$G_{\mathrm{Y}}(\xi/\bar{\xi}) = \varTheta(\xi - \bar{\xi})$	(22)
$w_{\mathrm{A}}(\xi) = w_{\mathrm{A}}(0) - \int_{0}^{1} d\bar{\xi}\, G_{\mathrm{A}}(\xi/\bar{\xi})\, \varepsilon_{\mathrm{L}}(\bar{\xi})\, Da_{\mathrm{A}}\, w_{\mathrm{A}}(\bar{\xi})\, w_{\mathrm{H}}(\bar{\xi})$	(19)	$G_{\mathrm{A}}(\xi/\bar{\xi}) = \varTheta(\xi - \bar{\xi})$	(23)
$w_{\mathrm{H}}(\xi) = w_{\mathrm{H}}(0) - \int_{0}^{1} d\bar{\xi}\, G_{\mathrm{H}}(\xi/\bar{\xi})\, \varepsilon_{\mathrm{G}}(\bar{\xi})\, St\, w_{\mathrm{H}}^{*}(\bar{\xi})$	(20)	$G_{\mathrm{H}}(\xi/\bar{\xi}) = \varTheta(\xi - \bar{\xi}) \exp\left[-\int_{\bar{\xi}}^{\xi} d\hat{\xi}\left\{\varepsilon_{\mathrm{G}}(\hat{\xi})\, St + \left[1 - \varepsilon_{\mathrm{G}}(\hat{\xi})\right] Da_{\mathrm{H}}\, w_{\mathrm{A}}(\hat{\xi})\right\}\right]$	(24)
Linear momentum balance			
$P(\xi) = P(0) - \int_{0}^{1} d\bar{\xi}\, G_{\mathrm{P}}(\xi/\bar{\xi})\,(L/D)\left\{2 f_{\mathrm{L}} \varPhi_{\mathrm{L}} Fr + \varepsilon_{\mathrm{L}}(\bar{\xi}) \cos\theta(\bar{\xi})\right\}$	(21)	$G_{\mathrm{P}}(\xi/\bar{\xi}) = \varTheta(\xi - \bar{\xi})$	(25)

Figure captions

Figure 1: Scheme of vertical multiphase catalytic turbulent reactor (TTR), assembled with consecutive sections of pairs of upflow and downflow stretches (A-configuration)

Figure 2: Steady-state cocurrent flow patterns map for hydrogen-soybean oil system in terms of ratio between the Froude number and minimum Froude number (Fr_{min}) needed to achieve the dispersed bubble pattern (DBP), for wide range of gas holdup. $Fr_{\mathrm{min}} = E_{\mathrm{Hmin}} \Big/ (2 f_{\mathrm{L}} \phi_{\mathrm{L}}^{2} V_{\mathrm{m}} / \rho_{\mathrm{m}})$ (Cantero *et al.*, 1996). Physical properties taken at 423 K and 0.2-0.8 MPa. Insert: Schematic representations of two-phase turbulent flow patterns along inclined tube