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Robust model predictive control with zone control

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Abstract: Model predictive control (MPC) is usually implemented as a control strategy where the system outputs are controlled within specified zones, instead of fixed set points. One strategy to implement the zone control is by means of the selection of different weights for the output error in the control cost function. A disadvantage of this approach is that closed-loop stability cannot be guaranteed, as a different linear controller may be activated at each time step. A way to implement a stable zone control is by means of the use of an infinite horizon cost in which the set point is an additional variable of the control problem. In this case, the set point is restricted to remain inside the output zone and an appropriate output slack variable is included in the optimisation problem to assure the recursive feasibility of the control optimisation problem. Following this approach, a robust MPC is developed for the case of multi-model uncertainty of open-loop stable systems. The controller is devoted to maintain the outputs within their corresponding feasible zone, while reaching the desired optimal input target. Simulation of a process of the oil refining industry illustrates the performance of the proposed strategy.

1 Introduction

MPC controllers are usually implemented as part of a multilevel hierarchy of control functions [1]. At intermediary levels of this control structure, the process unit optimiser computes an optimal economic steady state and passes this information to the MPC in a lower level for implementation. The MPC is expected to drive the plant to a more profitable operating point, when minimising the dynamic error along the path. In several cases, the aim of the MPC level is not to guide the controlled variables to set points or desired values, but only to maintain them inside appropriate ranges or zones. This is what is called zone control [2]. This strategy is desired, for instance, when the aim is to drive the feed rate to its maximum value subject to constraints. Also, the zone control is adopted in some systems, where there are highly correlated outputs to be controlled and where there are no inputs enough to control them independently. Controlling the dense- and dilute-phase temperatures on an FCC regenerator is an example of this class of problems. Although not covered by the method presented here, another class of zone control problems relates to using the surge capacity of tanks to smooth out the operation of a unit. In this case, it is desired to let the level of the tank float between limits, as necessary, to buffer disturbances between sections of a plant. The controller used in this case must be capable of accounting for integrating systems. The paper by Qin and Badgwell [3], which represents an excellent survey of the existing industrial MPC technology, describes a variety of industrial controllers and mentions that they always provide a zone control option. Other example of zone control can be found in [4], where the authors exemplify the application of this strategy in the real-time optimisation of an FCC system. Although this strategy shows to have an acceptable performance, stability cannot be proved, even if an infinite horizon is used, as the control system keeps switching from one controller to another throughout the continuous operation of the process.

In parallel to the zone control formulation, there is a number of research works that treat the problem of how to obtain a stable MPC with fixed output set points. Although stability of the closed loop is commonly achieved by means of an infinite prediction horizon, the problem of how to eliminate output steady-state offset when a supervisory layer produces optimal economic set points, and how to explicitly incorporate the model uncertainty into the control problem formulation for this case, remains an open issue. For the nominal model case, Rawlings [5], Pannochia and Rawlings [6] and Muske and Badgwell [7] show how to include disturbance models to assure that the inputs and states are led to the desired values without offset. Muske and Badgwell [7] and Pannochia and Rawlings [6] develop rank conditions to assure the detectability of the augmented model.

For the uncertain system, Odloak [8] develops a robust MPC for the multi-plant uncertainty (i.e. for a finite set of possible models) that uses a non-increasing cost constraint [9]. In this strategy, the MPC cost function to be minimised is computed using a nominal model, but the non-increasing cost constraint is settled for each of the models belonging to the set. The stability is then achieved by means of the recursive feasibility of the optimisation problem, instead of the optimality. On the other hand, there exist some recent MPC formulations that are based on the existence of a controlled Lyapunov function (CLF), which is independent of the control cost function. Although the construction of the CFL may not be a trivial task, these formulations also allow the explicit characterisation of the stability region subject to constraints and they do not need an infinite output horizon. Mhaskar *et al.* [10] explore this approach for the control of nominal nonlinear systems and Mhaskar [11] extends the approach for the case of model uncertainty and control actuator fault. Following a similar line, the same authors [12] applied the CLF-based MPC to the control of switched systems by designing a stable controller for each constituent mode in which the system operates and by incorporating constraints in the control problem that ensures that the transition between modes will result in a stable closed-loop system. They also extended the approach to the switched system with uncertainties in the switching times and model parameters, as well as to the presence of exogenous time-varying disturbances in the dynamics of the system [13].

The objective of this paper was to develop a robust MPC that adapts the non-increasing cost constraint strategy to the case of zone control, that is, to the case where it is desirable to guide the manipulated input to the target given by a supervisory stationary optimisation stage, while maintaining the controlled output in their corresponding zones, taking into account a finite set of possible models. This problem that seems to interchange an output tracking by an input-tracking formulation is not trivial, since once the output lies outside the corresponding zone (because of a disturbance, or a change in the output zones), the priority of the controller is again to control the outputs, even if this implies that the input must be settled apart from its targets.

2 System representation

Consider a stable system with nu inputs and ny outputs, and assume that the poles relating any input ui to any output yj are non-repeated. Odloak [8] considers the following state space model that is suitable to the implementation of MPC

$$\begin{bmatrix} x^s(k+1) \\ x^d(k+1) \end{bmatrix} = \begin{bmatrix} I_{ny} & 0 \\ 0 & F \end{bmatrix} \begin{bmatrix} x^s(k) \\ x^d(k) \end{bmatrix} + \begin{bmatrix} D^0 \\ D^d FN \end{bmatrix} \Delta u(k) \quad (1)$$

$$y(k) = [\, I_{ny} \quad \Psi \,] \begin{bmatrix} x^s(k) \\ x^d(k) \end{bmatrix} \quad (2)$$

where

$$x^s = [\, x_1 \quad \cdots \quad x_{ny} \,]^{\mathrm{T}}, \quad x^s \in \Re^{ny},$$

$$x^d = [\, x_{ny+1} \quad x_{ny+2} \quad \cdots \quad x_{ny+nd} \,]^{\mathrm{T}},$$

$$x^d \in C^{nd}, \qquad F \in C^{nd \times nd}$$

$$\boldsymbol{\Psi} = \begin{bmatrix} \boldsymbol{\Phi} & & 0 \\ & \ddots & \\ 0 & & \boldsymbol{\Phi} \end{bmatrix}, \quad \boldsymbol{\Psi} \in \Re^{ny \times nd},$$

$$\boldsymbol{\Phi} = [\, 1 \quad \cdots \quad 1 \,], \quad \boldsymbol{\Phi} \in \Re^{nuna}$$

In the state equation (1), the state components xs correspond to the (predicted) output steady state and components xd correspond to the stable modes of the system. Naturally, when the system approaches steady state, these last components tend to zero. F is a diagonal matrix with components of the form $e^{r_i T}$ where ri is a pole of the system and T is the sampling period. It is assumed that the system has nd stable poles and D^0 is the gain matrix of the system. To build up matrix Φ, it is also assumed that na is the number of poles associated to any input ui and any output yj.

With the model structure presented in (1) and (2), model uncertainty is related to uncertainty in matrices F, D^0 and D^d. There are several practical ways to represent model uncertainty in model-predictive control. One of the simple ways to represent model uncertainty is to consider the multi-plant system [9], where we have a discrete set Ω of plants, and the real plant is unknown, but it is assumed to be one of the members of this set. With this representation of model uncertainty, we can define the set of possible plants as $\Omega = \{\theta_1, \ldots, \theta_L\}$ where each θ_n corresponds to a particular plant $\theta_n = (F_n, D_n^0, D_n^d), \ n = 1, \ldots, L$.

Also, let us assume that the true plant which lies within the set Ω is designated as θT and there is a most likely plant that also lies in Ω and is designated as θN. In addition, it is assumed that the current estimated state corresponds to the true plant.

Badgwell [9] developed a robust linear quadratic regulator for stable systems with the multi-plant uncertainty. Later,

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Odloak [8] extended the method of Badgwell to the output tracking of stable systems considering the same kind of model uncertainty. These strategies include a new constraint corresponding to each of the models lying in Ω that prevents an increase in the true plant cost function at successive time steps. In this work, we combine the approach presented in Odloak [8] with the idea of including the output set point as a new restricted optimisation variable to develop a robust MPC for systems where the control objective is to maintain the outputs into their corresponding feasible zone, while reaching the desired optimal input target given by the supervisory stationary optimisation.

3 Control structure

In this work, we consider the control structure shown in Fig. 1. In this structure, the economic optimisation stage is dedicated to the calculation of the desired target, $u_{\text{des},k}$, for the input manipulated variables. This stage may be based on a rigorous stationary model and takes into account the process measurements and some economic parameters. In addition, this stage works with a smaller frequency than the low-level control stage, which allows a separation between the two stages. The low-level control stage, given by the MPC controller, is devoted to guide the manipulated input to the desired values given by the supervisory economic stage, while keeping the outputs within specified zones. In general, the target $u_{\text{des},k}$ will vary whenever the plant operation or the economic parameters changes and this target satisfies

$$u_{\min} \le u_{\text{des},k} \le u_{\max}$$
$$y_{\min} \le D^0(\theta_n)(u_{\text{des},k} - u(k-1)) + \hat{x}_n^s(k) \le y_{\max}, \quad n = 1, \ldots, L \tag{3}$$

where $u_{\min}$ and $u_{\max}$ represent the lower and upper bounds of the input, $y_{\min}$ and $y_{\max}$ represent the lower and upper bounds of the output, $D^0(\theta_n)$ is the gain corresponding to model θ_n and $\hat{x}_n^s(k)$ is the estimated steady-state values of the output corresponding to model θ_n. Note that from the control structure depicted in Fig. 1, one has one observer per each model, and the observer corresponding to the true model θ_T is based on the true model matrices. In all these cases, however, as the model structure adopted here has integral action (which is given by the incremental form of the input), the estimation of component $x_n^s(k)$ tends to the measured output at steady state for all the models lying in Ω. To clarify this point, consider the equation that defines the state observer corresponding to model θ_n, applied at time $\bar{k}$ large enough to approach steady state

$$\begin{bmatrix} \hat{x}_n^s(\bar{k}) \\ \hat{x}_n^d(\bar{k}) \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & F(\theta_n) \end{bmatrix} \begin{bmatrix} \hat{x}_n^s(\bar{k}) \\ \hat{x}_n^d(\bar{k}) \end{bmatrix} + \begin{bmatrix} D^0(\theta_n) \\ D^d(\theta_n)F(\theta_n)N \end{bmatrix} \Delta u(\bar{k}) + \begin{bmatrix} L_n^s \\ L_n^d \end{bmatrix} \times \left[y(\bar{k}) - [I_{ny} \quad \Psi] \left(\begin{bmatrix} I & 0 \\ 0 & F(\theta_n) \end{bmatrix} \begin{bmatrix} \hat{x}_n^s(\bar{k}) \\ \hat{x}_n^d(\bar{k}) \end{bmatrix} + \begin{bmatrix} D^0(\theta_n) \\ D^d(\theta_n)F(\theta_n)N \end{bmatrix} \Delta u(\bar{k}) \right) \right]$$

where $[L_n^{s\mathrm{T}} \quad L_n^{d\mathrm{T}}]^{\mathrm{T}}$ is the observer gain, $\hat{x}_n^s(\bar{k})$ and $\hat{x}_n^d(\bar{k})$ are the estimated states at time $\bar{k}$, corresponding to model θ_n, and $y(\bar{k})$ is the measured steady-state output corresponding to the actual plant. Assuming that $\Delta u(\bar{k}) = 0$ and knowing that $\hat{x}_n^d(\bar{k}) = 0$ at steady state as this last state corresponds to the stable modes of the system, the state observer for component x_n^s becomes

$$\hat{x}_n^s(\bar{k}) = \hat{x}_n^s(\bar{k}) + L_n^s[y(\bar{k}) - \hat{x}_n^s(\bar{k})]$$

The above relation implies that, if $L_n^s \in \Re^{ny \times ny}$ is full rank, then

$$\hat{x}_n^s(\bar{k}) = y(\bar{k}), \quad n = 1, \ldots, L \tag{4}$$

and the output predictions will be unbiased with respect to the measurements.

Therefore condition (3) assures that, for a large $\bar{k}$, the desired input should be such that

$$y_{\min} \le D^0(\theta_n)u_{\text{des},\bar{k}} + [y(\bar{k}) - D^0(\theta_n)u(\bar{k})] \le y_{\max}, \quad n = 1, \ldots, L$$
$$y_{\min} \le D^0(\theta_n)u_{\text{des},\bar{k}} + d_n(\bar{k}) \le y_{\max}, \quad n = 1, \ldots, L \tag{5}$$
$$y_{\min} \le y^c_{n,\text{des},\bar{k}} \le y_{\max}, \quad n = 1, \ldots, L$$

where $d_n(\bar{k})$ is the output bias based on the comparison between the actual output at steady state and the predicted output at steady state for each model. Note that, since $u(\bar{k}) = \sum_{j=0}^{\bar{k}} \Delta u(j)$, then the term $D^0(\theta_n)u(\bar{k})$ represents the output prediction based only on the past inputs.

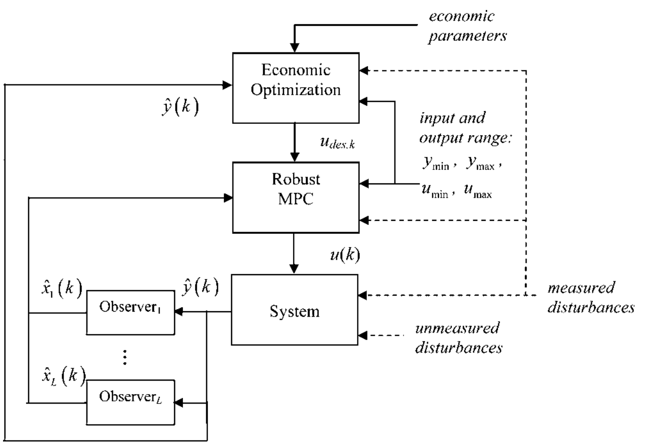

Figure 1 *Control structure*

4 Robust MPC with range control

One way to handle the range control strategy, that is, to maintain the controlled output inside its corresponding range, is by means of an appropriate choice of the output error penalisation in the conventional MPC cost function. In this case, the output weight is made equal to zero when the system output is inside the range, and the output weight is different from zero if the output prediction is violating any of the constraints, so that the output variable is strictly controlled only if it is outside the feasible range. In this way, the closed loop is guided to a feasible steady state. In [4], an algorithm assigns three possible values to the output set points used in the MPC controller: the upper bound of the output feasible range if the predicted output is larger than the upper bound; the lower bound of the output feasible range if the predicted output is smaller than this lower bound; and the predicted output itself, if the predicted output is inside the feasible range. However, a rigorous analysis of the stability of this strategy is not possible even when using an infinite output horizon. González *et al.* [14] describe a stable MPC based on the incremental model defined in (1) and (2) that takes into account a stationary optimisation of the plant operation. The controller was designed specifically for a heat exchanger network with a number of degrees of freedom larger than zero. In that work, the mismatch between the stationary and the dynamic model was treated by means of an appropriate choice of the weighting matrices in the control cost. However, stability and offset elimination were assured only when the model was perfect.

Based on the work of González *et al.* [14], we consider the following nominal cost function

$$V_k = \sum_{j=0}^{\infty} \{(y(k+j/k) - y_{sp,k})^{\mathrm{T}} Q_y (y(k+j/k) - y_{sp,k}) + (u(k+j/k) - u_{\mathrm{des},k})^{\mathrm{T}} Q_u (u(k+j/k) - u_{\mathrm{des},k})\} + \sum_{j=0}^{m-1} \Delta u(k+j/k)^{\mathrm{T}} R \Delta u(k+j/k) \quad (6)$$

where

$$y(k+j/k) = [I_{ny} \quad \Psi] \begin{bmatrix} I_{ny} & 0 \\ 0 & F^j \end{bmatrix} \begin{bmatrix} \hat{x}^s(k/k) \\ \hat{x}^d(k/k) \end{bmatrix} + [I_{ny} \quad \Psi] \times \begin{bmatrix} I_{ny} & 0 \\ 0 & F^{j-1} \end{bmatrix} \begin{bmatrix} D^0 \\ D^d FN \end{bmatrix} \Delta u(k/k) + \cdots + [I_{ny} \quad \Psi] \times \begin{bmatrix} I_{ny} & 0 \\ 0 & F^{j-m} \end{bmatrix} \begin{bmatrix} D^0 \\ D^d FN \end{bmatrix} \Delta u(k+m-1/k)$$

Here $\Delta u(k+j/k)$ is the control move computed at time k to be applied at time $k+j$, m is the control or input horizon, Q_y, Q_u, R are positive weighting matrices of appropriate dimensions, $\hat{x}^s(k/k)$ and $\hat{x}^d(k/k)$ are the estimated states at the present time and $y_{sp,k}$ and $u_{\mathrm{des},k}$ are the output and input targets, respectively. This cost explicitly incorporates an input deviation penalty that tries to accommodate the system at an optimal economic stationary point. However, as an infinite output horizon is used and the model used to perform the predictions has integral modes (it is an incremental model), a terminal constraint must be added to prevent the cost from becoming unbounded. These constraints can be written as [14]

$$x^s(k) + \tilde{D}^0 \Delta u_k - y_{sp,k} = 0 \quad (x^s(k+m/k) - y_{sp,k} = 0)$$
$$u(k-1) + \tilde{D}^u \Delta u_k - u_{\mathrm{des},k} = 0 \quad (u(k+m-1/k) - u_{\mathrm{des},k} = 0)$$

where

$$\Delta u_k = [\Delta u(k/k)^{\mathrm{T}} \quad \cdots \quad \Delta u(k+m-1/k)^{\mathrm{T}}]^{\mathrm{T}} \in \Re^{m.nu}$$
$$\tilde{D}^0 = \underbrace{[D^0 \quad \cdots \quad D^0]}_{m}$$
$$\tilde{D}^u = \underbrace{[I_{nu} \quad \cdots \quad I_{nu}]}_{m}$$

The above constraints assume that both the output and input errors will be null at the end of the control horizon m. As the input increments are generally bounded, the terminal constraints frequently result in infeasible problems, which means that it is not possible for the controller to achieve the constraints in m time steps, given that m is frequently small to reduce the computational cost. Then, we need to incorporate slack variables in these terminal constraints in order to guarantee that the control problem will be feasible. Besides, these slack variables must be included in the cost function with large weights to assure that the constraint violation will be minimised by the control actions. Thus, the cost function will be written as

$$V_k = \sum_{j=0}^{\infty} (y(k+j/k) - y_{sp,k} - \delta_k)^{\mathrm{T}} Q_y (y(k+j/k) - y_{sp,k} - \delta_k) + (u(k+j/k) - u_{\mathrm{des},k} - \delta_{k,u})^{\mathrm{T}} Q_u (u(k+j/k) - u_{\mathrm{des},k} - \delta_{k,u}) + \sum_{j=0}^{m-1} \Delta u(k+j/k)^{\mathrm{T}} R \Delta u(k+j/k) + \delta_{y,k}^{\mathrm{T}} S_y \delta_{y,k} + \delta_{u,k}^{\mathrm{T}} S_u \delta_{u,k}$$

and the terminal constraints become

$$x^s(k) + \tilde{D}^0 \Delta u_k - y_{sp,k} - \delta_{y,k} = 0$$
$$u(k-1) + \tilde{D}^u \Delta u_k - u_{\mathrm{des},k} - \delta_{u,k} = 0$$

where S_y, S_u are positive matrices of appropriate dimension and $\delta_{y,k} \in \Re^{ny}$, $\delta_{u,k} \in \Re^{nu}$ are the slack variables that eliminate any infeasibility problem. Now, we will focus our attention on the range control problem. In order to obtain a nominal stable MPC controller for the case of output

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range control, let us consider the following optimisation problem.

Problem P1:

$$\min_{\Delta u_k, y_{sp,k}, \delta_k, \delta_{k,u}} V_k = \sum_{j=0}^{\infty} \{(y(k+j/k) - y_{sp,k} - \delta_{y,k})^{\mathrm{T}} \times Q_y (y(k+j/k) - y_{sp,k} - \delta_{y,k}) + (u(k+j/k) - u_{\mathrm{des},k} - \delta_{u,k})^{\mathrm{T}} \times Q_u(u(k+j/k) - u_{\mathrm{des},k} - \delta_{u,k})\} + \sum_{j=0}^{m-1} \Delta u(k+j/k)^{\mathrm{T}} R \Delta u(k+j/k) + \delta_{y,k}^{\mathrm{T}} S_y \delta_{y,k} + \delta_{u,k}^{\mathrm{T}} S_u \delta_{u,k} \quad (7)$$

subject to

$$-\Delta u_{\max} \le \Delta u(k+j/k) \le \Delta u_{\max}, \quad j = 0, 1, \ldots, m-1 \quad (8)$$

$$\Delta u(k+j/k) = 0, \quad j \ge m \quad (9)$$

$$u_{\min} \le u(k-1) + \sum_{i=0}^{j} \Delta u(k+i/k) \le u_{\max}, \quad j = 0, 1, \ldots, m-1 \quad (10)$$

$$y_{\min} \le y_{sp,k} \le y_{\max} \quad (11)$$

$$x^s(k) + \tilde{D}^0 \Delta u_k - y_{sp,k} - \delta_{y,k} = 0 \quad (x^s(k+m/k) - y_{sp,k} - \delta_{y,k} = 0) \quad (12)$$

$$u(k-1) + \tilde{D}^u \Delta u_k - u_{\mathrm{des},k} - \delta_{u,k} = 0 \times (u(k+m-1/k) - u_{\mathrm{des},k} - \delta_{u,k} = 0) \quad (13)$$

In Problem P1, $y_{sp,k}$, which represents the output set point, is an additional optimisation variable. Note that, since variable $y_{sp,k}$ is restricted by constraint (11), the effective output set point is now the complete feasible zone. If the output bounds are settled so that the upper bound equals the lower bound, then the problem becomes the traditional set-point tracking problem. In the general case, the output slack variable, $\delta_{y,k}$, will be null if and only if the predicted steady state of the output is inside the feasible range.

Now, because of terminal constraints (12) and (13), the objective function defined in (7) can be written as

$$V_k = \sum_{j=0}^{m-1} \{(y(k+j/k) - y_{sp,k} - \delta_{y,k})^{\mathrm{T}} \times Q_y(y(k+j/k) - y_{sp,k} - \delta_{y,k}) + (u(k+j/k) - u_{\mathrm{des},k} - \delta_{u,k})^{\mathrm{T}} \, Q_u(u(k+j/k) - u_{\mathrm{des},k} - \delta_{u,k})\} + (x^d(k+m/k))^{\mathrm{T}} \bar{Q} x^d(k+m/k) + \sum_{j=0}^{m-1} \Delta u(k+j/k)^{\mathrm{T}} \times R \Delta u(k+j/k) + \delta_{y,k}^{\mathrm{T}} S_y \delta_{y,k} + \delta_{u,k}^{\mathrm{T}} S_u \delta_{u,k} \quad (14)$$

where matrix $\bar{Q}$ is computed by means of the following Lyapunov equation

$$\bar{Q} = \Psi^{\mathrm{T}} Q_y \Psi + F^{\mathrm{T}} \bar{Q} F$$

It can be shown that the controller produced through the solution of Problem P1 results in a stable closed-loop system for the nominal system. However, the aim here is to extend this formulation to the case of multi-model uncertainty. To this end, let us consider the following optimisation problem.

Problem P2:

$$\min_{\substack{\Delta u_k, y_{sp,k}(\theta_n), \\ \delta_{y,k}(\theta_n), \delta_{u,k}}} V_{1,k}(\theta_N) = \sum_{j=0}^{\infty} \{(y_N(k+j/k) - y_{sp,k}(\theta_N) - \delta_{y,k}(\theta_N))^{\mathrm{T}} \times Q_y(y_N(k+j/k) - y_{sp,k}(\theta_N) - \delta_{y,k}(\theta_N)) + (u(k+j/k) - u_{\mathrm{des},k} - \delta_{u,k})^{\mathrm{T}} \times Q_u(u(k+j/k) - u_{\mathrm{des},k} - \delta_{u,k})\} + \sum_{j=0}^{m-1} \Delta u(k+j/k)^{\mathrm{T}} R \Delta u(k+j/k) + \delta_{y,k}^{\mathrm{T}}(\theta_1) S_y \delta_{y,k}(\theta_1) + \cdots + \delta_{y,k}^{\mathrm{T}}(\theta_L) S_y \delta_{y,k}(\theta_L) + \delta_{u,k}^{\mathrm{T}} S_u \delta_{u,k} \quad (15)$$

subject to

$$-\Delta u_{\max} \le \Delta u(k+j/k) \le \Delta u_{\max}, \quad j = 0, 1, \ldots, m-1 \quad (16)$$

$$\Delta u(k+j/k) = 0, \quad j \ge m \quad (17)$$

$$u_{\min} \le u(k-1) + \sum_{i=0}^{j} \Delta u(k+i/k) \le u_{\max}, \quad j = 0, 1, \ldots, m-1 \quad (18)$$

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$$y_{\min} \leq y_{sp,k}(\theta_n) \leq y_{\max}, \quad n = 1, \ldots, L \tag{19}$$

$$\hat{x}_n^s(k) + \tilde{D}^0(\theta_n)\Delta u_k - y_{sp,k}(\theta_n) - \delta_{y,k}(\theta_n) = 0, \quad n = 1, \ldots, L \tag{20}$$

$$u(k-1) + \tilde{D}^u \Delta u_k - u_{\text{des},k} - \delta_{u,k} = 0 \tag{21}$$

$$V_{2,k}(\Delta u_k, \delta_{y,k}(\theta_n), \delta_{u,k}, y_{sp,k}(\theta_n), \theta_n) \leq V_{2,k}(\Delta\tilde{u}_k, \tilde{\delta}_{y,k}(\theta_n), \tilde{\delta}_{u,k}, \tilde{y}_{sp,k}(\theta_n), \theta_n), \quad n = 1, \ldots, L \tag{22}$$

where

$$\begin{aligned} V_{2,k} = \sum_{j=0}^{\infty} \{ & (y_n(k+j/k) - y_{sp,k}(\theta_n) - \delta_{y,k}(\theta_n))^{\mathrm{T}} \\ & \times Q_y(y_n(k+j/k) - y_{sp,k}(\theta_n) - \delta_{y,k}(\theta_n)) \\ & + (u(k+j/k) - u_{des,k} - \delta_{u,k})^{\mathrm{T}} \\ & \times Q_u(u(k+j/k) - u_{des,k} - \delta_{u,k})\} \\ & + \sum_{j=0}^{m-1} \Delta u(k+j/k)^{\mathrm{T}} R \Delta u(k+j/k) \\ & + \delta_{y,k}^{\mathrm{T}}(\theta_n) S_y \delta_{y,k}(\theta_n) + \delta_{u,k}^{\mathrm{T}} S_u \delta_{u,k} \end{aligned}$$

$$\Delta\tilde{u}_k = [\, \Delta u^*(k/k-1)^{\mathrm{T}} \quad \cdots \quad \Delta u^*(k+m-2/k-1)^{\mathrm{T}} \quad 0 \,]^{\mathrm{T}}$$

$\tilde{y}_{sp,k}(\theta_n) = y^*_{sp,k-1}(\theta_n)$, $\tilde{\delta}_{u,k}$ and $\tilde{\delta}_{y,k}(\theta_n)$ are such that

$$u(k-1) + \tilde{D}^u \Delta\tilde{u}_k - u_{\text{des},k} - \tilde{\delta}_{u,k} = 0 \tag{23}$$

$$\hat{x}_n^s(k) + \tilde{D}^0(\theta_n)\Delta\tilde{u}_k - \tilde{y}_{sp,k}(\theta_n) - \tilde{\delta}_{y,k}(\theta_n) = 0, \quad n = 1, \ldots, L \tag{24}$$

$\Delta u^*(\cdot/k-1)$, $\delta^*_{u,k-1}$ and $y^*_{sp,k-1}(\theta_n)$ are the optimal solutions to this optimisation problem at time step $k-1$. The output predictions obtained with each model can be computed as

$$\begin{aligned} y_n(k+j/k) = {} & [I_{ny} \quad \Psi] \begin{bmatrix} I_{ny} & 0 \\ 0 & F(\theta_n)^j \end{bmatrix} \\ & \times \begin{bmatrix} \hat{x}_n^s(k/k) \\ \hat{x}_n^d(k/k) \end{bmatrix} + [I_{ny} \quad \Psi] \\ & \times \left[\begin{bmatrix} I_{ny} & 0 \\ 0 & F(\theta_n)^{j-1} \end{bmatrix} \cdots \begin{bmatrix} I_{ny} & 0 \\ 0 & F(\theta_n)^{j-m} \end{bmatrix} \right] \\ & \times \begin{bmatrix} D^0(\theta_n) \\ D(\theta_n)F(\theta_n)N \end{bmatrix} \Delta u_k \end{aligned}$$

where $\hat{x}_n^s(k/k)$ and $\hat{x}_n^d(k/k)$ are the estimated states corresponding to model n, that is, the estimated state resulting from each state observer (see Fig. 1).

Remark 1: The output prediction $y_N(k+j/k)$ appearing in the definition of the cost function $V_{1,k}(\theta_N)$, is based on the nominal model. However, constraints (20) and (22) are imposed considering the estimated state of each model $\theta_n \in \Omega$. Constraint (22) is the non-increasing cost constraint that assures the convergence of the true state cost to zero.

Remark 2: Cost $V_{1,k}$, that is the cost to be minimised, contains penalisation terms for all of the output slack variables corresponding to the models lying in Ω. Cost $V_{2,k}$, used in constraint (22), contains only one penalisation term for the output slack variable corresponding to the model for which the constraint is written.

Remark 3: The introduction of L set-point variables allows the simultaneous zeroing of all the output slack variables. That is, because, whenever possible, the set-point variable $y_{sp,k}(\theta_n)$ will be equal to the output prediction at steady state represented by $x_n^s(k+m)$, the controller gains some flexibility to eliminate the output penalisation from the cost, when the individual output predictions are inside the output zones. This will allow the controller to better achieve the other control objectives.

Remark 4: Note that by hypothesis, one of the observers is based on the actual plant model, and if the initial and the final steady states are known, then the estimated state $\hat{x}_T(k)$ will be equal to the actual plant state at each time k.

Remark 5: Conditions (23) and (24) are used to update the pseudo-variables of constraint (22), by taking into account the current state estimation $\hat{x}_n^s(k)$ for each of the models lying in Ω, and the last change of the input target.

Lemma 1: If Problem P2 is feasible at time step k, it will remain feasible at any subsequent time step $k+j$, $j = 1, 2, \ldots$.

Proof: Let us first prove the recursive feasibility of the proposed controller. Assume that the output zones remain fixed, and also assume that

$$\Delta u_k^* = [\, \Delta u^*(k/k)^{\mathrm{T}} \quad \cdots \quad \Delta u^*(k+m-1/k)^{\mathrm{T}} \,]^{\mathrm{T}} \in \Re^{m.nu} \tag{25}$$

$$y^*_{sp,k}(\theta_1), \ldots, y^*_{sp,k}(\theta_L), \quad \delta^*_{y,k}(\theta_1), \ldots, \delta^*_{y,k}(\theta_L) \quad \text{and} \quad \delta^*_{u,k} \tag{26}$$

correspond to the optimal solution to Problem P2 at time k.

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Consider now the pseudo-variables $(\Delta\tilde{u}_{k+1}, \tilde{y}_{sp,k+1}(\theta_1), \ldots, \tilde{y}_{sp,k+1}(\theta_L), \tilde{\delta}_{y,k+1}(\theta_1), \ldots, \tilde{\delta}_{y,k+1}(\theta_L), \tilde{\delta}_{u,k+1})$ where

$$\Delta\tilde{u}_{k+1} = [\,\Delta u^*(k+1/k)^{\mathrm{T}} \quad \cdots \quad \Delta u^*(k+m-1/k)^{\mathrm{T}} \quad 0\,]^{\mathrm{T}} \tag{27}$$

$$\tilde{y}_{sp,k+1}(\theta_n) = y^*_{sp,k}(\theta_n),\ n = 1, \ldots, L,\ \tilde{\delta}_{u,k+1} \text{ and } \tilde{\delta}_{y,k+1}(\theta_n) \tag{28}$$

and

$$u(k) + \tilde{D}^u \Delta\tilde{u}_{k+1} - u_{\mathrm{des},k+1} - \tilde{\delta}_{u,k+1} = 0 \tag{29}$$

$$\hat{x}^s_n(k+1) + \tilde{D}^0(\theta_n)\Delta\tilde{u}_{k+1} - \tilde{y}_{sp,k+1}(\theta_n) - \tilde{\delta}_{y,k+1}(\theta_n) = 0 \quad n = 1, \ldots, L$$

We can show that the solution defined in (27) and (28) represent a feasible solution to Problem P2 at time $k+1$, which proves the recursive feasibility. This means that if Problem P2 is feasible at time step k, then, it will remain feasible at all the successive time steps $k+1, k+2, \ldots$.

The robust stability of the closed loop resulting from the later optimisation problem can be stated as follows.

Theorem 1: Suppose that the undisturbed system starts at a known steady state and one of the state observers is based on the actual plant model. Consider also that the input target is moved to a new value, or the boundaries of the output zones are modified. Then, if condition (3) is satisfied for each model $\theta_n \in \Omega$, the cost function of the undisturbed true system will converge to zero.

Proof: Suppose that, at time k the uncertain system starts at a steady state corresponding to output $y(k)$ and input $u(k-1)$. We have already shown that, with the model structure considered in this work, the model states corresponding to this initial steady state can be represented as

$$\hat{x}^s_n(k) = y(k), \quad \hat{x}^d_n(k) = 0, \quad n = 1, \ldots, L$$

At time k, the cost corresponding to the solution defined in (25) and (26) for the true model is given by

$$\begin{aligned} V^*_{2,k}(\theta_T) = \sum_{j=0}^{\infty} &\{(y^*_T(k+j/k) - y^*_{sp,k}(\theta_T) - \delta^*_{y,k}(\theta_T))^{\mathrm{T}} \\ &\times Q_y(y^*_T(k+j/k) - y^*_{sp,k}(\theta_T) - \delta^*_{y,k}(\theta_T)) \\ &+ (u^*(k+j/k) - u_{\mathrm{des},k} - \delta^*_{u,k})^{\mathrm{T}} \\ &\times Q_u(u^*(k+j/k) - u_{\mathrm{des},k} - \delta^*_{u,k})\} \\ &+ \sum_{j=0}^{m-1} \Delta u^*(k+j/k)^{\mathrm{T}} R \Delta u^*(k+j/k) \\ &+ \delta^{*\mathrm{T}}_{y,k}(\theta_T) S_y \delta^*_{y,k}(\theta_T) + \delta^{*\mathrm{T}}_{u,k} S_u \delta^*_{u,k} \end{aligned} \tag{30}$$

At time step $k+1$, the cost corresponding to the pseudo-variables defined in (27)–(29) for the true model is given by

$$\begin{aligned} \tilde{V}_{2,k+1}(\theta_T) = \sum_{j=0}^{\infty} &\{(y^*_T(k+j+1/k) - y^*_{sp,k}(\theta_T) - \delta^*_{y,k}(\theta_T))^{\mathrm{T}} \\ &\times Q_y(y^*_T(k+j+1/k) - y^*_{sp,k}(\theta_T) - \delta^*_{y,k}(\theta_T)) \\ &+ (u^*(k+j+1/k) - u_{\mathrm{des},k} - \delta^*_{u,k})^{\mathrm{T}} \\ &\times Q_u(u^*(k+j+1/k) - u_{\mathrm{des},k} - \delta^*_{u,k})\} \\ &+ \sum_{j=0}^{m-1} \Delta u^*(k+j+1/k)^{\mathrm{T}} R \Delta u^*(k+j+1/k) \\ &+ \delta^{*\mathrm{T}}_{y,k}(\theta_T) S_y \delta^*_{y,k}(\theta_T) + \delta^{*\mathrm{T}}_{u,k} S_u \delta^*_{u,k} \end{aligned} \tag{31}$$

Observe that, as the same input sequence is used and the current estimated state corresponding to the actual model is equal to the actual state, then the predicted state and output trajectory will be the same as the optimal predicted trajectories at time step k. That is, for any $j \geq 1$, we have

$$x_T(k+j/k+1) = x_T(k+j/k)$$

and

$$y_T(k+j/k+1) = y_T(k+j/k)$$

In addition, for the true model we have $\tilde{\delta}_{y,k+1}(\theta_T) = \delta^*_{y,k}(\theta_T)$ and $\tilde{\delta}_{u,k+1} = \delta^*_{u,k}$. However, the first of these later equalities, are not true for the other models, since in these cases $\hat{x}_n(k+1/k+1) \neq x_n(k+1/k)$ for $\theta_n \neq \theta_T$.

Now, subtracting (31) from (30) we have

$$\begin{aligned} V^*_{2,k}(\theta_T) - \tilde{V}_{2,k+1}(\theta_T) = &(y^*_T(k/k) - y^*_{sp,k}(\theta_T) - \delta^*_{y,k}(\theta_T))^{\mathrm{T}} \\ &\times Q_y(y^*_T(k/k) - y^*_{sp,k}(\theta_T) - \delta^*_{y,k}(\theta_T)) \\ &+ (u^*(k/k) - u_{\mathrm{des},k} - \delta^*_{u,k})^{\mathrm{T}} \\ &\times Q_u(u^*(k/k) - u_{\mathrm{des},k} - \delta^*_{u,k}) \\ &+ \Delta u^*(k)^{\mathrm{T}} R \Delta u^*(k) \end{aligned}$$

and, from constraint (22), we have

$$V^*_{2,k+1}(\theta_T) \leq \tilde{V}_{2,k+1}(\theta_T)$$

which finally implies

$$\begin{aligned} V^*_{2,k}(\theta_T) - V^*_{2,k+1}(\theta_T) \geq &(y^*_T(k/k) - y^*_{sp,k}(\theta_T) - \delta^*_{y,k}(\theta_T))^{\mathrm{T}} \\ &\times Q_y(y^*_T(k/k) - y^*_{sp,k}(\theta_T) - \delta^*_{y,k}(\theta_T)) + (u^*(k/k) - u_{\mathrm{des},k} \\ &- \delta^*_{u,k})^{\mathrm{T}} Q_u(u^*(k/k) - u_{\mathrm{des},k} - \delta^*_{u,k}) + \Delta u^*(k)^{\mathrm{T}} R \Delta u^*(k) \end{aligned} \tag{32}$$

Since the right-hand side of (32) is positive definite, the successive values of the cost will be strictly decreasing and for a large enough time $\bar{k}$, we will have $(V^*_{2,\bar{k}}(\theta_T) - V^*_{2,\bar{k}+1}(\theta_T)) = 0$, which proves the convergence of the cost.

The convergence of $V^*_{2,k}(\theta_T)$ means that, at steady state, the following relations should hold

$$y^*_T(\bar{k}/\bar{k}) - y^*_{sp,\bar{k}}(\theta_T) = \delta^*_{y,\bar{k}}(\theta_T)$$
$$u^*(\bar{k}/\bar{k}) - u_{\text{des},\bar{k}} = \delta^*_{u,\bar{k}}$$
$$\Delta u^*(\bar{k}) = 0$$

At steady state, the state is such that

$$\hat{x}_n(\bar{k}) = \begin{bmatrix} \hat{x}^s_n(\bar{k}) \\ \hat{x}^d_n(\bar{k}) \end{bmatrix} = \begin{bmatrix} y^s_n(\bar{k}) \\ \hat{x}^d_n(\bar{k}) \end{bmatrix} = \begin{bmatrix} y(\bar{k}) \\ 0 \end{bmatrix}$$

where $y(\bar{k})$ is the actual plant output. Note that the state component $\hat{x}^d_n(\bar{k})$ is null, as the input increment is null at steady state. Then, constraint (20) can be written as

$$\delta^*_{y,\bar{k}}(\theta_n) = y^*_n(\bar{k}/\bar{k}) - y^*_{sp,\bar{k}}(\theta_n) = y(\bar{k}) - y^*_{sp,\bar{k}}(\theta_n) \quad n = 1, \ldots, L \qquad (33)$$

This means that if the output of the true system is stabilised inside the output zone, then the set-point variable corresponding to each particular model will be placed by the optimiser exactly at the output predicted values. In this case, all the output slacks will be null. On the other hand, if the output of the true system is stabilised at a value outside the output zone, then the set-point variable corresponding to each particular model will be placed by the optimiser at the boundary of the zone. In this case, the output slack variables will be different from zero, but they will all have the same numerical value as can be seen from (33).

Now, to strictly prove the convergence of the input and output to their corresponding targets, we must show that slacks $\delta_{u,\bar{k}}$ and $\delta_{y,\bar{k}}(\theta_T)$ will converge to zero. Note that, since there are not fixed output set points, the desired input values may be exactly achieved by the true system, even in the presence of some bounded disturbances. In fact, with this formulation, the output set point corresponding to each model may follow in some sense the predicted output values. Let us now assume that the system is stabilised at a point where $\delta^*_{y,\bar{k}}(\theta_1) = \cdots = \delta^*_{y,\bar{k}}(\theta_L) \neq 0$ and $\delta_{u,\bar{k}} \neq 0$. In addition, assume that the desired input value is constant at $u_{\text{des},\bar{k}}$. Then, at time $\bar{k}$ large enough, the optimal cost will be reduced to

$$V^*_{1,\bar{k},u} = \delta^T_{y,\bar{k}}(\theta_1) S_y \delta_{y,\bar{k}}(\theta_1) + \cdots + \delta^T_{y,\bar{k}}(\theta_L) S_y \delta_{y,\bar{k}}(\theta_L) + \delta^T_{u,\bar{k}} S \delta_{u,\bar{k}} \qquad (34)$$

and constraints (20) and (21) become

$$\hat{x}^s_n(\bar{k}) - y_{sp,\bar{k}}(\theta_n) = \delta_{y,\bar{k}}(\theta_n), \quad n = 1, \ldots, L \qquad (35)$$

and

$$u(\bar{k} - 1) - u_{\text{des},\bar{k}} = \delta_{u,\bar{k}}$$

Since $\hat{x}^s_n(\bar{k}) = y(\bar{k})$, $n = 1, \ldots, L$, (35) can be written as

$$y(\bar{k}) - y_{sp,\bar{k}}(\theta_n) = \delta_{y,\bar{k}}(\theta_n), \quad n = 1, \ldots, L$$

Now, we want to show that if $u(k-1)$ and $u_{\text{des},\bar{k}}$ are not on the boundary of the input operating range, then it is possible to guide the system toward a point in which the slack variables $\delta_{y,k}(\theta_n)$ and $\delta_{u,k}$ are null, and this point have a smaller cost than the steady state defined above. Assume also for simplicity that $m = 1$. Then, at time $\bar{k}$ let us consider a candidate solution to Problem P2 defined by

$$\Delta\bar{u}(\bar{k}/\bar{k}) = u_{\text{des},\bar{k}} - u(\bar{k} - 1) = -\delta_{u,\bar{k}} \qquad (36)$$

and

$$\bar{y}_{sp,\bar{k}}(\theta_n) = y(\bar{k}) - D^0(\theta_n)\delta_{u,\bar{k}}, \quad n = 1, \ldots, L \qquad (37)$$

The set points given in (37) are the steady-state values of the outputs corresponding to the input increment given in (36). Note that these new set points are feasible because it is assumed that, at steady state, condition (3) holds true.

The variables defined in (36) and (37) must satisfy constraints (20) and (21) of Problem P2 for all the models lying in Ω or

$$y(\bar{k}) - D^0(\theta_n)\delta_{u,\bar{k}} - \bar{y}_{sp,\bar{k}}(\theta_n) - \bar{\delta}_{y,\bar{k}}(\theta_n) = 0, \quad n = 1, \ldots, L \qquad (38)$$

$$\overbrace{u(\bar{k} - 1) - \delta_{u,\bar{k}}}^{\bar{u}(\bar{k})} - u_{\text{des},\bar{k}} - \bar{\delta}_{u,\bar{k}} = 0 \qquad (39)$$

Note that $\bar{u}(\bar{k})$, $\bar{y}_{sp,\bar{k}}(\theta_n)$, $\bar{\delta}_{y,\bar{k}}(\theta_n)$ and $\bar{\delta}_{u,\bar{k}}$ are the solution variables that will result if the input increment defined in (36) is implemented at $\bar{k}$ instead of the null increment. Now, combining (36) and (39), we conclude that $\bar{\delta}_{u,\bar{k}} = 0$. This means that, the inputs will reach their target at steady state, or $\bar{u}(\bar{k}) = u_{\text{des},\bar{k}}$. Also, substituting (37) into (27) results in $\bar{\delta}_{y,\bar{k}}(\theta_n) = 0$, which means that the predicted output at steady state (for all the models) is inside the output zone.

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With these optimisation variables, the objective function defined in (15) becomes

$$\begin{aligned}\bar{V}_{1,\bar{k}} &= (y_N(\bar{k}/\bar{k}) - \bar{y}_{sp,\bar{k}}(\theta_N) - \bar{\delta}_{y,\bar{k}}(\theta_N))^{\mathrm{T}} \\ &\quad \times Q_y(y_N(\bar{k}/\bar{k}) - \bar{y}_{sp,\bar{k}}(\theta_N) - \bar{\delta}_{y,\bar{k}}(\theta_N)) \\ &\quad \underbrace{+ (u(\bar{k}/\bar{k}) - u_{\mathrm{des},\bar{k}} - \bar{\delta}_{u,k})^{\mathrm{T}} Q_u(u(\bar{k}/\bar{k}) - u_{\mathrm{des},\bar{k}} - \bar{\delta}_{u,\bar{k}})}_{=0} \\ &\quad + (x_N^d(\bar{k}+1/\bar{k}))^{\mathrm{T}} \bar{Q}(\theta_N) x_N^d(\bar{k}+1/\bar{k}) \\ &\quad + \overbrace{\Delta u(\bar{k}/\bar{k})^{\mathrm{T}} R \Delta u(\bar{k}/\bar{k})}^{\delta_{u,\bar{k}}^{\mathrm{T}} R \delta_{u,\bar{k}}} + \overbrace{\bar{\delta}_{y,\bar{k}}^{\mathrm{T}}(\theta_1) S_y \bar{\delta}_{y,\bar{k}}(\theta_1)}^{=0} \\ &\quad + \cdots + \overbrace{\bar{\delta}_{y,\bar{k}}^{\mathrm{T}}(\theta_L) S_y \bar{\delta}_{y,\bar{k}}(\theta_L)}^{=0} + \overbrace{\bar{\delta}_{u,\bar{k}}^{\mathrm{T}} S_u \bar{\delta}_{u,\bar{k}}}^{=0} \end{aligned} \tag{40}$$

Now, using model equations (1) and (2), we have

$$y_N(\bar{k}/\bar{k}) - \bar{y}_{sp,\bar{k}}(\theta_N) - \overbrace{\bar{\delta}_{y,\bar{k}}(\theta_N)}^{=0} = \hat{x}_N^s(\bar{k}) + \Psi \hat{x}_N^d(\bar{k}) - \bar{y}_{sp,\bar{k}}(\theta_N)$$

$$= y(\bar{k}) + \Psi \overbrace{\hat{x}_N^d(\bar{k})}^{=0} - \bar{y}_{sp,\bar{k}}(\theta_N) = D^0(\theta_N)\delta_{u,\bar{k}}$$

and

$$\begin{aligned} x_N^d(\bar{k}+1/\bar{k}) &= F(\theta_N)\hat{x}_N^d(\bar{k}) + D^d(\theta_N)F(\theta_N)N\Delta u(\bar{k}/\bar{k}) \\ &= F(\theta_N)\underbrace{\hat{x}_N^d(\bar{k})}_{=0} - D^d(\theta_N)F(\theta_N)N\delta_{u,\bar{k}} = -D^d(\theta_N)F(\theta_N)N\delta_{u,\bar{k}} \end{aligned}$$

Consequently, the cost represented in (40) can be written as

$$\bar{V}_{1,\bar{k},u} = \delta_{u,\bar{k}}^T S_u^{\min} \delta_{u,\bar{k}}$$

where

$$\begin{aligned} S_u^{\min} &= D^0(\theta_N)^{\mathrm{T}} Q_y D^0(\theta_N) + N^{\mathrm{T}} F(\theta_N)^{\mathrm{T}} D^d(\theta_N)^{\mathrm{T}} \\ &\quad \times \bar{Q}(\theta_N) D^d(\theta_N) F(\theta_N) N + R \end{aligned}$$

Then, if

$$S_u > S_u^{\min} \tag{41}$$

the cost corresponding to the decision variables defined in (36) and (37) will be smaller than the cost obtained in (34). This means that it is not possible for the system to remain at a point in which the slack variables $\delta_{y,k}(\theta_n)$, $n = 1, \ldots, L$ and $\delta_{u,k}$ are different from zero.

Thus, as long as the system remains controllable, condition (41) is sufficient to guarantee the convergence of the system inputs to their target while the system output will remain within the output zones.

Observe that only matrix Su is involved in condition (41) because condition (3) assures that the corrected output prediction, $y_{\mathrm{des},\bar{k}}^{\mathrm{c}}$, corresponding to the desired input values lies in the feasible zone. In this case, for all positive matrices Sy, the total cost can be reduced by making the set-point variable equal to the steady-state output prediction, which is a feasible solution and produces no additional cost. Matrix Sy, however, must be large enough to avoid any numerical problem in the optimisation solution.

Remark 6: We can prove the stability of the proposed zone controller under the same assumptions considered in the proof of the convergence. Output tracking stability means that for every $\gamma > 0$, there exists a $\rho(\gamma)$ such that if $\|\bar{x}_T(0)\| < \rho$, then $\|\bar{x}_T(k)\| < \gamma$ for all $k \geq 0$, where the extended state of the true system $\bar{x}_T(k)$ is defined as

$$\bar{x}_T(k) = \begin{bmatrix} e_T^s(k) \\ x_T^d(k) \\ e_u(k) \end{bmatrix} = \begin{bmatrix} \hat{x}_T^s(k) - y_{sp,k-1}^*(\theta_T) \\ \hat{x}_T^d(k) \\ u(k/k) - u_{\mathrm{des},k} \end{bmatrix}$$

As in the formulation used in this work, the set point is an optimisation variable (even if the input target remain constant) the first component of that new state is not a constant translation of coordinate of the original state $\hat{x}_T^s(k)$. However, as the set-point variable is restricted to be in the output zone (and the recursive feasibility of the closed loop is assured), and once inside this zone, we consider that the state $\hat{x}_T^s(k)$ has reached its target, this extended state is an appropriate state to analyse stability.

To simplify the proof, we still assume that $m = 1$, and suppose that the optimal solution obtained at step $k - 1$ is given by $\Delta u_{k-1}^* = \Delta u^*(k-1/k-1)$, $y_{sp,k-1}^*(\theta_1), \ldots, y_{sp,k-1}^*(\theta_L)$, $\delta_{y,k-1}^*(\theta_1), \ldots, \delta_{y,k-1}^*(\theta_L)$ and $\delta_{u,k-1}^*$.

A feasible solution to Problem P2 at time k is given by $\Delta\tilde{u}_k = 0$, $\tilde{y}_{sp,k}(\theta_n) = y_{sp,k-1}^*(\theta_n)$, $\tilde{\delta}_{u,k}$ and $\tilde{\delta}_{y,k}(\theta_n)$ are such that

$$u(k-1) + \tilde{D}^u \overbrace{\Delta\tilde{u}_k}^{=0} - u_{\mathrm{des},k} - \tilde{\delta}_{u,k} = 0 \tag{42}$$

$$\hat{x}_n^s(k) + \tilde{D}^0(\theta_n)\overbrace{\Delta\tilde{u}_k}^{=0} - \tilde{y}_{sp,k}(\theta_n) - \tilde{\delta}_{y,k}(\theta_n) = 0, \quad n = 1, \ldots, L \tag{43}$$

In the output tracking of the undisturbed system, slack variables $\tilde{\delta}_{y,k}(\theta_n)$ will be, in general, different from the previous one, except for the true model, in which case, since $\hat{x}_T(k) = A(\theta_T)\hat{x}_T(k-1) + B\Delta u(k-1)$, (43) implies $\tilde{\delta}_{y,k}(\theta_T) = \delta_{u,k-1}^*$. In addition, if the input target is not modified, then $\tilde{\delta}_{u,k} = \delta_{u,k-1}^*$.

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The cost for the actual model θ_T corresponding to this feasible solution is then

$$\begin{aligned}\tilde{V}_{2,k}(\theta_T) &= (y_T(k/k) - y^*_{sp,k-1}(\theta_T) - \delta^*_{y,k-1}(\theta_T))^{\mathrm{T}} \\ &\times Q_y(y_T(k/k) - y^*_{sp,k-1}(\theta_T) - \delta^*_{y,k-1}(\theta_T)) \\ &+ \underbrace{(u(k-1) - u_{\mathrm{des},k} - \delta^*_{u,k-1})^{\mathrm{T}} Q_u(u(k-1) - u_{\mathrm{des},k} - \delta^*_{u,k-1})}_{=0} \\ &+ (x^d_T(k+1/k))^{\mathrm{T}} \bar{Q}(\theta_T) x^d_T(k+1/k) \\ &+ \overbrace{\Delta u(k/k)^{\mathrm{T}} R \Delta u(k/k)}^{=0} + \delta^*_{y,k-1}(\theta_T)^{\mathrm{T}} S_y \delta^*_{y,k-1}(\theta_T) \\ &+ \delta^{*\mathrm{T}}_{u,k-1} S_u \delta^*_{u,k-1}\end{aligned} \tag{44}$$

Now from (20) and (21), we have

$$\delta^*_{y,k-1}(\theta_T) = \hat{x}^s_T(k) - y^*_{sp,k-1}(\theta_T) = C_0 \bar{x}_T(k)$$

$$\delta^*_{u,k-1} = u(k-1) - u_{\mathrm{des},k} = u(k/k) - u_{\mathrm{des},k} = C_1 \bar{x}_T(k)$$

where $C_0 = [\,I_{ny} \quad 0 \quad 0\,]$ and $C_1 = [\,0 \quad 0 \quad I_{nu}\,]$.

Now, considering the model equations (1) and (2) for the true model, we have

$$\begin{aligned}y_T(k/k) - y^*_{sp,k-1}(\theta_T) - \delta^*_{y,k-1}(\theta_T) &= \hat{x}^s_T(k) + \Psi \hat{x}^d_T(k) \\ - y^*_{sp,k-1}(\theta_T) - \delta^*_{y,k-1}(\theta_T) &= \Psi x^d_T(k) = C_2 \bar{x}_T(k)\end{aligned}$$

$$x^d_T(k+1/k) = F x^d_T(k) + D^d F N \overbrace{\Delta u(k/k)}^{=0} = C_3 \bar{x}_T(k)$$

where $C_2 = [\,0 \quad \Psi \quad 0\,]$ and $C_3 = [\,0 \quad F(\theta_T) \quad 0\,]$.

Thus, the cost defined in (44) can be written as

$$\tilde{V}_{2,k}(\theta_T) = \bar{x}_T(k)^{\mathrm{T}} H_1(\theta_T) \bar{x}_T(k) \tag{45}$$

where $H_1 = C_2^{\mathrm{T}} Q_y C_2 + C_3^{\mathrm{T}} \bar{Q}(\theta_T) C_3 + C_0^{\mathrm{T}} S_y C_0 + C_1^{\mathrm{T}} S_u C_1$.

Because of constraint (22), the optimal true cost (i.e. the cost based on the true model, considering the optimal solution that minimise the nominal cost at time k) will satisfy

$$V^*_{2,k}(\theta_T) \le \tilde{V}_{2,k}(\theta_T) \tag{46}$$

and

$$V^*_{2,k+n}(\theta_T) \le V^*_{2,k}(\theta_T) \quad \text{for any } n > 1 \tag{47}$$

By a similar procedure as above and based on the optimal solution at time $k+n$, we can find a feasible solution to Problem P1 at time $k+n+1$, for any $n > 1$, such that

$$\tilde{V}_{2,k+n+1}(\theta_T) \le V^*_{2,k+n}(\theta_T) \tag{48}$$

and from the definition of $\tilde{V}_{k+n+1}$ we have

$$\tilde{V}_{2,k+n+1}(\theta_T) = \bar{x}_T(k+n+1)^{\mathrm{T}} H_1(\theta_T) \bar{x}_T(k+n+1) \tag{49}$$

Therefore combining inequalities (45)–(49) results

$$\begin{aligned}&\bar{x}_T(k+n+1)^{\mathrm{T}} H_1(\theta_T) \bar{x}_T(k+n+1) \\ &\quad \le \bar{x}_T(k)^{\mathrm{T}} H_1(\theta_T) \bar{x}_T(k) \quad \forall n > 1\end{aligned}$$

As $H_1(\theta_T)$ is positive definite, it follows that

$$\|\bar{x}_T(k+n+1)\| \le \alpha(\theta_T) \|\bar{x}_T(k)\| \quad \forall n > 1$$

$$\alpha(\theta_T) = \left[\frac{\lambda_{\max}(H_1(\theta_T))}{\lambda_{\min}(H_1(\theta_T))}\right]^{1/2} \le \max_j \left[\frac{\lambda_{\max}(H_1(\theta_j))}{\lambda_{\min}(H_1(\theta_j))}\right]^{1/2}$$

If we restrict the state at time k to the set defined by

$$\|\bar{x}_T(k)\| < \rho$$

then the state at tine $k+n+1$ will be inside the set defined by

$$\|\bar{x}_T(k+n+1)\| < \alpha(\theta_T)\rho \quad \forall n > 1$$

which proves stability of the closed-loop system, as $\bar{x}_T$ will remain inside the ball $\|\bar{x}_T\| < \alpha(\theta_T)\rho$, where $\alpha(\theta_T)$ is limited, as long as the closed loop starts from a state inside the ball $\|\bar{x}_T\| < \rho$. Therefore as we have already proved the convergence of the closed loop, we can now assure that under the assumption of state controllability at the final equilibrium point, the proposed MPC is asymptotically stable.

Remark 7: It is important to observe that even if condition (3) cannot be satisfied by the input target, or the input target is such that one or more outputs need to be kept outside their zones, the proposed controller will still be stable. This is a consequence of the decreasing property of the cost function (inequality (32)) and the inclusion of appropriate slack variables into the optimisation problem. In this case, the system will evolve to an operating point in which the slack variables (that in steady state are the same for all the models) are as small as possible, but different from zero. This is an important aspect of the controller, as in practical applications a disturbance may move the system to a point from which it is not possible to reach a steady state that satisfies (3). When this happens, the controller will do the best to compensate the disturbance, while maintaining the system under control.

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4.1 General case (case in which condition (3) is not satisfied)

First, we define the global input feasible set ϑ_0, as the set given by

$$\vartheta_0 = \{u : u_{\min} \le u \le u_{\max}\}$$

In addition, we define the more restricted input feasible set, ϑ_u, which is computed taking into account the input and output constraints and the model gains, as

$$\vartheta_u = u : u_{\min} \le u \le u_{\max}$$
$$\text{and } y_{\min} \le D^0(\theta_n)u - \overbrace{D^0(\theta_n)u(\bar{k}) + \hat{x}^s_n(\bar{k})}^{d_n(\bar{k})} \le y_{\max}, \quad n = 1, \ldots, L \tag{50}$$

This set, which depends on the stationary point given by $(u(\bar{k}), \hat{x}^s_n(\bar{k}))$, is the intersection of several sets, each one corresponding to a model of set Ω. When the output zones are narrow, the restricted input feasible set is smaller than the global feasible set, defined solely by the input constraints. An intuitive diagram of the input feasible set is shown in Fig. 3, where only three models are used to represent the uncertainty set.

There are two different feasibility problems because of which condition (3) could not be satisfied. If ϑ_u is not null, the input target $u_{\text{des},k}$ could be within the global input feasible set ϑ_0, but outside the restricted input feasible set ϑ_u. In this case, the slack variables $\delta_{u,\bar{k}}$ and $\delta_{y,\bar{k}}(\theta_n)$ cannot be simultaneously zeroed, and the relative magnitude of matrices Sy and Su will decide the equilibrium point. If the priority is to maintain the output into the corresponding range, the choice must be $S_y \gg S_u$, while preserving $S_u > S_u^{\min}$. Then, the controller will guide the system to a point in which $\delta_{y,\bar{k}}(\theta_n) = 0$, $n = 1, \ldots, L$ and $\delta_{u,\bar{k}} \neq 0$. On the other hand, if ϑ_u is null, that is, if no input belonging to the global input feasible set ϑ_0 simultaneously satisfy all the zones for the models lying in Ω, then the slack variables $\delta_{y,\bar{k}}(\theta_n)$, $n = 1, \ldots, L$, cannot be zeroed, no matter the value of $\delta_{u,\bar{k}}$. In this case (considering that $S_y \gg S_u$), the slack variables $\delta_{y,\bar{k}}(\theta_n)$, $n = 1, \ldots, L$, will be made as small as possible, independent of the value of $\delta_{u,\bar{k}}$. Then, once the output slack is established, the input slack will be accommodated to satisfy these values of the outputs.

5 Simulation results

The system adopted to test the performance of the robust controller presented here is part of the FCC system presented in [15]. This is a typical example of the chemical process industry, and instead of output set points, this system has output zones. The objective of the controller is then to guide the manipulated inputs to the corresponding targets and to maintain the outputs (that are more numerous than the inputs) within the corresponding feasible zones. The system considered here has two inputs and three outputs. Three models constitute the multi-model set Ω on which the robust controller is based. The parameters corresponding to each of these models can be seen in the following transfer functions:

$$G(\theta_1) = \begin{bmatrix} \dfrac{0.4515}{2.9846s+1} & \dfrac{0.2033}{1.7187s+1} \\ \dfrac{1.5}{20s+1} & \dfrac{0.1886s-3.8087}{17.7347s^2+10.8348s+1} \\ \dfrac{1.7455}{9.1085s+1} & \dfrac{-6.1355}{10.9088s+1} \end{bmatrix}$$

$$G(\theta_2) = \begin{bmatrix} \dfrac{0.25}{3.5s+1} & \dfrac{0.135}{2.77s+1} \\ \dfrac{0.9}{25s+1} & \dfrac{0.1886s-2.8}{19.7347s^2+10.8348s+1} \\ \dfrac{1.25}{11.1085s+1} & \dfrac{-5}{12.9088s+1} \end{bmatrix}$$

$$G(\theta_3) = \begin{bmatrix} \dfrac{0.7}{1.98s+1} & \dfrac{0.5}{2.7s+1} \\ \dfrac{2.3}{25s+1} & \dfrac{0.1886s-4.8087}{15.7347s^2+10.8348s+1} \\ \dfrac{3}{7s+1} & \dfrac{-8.1355}{7.9088s+1} \end{bmatrix}$$

In this reduced system, the manipulated input variables are correspond to u_1 the air flow rate to the catalyst regenerator and u_2 the opening of the regenerated catalyst valve. The controlled outputs to y_1 the riser temperature, y_2 the regenerator dense phase temperature and y_3 the regenerator dilute phase temperature.

Following the controller hypothesis, model θ_1 is assumed to be the true model, whereas model θ_3 represents the nominal model that is used into the MPC cost. In the discussion that follows the adopted tuning parameters of the controller are $m = 3$, $T = 1$, $Q_y = 0.5 * \text{diag}(1 \quad 1 \quad 1)$, $Q_u = 0.8 * \text{diag}(1 \quad 1)$, $R = 0.05 * \text{diag}(1 \quad 1)$, $S_y = 10^5 * \text{diag}(1 \quad 1 \quad 1)$ and $S_u = 7.10^3 * \text{diag}(1 \quad 1)$. The input and output constraints, as well

Table 1 Output zones of the FCC system

Output	$y_{\min}$	$y_{\max}$
y_1, °C	510	530
y_2, °C	600	610
y_3, °C	530	590

Table 2 Input constraints of the FCC system

Input	$\Delta u_{\max}$	$u_{\min}$	$u_{\max}$
u_1, ton/h	25	75	250
u_2, %	25	25	101

as the maximum input increment, are shown in Tables 1 and 2.

Before starting the detailed analysis of the properties of the proposed robust controller, we find it useful to justify the need for a robust controller for this specific system. We compare the performance of the proposed robust controller defined through Problem P2, with the performance of the nominal MPC defined through Problem P1. We consider the same scenario described above except for the input targets that are not included (by simply making $Qu = 0$ and $Ru = 0$). This is a possible situation that may happen in practice when the controller is operating as a regulator. Figs. 2 and 3 show the output and input responses, respectively, for the two controllers when the system starts from a steady state where the outputs are outside their zones. It is clear that the conventional MPC cannot stabilise the plant corresponding to model θ_3 when the controller uses model θ_1 to calculate the output predictions. However, the proposed robust controller performs quite well and is able to bring the three outputs to their zones.

We now concentrate our analysis on the application of the proposed controller to the FCC system. As was defined in (50), each of the three models produces an input feasible set, whose intersection constitutes the restricted input feasible set of the controller. These sets have different shapes and sizes for different stationary operating points (since the disturbance $d_n(k)$ is included in (50)), except for the true model case, where the input feasible set remains unmodified as the estimated states exactly match the true states. The closed-loop simulation begins at $uss = [230.5977 \quad 60.2359]$ and $yss = [549.5011 \quad 704.2756 \quad 690.6233]$, which are values taken from the real FCC system. For such an operating point, the input feasible set corresponding to models 1, 2 and 3 are depicted in Fig. 4. These sets are quite distinct from each other, which results in an empty restricted feasible input set for the controller ($\vartheta_u = \vartheta_u(\theta_1) \cap \vartheta_u(\theta_2) \cap \vartheta_u(\theta_3)$). This means that it does not exist an input that taking into account all the models gain and all the estimated states satisfies the output constraints.

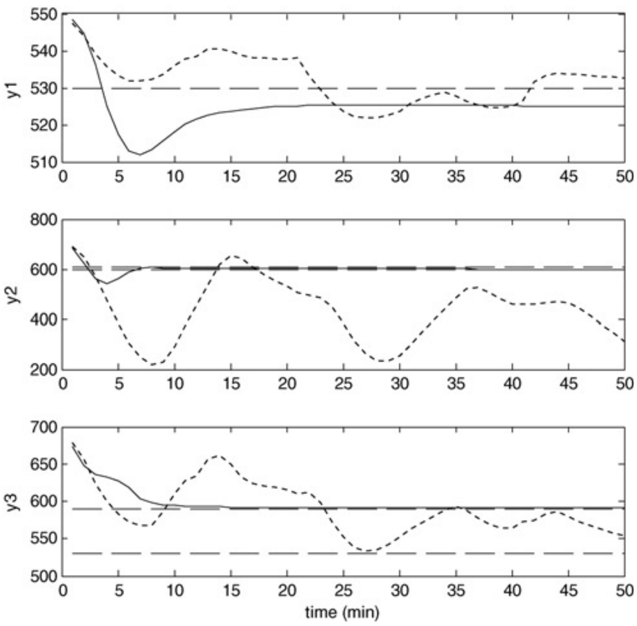

Figure 2 *Controlled outputs for the nominal (– – –) and robust (——) MPC*

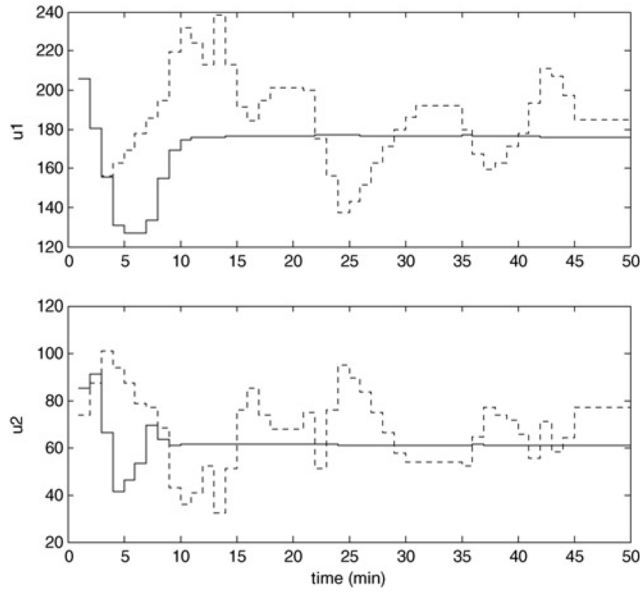

Figure 3 *Manipulated inputs for the nominal (– – –) and robust (——) MPC*

The first objective of the control simulation is to stabilise the system input at $u^1_{\text{des},k} = [165 \quad 60]$. This input corresponds to the output $y = [520 \quad 606.8 \quad 577.6]$ for the true system, which results in the input feasible sets shown in Fig. 5*a*. In this figure, it can be seen that the input feasible set corresponding to model 1 is the same as in Fig. 4, whereas

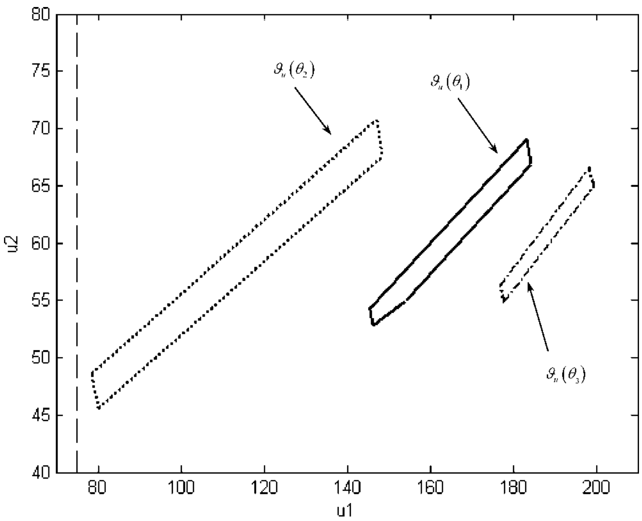

Figure 4 *Input feasible sets of the FCC system*

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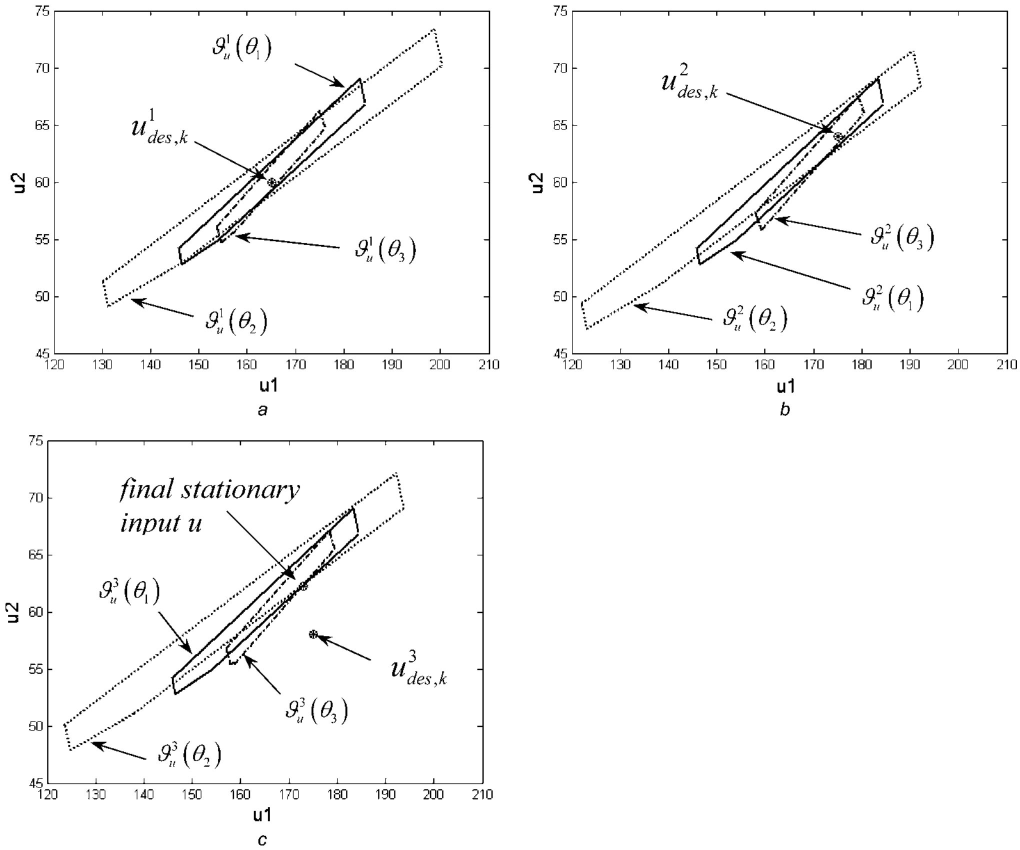

Figure 5 *Input feasible sets*

a Initial input feasible sets
b Input feasible sets when the first input target is changed
c Input feasible sets when the second input target is changed

the sets corresponding to the other models adapt their shape and size to the new steady state. Once the system is stabilised at this new steady state, we simulate a step change in the target of the input (at time step $k = 200$ min). The new target is given by $u^2_{\text{des},k} = [175 \quad 64]$ and the corresponding input feasible sets are shown in Fig. 5*b*. In this case, it can be seen that the new target remains inside the new input feasible set ϑ^2_u, which means that the cost can be guided to zero for the true model. Finally, at time step $k = 400$ min, when the system reaches the steady state, a different input target is introduced ($u^3_{\text{des},k} = [175 \quad 58]$). Different from the previous targets, this new target is

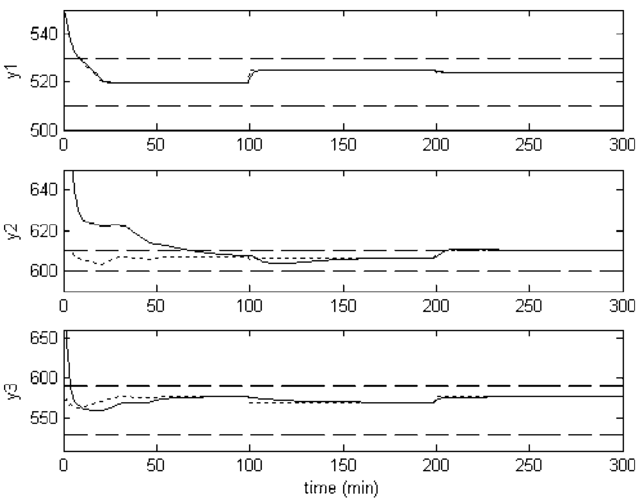

Figure 6 *Controlled outputs and set points for the FCC subsystem with modified input target*

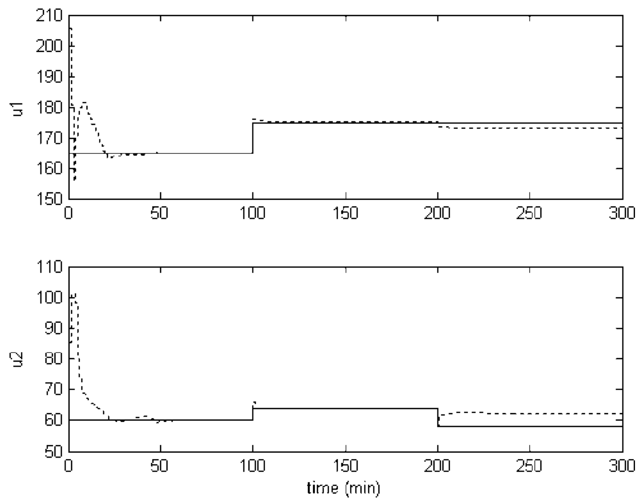

Figure 7 *Manipulated inputs for the FCC subsystem with different input target*

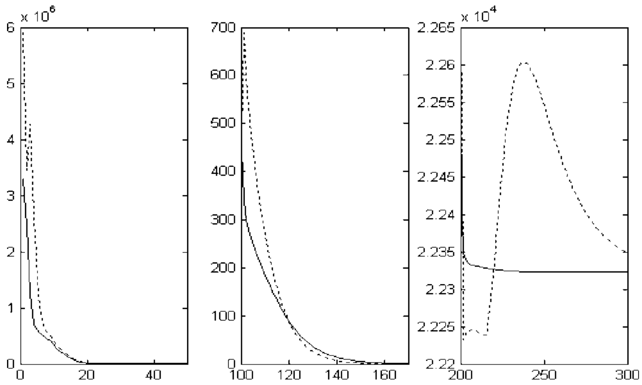

Figure 8 *Cost function corresponding to the true system (solid line) and cost function corresponding to model 3 (dotted line)*

outside the input feasible set ϑ_u^3, as can be seen in Fig. 5*c*. Since in this case, the cost cannot be guided to zero and the output requirements are more important than the input ones, the inputs are stabilised in a feasible point as close as possible to the desired target. This is an interesting property of the controller as such a change in the target is likely to occur in the real plant operation.

Fig. 6 shows the true system outputs (solid line), the set-point variables (dotted line) and the output zones (dashed line) for the complete sequence of changes. Fig. 7, on the other hand, shows the inputs (dotted line) and the input targets (solid line) for the same sequence. As was established in Theorem 1, the cost function corresponding to the true system is strictly decreasing, and this can be seen in Fig. 8. In this figure, the solid line represents the true cost function, whereas the dotted line represents the cost corresponding to model 3. It is interesting to observe that this last cost function is not necessarily decreasing, since the estimated state does not match exactly the true state. Note, in addition, that in the last period of time the cost does not reach zero, as the new target is not inside the input feasible set.

Next, we simulate a change in the output zones. The new bounds are given in Table 3. Corresponding to the new control zones, the input feasible set changes its dimension and shape significantly. In Fig. 9, $\vartheta_u^1(\theta_1)$ corresponds to the initial feasible set for the true model, and $\vartheta_u^4(\theta_1)$, $\vartheta_u^4(\theta_2)$ and $\vartheta_u^4(\theta_3)$ represent the new input feasible sets for the three models considered in the robust controller. Since the input target is outside the input feasible set $\vartheta_u^4 = \vartheta_u^4(\theta_1) \cap \vartheta_u^4(\theta_2) \cap \vartheta_u^4(\theta_3)$, it is not possible to guide

Table 3 New output zones for the FCC subsystem

Output	y_{min}	y_{max}
y_1, °C	510	550
y_2, °C	400	500
y_3, °C	350	500

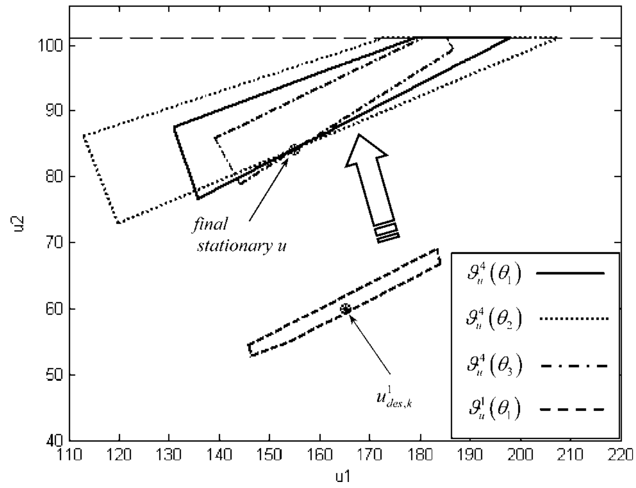

Figure 9 *Input feasible sets for the FCC subsystem when a change in the output zones is introduced*

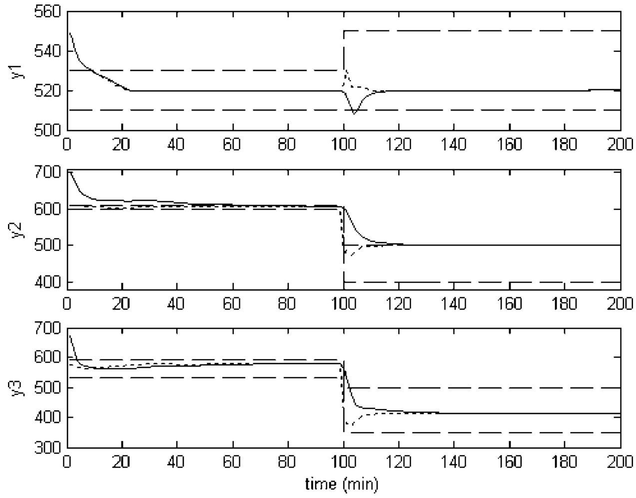

Figure 10 *Controlled outputs and set points for the FCC subsystem with modified zones*

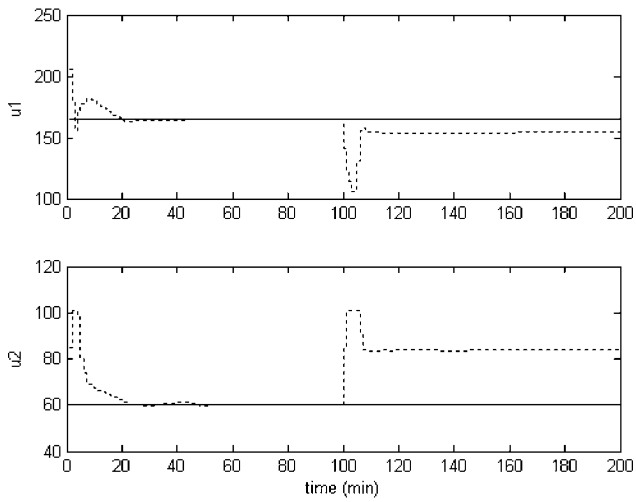

Figure 11 *Manipulated inputs for the FCC subsystem with modified output zones*

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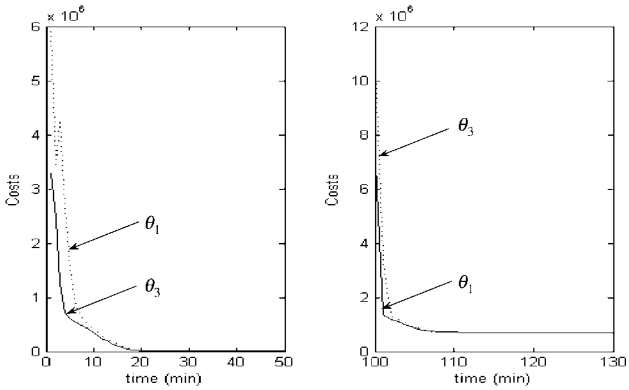

Figure 12 *Cost function for the FCC subsystem with modified zones*

True cost function (solid line)
Cost function of model 3 (dotted line)

the system to a point in which the control cost is null at the end of the simulation time. As long as the output weight Sy is kept larger than the input weight Su, all the outputs are guided to their corresponding zones, while the inputs show a steady state offset with respect to the target $u^1_{\text{des},k}$. The complete behaviour of the outputs and inputs of the FCC subsystem, as well as the output set points, can be seen in Figs. 10 and 11, respectively. The final stationary value of the input is $u = [155 \quad 84]$, which represents the closest feasible input value to the target $u^1_{\text{des},k}$. Finally, Fig. 12 shows the control cost of the two simulated time periods. Observe that in the last period of time (from 200 to 400 min) the true cost function does not reach zero since the change in the operating point prevents the input and output constraints to be satisfied simultaneously.

6 Conclusion

In this work, a robust MPC previously presented in the literature was extended to the case of output zone control. The control structure assumes that model uncertainty can be represented as a discrete set of models (multi-model uncertainty). The proposed approach assures both recursive feasibility and stability of the closed-loop system. The main idea consists in using an extended set of variables in the control optimisation problem, which includes the set point to each predicted output. This approach introduces additional degrees of freedom in the zone control problem. Stability is achieved by imposing non-increasing cost constraints that prevent the cost corresponding to the true plant to increase. The strategy was shown, by simulation, to have an adequate performance for a 2 × 3 subsystem of a typical industrial system.

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