Hedging late frost risk with American barrier options: An application to grape cultivation in Argentina

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Abstract The purpose of this paper is to model and to value a temperature derivative to hedge late frost risk in grape cultivation. Starting from historical data, we propose a continuous stochastic model to describe the minimum temperature behavior. We define an American binary option to hedge late frost risk, and present a numerical application to grape cultivation.
Keywords: Weather derivatives, minimum temperature, agriculture, late frost risk, American options.

JEL Classification code: 21A54 - 55P54

1 Introduction

A financial weather contract is defined as a contract in which the payments are contingent upon a future weather event. This type of agreement can take the form of a weather derivative or an insurance contract.

Weather derivatives are financial contracts with payoffs that depend upon a underlying climatological event, such as temperature, hail, rainfall or snowfall, and are mainly used by companies whose businesses are exposed to weather variations such as energy related companies (producers or distributors) and agriculture, to hedge potential weather risks. In 1997 the Chicago Stock Exchange started trading these kinds of derivatives. Since then, the use of weather derivatives and related markets has increased significantly. It is estimated that over 95% of the wheather derivatives traded are contracts on temperature indices, such as Heating Degree Days (HDD), Cooling Degree Days (CDD) and Cumulative Degree Days (CDD), defined in terms of daily average temperature, and are mainly used by companies in the energy sector. In [5] there is a thorough summary of the characteristics of the European market of weather derivatives and a comparison between traditional insurance policies and weather derivatives in terms of risk management applications.

In the case of the agriculture industry, the main differences between weather derivatives and insurance contracts are related to regulatory and legal terms [6]. For example, if an event of hail takes place in a wine production area, traditional insurance contracts will compensate producers once the damage is properly quantified and only if the percentage of the damages is higher than the one stipulated in the agreement. On the contrary, in the case of weather derivatives that rigidity does not exist as the payment is made when a weather event occurs, regardless of the degree of the damage. Furthermore, the use of weather derivatives would eliminate any moral hazard issues that occur with insurance contracts [15]. These two features would make weather derivatives attractive for the producer. However, the main disadvantage of weather derivatives is that they cannot be used as perfect hedges against risk [14], since the loss incurred in case of a weather event and the payment of the derivative itself may differ in "basis risk"; e.g. in the case that the temperature of the crops area differs from the temperature of the area where the meteorological station is located, the payment of the derivate my not cover the losses incurred by the producer.

Despite the fact that the late frost can be prevented by traditional methods such as fuel burning, sprinkler irrigation, and through heaters, these methods are not always effective or profitable [8]. The first method is the most common way to prevent frost, but the cost associated to the fuel used is relative high compared to total production's costs. The use of sprinkler irrigation only reduces damages. Furthermore, if the temperature is too low this method is ineffective.

In this paper we propose an American binary option on minimum temperature to hedge the risk of late frost faced by fruit producers, and present a numerical application to viticulture, using temperature data collected at meteorological stations in Mendoza, Argentina. An updated review on temperature models may be found in [2] and [10].

The remainder of this paper is organized as follows. In Section 2 we present the data analysis. Section 3 contains details of the minimum temperature model. In Section 4 we discussed the calibration of the model to data from Mendoza, Argentina. In Section 5 we intoduce the option. In Section 6 we show the numerical results for an application to viticulture. The last section contains the conclusions.

2 Theoretical Framework: Weather derivatives

A weather derivative is a contract between two parties (buyer and seller) that agree, at inception, the exchange of payments depending on the weather conditions throughout the contract period. Specifically, and according to Zeng (2000), in order to set up the derivative both parties must agree the duration, the weather index to be used as an underlying (W), the type of contract (put or call), an official weather station where they can get the data required to compile the index, the strike(X), the payment scheme, and finally, the contract premium.

Taking into account all the elements above, the payoff for a call and a put would be, respectively

$$P_{call} = k \max\left(W - X; 0\right),$$

$$P_{put} = k \max\left(X - W; 0\right),$$

where k is a constant factor defined at the beginning of the contract that determines the amount to be paid per weather unit W that exceeds the strike X.

The weather derivatives based on temperature indices are used primarily in the energy sector since the temperature is one of the main factors that determine energy consumption. The underlying of these contracts are the indices HDD (Heating Degree Days) or CDD (Cooling Degree Days), defined as

$$HDD_i = \max\left(18 - T_i; 0\right),$$

$$CDD_i = \max\left(T_i - 18; 0\right),$$

where

$$T_i = \frac{T_{i^{Max}} + T_i^{min}}{2},$$

T_i^{Max}, and T_i^{min} are the maximum and the minimum temperature for the day i, and 18°C is a reference temperature; i.e. the values of HDD and CDD values for a given day depend on the difference, in Celsius degrees, (°C) between T_i and the reference temperature. The choice of the reference value relies on the criteria established by the standards of the U.S. energy sector; the reason is that when the temperature is below 18°C people tend to consume more energy in order to heat homes, and when it is above 18°C a higher consumption of energy also occurs since they tend to use air conditioners.

Most traditional weather derivatives are based on cumulative $HDDs$, and $CDDs$ during a given period. In the particular case of this work, it is not appropriate to use these indices as underlying since the damage to the crop occurs when the temperature, and not their average, is below certain value.

3 Data Analysis

Our dataset consists of 11 years of daily minimum temperature data collected at Tunuyán station, in the province of Mendoza, the main area of fruit cultivation in Argentina. As 37 data were missing, our first step was to interpolate these missing observations. Following [12], we use the method of Principal Component Analysis (PCA) described in [7] to reconstruct 26 of them. For the remaining 11 data we use linear interpolation, since the records from neighbouring meteorological stations required to apply PCA were not available.

The temperature path can be seen as a combination of a deterministic trend together with random shocks. As it is observed in Figure 1, there is strong evidence of a periodic component and mean reversion towards this periodical variation.

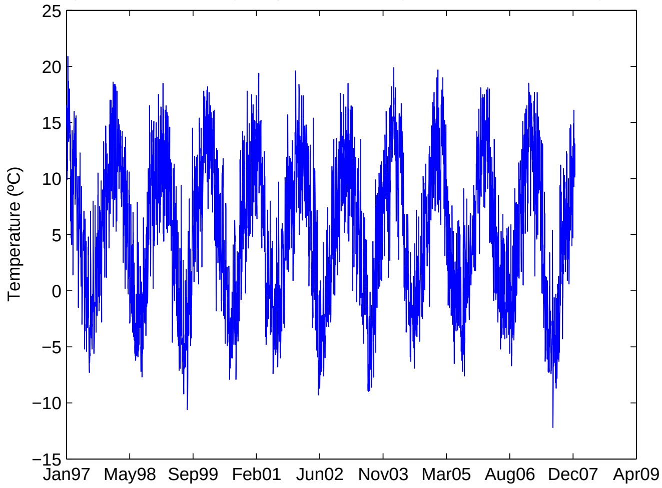


Figure 1: Minimum daily temperature − Tunuyán station, Mendoza, Argentina

Figure 2 shows a Normality test for the temperature returns, and we can observe a deviation from normality and evidence of fat tails (if the returns were Normally distributed we would get a straight line). In Figure 3 we have the evidence of a non negligible first order autocorrelation, as expected from the temperature seasonality (c.f. [13]).

However, the frequency distribution of the raw data shown in Figure 4 justifies the adoption of a Normal distribution to model the stochastic component.

4 The Temperature Model

We propose a model for the minimum temperature T_t^{min} similar to the first one-factor model introduced in [11]

$$T_t^{min} = f(t) + X_t, \tag{1}$$

$$dX_t = -\kappa X_t dt + \sigma_t dW_t, \tag{2}$$

where $f(t)$ is a totally predictable term, the dynamics of X_t is given by the SDE (2), $\kappa > 0$, $X(0) = X_0$, and dW_t is the increment to a standard Wiener process W_t; i.e., X_t follows a mean-reverting process, with a time dependent long-term reversion level, and reversion speed κ. Assuming that $f(t)$ satisfies

Figure 2: Normal probability test for temperature returns

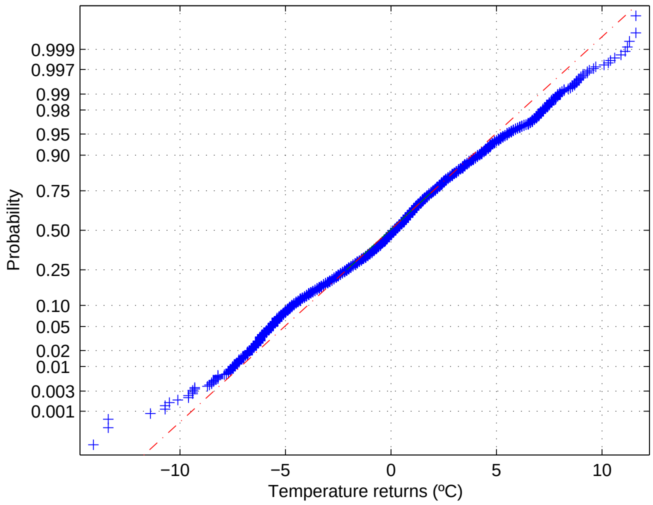
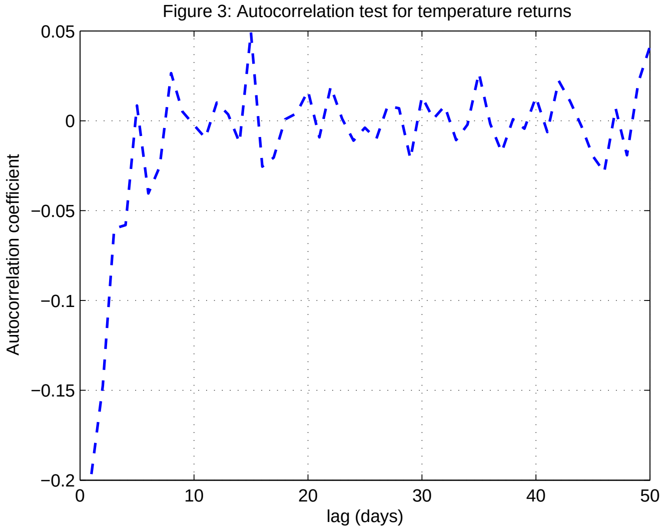


Figure 3: Autocorrelation test for temperature returns

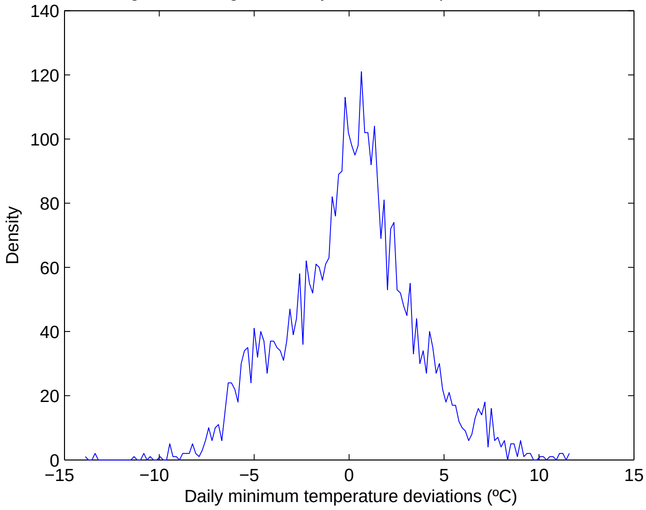


the required regularity conditions, from (1) and (2) we derive the stochastic differential equation for the minimum temperature

$$dT_t^{min} = \kappa\left(g(t) - T_t^{min}\right)dt + \sigma_t dW_t, \tag{3}$$

where

Figure 4: Histogram of daily minimum temperature deviations



$$g(t) \equiv \frac{1}{\kappa}\frac{\partial f(t)}{\partial t} - f(t). \qquad (4)$$

This model is similar to the extended Vasicek model for interest rates [9], and coincides with the model proposed in [1] for the daily average temperature.

Due to the non tradable nature of the temperature, to calculate the value of derivatives we should consider the market price of risk. Following [11], we obtain the risk neutral process for the variable T^m

$$dT_t^{min} = \kappa\left(\alpha(t) - T_t^{min}\right)dt + \sigma_t dW_t,$$

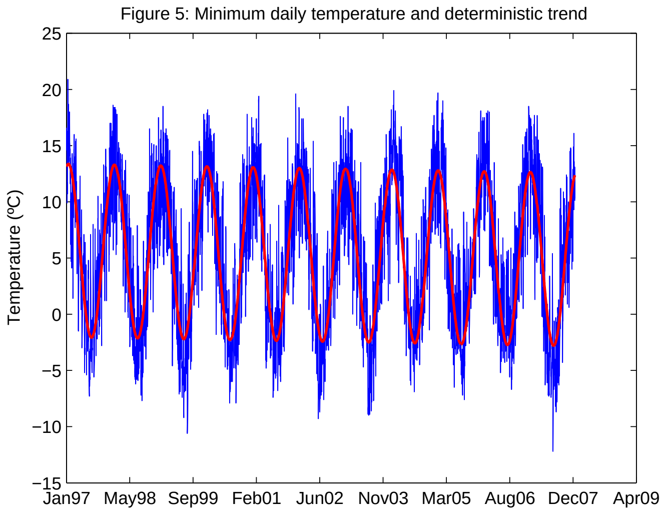
where $\alpha = \frac{\lambda\sigma}{\kappa}$, and λ is the market price of risk. Since there is not a wheather derivative market in Argentina to estimate the implicit market price of risk, our valuation results will be based on (3) and (4); i.e., under the restrictive assumption that the market price of risk equals zero or, equivalently, that the investors are risk neutral.

5 Calibration of the Model

We model the function $f(t)$ in (1) as

$$f(t) = A + Bt + Csin(\omega t + \phi),$$

where t es the time variable, $\omega = \frac{2\pi}{365}$ is the annual frequency. By applying ordinary least squares to estimate the parameters of the above equation we found $A = 5.67$, $B = -0.0002$, $C = 7.70$, $\phi = 0.262$



To estimate the rate of mean reversion κ, we need an estimate of the volatility. In [1] it is argued that the volatility varies throughout the months but remains approximately constant for each month; i.e.

$$\sigma_k = \begin{cases} \sigma_1 & \text{during January} \\ \sigma_2 & \text{during February} \\ \dots\dots\dots & \dots\dots\dots\dots\dots\dots \\ \sigma_{12} & \text{during December,} \end{cases}$$

with

$$\sigma_k = \frac{1}{N_k} \sum_{j=1}^{N_t - 1} \left(T_{j+1}^{min} - T_j^{min} \right)^2, \tag{5}$$

where the index $k = 1, 2, \dots 12$ runs throughout the months of the year, N_k is the number of days in the month t, and T_j is the minimum temperature registered on day j.

To estimate the speed of reversion we use the martingale estimation function method introduced in [4] (see also [1]). The unbiased estimator of the mean reversion speed is given by (see [4], Example 1.2)

$$\widetilde{\kappa} = -\log \frac{\sum_{i=1}^{n} \left[f(i-1) - T^{min}(i-1) \right] \left[T^{min}(i) - f(i) \right] / \sigma_{i-1}^2}{\sum_{i=1}^{n} \left[f(i-1) - T^{min}(i-1) \right] \left[T^{min}(i-1) - f(i-1) \right] / \sigma_{i-1}^2}$$

where n is the size of the sample, and σ_{i-1}^2 are the estimates (5) of the monthly volatility. We obtained $\widetilde{\kappa} = 0.517$.

Before applying the Normality tests to the deseasonalized returns we removed the positive outliers defined as the data with values greater than three times the average volatility. Only 10 positive outliers were found in 4017 data. It was not necessary to iterate the procedure since Jacque-Bera test at 5% performed on the deseasonalized returns confirm our initial assumption of normality. Compare Figures 2. and 6. to see the improvement in Normality.

Figure 6: Normal probability test for filtered returns

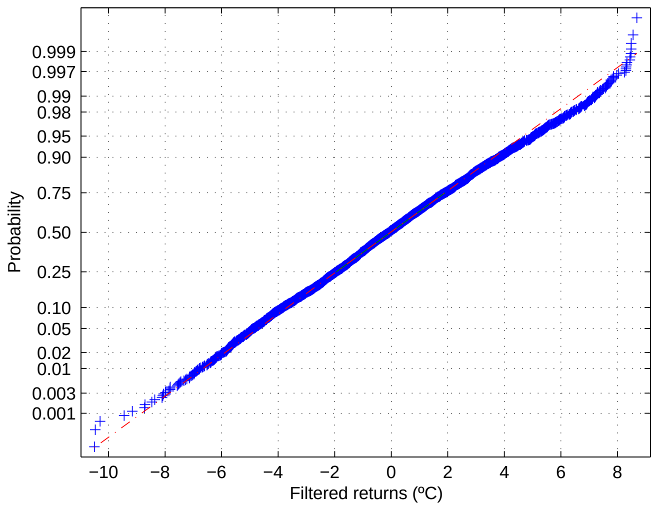
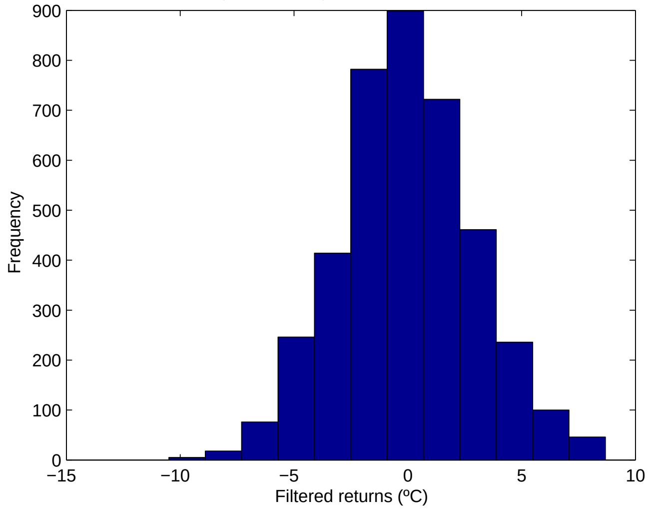
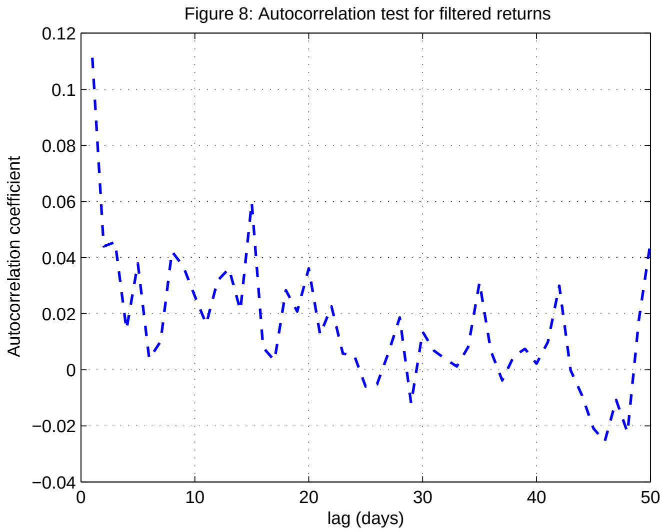


Figure 7: Histogram for filtered returns



The histogram for filtered returns is shown in Figure 7, and in Figure 8 we can observe the first fifty autocorrelation coefficients.



The plot of the volatility of the deseasonalized returns in Figure 9 suggests a stochastic behaviour of the monthly volatility.

Unlike the results in [3] and [12], where the behaviour of the volatility is described by a Vasicek model, our tests indicate that the monthly volatility can be modeled by

$$\sigma_t = \sigma_0 + \gamma \varepsilon_t. \tag{6}$$

where t varies throughout the months, σ_0 is a constant trend, and γ is the volatility of the volatility. From the tests performed on the difference $\sigma_t - \sigma_0$ we conclude that

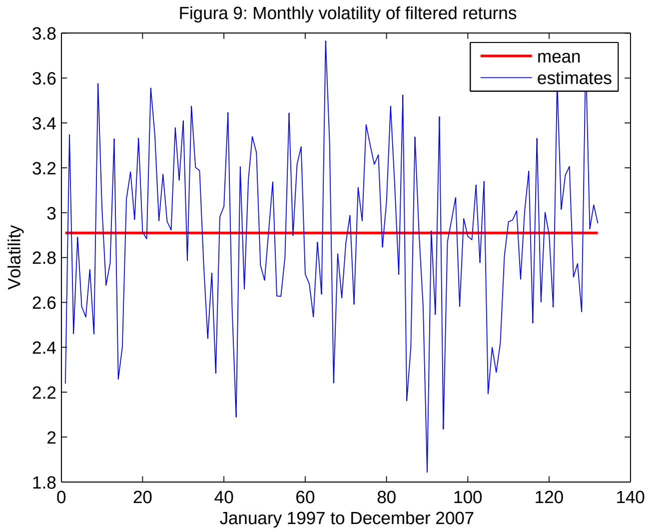
i the null hypothesis can not be rejected at 5% for Jacque-Bera and Lilliefords tests;

ii we can reject at 5 % the null hypothesis of ARCH effects in the Engels test on heteroscedasticity, thus we assume that the volatility residues are i.i.d. Gaussian perturbations.

The estimates computed for the parameters in (6) are $\sigma_0 = 2.91$, and $\gamma = 0.37$.

6 The Option

We propose an American option with binary payment (cash-or-nothing) defined by



$$\text{Payoff} = \mathcal{H}\left(K - T^{min}\right), \tag{7}$$

where $\mathcal{H}$ is the Heaviside function, and K is the critical minimum temperature for frost injuries; i.e., the payoff has value 1 when its argument is positive, and zero otherwise. It is always optimum to exercise the option as soon as K exceeds T^{min} (c.f. [17]). These contracts must be tailored to the individual location and species, since the ocurrence of frost damages depends on the phenological stages of the selected species and the strike is the critical temperature for each stage. Phenological data from the Department of Contingencies of Mendoza are availble for several fruit plants, and some of them are exhibited in Table 1.

	Colored closed buds	**Full bloom**	**Small green fruits**	**2 cm fruits**
Grapevine	-1.1	-0.6	-0.6	
Peach	-3.9	-2.8	-1.1	-3.0
Plum	-3.4	-2.2	-1.1	-2.0
Apple	-3.9	-2.2	-1.7	-4.0
Almond-tree	-3.3	-2.7	-1.1	-4.0
Walnut	-1.0	-2.2	-0.5	

Table 1: Critical Temperatures for Phenological Stages (°C)

The option value can be interpreted as a percentage, i.e. to receive 100% of the amount to be hedged, a premium equal to 100 times the option value of that amount must be payed.

7 Numerical Example: An application to grape cultivation

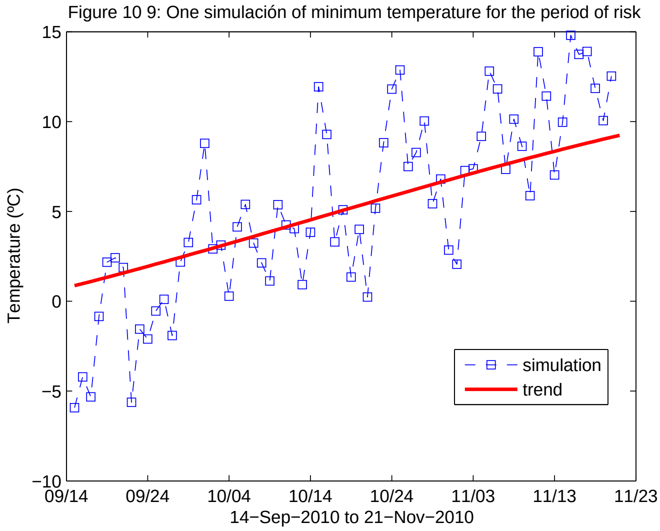
We choose the grape cultivation for a numerical example and use Monte Carlo method to valuate a contract with payoff given by (7), where the parameters are defined in terms of the phenological data shown in Table 2.

Phenological stage	Starting	Ending
Colored closed buds	14-Sep	3-Oct
Full bloom	4-Oct	16-Nov
Small green fruits	17-Nov	21-Nov

Table 2: Periods of late frost-risk for vineyards

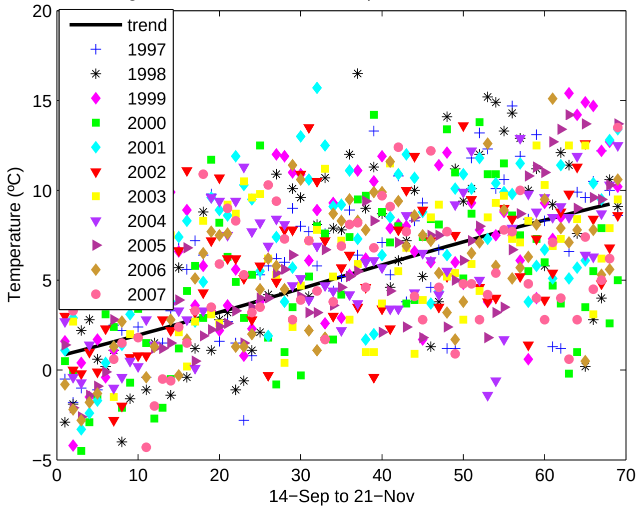
Figure 10 shows one simulation of the minimum temperature for the period September 14 to November 21, 2010. The value of the option to cover late frost risk for the whole risk period is 0.957; therefore, it is not a financially attractive alternative, but it is a result considered reasonable since the probability of frost is close to 1 during the first part of the risk period. We already had an indication of its order of magnitude from a valuation performed using historical data that gives an average option value of 0.91. In Figure 11 we can see eleven years of historical minimum temperature for the period of risk.

Figure 10 9: One simulación of minimum temperature for the period of risk



For shorter periods the value of the option decreases as expected. Table 3 shows the option value for different periods. For the first phenological stage the

Figure 11: Historical data for the period of risk, 1997 − 2007



value is 0.8964; for the period September 25 to November 21, it decreases to 0.81, and if the whole period of risk is divided into 10-day intervals, it ranges between 0.0219 and 0.7859. The dependence of the option value both on the length of the period of analysis and on the period starting date is consistent with the seasonal characteristic of the temperature.

Period	**Option value**
14 sep - 21 Nov (the whole period of risk)	0.9570
20 Sep - 21 Nov	0.9004
25 Sep - 21 Nov	0.8108
Phenological stage	
1st.	0.8964
2nd.	0.6269
3rd.	0.0111
10-day periods	
14 Sep - 23 Sep	0.7859
24 Sep - 3 Oct	0.5804
4 Oct - 13 Oct	0.3902
14 Oct - 23 Oct	0.2535
24 Oct - 2 Nov	0.1230
3 Nov - 12 Nov	0.0510
13 Nov - 21Nov	0.0219

Table 3: Sensitivity to the period of analysis

The decision about the amount to be hedged basically boils down to two possible choices: the value of production or the costs incurred up to the date on which the climatological event occurs. In the first case it is difficult to give an *a priori* estimate of that amount since the future price of production is uncertain. Even though there are future markets for products derived from grape, such as sulfited must, these markets are poorly developed. Moreover, the sulfited could be a wrong comparable because it is only used in the preparation of syrups, candies or grape juice. On the other hand, the data for the second choice are more accurate because each producer knows the costs incurred up to the date on which the frost occurs.

Table 4 (c.f. [16]) shows the production cost breakdown for grape cultivation at four ages of the plant. For three-year-old plants the costs related to the prevention of late frost have a significant share in the total production cost, approximately 58%, and are substantially reduced to 17% when the plant is in its adult stage.

Processes	**Year 1**		**Year 2**		**Year 3**		**Year 4**	
Implantation	**7,560**		**167**		**0**		**0**	
Raw Material	5,597		299					
Labor	1,750	94%	190	13%				
Machinary	213		22					
Tillage	**361**		**556**		**556**		**556**	
Raw Material	8		107		107		107	
Labor	210	4%	225	43%	225	16%	225	29%
Machinary	143		223		223		223	
Maintenance	**113**		**278**		**676**		**818**	
Raw Material	5		8		10		14	
Labor	109	1%	270	21%	665	19%	180	42%
Machinary							24	
Fertilization	**0**		**190**		**126**		**126**	
Raw Material			54		75		75	
Labor			93	15%	16	4%	16	7%
Machinary			43		36		36	
Sanitation	**31**		**104**		**131**		**91**	
Raw Material	15		42		59		19	
Labor	16	1%	16	8%	16	4%	16	5%
Machinary			45		56		56	
Late fros defense	**0**		**0**		**1,981**		**343**	
Raw Material					1,821		183	
Labor					75	57%	75	18%
Machinary					85		85	
Total	**8,065**		**1,294**		**3,470**		**1,934**	

Table 4: Grape - Cost breakdown - U$S per hectare

Depending on the effectiveness of traditional methods of late frosts defense, the producer could find a good financial solution by combining both hedging methods: using traditional methods of frost defense in the first part of the risk period, when the probability of frost is close to 1, and the weather derivative for the remainder of the risk period. Thus, the cost of both the traditional method and the derivative would decrease. The optimal combination depends on insurance costs for different subperiods within the whole period risk.

8 Conclusions

The option value obtained for grapes indicates that if the entire risk period (September 14 to November 21) is considered in the definition of the option, the derivative is not a suitable financial alternative since its value is very close to its payoff. This result is consistent with the fact that the probability of late frosts, especially in the first 10 days of the period, is close to 1. If we consider a shorter period of analysis, a combination of this financial alternative and the traditional methods of defense against frost could be used in order to reduce the costs of both methods, and the decision will rely on the search of an optimal combination.

The results obtained from tests for other crops, such as apple, with a critical temperature of -3.9°C in the first risk period, might be convenient from a financial standpoint. This is shown qualitatively in Table 5 that exhibits the sensitivity of the option value to the strike (critical temperature).

It is pertinent to point out that the minimum temperature shows a non-negligible decreasing trend. The slope of the function (5.4) fitted to 11 years of historical data is -2 * 10-4. Hence, the risk of late frosts increased considerably in 11 years; e.g. the values of the trend for 14th. September 1997 and 2007 are 1.5813 and 0.8561 respectively. Furthermore, the model was also calibrated for the last 5 years of data obtaining a slope of -7 * 10-4 which would predict a greater increase in the risk of late frosts.

Apart from the conclusions obtained for the particular case considered in this paper, the results on the behavior of the minimum temperature could have an economic impact: the model predicts an appreciable decreasing trend which is likely have consequences in a few years' time for crops in the area of concern. Preliminary tests carried out maximum daily temperature data gave an positive linear trend of 2.3 * 10-4. These estimates are consistent with the "global warming", whose effects are already being observed in the desertification of fertile areas and the transformation of desertic areas into fertile ones.

Acknowledgments

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