Comment on "General Non-Markovian Dynamics of Open Quantum Systems"

The existence of a "non-Markovian dissipationless" regime, characterized by long lived oscillations, was reported in Ref. [1] for a class of quantum open systems. It is claimed this could happen in the strong coupling regime, a surprising result that has attracted some attention. We show that this regime exists if and only if the total Hamiltonian is unbounded from below. This fact was not mentioned in Ref. [1], and casts serious doubts on the usefulness of this result. Having no ground state and no thermal state, unbounded Hamiltonians are thermodynamically unstable since they act as infinite sources of energy when weakly coupled to any other system.

The calculation in Ref. [1] is correct but overlooks this instability, whose existence can be easily shown. An oscillator $H_S = \Omega a^\dagger a$ couples to an environment $H_E = \sum_k \omega_k b_k^\dagger b_k$ through $H_I = \sum_k \lambda_k (a b_k^\dagger + a^\dagger b_k)$. The total Hamiltonian $H_T = H_S + H_E + H_I$ commutes with the number operator $N = a^\dagger a + \sum_k b_k^\dagger b_k$. Thus, an eigenstate in the single excitation sector can be written $|\phi\rangle = C^\dagger|0,0\rangle$, obtained by applying the creation operator $C^\dagger = c_s a^\dagger + \sum_k c_k b_k^\dagger$ to the vacuum $|0,0\rangle$. The equation $H_T|\phi\rangle = E|\phi\rangle$ implies $E c_s = \Omega c_s + \sum_k \lambda_k c_k$ and $E c_k = \omega_k c_k + \lambda_k c_s$ [2]. Thus E satisfies $E = \Omega + \sum_k \lambda_k^2/(E - \omega_k)$, which for $E = -|\omega_0| \leq 0$ becomes $\Omega + |\omega_0| = \sum_k \lambda_k^2/(|\omega_0| + \omega_k)$. This has solutions if and only if $\Omega + \delta\Omega < 0$ with $\delta\Omega = -\sum_k \lambda_k^2/\omega_k$. Thus, in this regime the total Hamiltonian acquires a negative eigenvalue $E < 0$. Moreover, as $[H_T, C^\dagger] = E C^\dagger$, the states $|\phi_n\rangle \propto (C^\dagger)^n|0,0\rangle$ satisfy $H_T|\phi_n\rangle = -n|\omega_0||\phi_n\rangle$. As its eigenvalues extend to $-\infty$, H_T is unbounded from below if and only if $\Omega + \delta\Omega < 0$.

We now follow the argument presented in Ref. [1] to show that the dissipationless regime at strong coupling is precisely the regime when $\Omega + \delta\Omega < 0$, and the total Hamiltonian is unbounded. The density matrix of the system satisfies [1,3] $\dot\rho = -i[\tilde\Omega(t) a^\dagger a, \rho] + \gamma(t)[1 + \tilde n(t)](2a\rho a^\dagger - a^\dagger a \rho - \rho a^\dagger a) + \gamma(t)\tilde n(t)(2a^\dagger \rho a - a a^\dagger \rho - \rho a a^\dagger)$. Coefficients depend on the Green's function that satisfies $\dot u(t) + i\Omega u(t) + \int_0^t ds \eta(t-s) u(s) = 0$, where the dissipation kernel is $\eta(s) = \int_0^\infty d\omega J(\omega)\exp[-i\omega s]$, and the spectral density is $J(\omega) = \sum_k \lambda_k^2 \delta(\omega - \omega_k)$. The frequency and damping rate satisfy $i\tilde\Omega(t) + \gamma(t) = -\dot u(t)/u(t)$ while $\gamma(t)\tilde n(t) = \frac{1}{2}\dot\xi(t) + \gamma(t)\xi(t)$, where $\xi(t) = \int_0^t d\tau \int_0^t ds u(\tau)\tilde\nu(s-\tau)u(s)^*$ and $\tilde\nu(s) = \int_0^\infty d\omega J(\omega)\exp[i\omega s]/[\exp(\omega/k_B T) - 1]$ (T is the temperature of the environment). A dissipationless regime exists when $u(t) \to r\exp[-i\omega_0 t]$ at long times. In this case $\Omega(t) \to \omega_0$ and $\gamma(t) = \gamma(t)\tilde n(t) \to 0$: the system evolves unitarily with Hamiltonian $\tilde H_S = \omega_0 a^\dagger a$. For $u(t)$ to behave in this way, its Laplace transform must have a purely imaginary pole, i.e., $\omega_0 - \Omega + i\hat\eta(-i\omega_0) = 0$, where the Laplace transform of $\eta(t)$ is $\hat\eta(s)$. The imaginary part of this equation is $J(\omega_0) = 0$. For spectral densities of any type (Ohmic, sub-Ohmic, etc.) satisfying $J(\omega) > 0$ for all $\omega > 0$, this condition is satisfied for $\omega_0 < 0$ [1]. With $\omega_0 = -|\omega_0|$ the real part gives $\Omega + |\omega_0| = \int_0^\infty d\omega J(\omega)/(\omega + |\omega_0|)$, which has solutions if and only if $\Omega + \delta\Omega < 0$, with $\delta\Omega = -\int_0^\infty d\omega J(\omega)/\omega$, i.e., the condition under which H_T becomes unbounded. Figure 1 shows $\gamma(t)$ and $\tilde\Omega(t)$ for the same parameters as in Fig. 2 of Ref. [1] [where $\Omega(t)$ is not shown]. We see $\tilde\Omega(t)$ approaches a negative value, making the renormalized Hamiltonian unbounded, while $\gamma(t)$ vanishes.

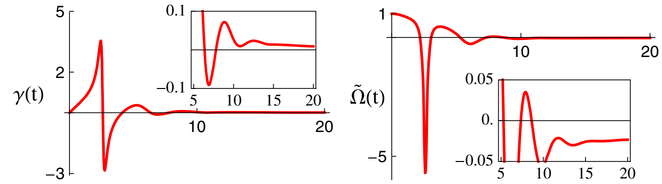

FIG. 1 (color online). Damping rate and renormalized frequency for the dissipationless sub-Ohmic model shown in Fig. 2 of Ref. [1].

Thus, the dissipationless regime of Ref. [1] for strong coupling exists if and only if the total Hamiltonian is unbounded from below, and therefore thermodynamically unstable. This raises serious doubts regarding the physical relevance of the Hamiltonian of Ref. [1] at strong coupling, since any residual counterrotating terms (however small) will cause the system to tend towards a state with infinite negative energy. An analogous (but more severe) instability is known for the famous model where the system Hamiltonian is $H_S = p^2/2m + \kappa x^2/2$, the environment is $H_E = \sum_k (p_k^2/2m_k + m_k\omega_k^2 q_k^2/2)$, and the interaction is $H_I = \sum_k \tilde\lambda_k x q_k$. Then, the total Hamiltonian is $\tilde H_T = H_R + \sum_k [p_k^2/2m_k + (m_k\omega_k^2 q_k + \tilde\lambda_k x)^2/2m_k\omega_k^2]$. Here, $H_R = p^2/2m + \kappa_R x^2/2$ with $\kappa_R = \kappa + \delta\kappa$ and $\delta\kappa = -\sum_k \tilde\lambda_k^2/m_k\omega_k^2$ [4]. Thus, when $\kappa_R < 0$, $\tilde H_T$ is unbounded. In this case the model is not only thermodynamically unstable but also dynamically unstable.

Dara P. S. McCutcheon, Juan Pablo Paz
and Augusto J. Roncaglia
Departamento de Física, FCEyN, UBA and IFIBA, Conicet
Pabellón 1, Ciudad Universitaria
1428 Buenos Aires, Argentina

Received 17 December 2013; revised manuscript received 21 October 2014; published 13 October 2015
DOI: 10.1103/PhysRevLett.115.168901
PACS numbers: 03.65.Yz, 03.65.Ta, 05.70.Ln, 42.50.Lc

[1] W.-M. Zhang, P.-Y. Lo, H.-N. Xiong, M. W.-Y. Tu, and F. Nori, Phys. Rev. Lett. **109**, 170402 (2012).

[2] There are various ways to show the unboundedness of H_T. Here, we follow A. Rancon and J. Bonart, Europhys. Lett. **104**, 50010 (2013).

[3] J. P. Paz and A. J. Roncaglia, Quantum Inf. Process. **8**, 535 (2009).

[4] E. A. Martinez and J. P. Paz, Phys. Rev. Lett. **110**, 130406 (2013).