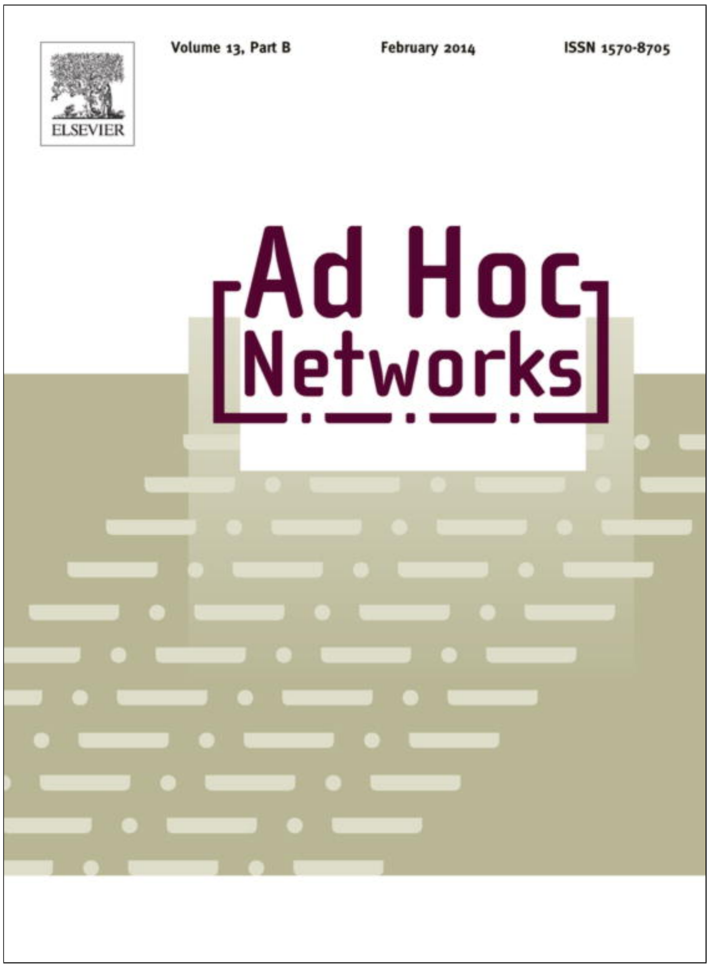
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Improved geographic routing in sensor networks subjected to localization errors

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ABSTRACT

Geographic routing strategies used in wireless communication networks require that each transmitting node is aware of its location, the locations of its neighbors, and the destination. With this information, the message is routed by choosing intermediate nodes, or relays, which allow the destination to be reached with the maximum possible transmitted information rate and with minimum delay. However, this strategy needs to take into account the uncertainties of the relays locations in order to avoid an important performance degradation of the link, or even a routing failure.

Taking into account the presence of uncertainties in the relays locations, each possible geographic routing strategy is able to recognize a subset of nodes that can be candidates for relays. Furthermore, the transmission range between nodes not only depends on the distance between them, but also the communication channel fading. Based on the effect that these uncertainties have on the link channel capacity, a minimization of a cost function is proposed to decide the next hop relay, which optimizes, in mean, the maximum rate of information transmitted with the minimum number of hops. Using the location statistics, this optimal strategy is applied for both one-hop decisions and two-hops decisions. Working expressions for on-line fast calculations are provided and used for results illustrations.

1. Introduction

The solid wireless sensor networks information processing approach is based on a canonical problem formulation of localizing and tracking moving objects [1]. Location-aware personal devices and location-based services have become ever more prominent in the past few years (see [2,3]). Moreover, geographic routing protocols show high performance and are considered as promising candidates for large-scale ad hoc networks. These protocols carry low overhead as they do not require a route management process. These routings use location information, so that each transmitting node is aware of its location, the locations of its neighbors – called relays – and the destination. The routing decision is made locally, where every node forwards the packet to the most promising neighbor towards the destination.

Geographic routing protocols use geographic forwarding by assuming ideal conditions [4]. However, under realistic situations such as location errors, obstacles, and radio irregularity the performance degrades or may lead to routing failures. In [5] the authors concludes that a reception-based forwarding strategies in realistic conditions are generally more effective than distance-based strategies but this could be at the cost of lower energy efficiency. The performance of geographic routing, based on the assumption that the location of each node is accurate, can be greatly improved when location uncertainties and channel transmission properties are taken into account.

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Taking into account the presence of uncertainties in the relays locations, and also the communication channel fading, in this work, a minimization of a cost function is proposed to decide the next hop relay. The factors to consider in defining appropriate metrics in the cost function are: (i) Low probability of data loss; (ii) maximum rate of information arrival to the destination; and (iii) minimum impact of signaling and control on effective data rate. In order to achieve an efficient relay selection that takes into account all these factors, in this paper the channel capacity, together with the uncertainties statistics, will be considered as a basis to derive formal expressions of two metrics. One is the probability of error, defined as the relative amount of information lost when nodes have uncertainty in position and are subject to fading noise. The other is the progress of information in terms of the expected distance between hops toward the destination. Since a greater progress information implies an increase in the probability of error, the optimal selection of the next relay toward the destination should be a trade off between the two metrics. In this paper, by using these two quantities, we define a cost, which weighs in a relative way both the probability of error and the progress of the information. Based on a subset of possible relays, the one which optimizes the proposed cost function is selected. In this way, we obtain optimal relay selections, in the sense of minimum probability of error and maximum information progress.

The remainder of this paper is organized as follows: In Section 2, a discussion of recent works that address strategies to improve geographic routing in networks subject to uncertainty, and the relationship with our approach, is performed. In Section 3, based on the location statistics and their influence on the channel capacity we define and derive the proposed metrics. In Section 4 we propose the criterion for the cost evaluation in order to decide optimally the relays. Also working equations useful to on line evaluate the metrics for one-hop and the two-hops cases are derived. The results for both selection cases using simulations for different uncertainties levels and different numbers of possible relay are shown in Section 5. Furthermore, these results are compared with other two different criteria, the greedy routing scheme (GRS), and the maximum expectation within transmission range (MER). Finally, we conclude the paper in Section 6.

2. Related work

The analysis in [6] shows that one of the main reasons of failures in face routing happen due inconsistency in the distance between two nodes caused by location errors. Their study shows that realistic location errors can in fact lead to incorrect (non-recoverable) behavior and noticeable degradation of performance on geographic routing. They find that in some cases, more than 10% storage failure of sensor network events can occur in the presence of 10% location error. These failures can be reduced by using improved protocols based on using extra local information exchange between nodes. They analyze and identify the error scenarios and propose modifications to eliminate the error and enhance the performance in both Greedy Perimeter Stateless Routing and in Geographic Hash Table protocols. In addition, to study the effect of location inaccuracy on greedy forwarding, an enhancement using mobility prediction models is proposed in [7]. Based on a stochastic decisions, in which each node performs on-line probing of its neighbors in order to decide the next hop, in [8] a strategy is proposed obtaining significant improvements with respect to the deterministic decision. In the cited references, the proposal to mitigate the problem of uncertainty in the location consists of reducing uncertainties either by adding predictive models, by improving measures, or by checking connectivity.

A different approach on how to mitigate the impact of location errors is discussed in [9]. Since it is assumed that each node knows its position and the position error variance, the authors propose to attach an error information field in a message for geographic routing and to announce the statistical characteristics of the location error to neighbors together with location information. With this information, they propose to choose the relay that maximizes the expected progress of information within the transmission range.

In this paper, we will also follow a stochastic approach, but we will consider different metrics and cost function, from those used in [9]. It is important to note the similarities and differences between both approaches. (i) As proposed in [9], we also consider that each node informs its neighbors about its own location error bound. Also, we use the progress in the cost function, but unlike [9], which uses the expected progress within the transmission range, our progress is the conditional expected value, given that the message was successfully received by the relay. This is a major difference, because the progress is calculated only considering the messages that are successfully received by the relay. (ii) Differently from [9], our cost includes a second metric, which is the probability that the message was actually received by the relay. This strategy allows us to get the optimal selection of relays that combine the maximum possible progress with the maximum number of messages successfully received. (iii) Additionally, our approach includes the uncertainty in the transmission range due to the fading of the communication channel. Formally speaking, the transmission range of a node, defined as the maximum distance within which a relay can receive messages, is given by the channel capacity of the link [11]. The channel capacity of the link depends on the signal-to-noise power ratio at the receiving node. The signal-to-noise ratio depends on the distance between the transmitter and the receiver, the thermal noise, the transmission power, the signal propagation constants, and also largely on the channel fading. In real conditions, the received signal strength at the relay is affected by multi-path or shadowing fading, which is usually modeled by a stochastic process with log-normal distribution. As a consequence, the transmission range must be considered as a stochastic variable for performance evaluation purposes. To this end, it is important to consider the channel capacity for each relay in order to evaluate the quality for routing information. In the presence of location uncertainties and fading, the channel capacity is a random variable from which both the probability that information reaches a given relay and the most probable distance reached, can be obtained. It is worth noting that, by considering our cost function together with the channel fading, the results differ considerably from [9], as it will be shown in the simulation section.

3. Metrics derivation

Let us assume a scenario where a sensor network is governed by a predetermined geographic routing protocol. If the locations of the nodes were perfectly known, the given protocol is designed to decide the next relay. However, if the positions are uncertain, the next relay decision is not uniquely defined and a decision within a subset of possible relays is required. Assuming that the statistics of the uncertainties are known, they can be used to select the next relay in the subset optimally.

Suppose that a source sends r messages per second, and the entropy of a message is H bits per message. The information rate is $R = rH$ bits/s. The Shannon's theorem states that (i) a given communication system has a maximum rate of information C, known as the channel capacity; (ii) if the information rate R is less than C, then one can approach arbitrarily small error probabilities by using intelligent coding techniques. In the case of Gaussian channels, the capacity is given in bits/s by

$$C = B\log_2(1 + SNR), \tag{1}$$

where B is the bandwidth of the channel and SNR is the signal to noise power ratio. In order to quantify the degradation of the channel in terms of uncertainties, we first analyze the channel capacity as a function of the distance between transmitter and receiver. We consider the case where the channel is affected by multipath or shadowing fading with log-normal distribution. The received power at distance d is governed by the following log-normal path loss model [17]:

$$Pl(d) = Pl(d_0) + 10\gamma\log_{10}(d/d_0) + \eta, \tag{2}$$

where $Pl(d)$ is the power loss in dB after the signal propagates through distance d; $Pl(d_0)$ is the power loss in dB at the reference distance d_0; γ is the path loss distance exponent (typically between 2 and 4); and η is a random variable expressed in dB representing the slow fading noise (shadow noise), which is normal distributed, $\eta \sim \mathcal{N}\left(0, \sigma_\eta^2\right)$.

The received power, $Pr(d)$, is equal to the transmitting power Pt minus the path loss $Pl(d)$, all in dB, such that $Pr(d) = Pt - Pl(d)$. By taking the reference distance equal to the unity, it can be written as

$$Pr(d) = \kappa - 10\gamma\log_{10}(d) - \eta, \tag{3}$$

where $\kappa = Pt - Pl(d_0)$ is a constant, and the Gaussian shadow noise η has zero mean and variance between 3 dB and 8 dB [3]. Thus, using these definitions, the signal-to-noise ratio in dB is given by

$$SNR^{dB} = Pr(d) - N = \kappa - 10\gamma\log_{10}(d) - \eta - N, \tag{4}$$

where N is the power in dB of the thermal additive white Gaussian noise $n(t)$. Due to the presence of $n(t)$ the communication channel is called AWGN channel. The signal-to-noise ratio in (4) is given by $SNR = 10^{SNR^{dB}/10}$, using the relationship above we have

$$SNR = 10^{(\kappa-N)/10} d^{-\gamma} 10^{-\eta/10} = \alpha d^{-\gamma} e^{-a\eta}, \tag{5}$$

where $a = \ln(10)/10$ and $\alpha = 10^{(\kappa-N)/10}$. The distance d is a *iid* variable, statistically independent from η, Rician distributed $d \sim \mathcal{R}(\mu, \sigma^2)$ (see [13,14]), with mean μ and variance σ^2. The Rician distribution is due to the assumption that, in the two dimensional case, coordinated (x,y) of both transmitter and receiver have *iid* uncertainties Normal distributed with $x_t \sim \mathcal{N}(\mu_{xt}, \sigma_t^2)$, $y_t \sim \mathcal{N}(\mu_{yt}, \sigma_t^2)$ for the transmitter, and $x_r \sim \mathcal{N}(\mu_{xr}, \sigma_r^2)$, $y_r \sim \mathcal{N}(\mu_{yr}, \sigma_r^2)$ for the receiver. Then, the euclidean distance $d = |x_t - x_r, y_t - y_r|$, is Rice distributed $d \sim \mathcal{R}(\mu, \sigma^2)$ with mean $\mu = |\mu_{xt} - \mu_{xr}, \mu_{yt} - \mu_{yr}|$and variance $\sigma^2 = \sigma_r^2 + \sigma_t^2$ (see [15,13]). Then, the statistics of distance d can be obtained from the mean and variance of the transmitter and the receiver location. These parameters depend on the method of estimating the position and are assumed to be known.

In what follows we will consider that (i) The nodes to be considered as possible relays are pre-selected using some protocol that ensures the forward direction of the message toward the destination. (ii) The average distance between the transmitter and relay μ is much larger than its standard deviation σ. For relationship $\mu/\sigma \geqslant 3$, using the Hankel approximation of the modified Bessel function of the first kind and zero order, see [16], it can be shown that the rice distribution is approximately equal to a normal one such that the following fulfills [14]:

$$\mathcal{R}(\mu, \sigma^2 | \mu/\sigma \geqslant 3) \approx \mathcal{N}(\tilde{\mu}, \sigma^2), \tag{6}$$

where $\tilde{\mu} = (\mu^2 + \sigma^2)^{1/2}$. Thus, we consider the uncertain distance as normal distributed on the imaginary line to the destination address constrained to $\mu/\sigma \geqslant 3$.

Assuming a value of bandwidth $B = \ln(2)$, using (5) in (1), the channel capacity can be expressed by

$$C = \ln(1 + \alpha d^{-\gamma} e^{-a\eta)}). \tag{7}$$

The capacity of a transmission channel gives us the limit, in terms of bits of information per unit of time, which cannot be exceeded without loss of information. We are interested in evaluating the link quality using (7) for efficient relay decisions. To do this, we will use two important metrics. The first is called *error probability*, which is the probability that the transmission rate (R) exceeds the capacity of the link (C). It is worth noting that if part of the transmitted data are used for channel signaling and control, it is possible to consider a smaller effective capacity. The other is the expected distance reached by the information at the chosen relay, which we call *progress*. In order to write such metrics formally let us first define, using (7), the constant z_c as which

$$z_c = \ln((e^C - 1)/\alpha) = -a\eta - \gamma\ln(d), \tag{8}$$

The constant z_c define a bound in the domain (η, d). Given a transmission rate R, the corresponding constant z_r is obtained by replacing R in place of C in (8). Note that z_r and z_c are negative definite values. Thus, z_r needs to be greater than or equal to, z_c so that information can be transmitted.

Let us consider the set $\mathcal{D} = \{(\eta, d)|z_r > z_c\}$, shown in Fig. 1 where the values of the couple (η, d) allow the information to reach the relay. Note that the critical distance $\bar{d}$ in the figure is the maximum distance, at $\eta = 0$, where transmission is possible, usually called transmission range, and is given by

$$\bar{d} = e^{-z_c/\gamma}. \tag{9}$$

The first metric we consider is the error probability P_e, defined as the volume of the density function bounded by the area defined by $\mathcal{D}$ in Fig. 1. Formally, we can express this probability as

$$P_e = \mathcal{P}\{z_r \leqslant z_c\} = 1 - \int\int_{\mathcal{D}} f_{\eta d}(\eta, d) d_\eta d_d, \tag{10}$$

where $f_{\eta d}(\eta, d)$ is the joint distribution of the random variables pair (η, d). Considering the independence between η and d, and calling $\mathcal{D}(\eta)$ the subset of D for a given value of η, the error probability can be obtained as

$$P_e = 1 - \int_{-\infty}^{\infty} f_\eta(\eta) \left[\int_{\mathcal{D}(\eta)} f_d(d) d_d \right] d_\eta, \tag{11}$$

where $f_\eta(\eta)$ and $f_d(d)$ are the distribution of variables η and d.

The second metric we consider is the progress P_r, which is defined as the expected value of the distance covered by the message subjected to the message has passed. It can be written as

$$P_r = \int_{-\infty}^{\infty} f_\eta(\eta) \left[\int_{\mathcal{D}(\eta)} d f_d(d) d_d \right] d_\eta / (1 - P_e). \tag{12}$$

We shall now see how to use these metrics to decide the relay within a subset of possible candidates. To this end, we need to define the decision criterion.

4. The optimality criterion

If the distance between the transmitter and the relay increases, it is more likely that the information does not reach the relay, and vice versa. Thus, both metrics, error probability and progress, are inversely related. In order to derive a criterion to optimally decide the best relay, we propose to weigh the error probability and the inverse of progress. The way to achieve the optimal relay decision, using these two metrics, is by minimizing the following cost function:

$$J(\rho) = Pe + \frac{\rho}{Pr}, \tag{13}$$

where ρ is a positive scalar. The value of ρ weighs the importance of both metrics for the optimum selection. Thus, the choice of the weight reflects the tradeoff between the requirements of low probability of error and progress information. Values of ρ close to zero prioritize a low probability of error at the expense of low progress. This will bring a greater number of hops to reach the destination but will ensure that the message reaches the destination with high probability. Conversely, a high value of ρ prioritizes the selection of remote relays, decreasing the number of hops, even if the probability that the message reaches the destination is low. However, it is important to note that the relay which minimizes the cost function, for any value of the weight, is always an optimal solution in the sense that there is no other relay to achieve more progress for the same error, and vice versa. Setting a value of ρ, it is possible to calculate the cost function for each one of the admissible relays.

Now, we show how to proceed for one-hop or two-hops decisions. Let us assume that each node is able to define a cone toward the destination in which a number of possible relays lies, as shown in Fig. 2. In the case of one-hop strategy, starting from A, an optimal decision must be made between B and C, which are within the transmission range of A. In general, by denoting $J_i(\rho)$ the cost function of each one of the N admissible relays that belong to the set $\mathcal{C}$, the optimal relay is given by

$$\arg\min_{i \in \mathcal{C}} \{J_i(\rho)\}. \tag{14}$$

For node A, the set is $\mathcal{C} = \{B, C\}$. To proceed with the next hop, if B was selected, the optimal relay decision must be made in the set $\mathcal{C} = \{D, E\}$. On the other hand, if it was C, a decision must be done in the set $\mathcal{C} = \{E, G, F\}$. We call this strategy *one-hope twice*, since each of the two hops are decided by using the strategy of one-hop used twice. Now, in the case of the joint two-hops strategy, in the example, the optimal decision must be made considering the set $\mathcal{C} = \{(A, B, D), (A, B, E); (A, C, E); (A, C, G); (A, C, F)\}$.

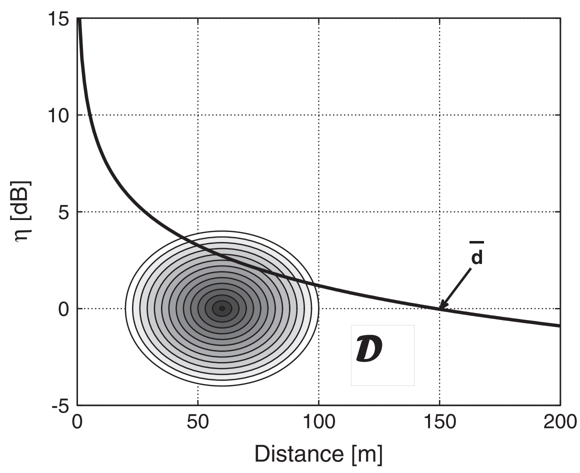

Fig. 1. Joint (η, d) distribution and set $\mathcal{D} = \{(\eta, d)|z_r > z_c\}$, the area below the curve which defines the probability that the information reaches the next relay. The concentric circles represent the levels of the relay uncertainty with normal distribution.

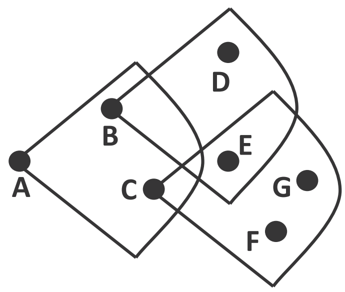

Fig. 2. Forwarding cone.

Now, we will obtain useful expressions for practical calculations of P_e and P_r, for two cases. The first one is for one-hop relay decision, and in the second one we extend the approach for two-hops relays decision.

4.1. One-hop case

Considering the independence between distance and fading, the metrics given by Eqs. (11) and (12), for the case of one-hop decision are well approximated as follows:

$$Pe \approx 1 - \frac{1}{2\pi\sigma\sigma_\eta}\int_{-\infty}^{\infty} e^{-\frac{1}{2}\left(\frac{\eta}{\sigma_\eta}\right)^2}\int_{-\infty}^{e^{(-a\eta-z_C)/\gamma}} e^{-\frac{1}{2}\left(\frac{d-\mu}{\sigma}\right)^2} d_d d_\eta, \quad (15)$$

$$Pr \approx \frac{1}{2\pi\sigma\sigma_\eta}\int_{-\infty}^{\infty} e^{-\frac{1}{2}\left(\frac{\eta}{\sigma_\eta}\right)^2}\int_{-\infty}^{e^{(-a\eta-z_C)/\gamma}} d e^{-\frac{1}{2}\left(\frac{d-\mu}{\sigma}\right)^2} d_d d_\eta/(1 - Pe), \quad (16)$$

where the lower limit of the integration interval extends from zero to $-\infty$ to be consistent with the gaussian approximation, and up to the bound given by (8). Is worth noting that, because we are considering that the uncertainty is bounded proportionally to the distance, the contribution to the metric when extending the integration interval from zero to $-\infty$ is negligible. In order to obtain working equations suitable for real time with low computational cost, we need to solve and obtain simple expressions for both metrics. In the appendix, we develop the approaches needed to obtain the following simple expressions:

$$Pe \approx 1 - \mathcal{C}\left(\tilde{\mu}, \sigma_g^2, \bar{d}\right), \quad (17)$$

$$Pr \approx \tilde{\mu} - \frac{\sigma^2}{(1 - Pe)}\mathcal{N}(\tilde{\mu}, \sigma_g^2, \bar{d}), \quad (18)$$

where $\mathcal{C}(\tilde{\mu}, \sigma_g^2, \bar{d})$ and $\mathcal{N}(\tilde{\mu}, \sigma_g^2, \bar{d})$ are the normal cumulative and the normal probability distribution function, with mean $\tilde{\mu}$ and variance $\sigma_g^2 = \sigma^2 + \frac{a^2\bar{d}^2}{\gamma^2}\sigma_\eta^2$, evaluated at distance $\bar{d}$. Note that the parameters required to calculate these expressions for each relay are $(C, \gamma, \sigma_\eta, \tilde{\mu}, \sigma)$, where (C, γ, σ_η) are channel parameters and $(\tilde{\mu}, \sigma)$ are obtained as it was described before using the mean and variance of the transmitter and the receiver location uncertainties. Moreover, for an efficient evaluation of the cumulative function in (18), there are abundant methods which range from tables to modern software, and fast calculation of normal pdf in (18) can be obtained directly from its mathematical expression.

4.2. Two-hops case

The joint normal distribution between two consecutive distances, one between the transmitter node and the first relay – that we call d_1 – and the other, between the first relay and the second – that we call d_2 – is as follows:

$$f_{d_1 d_2}(d_1, d_2) = \frac{1}{2\pi|\Sigma|^{1/2}} e^{-\frac{1}{2}\mathbf{h}\Sigma^{-1}\mathbf{h}^T}, \quad (19)$$

where $\mathbf{h} = [d_1 - \tilde{\mu}_1, d_2 - \tilde{\mu}_2]$, and Σ is the covariance matrix between both stochastic variables. It is assumed that the distance between the first relay and destination is greater than the distance between the second relay and destination. Thus, both quantities, d_1 and d_2, are positive scalar. Calling σ_1^2 the variance of distance d_1, σ_2^2 the variance of distance d_2, and σ_{12} the cross-covariance between d_1 and d_2, the covariance matrix is given by

$$\Sigma = \begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{bmatrix}. \quad (20)$$

In the case that σ_A^2, σ_B^2, and σ_C^2 are the variances of locations at nodes A, B, and C, the variances of distances d_1 and d_2 are given by $\sigma_1^2 = \sigma_A^2 + \sigma_B^2$, $\sigma_2^2 = \sigma_B^2 + \sigma_C^2$, and $\sigma_{12} = -\sigma_B^2$. Now, taking into account this joint distribution, the metrics (11) and (12) can be well approximated by

$$Pe \approx 1 - \int_{-\infty}^{\infty} \frac{e^{-\frac{1}{2}\left(\frac{\eta}{\sigma_\eta}\right)^2}}{\sqrt{2\pi}\sigma_\eta} \times \int\int_{-\infty}^{e^{(-a\eta-z_C)/\gamma}} f_{d_1 d_2}(d_1, d_2) d_{d1} d_{d2} d_\eta. \quad (21)$$

$$Pr \approx \int_{-\infty}^{\infty} \frac{e^{-\frac{1}{2}\left(\frac{\eta}{\sigma_\eta}\right)^2}}{\sqrt{2\pi}\sigma_\eta}\int\int_{-\infty}^{e^{(-a\eta-z_C)/\gamma}} (d_1 + d_2) f_{d_1 d_2}(d_1, d_2) d_{d1} d_{d2} d_\eta/(1 - Pe). \quad (22)$$

In the appendix, we derive the following simple expressions for fast computation of the metrics:

$$Pe \approx 1 - \mathcal{C}([\tilde{\mu}_1, \tilde{\mu}_2], \Sigma_g, [\bar{d}, \bar{d}]), \quad (23)$$

$$Pr \approx (\tilde{\mu}_1 + \tilde{\mu}_2) - \left(\theta_1\beta_1\mathcal{N}(\tilde{\mu}_1, \sigma_{g11}^2, \bar{d}) + \theta_2\beta_2\mathcal{N}(\tilde{\mu}_2, \sigma_{g22}^2, \bar{d})\right)/(1 - P_e). \quad (24)$$

where

$$\beta_1 = \mathcal{C}\left(\tilde{\mu}_2 - \frac{\sigma_{g12}^2}{\sigma_{g11}^2}(\bar{d} - \tilde{\mu}_1), \frac{|\Sigma_g|}{\sigma_{g11}^2}, \bar{d}\right),$$

$$\beta_2 = \mathcal{C}\left(\tilde{\mu}_1 - \frac{\sigma_{g21}^2}{\sigma_{g22}^2}(\bar{d} - \tilde{\mu}_2), \frac{|\Sigma_g|}{\sigma_{g22}^2}, \bar{d}\right),$$

$$[\theta_1, \theta_2] = [1, 1]\Sigma; \text{ and } \Sigma_g = \begin{pmatrix} \sigma_{g11}^2 & \sigma_{g12}^2 \\ \sigma_{g21}^2 & \sigma_{g22}^2 \end{pmatrix} = \Sigma + \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\left(\frac{\bar{d}a\sigma_\eta}{\gamma}\right)^2.$$

The parameters required to calculate these expressions for each relay, are $(C, \gamma, \sigma_\eta, \tilde{\mu}_1, \tilde{\mu}_2, \sigma_A, \sigma_B, \sigma_C)$. Note, first, that considering $\sigma_{g12} = \sigma_{g21} = 0$ both metrics one-hop twice and two-hops give the same decision. Secondly, note that the improvement obtained by considering the two-hops design, compared to the one-hop twice can be obtained analytically by subtracting Eq. (23), for the error performance, and (24), for the progress performance, from the same equations but with $\sigma_{g12} = \sigma_{g21} = 0$.

5. Simulation results and discussion

In this section, we present simulations to illustrate the performance of the method and also comparisons with other strategies. In the simulations, it is assumed that the admissible relays are pre-selected using a protocol that ensures the advance direction of the message toward the destination. Moreover, the used protocol is able to select relays that fulfill the relationship $\mu/\sigma \geqslant 3$. Any other relay that does not satisfy this condition is discarded, eliminating, in this way, the possibility of considering those that are farther away from the destination than the transmitter, i.e. behind the transmitter. We also assume that each node broadcasts its own location, both mean value and standard deviation, periodically and proactively.

The communication channel considered is Gaussian, and is affected by shadowing fading with log-normal distribution. The channel capacity between transmitter and relay, (7), is given by

$$C = B\ln(1 + \alpha d^{-\gamma} e^{-a\eta}), \tag{25}$$

where $B = 100/\ln(2)$ kHz, $\gamma = 2$, and $\alpha = 10^6$. Using these parameters, the signal-to-noise level at distance $\bar{d} = 250$ m, without fading, is $SNR^{dB} = 12\ dB$. The adopted transmission rate is $R = 200$ Kbits/s which corresponds to the channel capacity at distance $\bar{d}$, the transmission range without fading noise.

To illustrate the results, we will consider a wireless sensor network using two different simulation softwares. On the one hand, we developed a source, using MATLAB's built-in functions, which simulate random distances between nodes, linked by transmission channels affected by fading noise. On the other hand, we developed C++ functions inside the Network Simulator (NS2). In both cases, three strategies are compared, greedy routing scheme (GRS), the maximum expectation within the transmission range (MER), and our approach. For each criterion used a one-hop relay is chosen within the admissible set.

The next hop selection in GRS consists in choosing the relay that minimizes the distance to the destination within the transmission radius [12]. In the proposed scenario, considering N admissible relays, GRS chooses the relay which maximizes the distance $\bar{d} - d_i$, given $d_i < \bar{d}$, where d_i is the distance from transmitter to ith relay. Formally,

$$\arg\max_{i=1,\dots,N} \{\bar{d} - d_i\}; \quad \text{st} : d_i < \bar{d}. \tag{26}$$

The second strategy considered is the MER [9], which consists in choosing the relay that maximizes the expected progress, given by the following expression:

$$\arg\max_{i=1,\dots,N} \left\{ d_i \left(1 - e^{-u^2/2\sigma 2}\right) \right\}; \quad \text{st} : d_i < \bar{d}. \tag{27}$$
$$u = \min(d_i, \bar{d} - d_i)$$

The third strategy implies to choose the relay that minimizes the cost (13) for an adopted value of weight $\rho \geqslant 0$, as follows:

$$J(\rho) = \arg\min_{i=1,\cdots,N} \{J_i(\rho)\}; \quad \text{st} : d_i < \bar{d}. \tag{28}$$

5.1. Failure rate vs progress

First, the one-hop relay decision using MATLAB's built-in functions is simulated in several different scenarios. Within this setup, for each relay selected, the channel capacity is calculated by using Eq. (25). If the channel capacity is greater than R, the message is considered successfully received by the relay; otherwise, the message has failed. When the message has failed, no retransmission method is considered. The relationship between the number of lost messages and the number of messages sent gives the failure rate. The progress is computed as the averaged distances between transmitter and relay for each message successfully received at each hop.

In the simulations, each one of the N admissible relays are uniformly distributed within the distance interval $d \in [0, \bar{d}]$. The standard deviation σ_i, representing the distance uncertainty of each relay, is chosen randomly and proportionally to the distance d_i, as follows:

$$\sigma_i = qrd_i, \tag{29}$$

where $q \in [0, 0.2]$, so that the variance remains within the range $\mu/\sigma \geqslant 5$, and r is a random variable uniformly distributed in the interval $[0, 1]$. Three cases of fading with standard deviation noises of $\sigma_\eta = 0\ dB$, $\sigma_\eta = 1\ dB$, and $\sigma_\eta = 2\ dB$ are considered.

In Fig. 3, the failure rate versus progress for the one-hop decision case, considering 10 relays for each hope at different fading noises and uncertainty standard deviation levels, are depicted. These values were obtained by averaging 3000 transmitted packages. The same, but considering 20 relays, is shown in Fig. 4. It can be seen from the figures that for low fading the signal-to-noise ratio increases, and so does the channel capacity. Thus, for every selection criteria, when fading decreases, progress increases and failure rate decreases, as it was expected. In all cases MER, has a lower failure rate than GRS at the expense of a decrease in progress. However, there always exist weights ρ in our cost that improve the relay selection, as regards tradeoff between failure and progress, for both GRS and MER. Thus, for the same progress obtained with GRS or MER, our optimal selection reaches lower failures and for the same failure rate, more progress. With increasing uncertainties, the differences between our cost with respect to GRS and MER become significant. As expected, these differences are greater when more relays are considered to decide, as it can be seen from the figures for 10 and 20 relays. In the figures, only a limited range of weight values ρ in our cost is depicted. In Fig. 5, the performance of the optimal cost, for an extended rank of weights ρ, different number of relays, and fading noise are shown.

Now, we present the performance for the two-hops decision. In this case, it is assumed that the nodes have the location information of their neighbors beyond their transmission range, so that the metrics for two consecutive hops toward the destination can be calculated. The same scenario as in the one-hop case is considered. The two-hops decision performance is compared with the one-hop decision twice. In a first case, we consider two relays for the first hop and three, for the second hop. Each one of the first hop can reach two of the second hop. Thus, there

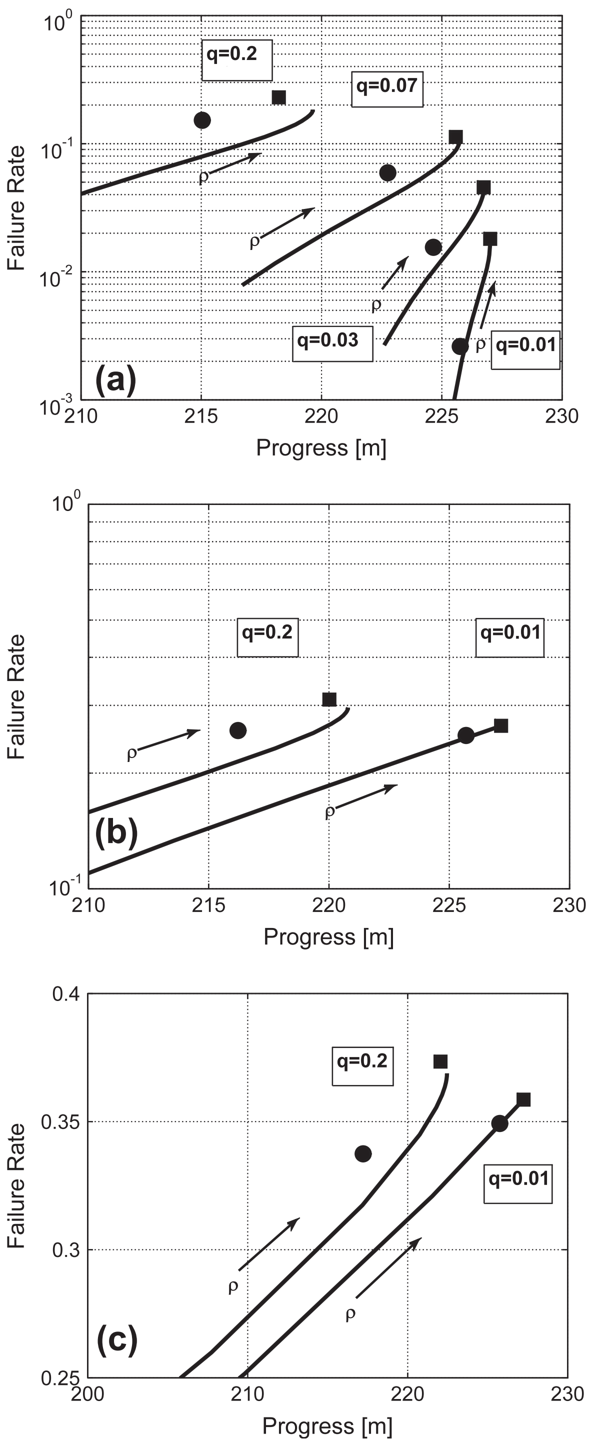

Fig. 3. Failure rate versus progress for 10 relays, (a) without fading noise $\sigma_\eta = 0$; (b) with fading noise standard deviation $\sigma_\eta = 1$; (c) with $\sigma_\eta = 2$. The square stands for GRD, the circle for MER, the solid line for optimal cost $J(\rho)$, for different values of q.

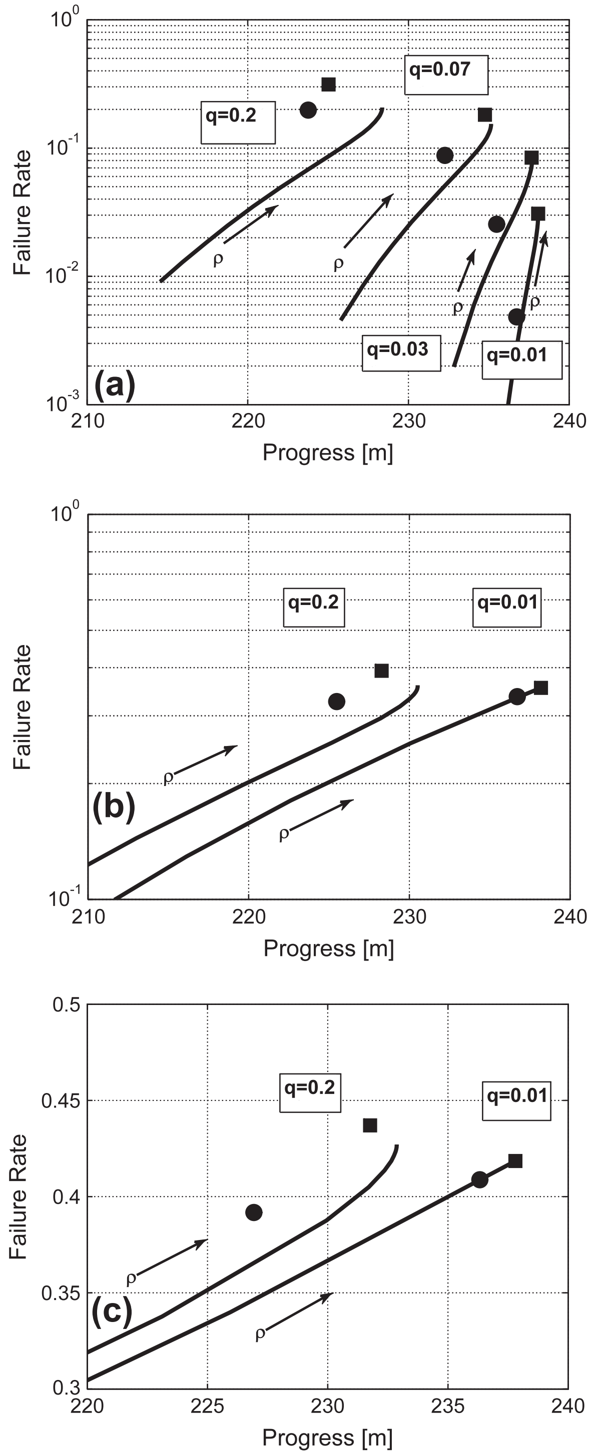

Fig. 4. Failure rate versus progress for 20 relays, (a) without fading noise $\sigma_\eta = 0$; (b) with fading noise standard deviation $\sigma_\eta = 1$; (c) with $\sigma_\eta = 2$. The square stands for GRD, the circle for MER, the solid line for optimal cost $J(\rho)$, for different values of q.

is one relay that is reached by both of the first hop. In Fig. 6a, failure rate versus progress is depicted. In a second case, the same scenario but using 10 relays in the first hop and 55 in the second, is shown in Fig. 6b. In this case, each one of the first hop can reach ten of the second hop. Thus, five relays are reached by two relays from the first hop. As was already explained in Section 4.2, the improvement between two-hops considered jointly and one-hop twice lies in how much the value of σ_{g12} affects the normal distribution and cumulative functions in metrics (23) and (24). In the case that $\sigma_{g12} = \sigma_{g12} = 0$, both cases give the same decision. In our example, $\sigma_{g12} = \sigma_{g12}$ is a random value within the interval $(0, 0.2d)$, which gives improvements up to 10% in the full range of ρ.

5.2. Network simulator 2

Our goal in this section is to perform comparative simulations between the three methods by using the network simulator NS2, developed by University of California at Berkeley. In order to complement the results obtained in the previous section, now a different scenario has been chosen. Settings include: MAC layer, IEEE 802.11; peer-to-peer 64 bytes packets; 914 MHz Lucent WaveLAN DSSS radio interface; node velocity in the interval 0–5 m/s; number of transmitted packages 1000; number of nodes 50 and 100; simulation area 300×600 m; random destination; protocol used to define the forward cone, DREAM (distance routing effect algorithm for mobility [18]); transmission range $\bar{d} = 40$ m; fading noise, $\sigma_\eta = 0$. The uncertainties in location follows the criterion given in (29) with $q = 0.2$. The metrics to be evaluated are the failure rate, the average delay from packet submission to reception at the destination (end-to-end transmission), and the average number of hops to reach destination. The latter is a metric proportional to the expected progress of the message. In Table 1, the comparative results between GRS, MER, and the optimal cost with $\rho = 1$, are presented.

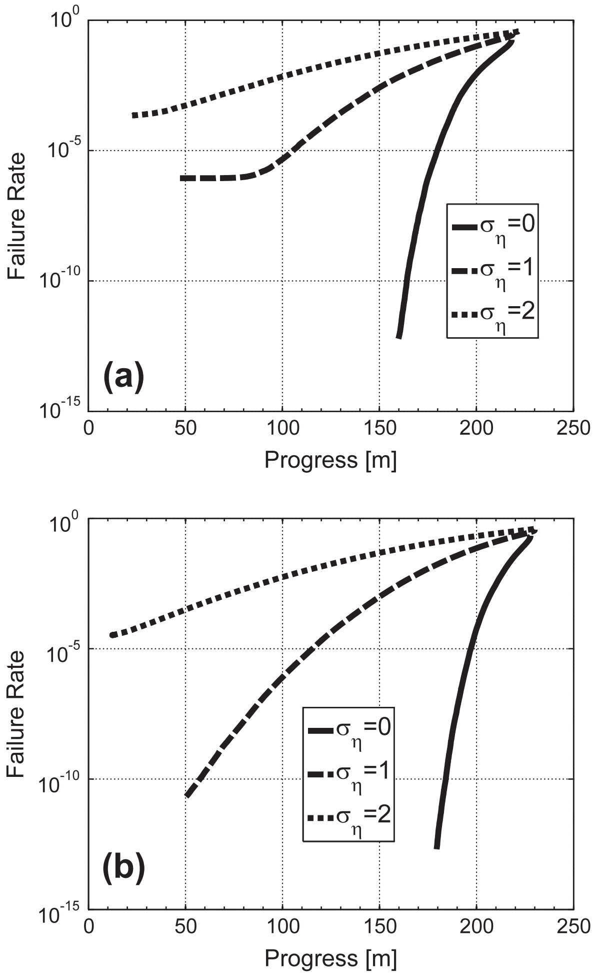

Fig. 5. Failure rate versus progress for $\rho \in [10^{-2}, 10^4]$, in cost $J(\rho)$, with fading noise $\sigma_\eta = 0$, $\sigma_\eta = 1$, and $\sigma_\eta = 2$. (a) For 10 relays; (b) For 20 relays.

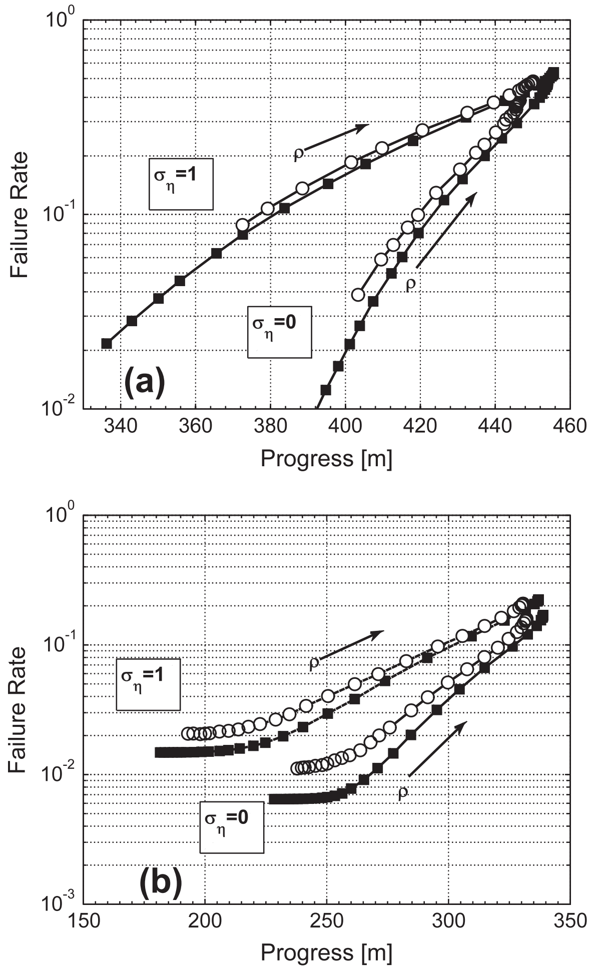

Fig. 6. Failure rate versus progress for (a) two relays for the first hop and three, for the second hop, each one of the first hop can reach two of the second hop; and (b) 10 realys in the first hop and 55 in the second, each one of the first hop can reach ten of the second hop. The square for two-hops selections and the circle for one-hop selection twice.

Table 1
Table I: NS2 simulation with \rho = 1, node velocity in the interval 0–5 m/s; simulation area 300×600 m; random destination; protocol DREAM; transmission range $\bar{d} = 40$ m; fading noise, $\sigma_\eta = 0; q = 0.2$; number of nodes (a) 50 and (b) 100.

Method	GRS	MER	J
a			
Failure rate	0.85	0.82	0.74
End to end delay (s)	0.21	0.19	0.19
Averaged number of hops	1.74	1.74	1.2
b			
Failure rate	0.82	0.80	0.66
End to end delay (s)	0.22	0.22	0.22
Averaged number of hops	1.7	1.68	1.4

From the tables, the same trends as in the previous simulations obtained with the Matlab's building-functions are observed. The MER improves the GRS but our proposed method improves MER. In the working scenario, we can see that the failure rate is high. This is because it has been considered a low transmission range, $d = 40$ m, thereby, many destinations that are far from the sender cannot be reached. As a consequence, progress, or equivalently the average number of hops, is also low. This improves when the number of nodes increases from 50 to 100. The end-to-end time delay are approximately the same for all the strategies.

6. Conclusion

Given a subset of possible relays for geographic routing subjected to location uncertainties and fading conditions, we have presented a selection strategy which consists in minimizing a criterion that weighs the probability of error and the progress of the transmitted information. Both metrics were calculated considering the channel capacity as a random variable due to the random characterization of the node location. The uncertainties considered are the fading and the location of the relay.

We analyzed two cases, one where the decision is made by using a one-hop statistic. The other, when deciding every two-hops by using joint statistics between the first and the second hope. The improvements from the one-hop design to the two-hops design can be known analytically. The information required for each decision is the mean and the variance of the location of each relay and also the degree of channel fading. Working expressions with low computational load for both cases were derived. The expressions are suitable to be used in real time computation in order to decide the optimal relays.

The proposed method was compared to other existing ones, the GRS, which does not take into consideration the uncertainties, and to MER, which does. The three approaches were compared by using two different simulators; one, using Matlab built-in functions; and the other, using Network Simulator 2. As expected, MER, which optimizes the progress within the transmission range outperforms GRS by reducing the failure rate. According to our theoretical results and the simulations, our proposal improves MER by optimizing both the rate of failure and the conditional expected progress, given that the message was successfully received by the relay, simultaneously.

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Appendix A

The solution of Eqs. (15) and (16) are computationally expensive for using in real time. In this section, suitable working approaches will be derived. Taking into account that the density function $f_{\eta d}$ is concentrated around $\eta = 0$, a convenient approximation is obtained by linearizing the integration limit of the inner integral in (11) around this value. The error, using this linearization, increases as we move away from $\eta = 0$, but at the same time, the density function decreases rapidly, so that when using the approximation, we can expect a very small error in the integral. The approximation is obtained by taking the linear term of the Taylor series expansion of the integration limits of d, at coordinates $\eta = 0$, accordingly with the following expression:

$$d = e^{(-a\eta - z_r)/\gamma} \approx \bar{d}(1 - \frac{a}{\gamma}\eta). \tag{30}$$

Then, taking into account that the left side of the shaded portion of Fig. 1 does not contribute significantly to the probability of error, the expression of P_e in (15) can be used. Making the following change of variable: $\tau = d + (\bar{d}a/\gamma)\eta$, the last term of the right side of the approximation (15) can be written, using matrix formulation, as

$$\frac{1}{2\pi|\Phi|^{1/2}} \int_{-\infty}^{\bar{d}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(h\mathbf{f}+\mathbf{b}\eta)\Phi^{-1}(h\mathbf{f}+\mathbf{b}\eta)^T} d_\eta d_\tau, \tag{31}$$

where |.|means the determinant and

$$h = \tau - \tilde{\mu}; \quad \mathbf{f} = [1, 0]; \quad \mathbf{b} = \left[-\frac{\bar{d}a}{\gamma}, 1\right]; \quad \Phi = \begin{pmatrix} \sigma^2 & 0 \\ 0 & \sigma_\eta^2 \end{pmatrix}. \tag{32}$$

To proceed, we need the following lemma:

Lemma 1. *Given row vectors X and Y, and scalars τ and t, the following equality holds:*

$$\int_{-\infty}^{t} e^{-\frac{1}{2}(X\tau+Y)\Phi^{-1}(X\tau+Y)^T} d\tau = e^{-\frac{1}{2}\left(\alpha_0 - \frac{\alpha_1^2}{\alpha_2}\right)} \times \sqrt{\frac{2\pi}{\alpha_2}} \mathcal{C}\left(-\frac{\alpha_1}{\alpha_2}, \frac{1}{\alpha_2}, t\right), \tag{33}$$

where

$$\alpha_0 = Y\Phi^{-1}Y^T; \quad \alpha_1 = X\Phi^{-1}Y^T; \quad \alpha_2 = X\Phi^{-1}X^T; \tag{34}$$

Proof. Eq. (33) can be written as

$$\int_{-\infty}^{t} e^{-\frac{1}{2}(\alpha_2\tau^2 + 2\alpha_1\tau + \alpha_0)} d\tau = \int_{-\infty}^{t} e^{-\frac{\alpha_2}{2}(\tau + \frac{\alpha_1}{\alpha_2})^2 - \frac{1}{2}\left(\alpha_0 - \frac{\alpha_1^2}{\alpha_2}\right)} d\tau \tag{35}$$

$$= e^{-\frac{1}{2}\left(\alpha_0 - \frac{\alpha_1^2}{\alpha_2}\right)} \int_{-\infty}^{t} e^{-\frac{\alpha_2}{2}(\tau + \frac{\alpha_1}{\alpha_2})^2} d\tau. \tag{36}$$

Taking into account that

$$\sqrt{\frac{\alpha_2}{2\pi}} \int_{-\infty}^{t} e^{-\frac{\alpha_2}{2}(\tau + \frac{\alpha_1}{\alpha_2})^2} d\tau = \mathcal{C}\left(-\frac{\alpha_1}{\alpha_2}, \frac{1}{\alpha_2}, t\right), \tag{37}$$

and replacing in (36), the equality (33) fulfils. □

By applying this lemma, the marginalization of the bi-variate pdf in (31) with respect to η is given by

$$\frac{1}{(2\pi|\Phi|\mathbf{b}\Phi^{-1}\mathbf{b}^T)^{1/2}} \int_{-\infty}^{\bar{d}} e^{-\frac{h^2}{2\sigma_g^2}} d\tau, \tag{38}$$

where

$$\sigma_g^{-2} = \mathbf{f}(\Phi^{-1} - \frac{\Phi^{-1}\mathbf{b}^T\mathbf{b}\Phi^{-1}}{\mathbf{b}\Phi^{-1}\mathbf{b}^T})\mathbf{f}^T. \tag{39}$$

By denoting the scalar $b = \mathbf{b}\mathbf{f}^T$, it follows that $\mathbf{f}\Phi^{-1}\mathbf{f}^T = \sigma^{-2}$, and $\mathbf{b}\Phi^{-1}\mathbf{f}^T = b\sigma^{-2}$. Thus, by replacing in (39), we find that $\sigma_g^2 = \sigma^2 + b^2\sigma_\eta^2$. Also, we find that

$$|\Phi|(\mathbf{b}\Phi^{-1}\mathbf{b}^T) = \sigma^2 + b^2\sigma_\eta^2 = \sigma_g^2. \tag{40}$$

By using this equality, we find that (38) is equal to $\mathcal{C}\left(\tilde{\mu}, \sigma_g^2, \bar{d}\right)$ and (17) succeeds.

In order to obtain a working expression for the progress, let us first use the derivative of the following exponential function with respect to d:

$$\left(e^{-\frac{1}{2}\frac{(d-\tilde{\mu})^2}{\sigma^2}}\right)' = -\frac{(d-\tilde{\mu})}{\sigma^2}e^{-\frac{1}{2}\frac{(d-\tilde{\mu})^2}{\sigma^2}}. \tag{41}$$

Dividing by $\sqrt{2\pi}\sigma$ in both sides and rearranging, the following equality holds

$$df_d(d) = \tilde{\mu}f_d(d) - \sigma^2 f_d'(d). \tag{42}$$

By replacing in (12) the first term of the right side of (42) gives

$$\int_{-\infty}^{\infty} f_\eta(\eta)\left[\int_{\mathcal{D}(\eta)} \tilde{\mu}f_d(d)d_d\right]d_\eta/(1-P_e), \tag{43}$$

which is equal to $\tilde{\mu}$ according with (11). Thus, the first term of expression in (18) is obtained. For the second term we use the linearized expression, as before, for the integral limit, as follows:

$$\begin{aligned}\int_{-\infty}^{\infty} f_\eta(\eta)\left[\int_{\mathcal{D}(\eta)} \sigma^2 f_d'(d)d_d\right]d_\eta \\ = \int_{-\infty}^{\infty} f_\eta(\eta)\sigma^2 f_d(d)|_{\mathcal{D}(\eta)}d_\eta \\ \approx \frac{\sigma}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f_\eta(\eta)e^{-\frac{1}{2}\frac{(\bar{d}(1-\frac{a}{\gamma}\eta)-\tilde{\mu})^2}{\sigma^2}}d_\eta.\end{aligned} \tag{44}$$

By denoting $\bar{h} = \bar{d} - \tilde{\mu}$ the above approximation can be written as

$$\frac{1}{2\pi\sigma\sigma_\eta}\int_{-\infty}^{\infty} e^{-\frac{1}{2}(\bar{h}\mathbf{f}+\mathbf{b}\eta)\Phi^{-1}(\bar{h}\mathbf{f}+\mathbf{b}\eta)^T}. \tag{45}$$

Using the lemma, it can be written as

$$\frac{\sigma^2}{\sqrt{2\pi}\sigma_g}e^{-\frac{1}{2}\frac{\bar{h}^2}{\sigma_g^2}}. \tag{46}$$

Finally, dividing by the factor $(1 - P_e)$, the second term of expression in (18) is obtained.

Working expressions can be derived for the two-hops case in a similar way as for one-hop case. By using the following change of variables, $\tau_1 = d_1 + (\bar{d}a/\gamma)\eta$ and $\tau_2 = d_2 + (\bar{d}a/\gamma)\eta$, the inner double integral of (21), similarly to (38) but now considering two dimensions, can be approximated by

$$\frac{1}{(2\pi|\Phi|\mathbf{b}\Phi^{-1}\mathbf{b}^T)^{1/2}}\int\int_{-\infty}^{\bar{d}} e^{-\frac{1}{2}\mathbf{h}\Sigma_g^{-1}\mathbf{h}^T}d\tau_1 d\tau_2, \tag{47}$$

where

$$\Phi = \begin{pmatrix}\Sigma & 0\\ 0 & \sigma_\eta^2\end{pmatrix}, \tag{48}$$

$$\Sigma_g^{-1} = F\left(\Phi^{-1} - \frac{\Phi^{-1}\mathbf{b}^T\mathbf{b}\Phi^{-1}}{\mathbf{b}\Phi^{-1}\mathbf{b}^T}\right)F^T, \tag{49}$$

$$\mathbf{b} = \left[-\frac{\bar{d}a}{\gamma}, -\frac{\bar{d}a}{\gamma}, 1\right], \tag{50}$$

$$\mathbf{h} = [\tau_1 - \tilde{\mu}_1, \tau_2 - \tilde{\mu}_2], \tag{51}$$

$$F = [I, \mathbf{0}]. \tag{52}$$

where I is the identity of dimension two and $\mathbf{0} = [0,0]^T$. By denoting $\bar{\mathbf{b}} = \left[-\frac{\bar{d}a}{\gamma}, -\frac{\bar{d}a}{\gamma}\right]$, it follows that $\mathbf{b}\Phi^{-1}F^T = \bar{\mathbf{b}}\Sigma^{-1}$, and taking into account that $F\Phi^{-1}F^T = \Sigma^{-1}$, we find that

$$\Sigma_g^{-1} = \Sigma^{-1} - \frac{\Sigma^{-1}\bar{\mathbf{b}}^T\bar{\mathbf{b}}\Sigma^{-1}}{\bar{\mathbf{b}}\Sigma^{-1}\bar{\mathbf{b}}^T + \sigma_\eta^{-2}} = \left(\Sigma + \bar{\mathbf{b}}^T\sigma_\eta^2\bar{\mathbf{b}}\right)^{-1}, \tag{53}$$

where the matrix inversion lemma was used to obtain the last equality. By using properties of the determinant, the following holds:

$$\begin{aligned}|\Phi|\mathbf{b}\Phi^{-1}\mathbf{b}^T &= |\Sigma|(\bar{\mathbf{b}}\Sigma^{-1}\bar{\mathbf{b}}^T\sigma_\eta^2 + 1) = |I + \sigma_\eta^2\bar{\mathbf{b}}^T\bar{\mathbf{b}}\Sigma^{-1}||\Sigma| \\ &= |\Sigma + \bar{\mathbf{b}}^T\sigma_\eta^2\bar{\mathbf{b}}| = |\Sigma_g|.\end{aligned} \tag{54}$$

By using (53) and (54) we find that (47) is equal to $\mathcal{C}([\tilde{\mu}_1, \tilde{\mu}_2], \Sigma_g, [\bar{d}, \bar{d}])$ and (23) succeeds.

In order to obtain the working expressions for progress in the two-hops case we need first to use the following derivatives:

$$\begin{aligned}&\theta_1\frac{d\left(e^{-\frac{1}{2}\mathbf{h}\Sigma^{-1}\mathbf{h}^T}\right)}{d\tau_1} + \theta_2\frac{d\left(e^{-\frac{1}{2}\mathbf{h}\Sigma^{-1}\mathbf{h}^T}\right)}{d\tau_2} \\ &= -e^{-\frac{1}{2}\mathbf{h}\Sigma^{-1}\mathbf{h}^T}[\theta_1, \theta_2]\Sigma^{-1}\mathbf{h}^T.\end{aligned} \tag{55}$$

Let $[\theta_1, \theta_2] = [1,1]\Sigma$ and define $\mathbf{d} = [d_1 - \tilde{\mu}_1, d_2 - \tilde{\mu}_2]$. Thus, the above can be written as

$$\begin{aligned}(d_1 + d_2)e^{-\frac{1}{2}\mathbf{d}\Sigma^{-1}\mathbf{d}^T} &= (\tilde{\mu}_1 + \tilde{\mu}_2)e^{-\frac{1}{2}\mathbf{d}\Sigma^{-1}\mathbf{d}^T} - \theta_1 \\ &\times \frac{d\left(e^{-\frac{1}{2}\mathbf{d}\Sigma^{-1}\mathbf{d}^T}\right)}{d\tau_1} - \theta_2 \\ &\times \frac{d\left(e^{-\frac{1}{2}\mathbf{d}\Sigma^{-1}\mathbf{d}^T}\right)}{d\tau 2}.\end{aligned} \tag{56}$$

Dividing both sides by $2\pi|\Sigma|$ the following holds:

$$\begin{aligned}(d_1 + d_2)f_{d_1d_2} &= (\tilde{\mu}_1 + \tilde{\mu}_2)f_{d_1d_2} - \theta_1\frac{d(f_{d_1d_2})}{d\tau_1} - \theta_2 \\ &\times \frac{d(f_{d_1d_2})}{d\tau_2}.\end{aligned} \tag{57}$$

By replacing in (22), for similarity with (21), the first term on the right is equal to $(\tilde{\mu}_1 + \tilde{\mu}_2)$. The second term can be solved in a similar way as it was done to obtain (47), giving

$$\theta_1 \int \int_{-\infty}^{\bar{d}} \frac{df_{d_1,d_2}(\tau_1,\tau_2)}{d\tau_1} d\tau_1 d\tau_2 = \frac{\theta_1}{2\pi|\Sigma_g|^{1/2}} \int_{-\infty}^{\bar{d}} e^{-\frac{1}{2}\bar{\mathbf{h}}\Sigma_g^{-1}\bar{\mathbf{h}}^T} d\tau_2, \quad (58)$$

where $\bar{\mathbf{h}} = [\bar{d} - \tilde{\mu}_1, \tau_2 - \tilde{\mu}_2]$. This integral can be solved by applying the lemma, which gives the second term of (24). Finally, using a similar procedure, the third term of (24) is obtained by replacing the third term of (57) in (12) and solving.

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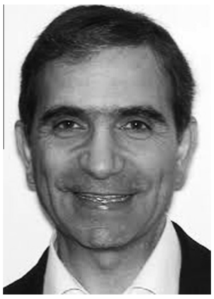

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