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Limits as $p(x) \to \infty$ of $p(x)$-harmonic functions

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ABSTRACT

In this note we study the limit as $p(x) \to \infty$ of solutions to $-\Delta_{p(x)}u = 0$ in a domain Ω, with Dirichlet boundary conditions. Our approach consists in considering sequences of variable exponents converging uniformly to $+\infty$ and analyzing how the corresponding solutions of the problem converge and which equation is satisfied by the limit.

1. Introduction

Our goal in this note is to look for the limit, as the exponent $p(x) \to \infty$, of solutions to

$$\begin{cases} -\Delta_{p(x)}u(x) = 0, & x \in \Omega \subset \mathbb{R}^N, \\ u(x) = f(x), & x \in \partial\Omega, \end{cases} \tag{1.1}$$

where $\Delta_{p(x)}u(x) := \operatorname{div}\left(|\nabla u(x)|^{p(x)-2}\nabla u(x)\right)$ is the $p(x)$-Laplacian operator with a variable exponent $p(x)$.

When p is constant in Ω, the limit of (1.1) as $p \to \infty$ has been extensively studied in the literature (see [1] and the survey [2]) and leads naturally to the infinity Laplacian

$$\Delta_\infty u := \left(D^2u\nabla u\right)\cdot\nabla u = \sum_{i,j=1}^N \frac{\partial u}{\partial x_i}\frac{\partial u}{\partial x_j}\frac{\partial^2 u}{\partial x_i x_j}.$$

Infinity harmonic functions (solutions to $-\Delta_\infty u = 0$) solve the optimal Lipschitz extension problem (see [3] and the survey paper [2]) and find applications in optimal transportation, image processing and tug-of-war games (see, e.g., [4–8] and the references therein). On the other hand, problems related to PDEs involving variable exponents became popular a few years ago due to applications in elasticity and the modeling of electrorheological fluids. Meanwhile, the underlying functional analytical tools have been extensively developed (cf. [9,10]) and new applications to image processing have kept the subject at the focus of an intensive research activity. Although a natural extension of the theory, the problem addressed here is a

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follow-up from a recent paper of the authors [11], where the case of a variable exponent that equals infinity in a subdomain of Ω is treated. Closely related to this work is [12], where the authors prove existence and uniqueness (via a comparison principle), as well as the validity of a Harnack inequality, for the solutions of our limit problem.

The approach in this paper is based on considering sequences $p_n(x)$ of variable exponents converging uniformly to $+\infty$ and analyzing how the corresponding solutions of the problem converge and which equation is satisfied by the limit. Before introducing our main result, let us state the assumptions on the data that will be assumed from now on.

- $\Omega \subset \mathbb{R}^N$ is a bounded smooth domain.
- f is a Lipschitz continuous function with Lipschitz constant less or equal than one.
- $p_n(x)$ is a sequence of C^1 functions in Ω such that

$$p_n(x) \to +\infty, \quad \text{uniformly in } \Omega; \tag{1.2}$$

$$p_n(x) \geq \alpha, \quad \text{for all } x \in \Omega; \tag{1.3}$$

$$\nabla \ln p_n(x) \longrightarrow \xi(x), \quad \text{uniformly in } \Omega, \tag{1.4}$$

for a constant $\alpha > N$ and a function $\xi \in C(\Omega)$.

We now present some examples of possible sequences $p_n(x)$. In each case, some smoothness assumptions have to be added, as well as conditions that guarantee that (1.3) holds. We are primarily interested in understanding (1.4) and hope the examples shed some light on the meaning of this assumption.

(1) $p_n(x) = n$; we have $\xi = 0$.
(2) $p_n(x) = p(x) + n$; we get again $\xi = 0$.
(3) $p_n(x) = np(x)$; we get a nontrivial vector field

$$\xi(x) = \nabla(\ln(p(x))).$$

This example is what motivates the study of the limit equation in [12].

(4) $p_n(x) = n^a p(x/n)$ [scaling in x]; in this case, we have

$$\nabla(\ln p_n(x)) = \frac{\nabla p}{p}(x/n)\frac{1}{n} \longrightarrow 0$$

and so $\xi = 0$. This also happens for $p_n(x) = n + p(x/n)$.

(5) $p_n(x) = n^a p(nx)$; we get

$$\nabla(\ln p_n(x)) = n\frac{\nabla p}{p}(nx),$$

which does not have a limit as $n \to \infty$. The same happens with $p_n(x) = n + p(nx)$, for which

$$\nabla(\ln p_n(x)) = \frac{n\nabla p(nx)}{n + p(nx)}$$

that does not have a uniform limit (although it is bounded).

(6) We can modify the previous example to get a nontrivial limit. Assume that $q(x)$ is a function of the angular variable and that $0 \notin \Omega$; then consider $p_n(x) = n + q(nx)$ to obtain

$$\nabla(\ln p_n(x)) = n\frac{\nabla q(\theta)}{n + q(nx)} \longrightarrow \nabla q(\theta).$$

(7) Finally, we can combine examples (3) and (6). Let $p_n(x) = np(x) + q(nx)$, with q and Ω as in (6). We get

$$\nabla(\ln p_n(x)) = \frac{n\nabla p(x) + n\nabla q(\theta)}{np(x) + q(nx)} \longrightarrow \frac{\nabla p(x) + \nabla q(\theta)}{p(x)}.$$

The following is the main result of this paper. We prove, under the above assumptions, that the limit of solutions of (1.1) with $p(x) = p_n(x)$ exists and can be characterized as the unique viscosity solution of a PDE that involves the ∞-Laplacian and an extra term in which the vector field $\xi(x) = \lim_n \nabla \ln p_n(x)$ appears.

Theorem 1.1. *Let u_n be the solution of (1.1) with $p(x) = p_n(x)$. Then*

$$u_n \longrightarrow u, \quad \text{uniformly in } \Omega, \tag{1.5}$$

where u is the unique viscosity solution of the problem

$$\begin{cases} -\Delta_\infty u - |\nabla u|^2 \ln|\nabla u| \langle \xi, \nabla u\rangle = 0, & \text{in } \Omega, \\ u = f, & \text{on } \partial\Omega. \end{cases} \tag{1.6}$$

Remark 1.2. Uniqueness of solutions to the limit problem (1.6) is a consequence of the results of [12]. See also [13].

Remark 1.3. Notice that we are taking $F(0) = 0$ for $F(s) = s^2 \ln(s)$, hence (1.6) makes sense when evaluated at a test function with vanishing gradient.

Remark 1.4. In dimension one, we get as the limit problem

$$\begin{cases} u''(x) + \ln|u'(x)|\langle \xi(x), u'(x)\rangle = 0, & x \in (0,1), \\ u(0) = f(0), \qquad u(1) = f(1), \end{cases} \tag{1.7}$$

which is uniquely and explicitly solvable. We just have to observe that we can assume, without loss of generality, that $f(0) = 0$ (just consider $v = u - f(0)$) and $f(1) > 0$. Then we solve the equation as follows:

$$u''(x) + \ln|u'(x)|\langle \xi(x), u'(x)\rangle = 0, \quad x \in (0,1)$$

is equivalent to

$$\int_0^t \frac{u''(x)}{\ln|u'(x)|u'(x)}\mathrm{d}x = -\int_0^t \xi(x)\mathrm{d}x,$$

that is,

$$\int_C^{u'(t)} \frac{1}{\ln(z)z}\mathrm{d}z = -\int_0^t \xi(x)\mathrm{d}x = -H(t).$$

This gives

$$\ln(\ln(u'(t))) = (C - H(t))$$

and thus

$$u(x) = \int_0^x \exp(\exp(C - H(t)))\mathrm{d}t.$$

Finally, we only have to choose C such that

$$f(1) = \int_0^1 \exp(\exp(C - H(t)))\mathrm{d}t.$$

The rest of the paper is organized as follows: in Section 2 we collect some properties of the approximate problems and in Section 3 we prove our main result, Theorem 1.1.

2. The approximate problem

We first consider the problem corresponding to (1.1) when $p(x)$ is replaced by $p_n(x)$. For convenience, we refer to this problem as $(1.1)_n$.

Lemma 2.1. *There exists a unique weak solution u_n to $(1.1)_n$, which is the unique minimizer of the functional*

$$F_n(u) = \int_\Omega \frac{|\nabla u|^{p_n(x)}}{p_n(x)}\mathrm{d}x \tag{2.1}$$

in the set

$$S_n = \left\{u \in W^{1,p_n(\cdot)}(\Omega) : u|_{\partial\Omega} = f\right\}. \tag{2.2}$$

Proof. Functions in the variable exponent Sobolev space $W^{1,p_n(\cdot)}(\Omega)$ are necessarily continuous thanks to the assumption $p_n(x) \geq \alpha > N$. Indeed, the continuous embedding in

$$W^{1,p_n(\cdot)}(\Omega) \hookrightarrow W^{1,\alpha}(\Omega) \subset C\left(\overline{\Omega}\right) \tag{2.3}$$

follows from [9, Theorem 2.8 and (3.2)].

We can then take the boundary condition $u|_{\partial\Omega} = f$ in the classical sense (recall that f is assumed to be Lipschitz) and the unique solvability is standard in view of the regularity of the variable exponent.

It is also standard that the minimizer of F_n in S_n is the unique weak solution of $(1.1)_n$, *i.e.*, $u_n = f$ on $\partial\Omega$ and

$$\int_\Omega |\nabla u_n|^{p_n(x)-2}\nabla u_n \cdot \nabla\varphi \mathrm{d}x = 0, \quad \forall \varphi \in C_0^\infty(\Omega). \quad \square \tag{2.4}$$

Let us now recall the definition of viscosity solution (cf. [14]) for a problem like (1.1) or (1.6). Assume we are given a continuous functions

$$F : \overline{\Omega} \times \mathbb{R}^N \times \mathbb{S}^{N\times N} \to \mathbb{R}.$$

Definition 2.2. Consider the problem

$$F(x, \nabla u, D^2 u) = 0 \quad \text{in } \Omega \tag{2.5}$$

with a boundary condition

$$u = f \quad \text{on } \partial\Omega. \tag{2.6}$$

A lower semi-continuous function u is a viscosity supersolution of (2.5) and (2.6) if $u \geq f$ on $\partial\Omega$ and for every $\phi \in C^2(\overline{\Omega})$ such that $u - \phi$ has a strict minimum at the point $x_0 \in \Omega$, with $u(x_0) = \phi(x_0)$, we have

$$F(x_0, \nabla\phi(x_0), D^2\phi(x_0)) \geq 0.$$

An upper semi-continuous function u is a viscosity subsolution of (2.5) and (2.6) if $u \leq f$ on $\partial\Omega$ and for every $\psi \in C^2(\overline{\Omega})$ such that $u - \psi$ has a strict maximum at the point $x_0 \in \Omega$, with $u(x_0) = \psi(x_0)$, we have

$$F(x_0, \nabla\psi(x_0), D^2\psi(x_0)) \leq 0.$$

Finally, u is a viscosity solution if it is both a viscosity supersolution and a viscosity subsolution.

In the sequel, we will use the notation as in the definition: ϕ will always stand for a test function touching the graph of u from below and ψ for a test function touching the graph of u from above.

Proposition 2.3. *Let u_n be a continuous weak solution of $(1.1)_n$. Then u_n is a viscosity solution of $(1.1)_n$ in the sense of Definition 2.2.*

Proof. The proof is contained in the proof of Proposition 2.4 in [11]. We reproduce it here for the sake of completeness and readability.

We omit the subscript n in this proof. Let $x_0 \in \Omega$ and a let ϕ be a test function such that $u(x_0) = \phi(x_0)$ and $u - \phi$ has a strict minimum at x_0. We want to show that

$$\begin{aligned} -\Delta_{p(x_0)}\phi(x_0) &= -|\nabla\phi(x_0)|^{p(x_0)-2}\Delta\phi(x_0) - (p(x_0) - 2)|\nabla\phi(x_0)|^{p(x_0)-4}\Delta_\infty\phi(x_0) \\ &\quad - |\nabla\phi(x_0)|^{p(x_0)-2}\ln(|\nabla\phi|)(x_0)\,\langle\nabla\phi(x_0), \nabla p(x_0)\rangle \\ &\geq 0. \end{aligned}$$

Assume, *ad contrarium*, that this is not the case; then there exists a radius $r > 0$ such that $B(x_0, r) \subset \Omega$ and

$$\begin{aligned} -\Delta_{p(x)}\phi(x) &= -|\nabla\phi(x)|^{p(x)-2}\Delta\phi(x) - (p(x) - 2)|\nabla\phi(x)|^{p(x)-4}\Delta_\infty\phi(x) \\ &\quad - |\nabla\phi(x)|^{p(x)-2}\ln(|\nabla\phi|)(x)\langle\nabla\phi(x), \nabla p(x)\rangle \\ &< 0, \end{aligned}$$

for every $x \in B(x_0, r)$. Set

$$m = \inf_{|x-x_0|=r} (u - \phi)(x)$$

and let $\Phi(x) = \phi(x) + m/2$. This function Φ verifies $\Phi(x_0) > u(x_0)$ and

$$-\Delta_{p(x)}\Phi = -\mathrm{div}(|\nabla\Phi|^{p(x)-2}\nabla\Phi) < 0 \quad \text{in } B(x_0, r). \tag{2.7}$$

Multiplying (2.7) by $(\Phi - u)^+$, which vanishes on the boundary of $B(x_0, r)$, we get

$$\int_{B(x_0,r)\cap\{\Phi>u\}} |\nabla\Phi|^{p(x)-2}\nabla\Phi \cdot \nabla(\Phi - u)\mathrm{d}x < 0.$$

On the other hand, taking $(\Phi - u)^+$, extended by zero outside $B(x_0, r)$, as test function in the weak formulation of $(1.1)_n$, we obtain

$$\int_{B(x_0,r)\cap\{\Phi>u\}} |\nabla u|^{p(x)-2}\nabla u \cdot \nabla(\Phi - u)\mathrm{d}x = 0.$$

Upon subtraction and using a well known inequality, we conclude

$$0 > \int_{B(x_0,r)\cap\{\Phi>u\}} \left(|\nabla\Phi|^{p(x)-2}\nabla\Phi - |\nabla u|^{p(x)-2}\nabla u\right)\cdot\nabla(\Phi-u)\mathrm{d}x$$
$$\geq c\int_{B(x_0,r)\cap\{\Phi>u\}} |\nabla\Phi-\nabla u|^{p(x)}\mathrm{d}x,$$

a contradiction.

This proves that u is a viscosity supersolution. The proof that u is a viscosity subsolution runs as above and we omit the details. □

We next obtain uniform estimates (independent of n) for the sequence $(u_n)_n$.

Proposition 2.4. *The minimizer of F_n in S_n, u_n, satisfies*

$$F_n(u_n) = \int_\Omega \frac{|\nabla u_n|^{p_n(x)}}{p_n(x)}\mathrm{d}x \leq |\Omega|.$$

Hence, the sequence $(F_n(u_n))_n$ is uniformly bounded and the sequence $(u_n)_n$ is uniformly bounded in $W^{1,\alpha}(\Omega)$ and equicontinuous.

Proof. Note that, since f has a Lipschitz constant less or equal than one, the set

$$S = \left\{u \in W^{1,\infty}(\Omega) : \|\nabla u\|_{L^\infty(\Omega)} \leq 1 \text{ and } u|_{\partial\Omega} = f\right\} \tag{2.8}$$

is nonempty. Recalling (2.2), the definition of S_n, observe that $S \subset S_n$, for every n. Since u_n is a minimizer, we have

$$F_n(u_n) \leq F_n(v), \quad \forall v \in S.$$

Hence, picking an element $v \in S \neq \emptyset$,

$$F_n(u_n) = \int_\Omega \frac{|\nabla u_n|^{p_n(x)}}{p_n(x)}\mathrm{d}x \leq \int_\Omega \frac{|\nabla v|^{p_n(x)}}{p_n(x)}\mathrm{d}x \leq |\Omega|.$$

In order to estimate the Sobolev norm, we first use Poincaré inequality and the boundary data, to obtain

$$\begin{aligned}\|u_n\|_{W^{1,\alpha}(\Omega)} &\leq \|u_n - f\|_{W_0^{1,\alpha}(\Omega)} + \|f\|_{W^{1,\alpha}(\Omega)}\\ &\leq C\,\|\nabla(u_n-f)\|_{L^\alpha(\Omega)} + \|f\|_{W^{1,\infty}(\Omega)}\\ &\leq C\,\|\nabla u_n\|_{L^\alpha(\Omega)} + (C+1)\|f\|_{W^{1,\infty}(\Omega)}.\end{aligned}$$

Let us denote by p_n^- and p_n^+ the minimum and the maximum of $p_n(x)$,

$$p_n^- = \min_{x\in\overline{\Omega}} p_n(x), \qquad p_n^+ = \max_{x\in\overline{\Omega}} p_n(x).$$

Now, we proceed, using the Hölder inequality and elementary computations, to obtain

$$\begin{aligned}\|\nabla u_n\|_{L^\alpha(\Omega)} &= \left(\int_\Omega |\nabla u_n|^\alpha \mathrm{d}x\right)^{1/\alpha}\\ &\leq |\Omega|^{\frac{1}{\alpha}-\frac{1}{p_n^-}}\left(\int_\Omega |\nabla u_n|^{p_n^-}\mathrm{d}x\right)^{1/p_n^-}\\ &\leq (1+|\Omega|)\left\{|\Omega| + \left(p_n^+\int_\Omega \frac{|\nabla u_n|^{p_n(x)}}{p_n(x)}\mathrm{d}x\right)^{1/p_n^-}\right\}\\ &\leq (1+|\Omega|)\left\{|\Omega| + \left(p_n^+F_n(u_n)\right)^{1/p_n^-}\right\}\\ &\leq (1+|\Omega|)\,|\Omega|\left\{1+\left(p_n^+\right)^{1/p_n^-}\right\} \leq C,\end{aligned}$$

since we have the bound

$$\left(p_n^+\right)^{1/p_n^-} \leq C \tag{2.9}$$

due to assumption (1.4).

Indeed, one can prove a Harnack inequality for a nonnegative function θ such that $\nabla \ln \theta$ is bounded. For completeness, we include here the argument. Suppose $\theta \geq 0$ and $|\nabla \ln \theta| \leq K$ in Ω, and take arbitrary points $x, y \in \Omega$. By the mean value theorem,

$$\ln \frac{\theta(x)}{\theta(y)} \leq |\ln \theta(x) - \ln \theta(y)| \leq K|x-y|$$

and thus

$$\frac{\theta(x)}{\theta(y)} \leq e^{K|x-y|}.$$

Since Ω is bounded,

$$\theta(x) \leq e^{K|x-y|}\theta(y) = C\theta(y).$$

Applying this reasoning to each p_n, (2.9) follows from the uniform boundedness of $|\nabla \ln p_n|$.

We conclude that $(u_n)_n$ is uniformly bounded in $W^{1,\alpha}(\Omega)$ and, recalling the embedding in (2.3), that it is equicontinuous. □

Remark 2.5. If f has Lipschitz constant greater than one then $F_n(u_n)$ is unbounded, see [11].

3. Passing to the limit: Proof of Theorem 1.1

Owing to Proposition 2.4, it follows from Ascoli's theorem, extracting a subsequence if necessary, that

$$u_n \longrightarrow u, \quad \text{uniformly in } \Omega,$$

for a certain continuous function u. Since $u_n = f$ on $\partial\Omega$ we have that $u = f$ on $\partial\Omega$.

To prove that u is a viscosity supersolution of (1.6), let ϕ be such that $u - \phi$ has a strict local minimum at $x_0 \in \Omega$, with $\phi(x_0) = u(x_0)$. We want to prove that

$$-\Delta_\infty \phi(x_0) - |\nabla\phi(x_0)|^2 \ln|\nabla\phi(x_0)| \langle \xi(x_0), \nabla\phi(x_0)\rangle \geq 0. \tag{3.1}$$

Since $u_n \to u$ uniformly, there is a sequence $(x_n)_n$ such that $x_n \to x_0$ and $u_n - \phi$ has a local minimum at x_n. As u_n is a viscosity solution of $(1.1)_n$ (cf. Proposition 2.3), we have

$$-\frac{|\nabla\phi(x_n)|^2 \Delta\phi(x_n)}{p_n(x_n)-2} - \Delta_\infty\phi(x_n) - |\nabla\phi(x_n)|^2 \ln|\nabla\phi(x_n)| \left\langle \nabla\phi(x_n), \frac{\nabla p_n(x)}{p_n(x_n)-2} \right\rangle \geq 0.$$

Using the fact that $x_n \to x_0$ and the assumptions (1.2) and (1.4), we obtain

$$\frac{|\nabla\phi(x_n)|^2 \Delta\phi(x_n)}{p_n(x_n)-2} \longrightarrow 0,$$
$$\Delta_\infty\phi(x_n) \longrightarrow \Delta_\infty\phi(x_0),$$
$$|\nabla\phi(x_n)|^2 \ln(|\nabla\phi(x_n)|) \longrightarrow |\nabla\phi(x_0)|^2 \ln(|\nabla\phi(x_0)|),$$
$$\left\langle \nabla\phi(x_n), \frac{\nabla p_n(x)}{p_n(x_n)-2} \right\rangle \longrightarrow \langle \nabla\phi(x_0), \xi(x_0)\rangle$$

and (3.1) follows.

This proves that u is a viscosity supersolution; the fact that it is also a viscosity subsolution follows analogously. □

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