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Efficient Approach for Scheduling Crude Oil Operations in Marine-Access Refineries

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Supporting Information

ABSTRACT: Due to the impact of crude oil prices on refinery revenues, the petroleum industry has switched to processing low-cost crude oils. They are blended with high-quality crude oils to feed the crude distillation units (CDUs) with reliable feedstock. The blending process takes place in storage tanks receiving crude parcels from ultralarge carriers and by mixing feed streams supplied to CDUs from multiple tanks. The crude blending and scheduling problem is usually represented by a large nonconvex mixed-integer nonlinear programming (MINLP) model. This work introduces an effective MINLP continuous-time formulation based on global-precedence sequencing variables to arrange loading and unloading operations in every tank. In addition, synchronized time slots of adjustable length permit to model the sequence of feedstock for each CDU. The basic solution approach consists of sequentially solving a very tight mixed-integer linear programming (MILP) model and a nonlinear programming (NLP) formulation that uses the MILP-solution as the starting point. For large problems, it has been developed a novel solution strategy that incorporates a time-partitioning scheme using the notion of vessel-blocks. The proposed approach has been applied to a series of large examples studied by other authors finding near-optimal schedules at much lower CPU times.

1. INTRODUCTION

Petroleum refineries are continuous manufacturing facilities that transform crude oil into a large number of refined products. At the front-end of the refining process, parcels of crude oil are unloaded from very large crude carriers (VLCCs) into multiple storage tanks. These tanks can either directly feed the crude distillation units (CDUs) or supply lots of crude oil to charging tanks, which in turn provide the feedstock for the CDUs. Petroleum fractions separated by the CDUs are afterward converted into finished products through a series of downstream physical and chemical processes. Storage and charging tanks hold blends of different crudes with compositions changing with time. In this work, a marine-access oil refinery is considered with no charging tanks, i.e. the storage tanks directly feed the CDUs.

Different types of crude oils present significant variations in compositions, properties, product yields, and prices. As the crude oil cost has a significant impact on the refinery revenues, the petroleum industry is switching to processing low-cost crude oils to increase profits. But cheaper crudes are richer in undesirable components such as sulfur, carbon residue, aromatics, etc., that originate processing and product quality problems in CDUs and downstream units. Then, refineries have to blend them with premium crude oils to get a reliable feedstock for the CDUs. The blending process can occur either in storage tanks receiving different crudes from the VLCCs or by mixing the flows of crude blends provided by two or more tanks to get the feedstock for a particular CDU. As a result, major issues for the scheduler are the types and volumes of crude oils to be purchased, and the optimal blends of crudes to be processed over time in order to maximize revenues. Given the amounts and compositions of the crude mix initially available in storage tanks and the parcels of crude oil to be unloaded from the arriving vessels, a substantial improvement in refinery profits and plant reliability can surely be achieved by applying advanced decision-making tools for the scheduling of crude oil operations. Front-end refinery operations include the unloading of crude oils into storage tanks from the VLCCs, the mixing of different crudes in storage tanks, and the blending of crude streams coming from various storage tanks and allocated to the same CDU. As stated by Kelly and Mann,[1,2] better planning and scheduling of front-end activities can save millions of dollars per year.

At the planning level, it is decided about the types and volumes of crudes to be procured and the mix of finished products to be obtained over the next several months. In turn, scheduling is concerned with the sequencing and timing of the front-end operations over a shorter time horizon. Good scheduling depends on some key assignment decisions. Compositions of crude blends in storage tanks depend on how the arriving crude parcels are allocated to those units. Besides, the quality of the feedstock for each CDU is determined by the choice of the storage tanks providing the feed streams and their flow-rates. The amount of crude mix and the quality of the feedstock to be processed in every CDU are decisions already made at the planning level. The quality of the feedstock is given by specifying the allowable ranges of certain properties like sulfur, aromatics, narrow fraction yields, etc. In order to meet the total crude demand of each CDU, a sequence of different feedstocks all satisfying the quality specifications are usually supplied to a CDU along the scheduling horizon. A switch of the feedstock occurs when the storage tanks supplying crude blends to the CDU and/or their feed rates change, and

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consequently the feedstock composition is modified. During this event, lasting a few hours, some amounts of off-spec products are generated, thus reducing the refinery profits.

Oil refineries process several kinds of crudes that arrive in two types of ships: very large tankers carrying parcels of different crudes, and small single-parcel vessels. Very large crude carriers (VLCCs) have multiple compartments to carry large parcels of crudes and must dock offshore at a single-buoy mooring (SBM) station. The SBM serves as a mooring point and is connected to the storage facilities at the refinery through a SBM pipeline. Usually, there is only one SBM terminal and a single SBM pipeline. As a result, parcels of crude oils must be sequentially unloaded from the tankers. Moreover, at most a single storage tank can receive crude from the VLCCs at any moment. Nonetheless, a parcel of crude can be sequentially allocated to several storage tanks. The cargo of a tanker must be completely unloaded before starting the discharge of another VLCC. The SBM line has a significant holdup capacity generally containing a portion of the crude parcel last unloaded. In practice, the holdup is usually regarded as another parcel that is ejected from the line when a new parcel is unloaded from a tanker.[3] Crude oil also arrives in small vessels carrying a single parcel. Due to its size, they can dock at the jetties. Onshore offloading facilities can comprise one or several jetties. In contrast to the SBM line, the pipeline connecting every jetty to the refinery tank farm has a small holdup that is usually neglected.

The crude scheduling problem seeks to maximize the refinery gross profit by properly scheduling both the unloading of crude oil parcels from VLCCs/vessels and the charging of crude blends to the CDUs. One of the major difficulties in solving this problem is the need to cope with bilinear constraints determining the compositions of (i) the crude blends available in storage tanks and (ii) the feedstock charged to the CDUs. Because of such bilinear constraints, one should represent the problem through a nonconvex mixed-integer nonlinear programming (MINLP) model. This work introduces a new MINLP formulation for the scheduling of crude oil operations in a marine-access refinery with no separate charging tanks. It is the first crude scheduling model based on global-precedence sequencing variables to arrange loading and unloading operations in the storage tanks.

2. PREVIOUS WORKS

Most contributions on the scheduling of front-end refinery operations assumed an inland oil refinery with separate storage and charging tanks (Lee et al.,[4] Jia et al.,[5] Furman et al.,[6] Karuppiah et al.,[7] Saharidis et al.,[8] Mouret et al.,[9−11] Chen et al.,[12] Castro and Grossmann,[13] and Yadav and Shaik[14]). When charging tanks are used, each CDU unit is fed by only one tank at any time. The charging schedule is determined by selecting the sequence of charging tanks connected to each CDU, the connection period and the volumes of crude blends supplied by the charging tanks. When the refinery configuration excludes charging tanks, several storage tanks can simultaneously feed a particular CDU. In that case, the quality of the feedstock not only depends on the compositions of the crude blends simultaneously supplied by the assigned storage tanks but also on their flow-rates. To have a control on the feedstock composition, the supply of crude blends from the source tanks must start and end at the same time and at constant rates while such tanks are concurrently feeding the CDU.

Fewer works have been focused on the crude scheduling problem for marine-access oil refineries with no separate charging tanks. Pinto et al.[15] addressed the problem of crude oil inventory management in a real world refinery that receives several types of crude oil. They proposed an MILP formulation based on a continuous-time representation with a scheduling horizon divided into a series of time slots with variable length. Linear bounding constraints instead of bilinear relationships were used to determine the crude mix composition in storage tanks and CDU feed streams. Reddy et al.[3] presented a continuous-time MILP approach where the scheduling horizon was divided into several blocks using fixed-time events like the scheduled arrival times and the expected departure times of VLCCs. In addition, each block comprises a series of synchronized time slots of variable-length that are common for all storage tanks. Reddy et al.'s algorithm (RA) seeks to eliminate the MILP composition discrepancy without solving an NLP model by recognizing that the linear bounding approximations are exact only if the compositions of crude blends in the storage tanks supplying the feedstock to the CDUs are known and constant. With this idea, they applied a rolling-horizon strategy that consists of iteratively solving a series of MILPs. Though it usually provides an operational schedule with no composition discrepancy, the RA procedure may fail to get a feasible schedule. Moreover, it lacks a mechanism to recover from such failures. Reddy et al.[16] also developed the discrete version of the RA approach with time slots of fixed-length. When both versions of the RA approach were applied to the same set of examples, they showed almost the same performance in solution quality and CPU time.

Li et al.[17] presented a simple way to recover the discrete RA approach from a failure by identifying and removing the infeasible combinations of binary variables through using the well-known integer cut of Balas and Jeroslow.[18] The revised Reddy's algorithm (RRA) applied a backtracking strategy that removes infeasible combinations of frozen binary variables and proceeds forward to obtain a feasible solution. The time horizon is divided into a number of blocks, called vessel-blocks, that are defined based on the scheduled arrival times of the tankers. To improve the performance of the RRA approach, Li et al.[17] applied a partial relaxation strategy where the binary variables beyond the block where the infeasibility arises are treated as continuous variables. This is the RRA-P approach. Because a variety of crude properties are used in industry to characterize the crude quality, Li et al.[17] extended the notion of key component concentration to consider several quality specifications. To this end, they reported the linear additive indices used in industry to compute those properties.

Liang et al.[19] considered a multipipeline crude oil blending system composed of storage tanks, a limited number of transfer pipelines to unload lots of crude oil from storage tanks, and mixing pipelines to charge the CDUs. Sulfur concentration limits and the desired true boiling point (TBP) curve for the CDU feedstock are problem specifications. The authors tackled the problem using a two-level strategy with the outer level allocating and sequencing storage tanks to pipelines, and the inner level optimizing the blending flow rates for the selected sequence. Such decisions are made using a pair of metaheuristics, namely the constrained ordinal optimization (COO) and the evolutionary algorithm (EA). More recently, Li et al.[20] developed a novel unit-specific event-based continuous-time MINLP formulation. The approach exploited recent advances in piecewise-linear underestimation of bilinear terms

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within a branch-and-bound algorithm to find a global optimum of the MINLP model. In addition, 15 important crude property indices are considered in one of the examples to ensure the quality of the feedstock. The approach provided better solutions when applied to several examples taken from the work of Li et al.[17] Pan et al.[21] used heuristic rules collected from refinery personnel experience to choose: (i) the receiving storage tanks for each parcel of crude oil and (ii) the volume of crude to be discharged into the assigned tank. They also assumed that no input operation can occur between two output activities to fix the values of the key component concentration in the storage tanks. After selecting the unloading operations and fixing the blend compositions in the storage tanks to linearize the bilinear terms, the resulting MILP model just arranges the charging operations of tanks to CDUs. The approach is able to find very good solutions for small-size problems.

3. PROBLEM DESCRIPTION

A schematic of front-end operations in a typical marine-access refinery without separate charging tanks is shown in Figure 1.

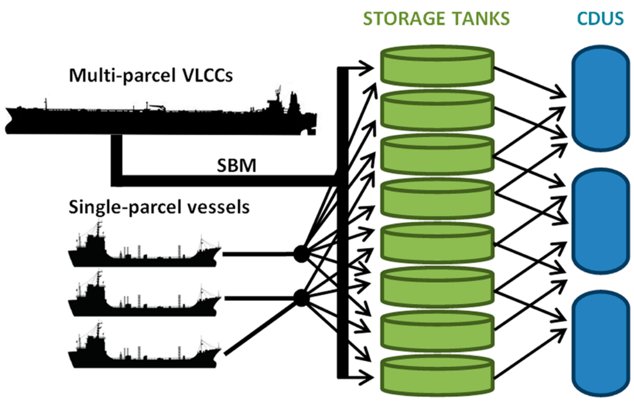

Figure 1. Schematic of front-end operations in marine-access oil refineries.

The system comprises (1) offshore facilities consisting of a single buoy mooring (SBM) station and a SBM pipeline for unloading crude oil from the tankers into the storage tanks, (2) onshore facilities for small vessels consisting of one or more jetties and their associated jetty-tank pipelines, (3) very large crude carriers (VLCCs) and small vessels arriving at different times, (4) multiple storage tanks holding crude blends of variable composition, and (5) crude distillation units (CDUs) processing the feedstock coming from the storage tanks. Besides, the front-end refinery operations involve the unloading of crude parcels into the storage tanks, the blending of crudes in storage tanks, and the supply of crude blends to CDUs from various tanks at different rates over time.

Available data to solve the problem include (a) the estimated arrival/departure times of VLCCs and the features of the crude parcels (crude type and volume); (b) the configuration of unloading facilities such as the numbers of SBM stations and jetties and the associated pipelines, the unloading rate limits, the SBM pipeline holdup, and the current crude resident in the pipeline; (c) the storage tank farm data including the number of tanks, their capacities and initial crude stocks and compositions, as well as the unloading rate limits from tanks to CDUs; (d) the CDU data including the processing rates and the quality specifications for the feedstock; (e) the selected mode of crude oil segregation in storage and processing units; and (f) economic data such as the demurrage costs for late vessel departures, changeover costs for changing the feedstock supplied to CDUs, penalties for safety stock violations, crude margins, and CDU demands. Without loss of generality, we assume that only one key component (typically sulfur) characterizes the crude parcels carried by the VLCCs and the crude blends in storage tanks. The problem goal seeks to maximize the total gross profit by optimizing (i) the schedule of the operations unloading crude oil from every vessel, (ii) inventory and composition profiles of crude blends in storage tanks over the time horizon, and (iii) the crude charging schedule for every CDU. The crude charging schedule specifies the tanks feeding each CDU at any moment, the feed flow-rates, and the time interval during which storage tanks concurrently supply feedstock to the distillation units. The total gross profit to be maximized is given by the difference between the total netback from processing crude oil and the operating costs (including demurrage, changeover, and safety stock penalties).

4. MODEL ASSUMPTIONS

The proposed MINLP formulation is based on the following assumptions:

(1) The amount and composition of the crude oil transported by the tankers as well as their arrival times and expected departure times are known data.
(2) The configuration of offshore and onshore facilities for unloading VLCCs and small vessels, the storage tank farm, and the CDUs are also known.
(3) Each tanker carries parcels of different types of crude oil stored in multiple compartments.
(4) There is only one SBM pipeline connecting the SBM with the storage tanks, so parcels must be unloaded from VLCCs one by one.
(5) A single jetty is available to receive small vessels carrying single crude parcels.
(6) Very large crude carriers and vessels are serviced following the first-come first served (FCFS) rule.
(7) The unloading sequence of parcels from a particular VLCC is known a priori. A parcel must be fully unloaded before starting the discharge of the next parcel in the list.
(8) The total amount of crude oil contained in each tanker should be unloaded not beyond its expected departure time. Otherwise, demurrage costs are to be paid.
(9) Parcels unloaded from a vessel can be sent into two or more storage tanks but there is at most one receiving tank connected to the tanker at any time. Then, storage tanks are sequentially loaded.
(10) The crude oil in the SBM pipeline is regarded as another parcel that is ejected into a storage tank when starting the unloading of the next parcel.
(11) The pipeline connecting the jetty to the refinery tank farm has a negligible volume.
(12) A storage tank cannot receive crude from a tanker and feed a CDU at the same time. After receiving a lot of crude oil, a storage tank should stay idle during some time for brine settling and removal before feeding some CDU.
(13) At most two storage tanks are allowed to concurrently feed a CDU.

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(14) A storage tank can at most feed two CDUs simultaneously.
(15) Times for perfect crude mixing in storage tanks and CDU changeovers are negligible. Besides, a perfect mixing of the crude blends supplied to a CDU from different storage tanks occurs in the mixing pipeline.
(16) A crude distillation unit must continually process feedstock coming from storage tanks.

5. MINLP MATHEMATICAL MODEL

In this paper, a new nonconvex MINLP formulation for the short-term scheduling of marine-access oil refineries is presented. It is a continuous-time approach that uses global-precedence sequencing variables to establish the ordering of loading and unloading operations in the storage tanks. In addition, it applies an efficient mode of tracking the composition and the inventory of crude mix in those tanks. A sequence of feed streams with different compositions is continually supplied to each CDU along the planning horizon. Each feed stream may be obtained by mixing crude flows coming from two different storage tanks. Such crude flows should be synchronized in the sense that the associated unloading operations must have the same starting and completion times. Therefore, synchronized time slots of variable length are used to model the sequence of feed streams for each CDU. Interestingly, the proposed time slots are unit-specific because their initial/end times may change with the CDU. In short, it is proposed a hybrid approach involving a set of general-precedence variables to sequence the operations in every storage tank and a set of time slots to define the sequence of storage tanks providing the feedstock to every CDU along the time horizon.

5.1. Model Variables. To avoid symmetrical solutions, the sets of batches to be unloaded from marine vessels ($iv \in \mathbf{IV}$) and storage tanks ($ik \in \mathbf{IK}$) are chronologically ordered. Then, there is only one set of batches **IV** to unload crude parcels $p \in \mathbf{P}$ from tankers and a single set of lots of crude blends **IK** to feed CDUs from storage tanks. The number of elements in each set must be postulated by the user. Batches are said to be chronologically ordered in the set **IV** because the unloading of the element $(iv + 1)$ never begins before completing the discharge of the element iv. If the elements ik and $ik' > ik$ in the set **IK** are unloaded from the same or different storage tanks, then the unloading of lot ik' never begins before starting the discharge of batch ik. Binary variables $WP_{iv,p}$ and $WK_{ik,k}$ assigning lots $iv \in \mathbf{IV}$ to crude parcels and batches of crude blend $ik \in \mathbf{IK}$ to storage tanks, respectively, are considered. A lot of crude oil iv does exist only if one of the related assignment variables $WP_{iv,p}$ is equal to one. If $WP_{iv,p} = 1$, the lot iv contains a portion or the whole parcel p, while the condition $\sum_{p\in\mathbf{P}} WP_{iv,p} = 0$ denotes that iv is a fictitious element. Moreover, any existing lot of crude oil unloaded from a vessel must have a single destination. Then, additional binary variables $\{YV_{iv,p,k}\}$ are defined to assign a destination to every lot iv. If $YV_{iv,p,k} = 1$, the lot iv containing a portion or the whole parcel p is loaded into the storage tank k. If instead iv is a dummy parcel, then $\sum_{p\in\mathbf{P}} WP_{iv,p} = 0$ and no parcel and destination are assigned to it, i.e. $\sum_{k\in\mathbf{K}} \sum_{p\in\mathbf{P}} YV_{iv,p,k} = 0$. On the other hand, the set **T** comprises all the available crude distillation units in the refinery.

To sequence the feed streams supplied to every CDU, the proposed MINLP uses a continuous-time representation with the scheduling horizon divided into a series of time slots of variable length $s \in \mathbf{S}$. The binary variable $YK_{ik,k,s,t}$ denotes that the batch of crude mix ik effectively unloaded from the storage tank $k \in \mathbf{K}$ is supplied to the distillation unit t during the entire time slot s when $YK_{ik,k,s,t} = 1$. In addition, the proposed model includes global-precedence sequencing variables $\{XK_{iv,ik,k}\}$ to control the ordering of loading and unloading operations in every storage tank. Assuming that $XK_{iv,ik,k} = 1$, the lot iv is loaded before unloading the batch ik from the storage tank k if both operations occur in tank k, i.e. $WK_{ik,k} = \sum_{p\in\mathbf{P}} YV_{iv,p,k} = 1$. The value of $XK_{iv,ik,k}$ is meaningless when $WK_{ik,k} + \sum_{p\in\mathbf{P}} YV_{iv,p,k} < 2$. If $XK_{iv,ik,k} = 0$ and both operations take place at the storage tank k, the batch ik is unloaded before receiving lot iv.

The MINLP approach also includes different sets of continuous variables standing for (a) the size of batches containing crude parcels coming from marine vessels, given by $QV_{iv,p}$; (b) the size of baches unloaded from storage tanks to feed CDUs ($QK_{ik,k}$); (c) the starting times $\{SV_{iv}, SK_{ik}\}$ and the end times $\{CV_{iv}, CK_{ik}\}$ of loading and unloading operations in storage tanks; (d) the starting and end times $\{ST_{s,t}, CT_{s,t}\}$ of the time slots for CDU t; (e) the cumulative amount of crude filled into the storage tank k just after loading the batch iv or after unloading the lot ik, given by $\{LKV_{k,iv}, LKK_{k,ik}\}$; (f) the cumulative amount of crude unloaded from the storage tank k at the times CV_{iv} and CK_{ik}, given by $\{UKV_{k,iv}, UKK_{k,ik}\}$; (g) the stock of crude mix in the storage tank k after unloading the lot ik, given by $\{IKK_{k,ik}\}$; and (h) the inventory of crude blend in the storage tank k after loading the lot iv, given by $\{IKV_{k,iv}\}$. To monitor the component concentration in every tank, the continuous variable $CJK_{k,ik}$ denoting the concentration in the storage tank k at the time CK_{ik} is also defined. Similarly, the variable $CJT_{s,t}$ represents the composition of the feedstock processed in the distillation column t during the time slot s. If lots ik and ik' come from different tanks k and k', and feed the same CDU t during the entire time slot s, then the related unloading operations should start and end at the same times, i.e. $SK_{ik} = SK_{ik'} = ST_{s,t}$ and $CK_{ik} = CK_{ik'} = CT_{s,t}$.

5.2. Illustrating the Method of Tracking the Stock of Crude Oil in Storage Tanks. A simple example is presented to illustrate the way of tracking the inventory of crude oil in a storage tank at the completion times of loading and unloading operations. The procedure is based on the use of the chronologically ordered sets **IV** and **IK**, and the global-precedence sequencing variables $XK_{iv,ik,k}$. Let us assume that $\mathbf{IV} = \{iv_1, iv_2\}$ and $\mathbf{IK} = \{ik_1, ik_2\}$ represent the batches of crude oil loaded into and withdrawn from the storage tank k, respectively. Lots in the set **IV** are assumed to come from a single vessel v. The batch sizes are $QV_{iv1,v} = 80$ kbbl, $QV_{iv2,v} = 20$ kbbl, $QK_{ik1,k} = 50$ kbbl, and $QK_{ik2,k} = 30$ kbbl. Besides, the values of the sequencing variables are $XK_{iv1,ik,k} = 1$ for any $ik \in \mathbf{IK}$, $XK_{iv2,ik1,k} = 0$ and $XK_{iv2,ik2,k} = 1$. In other words, the loading of iv_1 into the tank k is the first operation and the loading of iv_2 occurs after delivering the lot ik_1 and before unloading ik_2. Because **IV** and **IK** are chronologically ordered sets, lot ik_2 is unloaded after ik_1 and batch iv_1 is loaded before iv_2 into tank k. In this way, the sequence of operations performed in tank k is given by $\{iv_1, ik_1, iv_2, ik_2\}$. Lots are loaded/unloaded at a flowrate of 20 kbbl/h, and the initial inventory iik_k^0 is 10 kbbl. Moreover, the operations are performed one after the other without delay. Then, $CV_{iv1} = 4.0$ h, $CK_{ik1} = 6.5$ h, $CV_{iv2} = 7.5$ h, and $CK_{ik2} = 9.0$ h as shown in Figure 2.

The total amount of crude oil ($LKK_{k,ik1}$) filled into the tank k up to the time CK_{ik1} (i.e., after completing the unloading of lot

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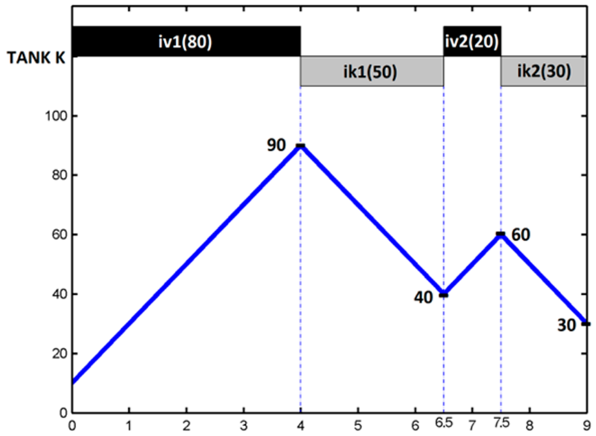

Figure 2. Simple example illustrating the method of tracking tank inventories.

ik_1) is obtained by summing up the sizes of the batches loaded into the tank k from vessel v before unloading the lot ik_1. The lot iv_1 is the only batch satisfying that condition (i.e., $XK_{iv,ik1,k} = 1$). Therefore,

$$LKK_{k,ik1} = \sum_{\substack{iv \in \mathbf{IV} \\ (XK_{iv,ik1,k}=1)}} QV_{iv,v} = QV_{iv1,v} = 80 \text{ kbbl}$$

In turn, the cumulative amount of crude mix withdrawn from tank k up to time CK_{ik1} is obtained by summing up the sizes of the batches unloaded from tank k before ik_1 plus the size of lot ik_1.

$$UKK_{k,ik1} = \sum_{\substack{ik \in IK \\ (ik \le ik1)}} QK_{ik,k} = QK_{ik1,k} = 50 \text{ bbl}$$

In this way, the model will calculate the inventory level in tank k at time CK_{ik1} by adding the difference ($LKK_{k,ik1} - UKK_{k,ik1}$) to the initial inventory iik_k^0. A similar procedure is applied to calculate the inventory of the key component in the storage tanks at the completion times of loading and unloading operations.

$$\begin{aligned} IKK_{k,ik1} &= iik_k^0 + (LKK_{k,ik1} - UKK_{k,ik1}) \\ &= 10 + (80 - 50) \\ &= 40 \text{ kbbl} \end{aligned}$$

5.3. Model Constraints. The model constraints have been grouped into six blocks that are concerned with (1) the discharge of crude parcels from the tankers, (2) the supply of crude mix to CDUs, (3) the allocation of lots of crude blends to time slots of CDUs, (4) tracking the inventory of crude mix in storage tanks, (5) monitoring the key component concentration in storage tanks, and (6) monitoring the key component concentration in the feedstock for the CDUs.

5.3.1. Unloading Parcels of Crude Oil into Storage Tanks from VLCCs. Allocating Parcels of Crude Oil to Generic Batches. A very large crude carrier (VLCC) with multiple compartments transports several parcels of different crude oils. The set **P** comprises all the parcels of crude oil to be unloaded from the arriving VLCCs. They are listed in the same order that they must be unloaded from the VLCCs. In other words, the parcel p must be unloaded before parcel p' if $p < p'$. Besides, multiple lots can be assigned to a given parcel to sequentially discharge it into several storage tanks. Equation 1 states that the lot iv can at most discharge, partially or entirely, a single parcel of crude oil.

$$\sum_{p \in \mathbf{P}} WP_{iv,p} \le 1 \qquad \forall iv \in \mathbf{IV} \tag{1}$$

Assigning Storage Tanks to Generic Batches of Crude Oil. Equation 2 states that an existing lot iv should be assigned to a single storage tank. The set $\mathbf{K}_p$ includes the storage tanks than can receive the parcel p.

$$\sum_{k \in \mathbf{K}_p} YV_{iv,p,k} = WP_{iv,p} \qquad \forall\ iv \in \mathbf{IV},\ p \in \mathbf{P} \tag{2}$$

Sizing Lots of Crude Oil Unloaded from VLCCs. If lot iv does exist (i.e., $\sum_{p\in\mathbf{P}} WP_{iv,p} = 1$), it will have a finite size. By eq 3, the size of an existing lot iv assigned to parcel p should belong to the range $\{sv_{\min}, a_p\}$, where $sv_{\min}$ is the minimum allowed size and a_p is the size of parcel p. Dummy lots iv featuring $\sum_{p\in\mathbf{P}} WP_{iv,p} = 0$ have a null size.

$$sv_{\min} WP_{iv,p} \le QV_{iv,p} \le a_p WP_{iv,p} \qquad \forall\ iv \in \mathbf{IV},\ p \in \mathbf{P} \tag{3}$$

Every Parcel of Crude Oil Should Be Completely Unloaded. Every parcel p should be completely unloaded before the end of the scheduling horizon. Then, the total amount of crude oil unloaded from parcel p along the time horizon should be equal to a_p by eq 4.

$$\sum_{iv \in \mathbf{IV}} QV_{iv,p} = a_p \qquad \forall\ p \in \mathbf{P} \tag{4}$$

Amount of Crude Oil in the Lot iv Sent to the Storage Tank k. The continuous variable $FV_{iv,p,k}$ denotes the amount of crude oil in lot iv coming from parcel p that is sent to the storage tank $k \in \mathbf{K}_p$. The value of $FV_{iv,p,k}$ is determined by eqs 5a–5b. If batch iv is not assigned to tank k, $FV_{iv,p,k}$ is equal to zero for any parcel p. Otherwise, it equals the size of lot iv given by $QV_{iv,p}$. At most one term of the summation on the LHS of eq 5b will be finite. Such a term is associated with the lot iv assigned to parcel p and the storage tank $k \in \mathbf{K}_p$ if $YV_{iv,p,k}$ is equal to 1.

$$FV_{iv,p,k} \le a_p YV_{iv,p,k} \qquad \forall\ iv \in \mathbf{IV},\ k \in \mathbf{K}_p,\ p \in \mathbf{P} \tag{5a}$$

$$\sum_{k \in \mathbf{K}_p} FV_{iv,p,k} = QV_{iv,p} \qquad \forall\ iv \in \mathbf{IV},\ p \in \mathbf{P} \tag{5b}$$

Length of Vessel Unloading Operations. The continuous variable LV_{iv} be the length of the operation discharging the lot iv containing a portion or the whole parcel p. If the parameters $\{rv_{\min}, rv_{\max}\}$ stand for the minimum/maximum unloading rates, then the relationship between LV_{iv} and the size of lot iv ($\sum_{p\in\mathbf{P}} QV_{iv,p}$) is given by eq 6.

$$rv_{\min} LV_{iv} \le \sum_{p \in \mathbf{P}} QV_{iv,p} \le rv_{\max} LV_{iv} \qquad \forall\ iv \in \mathbf{IV} \tag{6}$$

Starting Times for Vessel Unloading Operations. The discharge of lot iv containing a parcel of crude oil from the tanker v cannot start before the arrival of that vessel at the docking station. The parameter at_v denotes the arrival time of the marine vessel v, and P_v is the subset of parcels carried by vessel v.

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$$SV_{iv} \geq at_v(\sum_{p \in \mathbf{P}_v} WP_{iv,p}) \qquad \forall\ iv \in \mathbf{IV},\ v \in \mathbf{V} \tag{7}$$

Parcels of Crude Oil Are Discharged One at a Time in a Prespecified Order. Lots have to be discharged from vessels one by one because a single buoy mooring pipeline is available. To avoid symmetrical solutions, lot iv is discharged before lot iv' if $iv < iv'$ by eq 8a. As stated by eq 8b, all the parcels should be unloaded before the end of the planning horizon. The parameter H is the length of the scheduling horizon. By eq 9, the difference between the completion and the starting time for the operation unloading lot iv is its length LV_{iv}.

$$CV_{iv} \leq SV_{iv'} \qquad \forall\ (iv,\ iv') \in \mathbf{IV}(iv < iv') \tag{8a}$$

$$CV_{iv} \leq H \qquad \forall\ iv \in \mathbf{IV} \tag{8b}$$

$$CV_{iv} = SV_{iv} + LV_{iv} \qquad \forall\ iv \in I\mathbf{V} \tag{9}$$

Fictitious Lots. Lots assigned to parcel p should be unloaded after those allocated to parcel $(p-1)$. Then, eq 10 states that lot iv can be assigned to parcel p only if the precedence lot $(iv - 1)$ has been allocated to either parcel $(p-1)$ or parcel p. As a result, dummy lots with no assigned parcels will be the last elements of **IV**.

$$WP_{iv,p} \leq WP_{(iv-1),(p-1)} + WP_{(iv-1),p}$$
$$\forall\ (iv-1),\ iv \in \mathbf{IV},\ (p-1),\ p \in \mathbf{P} \tag{10}$$

Unloading of a Marine Vessel v Should Be Completed before Its Expected Departure Time edt_v. If the lot of crude oil iv comes from the marine vessel v, it should be unloaded before the expected departure time of v, i.e. edt_v. The continuous variable DT_v denotes the departure time of vessel v. If vessel v departs later (i.e., $DT_v > edt_v$) because of some delay in the unloading operations, it should pay a penalty that will be proportional to the departure tardiness TD_v. The values of DT_v and TD_v are defined by eqs 11a–11b.

$$CV_{iv} \leq DT_v + H(1 - WP_{iv,p})$$
$$\forall\ iv \in \mathbf{IV},\ v \in \mathbf{V},\ p \in \mathbf{P_v} \tag{11a}$$

$$TD_v \geq DT_v - edt_v \qquad \forall\ v \in \mathbf{V} \tag{11b}$$

5.3.2. Supplying Lots of Mixed Oils to CDUs from Storage Tanks. Batch of Crude Mix Should at Most Come from a Single Storage Tank. By eq 12, the lot ik comes from at most a single storage tank. It is unloaded from the tank k only if the allocation variable $WK_{ik,k}$ is equal to one. Otherwise, the lot ik comes from another storage tank k' or is a fictitious batch. To avoid symmetrical solutions, the elements of the set **IK** have been preordered in such a way that the unloading of lot ik' from some storage tank never begins before starting the discharge of lot ik whenever $ik' > ik$. Such a preordering condition is given by eq 13.

$$\sum_{k \in \mathbf{K}} WK_{ik,k} \leq 1 \qquad \forall\ ik \in \mathbf{IK} \tag{12}$$

$$SK_{ik} \leq SK_{ik'} \qquad \forall\ ik,\ ik' \in IK(ik < ik') \tag{13}$$

Sizing Lots of Crude Mix Unloaded from Storage Tanks. The continuous variable $QK_{ik,k}$ stands for the size of lot ik coming from the storage tank k. If lot ik exists (i.e., $\sum_{k \in \mathbf{K}} WK_{ik,k} = 1$), then it has a finite size by eq 14. The parameters $\{sk_{min}$, sk_{max} stand for the minimum and maximum allowed sizes for lots $ik \in$ **IK**.

$$sk_{min}WK_{ik,k} \leq QK_{ik,k} \leq sk_{max}WK_{ik,k}$$
$$\forall\ ik \in \mathbf{IK},\ k \in \mathbf{K} \tag{14}$$

Length of Storage Tank Unloading Operations. The continuous variable LK_{ik} represents the length of the operation unloading lot ik from a storage tank. The value of LK_{ik} is determined by eq 15 where the parameters $\{rk_{min}$, rk_{max} stand for the minimum and maximum unloading rates of crude mix from the storage tanks. Moreover, the completion time for the unloading of lot ik is determined by eqs 16a–16b.

$$rk_{min}LK_{ik} \leq \sum_{k \in \mathbf{K}} QK_{ik,k} \leq rk_{max}LK_{ik} \qquad \forall\ ik \in \mathbf{IK} \tag{15}$$

$$CK_{ik} = SK_{ik} + LK_{ik} \qquad \forall\ ik \in \mathbf{IK} \tag{16a}$$

$$CK_{ik} \leq H \qquad \forall\ ik \in \mathbf{IK} \tag{16b}$$

Sequencing Loading and Unloading Operations in Storage Tanks. The sequencing variable $XK_{iv,ik,k}$ indicates that the unloading of lot ik from the storage tank k does not start before completing both the loading of lot iv into that tank and the settling period sst if $XK_{iv,ik,k} = 1$. Such operations will take place only if $WK_{ik,k} = \sum_{p \in \mathbf{P}_k} YV_{iv,p,k} = 1$. In contrast, the unloading of lot ik will occur before receiving lot iv if $XK_{iv,ik,k} = 0$ and $WK_{ik,k} = \sum_{p \in \mathbf{P}_k} YV_{iv,p,k} = 1$. These conditions are imposed by the sequencing constraints 17a–17b. If the lot iv and/or the batch ik have not been assigned to the storage tank k, then $XK_{iv,ik,k} = 0$ by eqs 17c–17d.

$$CV_{iv} \leq SK_{ik} - sst + H(1 - XK_{iv,ik,k})$$
$$\forall\ iv \in \mathbf{IV},\ ik \in \mathbf{IK},\ k \in \mathbf{K} \tag{17a}$$

$$CK_{ik} \leq SV_{iv} + HXK_{iv,ik,k} + H(2 - \sum_{p \in \mathbf{P}_k} YV_{iv,p,k} - WK_{ik,k})$$
$$\forall\ iv \in \mathbf{IV},\ ik \in \mathbf{IK},\ k \in \mathbf{K} \tag{17b}$$

$$XK_{iv,ik,k} \leq \sum_{p \in \mathbf{P}_k} YV_{iv,p,k} \qquad \forall\ iv \in \mathbf{IV},\ ik \in \mathbf{IK},\ k \in \mathbf{K} \tag{17c}$$

$$XK_{iv,ik,k} \leq WK_{ik,k} \qquad \forall\ iv \in \mathbf{IV},\ ik \in \mathbf{IK},\ k \in \mathbf{K} \tag{17d}$$

*Last Elements of the Set **IK** Are Reserved for Fictitious Lots.* If there are some dummy lots, then they will be the last elements of the set **IK** by eq 18.

$$\sum_{k \in \mathbf{K}} WK_{ik,k} \leq \sum_{k' \in \mathbf{K}} WK_{(ik-1),k'} \qquad \forall\ (ik-1),\ ik \in \mathbf{IK} \tag{18}$$

5.3.3. Allocating Lots of Crude Mix to CDUs. Lot of Crude Mix Withdrawn from a Storage Tank Should Be Supplied to Just One CDU. Because a series of feed streams can be supplied to each CDU along the time horizon, the notion of unit-specific time slots of variable length is considered. The elements in the set of time slots **S** are preordered in such way that the time slot s occurs before s' whenever $s < s'$. Moreover, unit-specific slots (s,t) are defined because their starting/completion times depend on the associated CDU t. Therefore, each feed stream to a particular distillation unit t should be allocated to one of its slots s, i.e. the slot (s,t). The binary variable $YK_{ik,k,s,t}$ allocates the

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lot ik coming from tank k to $CDU\ t$ during the slot s. Equation 19 indicates that an existing lot ik should be allocated to a single time slot of a particular CDU. The set $\mathbf{T}_k$ only includes the CDUs that can receive crude mix from the storage tank k.

$$\sum_{s\in\mathbf{S}}\sum_{t\in\mathbf{T}_k} YK_{ik,k,s,t} = WK_{ik,k} \qquad \forall\ ik \in \mathbf{IK},\ k \in \mathbf{K} \tag{19}$$

Sequencing Unloading Operations of Batches Supplied to the Same CDU. According to eq 20, the unloading of lot ik from some storage tank should be completed before starting the discharge of lot ik' if they have been assigned to slots (s, s') of the same $CDU\ t$, with $ik < ik'$ and $s < s'$.

$$CK_{ik} \le SK_{ik'} + H(2 - YK_{ik,k,s,t} - YK_{ik',k',s',t})$$
$$\forall\ ik,\ ik' \in \mathbf{IK}(ik < ik'),\ k,\ k' \in \mathbf{K},\ s,\ s' \in \mathbf{S}(s < s'),\ t \in \mathbf{T}_{k,k'} \tag{20}$$

Every Lot of Crude Mix Can at Most Be Supplied by a Storage Tank to a Particular CDU at Any Time Instant. By eq 21, at most a single lot coming from the storage tank k can be assigned to the time slot s of CDU t.

$$\sum_{ik\in\mathbf{IK}} YK_{ik,k,s,t} \le 1 \qquad \forall\ s \in \mathbf{S},\ k \in \mathbf{K},\ t \in \mathbf{T}_k \tag{21}$$

Every Storage Tank Can Simultaneously Feed at Most Two CDUs. In practice, a storage tank cannot feed more than two CDUs at the same time. By using a segregated mode of processing crude blends in the CDUs, this condition is automatically satisfied by specifying the CDUs that can be fed by every storage tank.

Multiple Tanks Can Feed the Same CDU. For controllability reasons, at most two tanks can usually feed the same CDU at any time instant. In eq 22, the binary variable $YS_{s,t}$ denotes the existence of the time slot (s,t). No lot of crude blend can be assigned to the slot s of CDU t if $YS_{s,t} = 0$.

$$\sum_{ik\in\mathbf{IK}}\sum_{k\in\mathbf{K}_t} YK_{ik,k,s,t} \le 2YS_{s,t} \qquad \forall\ s \in \mathbf{S},\ t \in \mathbf{T} \tag{22}$$

Amount of Crude Mix in the Lot ik Supplied to CDU t over the Entire Time Slot s. The continuous variable $FK_{ik,k,s,t}$ stands for the volume of crude oil in the lot ik coming from tank $k \in \mathbf{K}_t$ supplied to the CDU t during the time slot s. By eqs 23a–23b, $FK_{ik,k,s,t} = 0$ if $YK_{ik,k,s,t} = 0$, and $FK_{ik,k,s,t} = QK_{ik,k}$ whenever $YK_{ik,k,s,t} = 1$.

$$FK_{ik,k,s,t} \le sk_{\max} YK_{ik,k,s,t}$$
$$\forall\ ik \in \mathbf{IK},\ s \in \mathbf{S},\ t \in \mathbf{T}_k,\ k \in \mathbf{K} \tag{23a}$$

$$\sum_{s\in\mathbf{S}}\sum_{t\in\mathbf{T}_k} FK_{ik,k,s,t} = QK_{ik,k} \qquad \forall\ ik \in \mathbf{IK},\ k \in \mathbf{K} \tag{23b}$$

Length of the Time Slot (s,t). The continuous variable $LT_{s,t}$ represents the length of the time slot s associated with the CDU t. Its value becomes determined by eq 24, where the parameters $(rt_{\min}, rt_{\max})$ stand for the minimum and maximum processing rates of every CDU.

$$rt_{\min} LT_{s,t} \le \sum_{ik\in\mathbf{IK}}\sum_{k\in\mathbf{K}_t} FK_{ik,k,s,t} \le rt_{\max} LT_{s,t}$$
$$\forall\ s \in \mathbf{S},\ t \in \mathbf{T} \tag{24}$$

Sequencing the Time Slots Associated with CDU t. If $s < s'$, then eq 25a states that the slot (s,t) must finish before starting the succeeding slot (s',t). Moreover, the last elements of the set $\mathbf{S}$ are reserved for fictitious time slots by eq 25b.

$$CT_{s,t} \le ST_{s',t} \qquad \forall\ s,\ s' \in \mathbf{S}(s < s'),\ t \in \mathbf{T} \tag{25a}$$

$$YS_{s,t} \le YS_{(s-1),t} \qquad \forall\ (s-1),\ s \in \mathbf{S},\ t \in \mathbf{T} \tag{25b}$$

Starting and End Times of the Slot (s,t). Equations 26a–26d are incorporated to guarantee that the unloading of crude lots assigned to CDU t during the slot s begins/finishes at the starting/end times of the time slot (s,t).

$$ST_{s,t} \le SK_{ik} + H\Big(1 - \sum_{k\in\mathbf{K}_t} YK_{ik,k,s,t}\Big)$$
$$\forall\ ik \in IK,\ s \in \mathbf{S},\ t \in \mathbf{T} \tag{26a}$$

$$CT_{s,t} \ge CK_{ik} - H\Big(1 - \sum_{k\in\mathbf{K}_t} YK_{ik,k,s,t}\Big)$$
$$\forall\ ik \in \mathbf{IK},\ s \in \mathbf{S},\ t \in \mathbf{T} \tag{26b}$$

$$LT_{s,t} = CT_{s,t} - ST_{s,t} \qquad \forall\ s \in \mathbf{S},\ t \in \mathbf{T} \tag{26c}$$

$$LK_{ik} \ge LT_{s,t} - H\Big(1 - \sum_{k\in\mathbf{K}_t} YK_{ik,k,s,t}\Big)$$
$$\forall\ ik \in \mathbf{IK},\ s \in \mathbf{S},\ t \in \mathbf{T} \tag{26d}$$

Crude Distillation Units Should Be Continually Processing Crude Blends. The total length of the time slots of every CDU must be equal to the length of the scheduling horizon. eq 27 guarantees a continuous supply of crude mix to every CDU all over the scheduling horizon.

$$\sum_{s\in\mathbf{S}} LT_{s,t} = H \qquad \forall\ t \in \mathbf{T} \tag{27}$$

Demand of Crude Mix from Each CDU Should Be Satisfied. The parameter dem_t represents the total amount of mixed oils that should be processed in CDU t. Equation 28 guarantees that the crude demand of every CDU will be satisfied within the time horizon.

$$\sum_{ik\in\mathbf{IK}}\sum_{k\in\mathbf{K}_t}\sum_{s\in\mathbf{S}} FK_{ik,k,s,t} = dem_t \qquad \forall\ t \in \mathbf{T} \tag{28}$$

5.3.4. Controlling the Inventory Level of Crude Mix in Storage Tanks. The inventory of crude oil in storage tanks should be monitored over time to prevent from overloading after a loading operation and from running out conditions after discharging a lot of crude blend from a tank.

Cumulative Amount of Crude Oil Discharged into a Storage Tank at the Completion Times of Loading and Unloading Operations. For monitoring the inventory level in storage tanks, the important events are the times at which loading and unloading operations are completed. Overloading and running out conditions can be avoided by properly controlling inventory levels at such events. The continuous variable $LKV_{k,iv}$ represents the cumulative amount of crude oil loaded into the storage tank k at time CV_{iv}. Its value given by eq 29 is obtained by summing up the amounts of crude oil in lots $iv' \le iv$ discharged from VLCCs into the storage tank k.

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$$LKV_{k,iv} = \sum_{\substack{iv' \in \mathbf{IV} \\ iv' \leq iv}} \sum_{p \in \mathbf{P}_k} FV_{iv',p,k} \qquad \forall\ iv \in \mathbf{IV},\ k \in \mathbf{K} \tag{29}$$

In turn, $LKK_{k,ik}$ represents the cumulative amount of crude oil loaded into the storage tank k at time CK_{ik}. Its value will never be (i) smaller than $LKV_{k,iv}$ if the loading of lot iv occurs before the unloading of lot ik from the same storage tank k (i.e., if $XK_{iv,ik,k} = 1$) or (ii) greater than $LKV_{k,iv}$ minus the size of lot iv when $XK_{iv,ik,k} = 0$. The parameter M is a large number.

$$LKV_{k,iv} \leq LKK_{k,ik} + M(1 - XK_{iv,ik,k}) \qquad \forall\ iv \in \mathbf{IV},\ ik \in \mathbf{IK},\ k \in \mathbf{K} \tag{30a}$$

$$LKK_{k,ik} \leq LKV_{k,iv} - \sum_{p \in \mathbf{P}_k} FV_{iv,p,k} + M\ XK_{iv,ik,k} + M(2 - \sum_{p \in \mathbf{P}_k} YV_{iv,p,k} - WK_{ik,k}) \qquad \forall\ iv \in \mathbf{IV},\ ik \in \mathbf{IK},\ k \in \mathbf{K} \tag{30b}$$

Moreover, the value of $LKK_{k,ik}$ must never be greater than the total amount of crude oil transferred from the vessels to the storage tank $k \in \mathbf{K}$ over the time horizon. Constraint 31 is relevant in case the unloading of lot ik is the last operation in the storage tank k.

$$LKK_{k,ik} \leq \sum_{iv \in \mathbf{IV}} \sum_{p \in \mathbf{P}_k} FV_{iv,p,k} \qquad \forall\ ik \in \mathbf{IK},\ k \in \mathbf{K} \tag{31}$$

Cumulative Amount of Crude Mix Unloaded from a Storage Tank at Key Time Events. The continuous variable $UKK_{k,ik}$ represents the cumulative amount of crude mix unloaded from the storage tank k at time CK_{ik}. If lot ik has been discharged from tank k, then every lot ik' (with $ik' \leq ik$) coming from the storage tank k has already been unloaded at time CK_{ik}. Then,

$$UKK_{k,ik} = \sum_{\substack{ik' \in \mathbf{IK} \\ ik' \leq ik}} QK_{ik',k} \qquad \forall\ ik \in \mathbf{IK},\ k \in \mathbf{K} \tag{32}$$

In turn, $UKV_{k,iv}$ stands for the cumulative amount of crude blend unloaded from the storage tank k at time CV_{iv}. Its value given by eqs 33a–33b will never be (i) greater than $UKK_{k,ik}$ minus the size of lot ik if the loading of lot iv is completed before unloading ik (i.e., when $XK_{iv,ik,k} = 1$) or (ii) smaller than $UKK_{k,ik}$ if $XK_{iv,ik,k} = 0$.

$$UKV_{k,iv} \leq UKK_{k,ik} - QK_{ik,k} + M(1 - XK_{iv,ik,k}) \qquad \forall\ iv \in \mathbf{IV},\ ik \in \mathbf{IK},\ k \in \mathbf{K} \tag{33a}$$

$$UKK_{k,ik} \leq UKV_{k,iv} + M\ XK_{iv,ik,k} + M(2 - \sum_{p \in \mathbf{P}_k} YV_{iv,p,k} - WK_{ik,k}) \qquad \forall\ iv \in \mathbf{IV},\ ik \in \mathbf{IK},\ k \in \mathbf{K} \tag{33b}$$

Moreover, the value of $UKV_{k,iv}$ can never be greater than the total amount of crude mix transferred from the storage tank k to CDUs over the time horizon.

$$UKV_{k,iv} \leq \sum_{ik \in \mathbf{IK}} QK_{ik,k} \qquad \forall\ iv \in \mathbf{IV},\ k \in \mathbf{K} \tag{34}$$

Inventories of Crude Mix in the Storage Tanks at the Completion Times of Loading and Unloading Operations. The continuous variables $IKV_{k,iv}$ and $IKK_{k,ik}$ denote the inventory level in the storage tank k at the times CV_{iv} and CK_{ik}, respectively. The value $IKV_{k,iv}$ must never exceed the capacity of the storage tank k given by the parameter $capk_k$. In turn, $IKK_{k,ik}$ can never be lower than the minimum allowed inventory of crude mix in the storage tank k ($invk_{k,\min}$). In eqs 35a–35b, the parameter iik_k^0 represents the initial inventory of crude mix in tank k.

$$IKV_{k,iv} = iik_k^0 + LKV_{k,iv} - UKV_{k,iv} \leq capk_k \qquad \forall\ iv \in \mathbf{IV},\ k \in \mathbf{K} \tag{35a}$$

$$IKK_{k,ik} = iik_k^0 + LKK_{k,ik} - UKK_{k,ik} \geq invk_{k,\min} \qquad \forall\ ik \in \mathbf{IK},\ k \in \mathbf{K} \tag{35b}$$

5.3.5. Monitoring the Concentration of the Key Component in Storage Tanks. To determine the concentration of the key component in the storage tanks during unloading operations, it is necessary to first establish the inventories of the key components available at the key time events. To this purpose, it will be made a mathematical treatment similar to the one applied to track the total inventory of crude mix in storage tanks along the scheduling horizon.

Amount of Key Component Transferred to the Storage Tank k through the Lot iv. The continuous variable $FVJ_{iv,k}$ represents the amount of key component in the lot iv transferred from a VLCC to the storage tank k. In eq 36, the parameter cjp_p stands for the concentration of the key component in the crude parcel p whose value depends on the type of crude oil contained in parcel p.

$$FVJ_{iv,k} = \sum_{p \in \mathbf{P}_k} cjp_p FV_{iv,p,k} \qquad \forall\ iv \in \mathbf{IV},\ k \in \mathbf{K} \tag{36}$$

Cumulative Amount of Key Component Already Discharged into a Storage Tank at the Completion Times of Loading and Unloading Operations. The continuous variables $LKKJ_{k,ik}$ and $LKVJ_{k,iv}$ represent the cumulative amounts of key component loaded into the storage tank k up to the times CK_{ik} and CV_{iv}, respectively. Their values are given by eqs 37–39. In eqs 38a–38b, the parameter MJ_k is a relatively large number. It is recommended to choose: $MJ_k = M\,cjku_k$, where $cjku_k$ is an upper bound on the concentration of the key component in any storage tank k. A criterion for selecting the value of $cjku_k$ is given in section 6.

$$LKVJ_{k,iv} = \sum_{\substack{iv' \in IV \\ iv' \leq iv}} FVJ_{iv',k} \qquad \forall\ iv \in \mathbf{IV},\ k \in \mathbf{K} \tag{37}$$

$$LKVJ_{k,iv} \leq LKKJ_{k,ik} + MJ_k(1 - XK_{iv,ik,k}) \qquad \forall\ iv \in \mathbf{IV},\ ik \in \mathbf{IK},\ k \in \mathbf{K} \tag{38a}$$

$$LKKJ_{k,ik} \leq LKVJ_{k,iv} - FVJ_{iv,k} + MJ_k XK_{iv,ik,k} + MJ_k(2 - \sum_{p \in \mathbf{P}_k} YV_{iv,p,k} - WK_{ik,k}) \qquad \forall\ iv \in \mathbf{IV},\ ik \in \mathbf{IK},\ k \in \mathbf{K} \tag{38b}$$

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$$LKKJ_{k,ik} \le \sum_{iv \in \mathbf{IV}} FVJ_{iv,k} \qquad \forall\ ik \in \mathbf{IK},\ k \in \mathbf{K} \tag{39}$$

Amount of Key Component in the Lot ik Unloaded from the Storage Tank k. The continuous variable $QKJ_{ik,k}$ denotes the amount of key component contained in the lot ik that is transferred from the storage tank k to some CDU. In turn, $CJK_{k,ik}$ represents the concentration of the key component in the storage tank k during the discharge of lot ik. Both variables are related by the nonlinear eq 40.

$$QKJ_{ik,k} = CJK_{k,ik} QK_{ik,k} \qquad \forall\ ik \in \mathbf{IK},\ k \in \mathbf{K} \tag{40}$$

Cumulative Amount of Key Component Unloaded from the Storage Tank k up to the Times CK_{ik} and CV_{iv}. The continuous variables $UKKJ_{k,ik}$ and $UKVJ_{k,iv}$ represent the cumulative amounts of key component unloaded from the storage tank k up to the times CK_{ik} and CV_{iv}, respectively. Their values are given by eqs 41–43.

$$UKKJ_{k,ik} = \sum_{\substack{ik' \in \mathbf{IK} \\ ik' \le ik}} QKJ_{ik',k} \qquad \forall\ ik \in \mathbf{IK},\ k \in \mathbf{K} \tag{41}$$

$$UKVJ_{k,iv} \le UKKJ_{k,ik} - QKJ_{ik,k} + MJ_k(1 - XK_{iv,ik,k}) \qquad \forall\ iv \in \mathbf{IV},\ ik \in \mathbf{IK},\ k \in \mathbf{K} \tag{42a}$$

$$UKKJ_{k,ik} \le UKVJ_{k,iv} + MJ_k XK_{iv,ik,k} + MJ_k\Big(2 - \sum_{p \in \mathbf{P_k}} YV_{iv,p,k} - WK_{ik,k}\Big) \qquad \forall\ iv \in \mathbf{IV},\ ik \in \mathbf{IK},\ k \in \mathbf{K} \tag{42b}$$

$$UKVJ_{k,iv} \le \sum_{ik \in \mathbf{IK}} QKJ_{ik,k} \qquad \forall\ iv \in \mathbf{IV},\ k \in \mathbf{K} \tag{43}$$

Inventory of the Key Component in Storage Tanks at the Completion of Loading and Unloading Operations. The continuous variables $IKVJ_{k,iv}$ and $IKKJ_{k,ik}$ denote the inventories of the key component in the storage tank k at the times CV_{iv} and CK_{ik}, respectively. Their values are given by eqs 44a–44b, respectively. The concentration of the key component in the storage tank k during unloading operations is defined by eq 45. The term cjk_k^0 is the initial concentration of the key component in the storage tank k.

$$IKVJ_{k,iv} = ck_k^0 iik_k^0 + LKVJ_{k,iv} - UKVJ_{k,iv} \qquad \forall\ iv \in \mathbf{IV},\ k \in \mathbf{K} \tag{44a}$$

$$IKKJ_{k,ik} = ck_k^0 iik_k^0 + LKKJ_{k,ik} - UKKJ_{k,ik} \qquad \forall\ ik \in \mathbf{IK},\ k \in \mathbf{K} \tag{44b}$$

$$IKKJ_{k,ik} = CJK_{k,ik} IKK_{k,ik} \qquad \forall\ ik \in \mathbf{IK},\ k \in \mathbf{K} \tag{45}$$

5.3.6. Concentration of the Key Component in the Feedstock for the CDUs. Amount of Key Component Transferred from Storage Tank k to the CDU t during the Time Slot (s,t). The continuous variable $FKJ_{ik,k,s,t}$ represents the amount of key component in lot ik delivered from tank k to the CDU t during the time slot (s,t). From eqs 46a–46b, the value of $FKJ_{ik,k,s,t}$ will be greater than zero only if lot ik has been assigned to the slot (s,t) (i.e., if $YK_{ik,k,s,t} = 1$). If so, $FKJ_{ik,k,s,t}$ is equal to $QKJ_{ik,k}$. Otherwise, $FKJ_{ik,k,s,t} = 0$.

$$FKJ_{ik,k,s,t} \le MJ_k YK_{ik,k,s,t} \qquad \forall\ ik \in \mathbf{IK},\ k \in \mathbf{K}_t,\ s \in \mathbf{S},\ t \in \mathbf{T} \tag{46a}$$

$$\sum_{s \in \mathbf{S}} \sum_{t \in \mathbf{T}_k} FKJ_{ik,k,s,t} = QKJ_{ik,k} \qquad \forall\ ik \in \mathbf{IK},\ k \in \mathbf{K} \tag{46b}$$

Concentration of the Key Component in the Feedstock for the CDUs. The feedstock for CDU t during the time slot (s,t) can result from the mixing of at most two lots of crude oils supplied by two different storage tanks. The composition of the feed stream supplied to CDU t during the time slot s ($CJT_{s,t}$) is given by eqs 47a–47b. It should belong to the acceptable range $[cjtl_t, cjtu_t]$ specified at the planning level.

$$FKJ_{ik,k,s,t} = CJK_{k,ik} FK_{ik,k,s,t} \qquad \forall\ ik \in \mathbf{IK},\ k \in \mathbf{K}_t,\ s \in \mathbf{S},\ t \in \mathbf{T} \tag{47a}$$

$$\sum_{ik \in \mathbf{IK}} \sum_{k \in \mathbf{K}_t} FKJ_{ik,k,s,t} = CJT_{s,t}\Big(\sum_{ik \in \mathbf{IK}} \sum_{k \in \mathbf{K}_t} FK_{ik,k,s,t}\Big) \qquad \forall\ s \in \mathbf{S},\ t \in \mathbf{T} \tag{47b}$$

$$cjtl_k \le CJT_{s,t} \le cjtu_k \qquad \forall\ k \in \mathbf{K} \tag{48}$$

5.4. Objective Function. The problem target given by eq 49 is the maximization of the gross profit obtained by processing different crudes in the CDUs. At any time, every storage tank contains a mix of different crude oils but their relative proportions change as loading and unloading operations take place. As a result, the unit netback of the crude mix available in a storage tank is a time-dependent continuous variable. Consequently, nonlinear terms also arise in the objective function. The continuous variable $UNK_{k,ik}$ represents the netback per barrel of crude mix coming from the storage tank k at time CK_{ik}. To track its value along the planning horizon, nonlinear relationships similar to those proposed for determining the key component concentration in storage tanks are used. Such equations are given as Supporting Information. In turn, the operating costs comprise the changeover loss and the demurrage cost. The parameter cch stands for the cost of any changeover, while cwv represents the demurrage cost per hour. The expression between parentheses in the second term of eq 49 gives the number of changeovers, while the last term provides the penalty for late vessel departures.

$$\min z = \sum_{ik \in \mathbf{IK}} \sum_{k \in \mathbf{K}} \sum_{s \in \mathbf{S}} \sum_{t \in \mathbf{T}_k} UNK_{k,ik} FK_{ik,k,s,t} - cch\Big(\sum_{s \in \mathbf{S}} \sum_{t \in \mathbf{T}} YS_{s,t} - \mathrm{card}(t)\Big) - cwv \sum_{v \in \mathbf{V}} TD_v\Big) \tag{49}$$

It should be noted that nonlinear terms in the proposed MINLP formulation just appear in the constraints 40, 45, 47a–47b, and in the first term of the objective function, 49.

6. APPROXIMATE MILP AND NLP MODELS

The proposed solution procedure to find a very good solution of the MINLP model consists on sequentially solving just a pair of mathematical programming formulations: (a) an MILP continuous-time model that is a tight approximation of the MINLP rigorous formulation and (b) an NLP model that

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includes the nonlinear terms arising in the rigorous expressions of the key component mass balances and the objective function.

6.1. Approximate MILP formulation. To avoid nonlinear terms in the approximate MILP model, the mass balance constraints for determining the key component concentration in storage tanks and CDU feed streams are replaced by linear bounding constraints. Besides, the unit margin of the crude mix coming from a storage tank is assumed to keep unchanged over the time horizon despite the loading/unloading operations. Then, it remains equal to un_k^0. It is usually a suitable approximation because the unit netback slightly varies with the type of crude. In this way, good initial values for the key component concentration in the storage tanks and also in the feedstock for the CDUs can easily be determined. It is usually a very good feasible solution for the NLP model to be subsequently solved. As a result, starting from that point the best NLP solution is found in less than one second of CPU time and the computational cost is mostly allocated to the MILP. Therefore, a tight MILP model is derived from the MINLP by replacing the nonlinear constraints 40, 45, and 47a–47b by tailor-made bounding constraints. Besides that, it is considered a set of valid cuts and $UNK_{k,ik} = un_k^0$ is assumed. Equation 40 is replaced by the linear constraints 50a–50b.

$$QKJ_{ik,k} \le cjku_k QK_{ik,k} \qquad \forall\ ik \in \mathbf{IK},\ k \in \mathbf{K} \tag{50a}$$

$$QKJ_{ik,k} \ge cjkl_k QK_{ik,k} \qquad \forall\ ik \in \mathbf{IK},\ k \in \mathbf{K} \tag{50b}$$

The parameters ($cjkl_k$,$cjku_k$) stand for the minimum/maximum allowed concentrations of the key component in the storage tank k. Such bounds can be defined as a means to guarantee that the feed streams supplied to the CDUs at later time slots can satisfy the feasible composition range specified at the planning level. Criteria for choosing the values of ($cjkl_k$,$cjku_k$) are given by eqs 51a–51b.

$$cjkl_k = \min(\min_{cr\in CR_k} cjcr_{cr},\ \min_{t\in \mathbf{T}_k} cjtl_t) \qquad k \in \mathbf{K} \tag{51a}$$

$$cjku_k = \mu \max(\max_{cr\in CR_k} cjcr_{cr},\ \max_{t\in \mathbf{T}_k} cjtu_t) \qquad k \in \mathbf{K} \tag{51b}$$

The parameters $cjtl_t$ and $cjtu_t$ represent the minimum and maximum allowed concentration of the key component in the feedstock for CDU$_t$, $cjcr_{cr}$ is the key component concentration in the crude cr, and $\mu < 1$ is a parameter selected by the user. Lower values for μ are recommended when ($\max_{cr\in CR_k} cjcr_{cr}$) is substantially larger than ($\max_{t\in \mathbf{T}_k} cjtu_t$). Besides, the nonlinear constraint 45 is substituted by the linear eqs 52a–52b that intend to keep the concentration of the key component in the storage tank k within the acceptable interval ($cjkl_k$,$cjku_k$). In eq 52a, the set CR_k comprises the crude types that can be stored in tank k.

$$IKKJ_{k,ik} \le cjku_k IKK_{k,ik} \qquad \forall\ ik \in \mathbf{IK},\ k \in \mathbf{K} \tag{52a}$$

$$IKKJ_{k,ik} \ge cjkl_k IKK_{k,ik} \qquad \forall\ ik \in \mathbf{IK},\ k \in \mathbf{K} \tag{52b}$$

Equations 47a and 47b are replaced by the linear constraints 53a–53b and 54a–54b, respectively.

$$FKJ_{ik,k,s,t} \le cjku_k FK_{ik,k,s,t} \qquad \forall\ ik \in \mathbf{IK},\ k \in \mathbf{K}_t,\ s \in \mathbf{S},\ t \in \mathbf{T} \tag{53a}$$

$$FKJ_{ik,k,s,t} \ge cjkl_k FK_{ik,k,s,t} \qquad \forall\ ik \in \mathbf{IK},\ k \in \mathbf{K}_t,\ s \in \mathbf{S},\ t \in \mathbf{T} \tag{53b}$$

$$\sum_{ik\in\mathbf{IK}} \sum_{k\in\mathbf{K}_t} FKJ_{ik,k,s,t} \le cjtu_t \sum_{ik\in\mathbf{IK}} \sum_{k\in\mathbf{K}_t} FK_{ik,k,s,t} \qquad \forall\ s \in \mathbf{S},\ t \in \mathbf{T} \tag{54a}$$

$$\sum_{ik\in\mathbf{IK}} \sum_{k\in\mathbf{K}_t} FKJ_{ik,k,s,t} \ge cjtl_t \sum_{ik\in\mathbf{IK}} \sum_{k\in\mathbf{K}_t} FK_{ik,k,s,t} \qquad \forall\ s \in \mathbf{S},\ t \in \mathbf{T} \tag{54b}$$

To make linear the objective function 49, the netback per barrel of crude mix in the lot ik coming from the storage tank k ($UNK_{k,ik}$) is assumed to remain constant with time and equal to the unit netback for the crude blend initially available in tank k (un_k^0) given by eq 55. The parameter $uncr_{cr}$ is the netback obtained by processing a barrel of crude cr, and $iicr_{cr,k}$ is the initial amount of crude cr in the storage tank k.

$$un_k^0 = \sum_{cr\in CR} uncr_{cr} iicr_{cr,k} \qquad \forall\ k \in \mathbf{K} \tag{55}$$

6.2. Valid Cuts. Often, the linear approximation of the bilinear terms leads to a mismatch between the composition of the crude mix delivered by a pair of storage tanks to a CDU and that actually received by the CDU according to the MILP solution. Such a mismatch was called composition discrepancy by Li et al.[17] To mitigate that discrepancy, some valid cuts are incorporated in the MILP model. They are based on the following facts: (1) the composition of the crude mix supplied from tank k to a distillation column t during the first time slot (s = 1) is equal to the initial concentration cjk_k^0; (2) a storage tank whose initial composition cjk_k^0 does not belong to the feasible range for a CDU t (i.e., $cjk_k^0 > cjtu_t$) cannot initially feed that distillation unit alone but together with some other tank k' featuring $cjk_{k'}^0 < cjtu_t$; (3) A lot of crude mix delivered by the storage tank k still features the initial composition cjk_k^0 if the tank k has not previously received any crude oil from some vessels, i.e. if $XK_{iv,ik,k} = 0$ for any lot iv. If in addition $cjk_k^0 > cjtu_t$, the tank k cannot supply the lot ik to the CDU t alone during any time slot s. The mathematical expressions for these valid cuts are given as Supporting Information.

6.3. NLP Model. After solving the approximate MILP model presented in section 6.1, an NLP model is obtained from the original MINLP formulation by fixing the binary variables $\{WP_{iv,p}, YV_{iv,p,k}, WK_{ik,k}, YK_{ik,k,s,t}, YS_{s,t}, XK_{iv,ik,k}\}$ to their optimal MILP values. Good initial values for the variables $CJK_{k,ik}$ and $CJT_{s,t}$ can be derived from the MILP-solution through an exact calculation of the key component concentrations in storage tanks during unloading operations. Note that the concentrations $CJK_{k,ik}$ and $CJT_{s,t}$ do not explicitly appear in the MILP model. Moreover, the set of equations defining the exact value of the netback $UNK_{k,ik}$ are given as Supporting Information.

7. MINLP SOLUTION PROCEDURE

7.1. Basic Solution Scheme. The basic two-step solution scheme for the MINLP problem formulation consists of first solving an approximate MILP, subsequently fixing the binary variables at their current values and finally solving the resulting NLP model. This procedure usually provides very good MINLP feasible solutions at low computational cost for examples involving the unloading of 1 or 2 VLCCs with

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three crude parcels each and a horizon length of 3–5 days. In such cases, most of the storage tanks feeding the CDUs feature a known and constant composition equal to the initial one. In other words, the storage tanks supplying crude blends to the CDUs do not receive crude lots from the tankers within the scheduling horizon.

To improve the computational performance of this basic solution scheme, some simple rules for choosing the cardinalities of the sets **IV** and **IK** are given. First, the conflicting crude parcels featuring key component concentrations above the maximum value allowed for the assigned CDUs should first be identified. For such poor quality crude parcels, 2 or 3 generic lots $iv \in \mathbf{IV}$ are postulated to eventually distribute them among multiple storage tanks. In this way, the quality of the crude blends available in the storage tanks can be fairly preserved. In contrast, a single lot is proposed for each other parcel, unless they are too large for the initial idle capacity of the receiving tanks. As the ordering followed to unload the crude parcels from a tanker is known beforehand, the postulated lots can also be preassigned to the crude parcels. In other words, the values of the variables $WP_{iv,p}$ can be prefixed to reduce the MILP-model size. However, the assignment of crude lots to storage tanks and the lot sizes are still model decisions.

In turn, the value of $|\mathbf{IK}|$ depends on the postulated number of time slots $|\mathbf{S}|$. The longer the scheduling horizon and closer the processing limits for the CDU feedstock, the higher the value of $|\mathbf{S}|$. For shorter horizons and loose processing limits, it is initially selected: $|\mathbf{S}| = 2$. Otherwise, the starting choice is $|\mathbf{S}| = 3$. Additional time slots are considered if either the MILP solver fails to find a feasible solution within the CPU time limit or the MILP is infeasible. After choosing the value of $|\mathbf{S}|$, the following criterion is applied to select the cardinality of the set **IK**: $|\mathbf{IK}| = (|\mathbf{T}|*|\mathbf{S}|) + [(|\mathbf{V}| - 1)/(|\mathbf{S}| - 1)]*(|\mathbf{P}|/|\mathbf{V}|)$, where $|\mathbf{T}|$ is the number of CDUs and $|\mathbf{P}|$ is the total number of crude parcels to be unloaded. In case $|\mathbf{T}| = 3$, $|\mathbf{S}| = 2$, $|\mathbf{P}| = 4$, and $|\mathbf{V}| = 1$, then $|\mathbf{IK}| = 6$. If the NLP solver fails to find a feasible solution from the starting point provided by the approximate MILP, $|\mathbf{IK}|$ should be further increased without modifying the cardinality of **S**.

7.2. Advanced Solution Scheme. When the scheduling horizon has a length of 2 or 3 weeks, a higher number of time slots should be used. As a result, the size of the approximate MILP representation substantially grows and no feasible solution for the MILP is usually discovered within the CPU time limit. Then, it is necessary to change the way of seeking a good feasible solution for the MINLP formulation. The new MINLP solution scheme relies on some interesting features of the crude scheduling problem: (1) VLCCs can dock one at a time at the SBM station; (2) the time between the arrivals of two consecutive VLCCs is rather significant because of the limited capacity of the storage tanks; (3) the supply of crude blends to the CDUs can never be interrupted; (4) until the arrival of the next VLCC, the CDUs must be fed with the crude parcels coming from the vessel last docked plus the stock of crude blends available in the tanks just before unloading that vessel; and (5) the unloading rate of a VLCC is much higher than both the tank–CDU transfer rate and the CDU processing rate. From those problem features, one can view the scheduling horizon as composed of a number of time segments equal to the number of arriving tankers $|\mathbf{V}|$. Such time segments defined by the vessel arrival times were called vessel-blocks by Li et al.[17] However, the definition of the vessel-blocks in this paper is slightly different.

The first vessel block goes from time $t = 0$ to the arrival time of the second scheduled VLCC, the intermediate blocks run between consecutive arrival times, and the last block goes from the last arrival time to the horizon end. The total amount of crude blends to be processed in each CDU over the entire horizon can then be divided into $|\mathbf{V}|$ portions, with each one allocated to a single vessel-block. Then, the partial demand of a CDU during the first vessel-block is satisfied using the initial stocks and the crude parcels unloaded from the first arriving VLCC. By applying the same reasoning, the partial demands of the CDUs during the first n vessel-blocks are fulfilled by allocating the initial stocks and the crude parcels carried by the first n VLCCs, with n varying from 1 to $|\mathbf{V}|$. Besides, the partial demand of a CDU during a vessel-block should be proportional to its length (L_v) but never lower than the value obtained from multiplying the minimum processing rate of CDUs ($rt_{\min}$) by the length of the vessel block. For instance, if $H = 100$ h, $L_v = 40$ h, $dem = 200$ kbbl, and $rt_{\min} = 3.0$ (kbbl/h), then the partial demand of a CDU during the vessel-block v is given by $\max[200*(40/100), 3.0*40] = 120$. An important conclusion is that tank–CDU assignment decisions during the vessel-block n have usually a minor influence on similar decisions for vessel-blocks 1, 2, ..., $(n - 1)$. Based on this conclusion, a new solution scheme is proposed to solve the MILNP formulation that includes the following steps:

(1) Divide the time horizon into $|\mathbf{V}|$ vessel-blocks defined by the scheduled arrival times of the vessels.

(2) Assign a partial CDU demand of crude mix to each vessel block ($pdem_{t,v}$) using the formula: $pdem_{t,v} = \max[dem_t*(L_v/H), rt_{\min}*L_v]$. To guarantee the minimum feed flowrate for CDUs in every vessel-block, an iterative calculation of $pdem_{t,v}$ is usually needed.

(3) Solve the crude scheduling problem using the MILP-NLP strategy for the first vessel-block ($n = 1$) by just considering the unloading of the first arriving vessel and the demands of the CDUs assigned to the first vessel-block.

(4) Solve the crude scheduling problem using the MILP-NLP strategy for the first n vessel-blocks accounting for the unloading of the first n arriving tankers, the initial crude stocks and the total demand of the CDUs during the first n vessel-blocks. Before solving the problem, prefix tank-CDU assignment variables to their values found for the first $(n - 1)$ vessel-blocks.

(5) Repeat the procedure until considering the $|\mathbf{V}|$ vessel-blocks to find a good feasible solution for the original MINLP crude scheduling formulation.

8. RESULTS AND DISCUSSION

The proposed MINLP crude scheduling problem formulation has been applied to a series of 11 examples previously studied by other authors.[3,17] All of them involve eight different crude types (cr_1–cr_8), eight storage tanks (T_1–T_8), and three crude distillation units (CDU_1–CDU_3). Crude oils are grouped into two classes accounting for their quality, and each class has been assigned to a different subset of storage tanks and CDUs. Crude oils cr_1–cr_4 can be stored in tanks (T_1, T_6–T_8) and processed in CDU_3. In turn, crude oils cr_5–cr_8 are stored in tanks T_2–T_5 and processed in distillation units CDU_1–CDU_2. Parcels of crude oils are carried by one to three VLCCs docking at a single SBM

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terminal connected to storage facilities through just one SBM pipeline. In any case, the VLCCs carry three parcels of crude oils. A fourth parcel called the SBM parcel to be first unloaded accounts for the SBM pipeline holdup. Horizon lengths range from 72 to 336 h and CDU crude demands go from 300 to 1000 kbbl. Tables containing all the data for the 11 examples are provided as Supporting Information. Expected departure times of the VLCCs result from allowing a maximum unloading time of 20−35 h depending on the example. Moreover, a settling time of 8 h is adopted in all the examples. Using the advanced solution procedure, examples 2−10 involving two or more VLCCs were also solved by considering the first n arriving tankers and a time horizon comprising the first n vessel-blocks, with n = 1, ..., (|V| − 1). Partial demands to be satisfied during the first n vessel-blocks and the related horizon lengths are given as Supporting Information. All the examples have been solved using GAMS/CPLEX 24.2 for MILPs and GAMS/CONOPT for NLPs on an Intel(R) Core i7 3632QM 2.20 GHz one-processor PC with 12 GB RAM and four cores. The relative gap tolerance has been fixed at 10^{-2} and a maximum CPU time of 3600 s was allowed. The best solutions found and the model sizes for examples 1−11 are all provided as Supporting Information. In addition, some of the results are also included in this section.

8.1. Solving Examples 1−10 by Considering the First Arriving VLCC. Ten examples involving the unloading of a single VLCC and a scheduling horizon comprising only the first vessel-block have all been solved. Example 1 was originally defined as a one-VLCC case study by Reddy et al.,[3] while the other ones were converted into a single-VLCC problem by just considering the first arriving VLCC. Partial demands during the first vessel-block and the length of the scheduling horizon are given as Supporting Information. Two versions of example 1, called examples 1A and 1B, have been considered. Example 1A is the one proposed by Reddy et al.,[3] while example 1B features larger CDU demands and a longer time horizon. Example 1 involves the unloading of a large parcel p_2 containing crude cr_4 that can exceed the free capacity available in any receiving tank. Then, two generic lots are preassigned to parcel p_2 to allow its distribution between a pair of storage tanks, and only one generic lot for the remaining parcels. As a result, it is adopted: |**IV**| = 5, |**S**| = 2, |**IK**| = 6, and edt_v = 40 h.

The best solution for example 1A illustrated in Figure 3 has been found in 196.8 s of CPU time and features a net profit of $1,768,260 (see Table 1). Loading and unloading operations in storage tanks and the time profiles for crude inventories, unit margins of crude blends, key component concentrations in storage tanks, and CDU processing rates are all shown in Figure 3. More than 99% of the CPU time was devoted to solve the MILP approximate representation. Instead, the NLP model only requires less than one second. This distribution of the CPU time is repeated in all the examples. Demands of the distillation units CDU_1 and CDU_2 are satisfied by allocating crude blends from storage tanks T_2 and T_4 over the entire time horizon. Then, tanks T_2 and T_4 cannot receive crude oil from VLCC-1 and their contents have the starting, known compositions. Crude mix from T_4 has initially the highest unit margin ($1,557.90 per kbbl) but its key component concentration (0.0125) exceeds the maximum allowed concentration for the feedstock of CDU_2 (0.0120). Then, it is mixed with a high-quality crude blend coming from T_2 featuring a lower concentration equal to 0.01087. As the initial stocks in

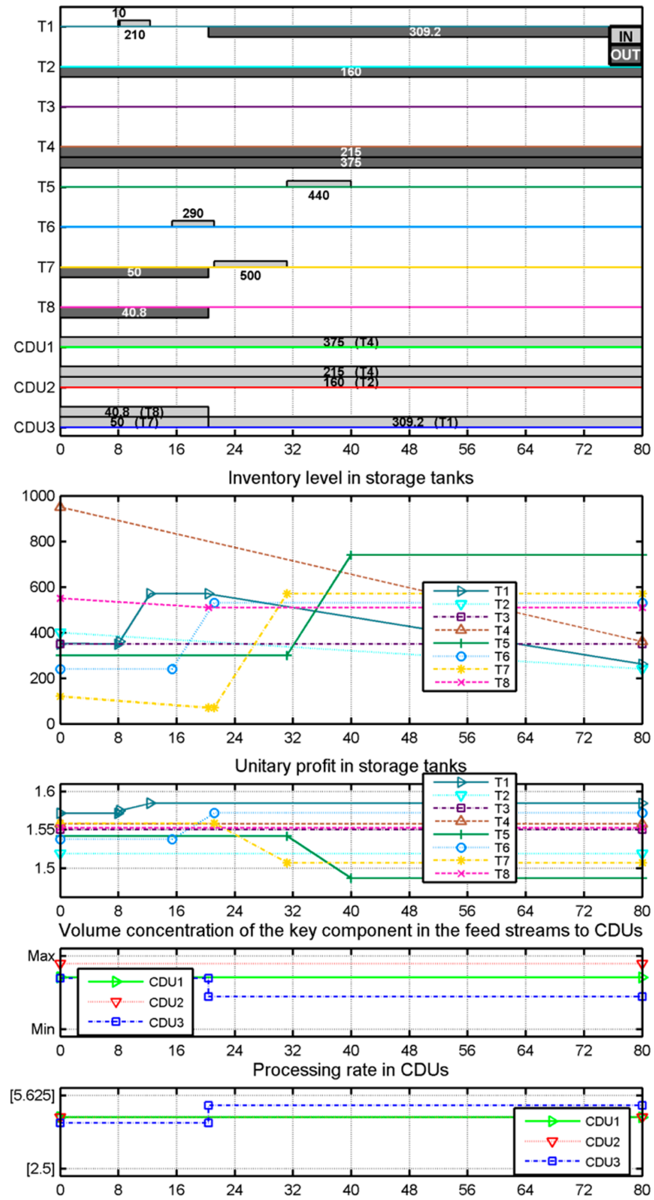

Figure 3. Best solution found for example 1A.

tanks T_4 and T_5 are enough to meet the demands of CDU_1 and CDU_2, no changeover is necessary.

On the other hand, tank T_1 contains the crude blend with the highest unit margin ($1,584.20 per kbbl) after receiving parcel p_1 and a portion of parcel p_2 from VLCC-1. That is why most of the demand of CDU_3 is satisfied by allocating crude blend from tank T_1 (309.2 kbbls) over the second slot going from time = 20.4 h to the horizon end (80 h). During the first time interval (0.0−20.4 h), CDU_3 is concurrently fed by tanks T_7 and T_8 that initially contain crude blends with high unit margins. Then, there is a single changeover in CDU_3. The best solution makes use of all elements of the set **IK**. The crude operational schedule for example 1A provided by the continuous version of the RA algorithm proposed by Reddy et al.[3,17] yields almost the same net profit (see Table 2). Pan et al.[21] found a slightly better schedule with a net profit amounting to $1,771,700.

A larger version of example 1, called example 1B, was generated by increasing the demand of each CDU from (375, 375, 400) to a common value of 450 kbbl and the length of the

Table 1. Computational Results for Examples 1−9 when Considering the First VLCC

			MILP Formulation		NLP Formulation	
no. (one-VLCC)	CDU demand (kbbl)	horizon length (h)	profit ($)	CPU time (s)	profit ($)	CPU time (s)
1A	375, 375, 400	80	1,764,340	196.8	1,768,260	0.03
1B	450. 450, 450	120	2,020,120	298.4	2,013,110	0.01
Examples 2−8 Only Considering the First Arriving Vessel VLCC-1						
2A	400, 400, 400	100	1,864,350	12.8	1,865,290	0.09
2B[a]	300, 300, 300	72	1,408,070	0.2	1,408,070	0.05
3A	400, 400, 400	104	1,855,350	2.3	1,851,120	0.02
4	360, 360, 360	120	1,689,680	0.8	1,689,680	0.09
5	360, 360, 360	120	1,689,680	0.6	1,689,680	0.08
6	300, 300, 300	120	1,406,670	0.5	1,406,670	0.08
7	380, 380, 340	120	1,720,440	2.7	1,720,440	0.09
8	500, 500, 500	160	2,300,730	33.3	2,290,300	0.02
9	400, 400, 400	144	1,890,070	13.2	1,878,850	0.03
10	400, 400, 400	120	1,867,400	27.4	1,867,760	0.09

[a]Using the CDU upper concentration limits of Reddy et al.[3] given by (0.0140, 0.0135, 0.0040).

time horizon from 80 to 120 h. By initially adopting: |**IV**| = 5, |**S**| = 2, and |**IK**| = 7, example 1B was successfully solved in 298.4 s and the net profit grows to $2,013,110 (see Table 1). In this case, the cardinality of **IK** is increased to 7 to get a MINLP feasible solution. A figure displaying the best solution for example 1B is provided as Supporting Information. None of the previous works deal with example 1B.

Example 2 also introduced by Reddy et al.[3] was solved by just considering the first arriving VLCC-1. This tanker carries a conflicting parcel p_3 featuring a key component concentration above the maximum value allowed by CDU_3. Then, two lots are made available to discharge p_3 and only one for the other parcels. Two versions of example 2 with a single tanker VLCC-1 were solved. Example 2A involves an horizon length given by the arrival time of VLCC-2 (i.e., 100 h) and a partial demand of 400 kbbl for each CDU. Moreover, the upper concentration limits for the CDU feedstock are more restricted (0.0130, 0.0125, 0.035). To solve this example, it was initially adopted: |**IV**| = 5, |**S**| = 2, |**IK**| = 6, and edt_v = 20. However, the starting point provided by the MILP formulation leads to an infeasible solution for the MINLP. Then, it is necessary to increase |**IK**| by one to get the solution shown in Figure 4. It was discovered in 13 s and features a total profit of $1,865,290. Model sizes are given as Supporting Information.

In example 2A, the CDUs mostly receive crude blends from storage tanks T_4 and T_8 during the first time slot. Those tanks initially contain crude blends with the highest unit margins, i.e. T_4 for CDU_1 and CDU_2 and T_8 for CDU_3. But the key component concentration in T_4 (0.0125789) exceeds the maximum allowed concentration for CDU_2 (0.01250). Consequently, it is mixed with crude blend coming from T_5 to obtain a feasible concentration of 0.012456. Tank T_5 is the only source for CDU_2 with an initial concentration lower than 0.0125. Because the initial inventories in tanks T_5 and T_8 are not enough to meet the demands of CDU_2 and CDU_3, respectively, a changeover is required in such distillation units. Instead, no changeover is needed in CDU_1. During the scheduling horizon, processing rates in CDUs fluctuate from 2.0 to 4.35 kbbl/h, i.e. within the feasible range 2.0−6.0.

In turn, example 2B, previously solved by Reddy et al.,[3] considers again the carrier VLCC-1, but with a demand of 300 kbbls for each CDU and a horizon length of 72 h. Moreover, the allowed concentration range for each CDU is less restricted than the one specified in example 2A. In this case, they are [0.001−0.0140] for CDU_1, [0.001−0.0130] for CDU_2, and [0.001−0.0040] for CDU_3. Example 2B was indeed the first case study solved by Reddy et al.[3] involving a single carrier. Our best solution found in 0.25 s is reported in Table 1 and Figure 5. It is rather similar to the solution for example 2A but changeovers are no longer required because of the lower CDU demands. In this case, four lots of crude blends are delivered to the CDUs to meet their demands. As shown in Table 2, the proposed solution features a total profit almost similar to the ones reported by Reddy et al.[3] and Pan et al.[21]

We also solve examples 3−10 taking account just the first arriving VLCC and a horizon length determined by the end of the first vessel-block. Crude demands and horizon lengths are given as Supporting Information. Compared with examples 1 and 2, a tighter feasible range for the CDU processing rate is prescribed. It is reduced from [2.0−6.0] to [2.5−5.0] kbbl/h to make a more restricted crude scheduling problem. As a result, more time slots may be required to get an MILP feasible solution because it is more likely that a single storage tank feeds each CDU during most of the time horizon. Consequently, it occurs an increase of the model size as shown in the Supporting

Table 2. Comparing the Results for 1 and 2 VLCC Examples with Previous Works

	our approach		Reddy et al.[3]		Li et al.[21]	
no.	profit ($)	CPU time (s)	profit ($)	CPU time (s)	profit ($)	CPU time (s)
Examples with a Single Arriving VLCC						
1A	1,768,260	196.9	1,766,400	814.4		
2B[a]	1,408,070	0.21	1,409,300	68.5		
Examples with Two Arriving VLCCs						
2A	3,252,210	21.0	3,252,900	3242.4		
3A	3,470,540	9.6			3,483,650	4012
3B	3,096,310	7.5			3,132,660	22 500
3C	3,345,000	10.7			3,355,000	445

[a]Proposed by Reddy et al.[3] with CDU upper concentration limits: (0.0140, 0.0130, 0.0040).

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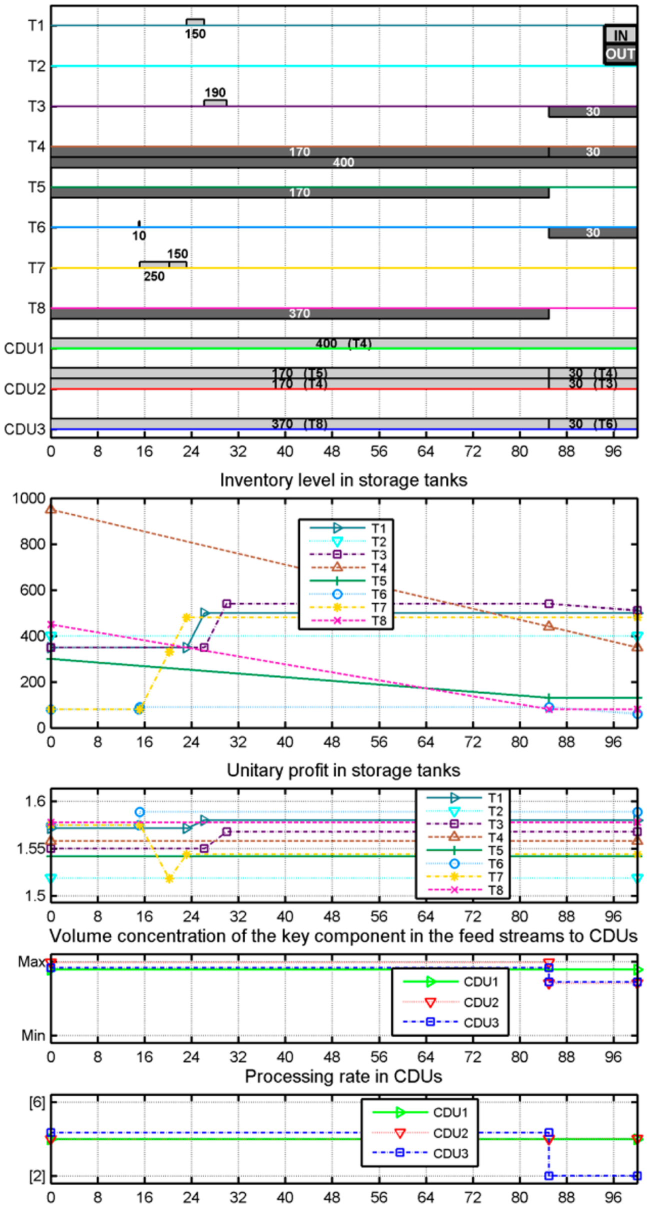

Figure 4. Best solution found for example 2A involving a single tanker.

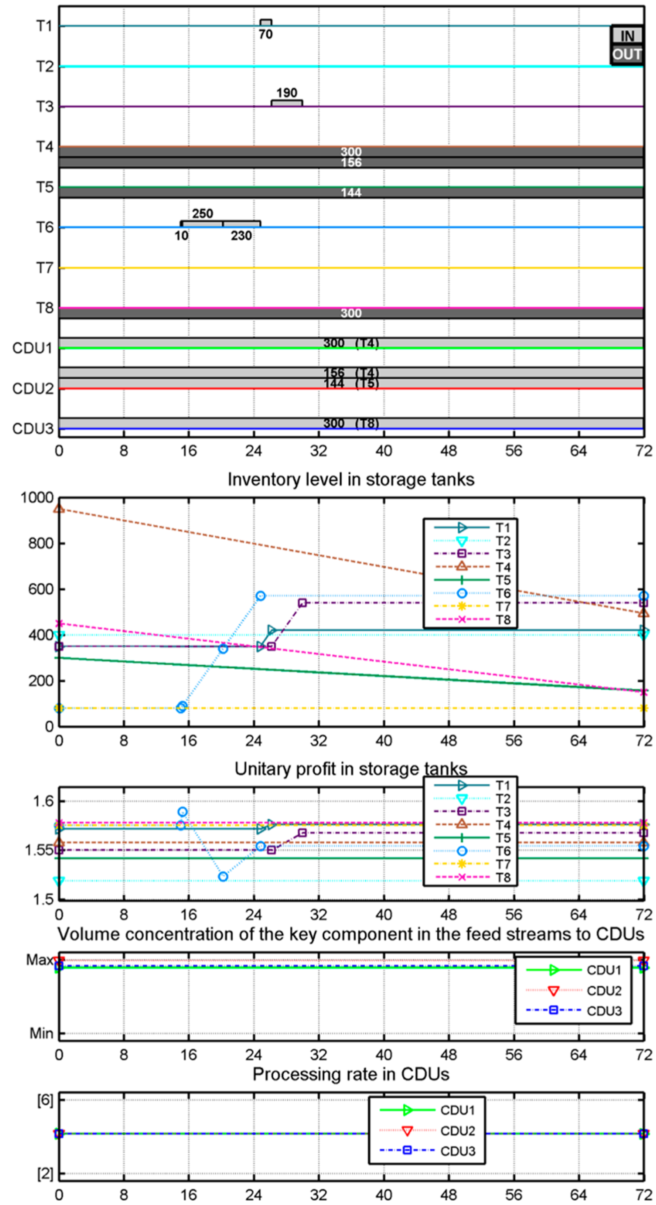

Figure 5. Best solution for example 2B with one arriving VLCC.

Information. In all cases, VLCC-1 carries a conflicting parcel p_4 and consequently two lots are preassigned to discharge p_4. Then, it was initially postulated: $|\mathbf{IV}| = 5$, $|\mathbf{S}| = 2$, $|\mathbf{IK}| = 6$, and $edt_v = at_v + 20$. Such choices directly lead to a good feasible solution for the MINLP in all but one of the cases. Example 8 was the only one requiring additional time slots and more lots of crude blends for the CDUs to make the MILP feasible. The new choices for example 8, $|\mathbf{S}| = 3$ and $|\mathbf{IK}| = 9$, produce a significant increase in both the model size and the CPU time. To make things easier, one can first allow a wider range for the CDU processing rate like for instance [2.0−6.0]. By postulating $|\mathbf{S}| = 2$ and $|\mathbf{IK}| = 6$, a feasible solution for the MINLP is found in 58 s of CPU time. At the first time slot s_1, CDU_1 receives crude blend from T_2, CDU_2 from T_4 and CDU_3 from tank T_1, i.e. $YK(ik_1,k_2,s_1,t_1) = YK(ik_2,k_4,s_1,t_2) = YK(ik_3,k_1,s_1,t_3) = 1$. They all have the initial, known compositions. Based on this information, a MINLP feasible solution for the restricted processing limits [2.5−5.0] can be obtained much faster by prefixing such binary variables to 1 before solving the approximate MILP formulation. In this way, a good MINLP solution for the restricted processing rate is found by solving the MILP in 33.3 s. Computational results for examples 3−10 involving a single VLCC are all shown in Table 1. Figures illustrating the best solutions for them are provided as Supporting Information. Previous works did not solve examples 3−10 by considering just the first arriving VLCC. Then, no comparison can be made.

8.2. Solving Examples 2−10 by Considering the First Two Arriving VLCCS.

Examples 2−10 are revisited, but this time the first two arriving VLCCs and a time horizon comprising the first two vessel-blocks are considered. Partial demands of CDUs and horizon lengths are given as Supporting Information. Example 2 first introduced by Reddy et al.[3] originally involves the unloading of two arriving VLCCs and a horizon length of 160 h. The vessels carry two conflicting parcels for the CDUs: p_4 in VLCC-1, and p_8 in VLCC-2. Hence, a pair of lots are proposed to distribute each of them, if necessary, between two receiving tanks. Because a wide feasible range [2.0–6.0] for the CDU processing rate is specified and eight crude parcels are to be unloaded, we adopt |**IV**| = 10, |**S**| = 2, and $edt_v = at_v + 20$ h. Therefore, |**IK**| = (|**T**|*|**S**|) + [(|**V**| − 1)/(|**S**| − 1)]*(|**P**|/|**V**|) = 3*2 + (8/2) = 10. Model sizes are given as Supporting Information. The best solution for the 2-VLCC version of example 2 was found in 21.1 s and depicted in Figure 6.

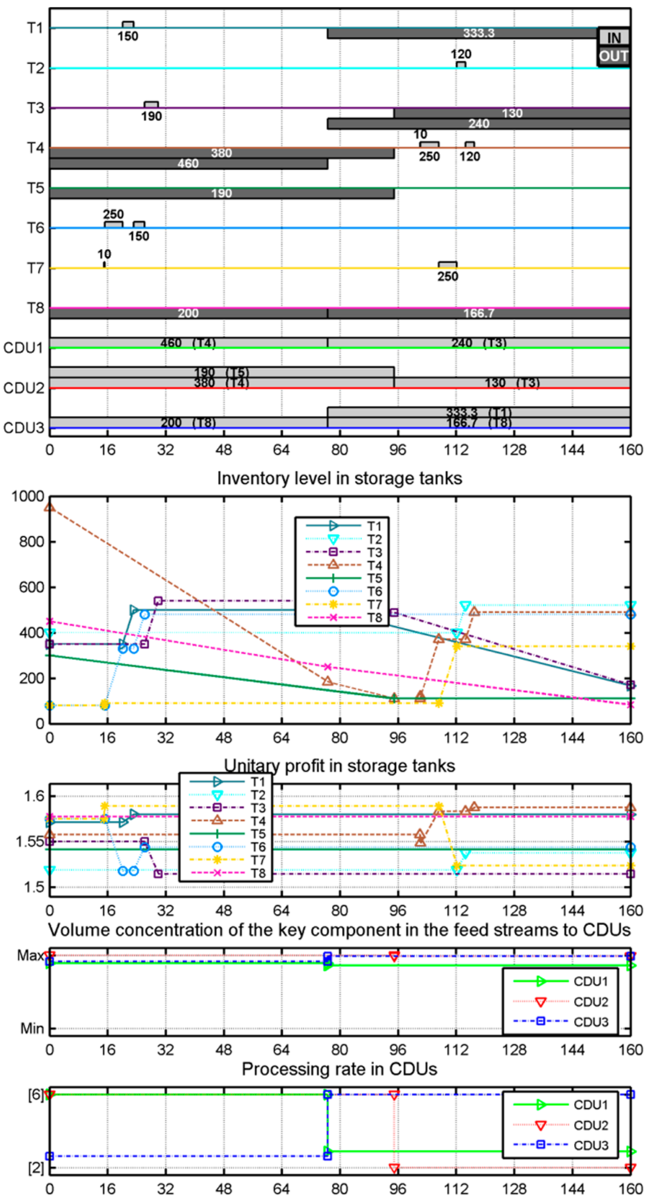

Figure 6. Best solution found for example 2 with two arriving VLCCs.

Eight lots of crude mix are supplied to CDUs to meet their demands. As shown in Table 3, most of the solution time is consumed by the MILP and the total profit amounts to \$3,252,210. Nearly similar results were found by Reddy et al.[3] (see Table 2). Pan et al.[21] reported a better solution but the feedstock for CDU_2 initially supplied by tanks T_2 (320 kbbl) and T_4 (380 kbbl) presents a key component concentration of 0.0125236 above the upper limit (0.0125). The initial concentrations in those tanks are T_2 (0.01275) and T_4 (0.012333).

Table 3. Computational Results for Examples 2−10 when Considering the First Two Tankers

			MILP formulation		NLP model	
no. (2-VLCC)	CDU demand (kbbl)	horizon length (h)	profit (\$)	CPU time (s)	profit (\$)	CPU time (s)
2	700, 700, 700	160	3,262,050	21.0	3,252,210	0.11
3A	750, 750, 750	160	3,484,630	9.5	3,470,540	0.08
3B	750, 750, 750	160	3,049,410	7.4	3,096,310	0.11
3C	750, 750, 750	160	3,345,000	10.6	3,345,000	0.06
4	680, 680, 680	216	3,157,780	1.2	3,151,030	0.09
5	680, 680, 680	216	3,156,350	2.3	3,149,690	0.12
6	580, 580, 580	216	2,686,140	53.3	2,668,710	0.03
7	680, 680, 600	216	3,033,440	47.1	3,041,860	0.11
8	690, 690, 690	256	3,160,490	189.2	3,129,850	0.08
9	600, 600, 600	240	2,813,780	38.5	2,811,450	0.03
10	680, 680, 680	216	3,153,870	1.5	3,137,740	0.11

The three versions of example 3 proposed by Li et al.[17] involving two VLCCs have also been solved. Examples 3A, 3B, and 3C differ in the unit margin associated with each crude oil. The VLCCs carry two complicating parcels containing crudes cr_4 and cr_8. Then, it is adopted: |**IV**| = 10, |**S**| = 2, and |**IK**| = 10. The best solutions are shown as Supporting Information. Comparison with the results reported by Li et al.[20] are given in Table 2. A minor decrease in the net profit is widely compensated by the important savings in CPU time when using our approach. The advanced solution procedure was also applied to example 4 by just considering the first two arriving VLCCs and the first two vessel-blocks. Similarly to example 3, the following was postulated: |**IV**| = 10, |**S**| = 2, and |**IK**| = 10. In this case, taking into account the selected tank-CDU assignments for the first vessel-block during which the tanker VLCC-1 is unloaded, the binary variables $YK(ik_1,k_4,s_1,t_1)$, $YK(ik_2,k_4,s_1,t_2)$, and $YK(ik_3,k_8,s_1,t_3)$ were prefixed to 1 before solving the approximate MILP for the two vessel-blocks. The best MINLP solution for the 2-VLCC case of example 4 was found in 1.3 s (see Table 3) and is shown as Supporting Information.

Using the same solution methodology, the 2-VLCC versions of examples 5−10 were also solved. Computational results are included in Table 3, and the figures illustrating the best solutions are given as Supporting Information. In all but one of the examples, the number of lots of crude blends supplied to CDUs was lower than |**IK**| = 10. The exception was the 2-VLCC version of example 8 where two additional time slots were necessary (|**S**| = 4) and consequently |**IK**| should be increased to 12 to get an MINLP feasible solution.

8.3. Solving Examples 4−10 by Considering the Three Arriving VLCCs. Examples 4−10 introduced by Li et al.[17] involve the unloading of three VLCCs over a scheduling horizon of 2 weeks (336 h) and CDU-demands varying from 900 to 1000 kbbl (see Table 1). In all of these examples, the vessels carry two conflicting parcels. Consequently, 14 lots are

Table 4. Computational Results for Examples 4−11 when Considering all VLCCs/Vessels

			MILP formulation		NLP model	
no.	CDU demand (kbbls)	horizon length (h)	profit ($)	CPU time (s)	profit ($)	CPU time (s)
4	1000, 1000, 1000	336	4,625,260	318.0	4,645,560	0.08
5	1000, 1000, 1000	336	4,616,890	1065.4	4,620,920	0.13
6	900, 900, 900	336	4,156,360	102.4	4,159,750	0.22
7	1000, 1000, 900	336	4,465,490	295.0	4,455,230	0.06
8	900, 900, 900	336	4,113,930	476.4	4,081,810	0.16
9	960, 960, 960	336	4,538,480	128.5	4,571,890	0.14
10	1000, 1000, 1000	336	4,625,540	81.9	4,651,940	0.14
11	1000,1000, 1000	336	4,678,200	546.5	4,743,080	0.16

made available to unload the crude parcels from the VLCCs. Moreover, it was initially adopted: $|S| = 3$, $|IK| = 3*3 + (2/2)*(12/3) = 13$ and $edt_v = at_v + 20$ h. When attempting to solve example 4, as originally proposed by Li et al.[17] with key component concentration ranges given by [0.001−0.0135] for CDU_1, [0.001−0.0130] for CDU_2, and [0.001−0.0040] for CDU_3, the MILP formulation is infeasible even if the first tanker VLCC-1 is only considered. To avoid infeasibility problems, we decided to use broader concentration ranges for CDU_1 and CDU_2 in example 4, given by [0.001−0.0140] for CDU_1 and [0.001−0.0135] for CDU_2. Similar concentration ranges were previously adopted when solving Example 4 with one and two tankers, respectively. Based on the results for the 2-VLCC version of example 4, the binary variables $YK(ik_1,k_4,s_1,t_1)$, $YK(ik_2,k_4,s_1,t_2)$, $YK(ik_3,k_8,s_1,t_3)$, $YK(ik_4,k_2,s_2,t_1)$, and $YK(ik_5,k_7,s_2,t_3)$ were prefixed to 1 before solving the MILP. Model sizes are given as Supporting Information. The best solution for example 4 featuring a net profit of $ 4,645,560 was found in 318.1 s (see Table 4). The CPU time rises to 320.3 s if the computational effort for the two previous steps of the advanced solution scheme amounting to 2.2 s is also considered. A detailed description of the operational schedule is given in Figure 7.

Computational results for the other 3-VLCC examples are given in Table 4. They favorably compare with the results reported by Li et al.[17] using the RRA-P algorithm in both solution quality and CPU time requirements (see Table 5). Using a global optimization technique, Li et al.[20] also solved example 10 discovering a solution slightly better featuring a net profit of $4,670,100 against $4,651,900 with our approach. Three time-slots were enough to find the best solutions for Examples 4, 6, 7, and 9. In contrast, it is necessary one and two additional slots (i.e., $|S| = 4$ and 5) to discover an MINLP feasible solution for examples 5 and 8, respectively.

In example 8, the crude demand of each CDU amounts to 900 kbbls and the length of the scheduling horizon is 336 h. This implies an average feed flow-rate of (900/336) = 2.678 kbbl/h. As a result, most of the time every CDU receives crude blend from a single storage tank because the minimum feed rate allowed for a CDU is 2.5 kbbl/h. Consequently, five time slots are necessary. Accounting for the solution of the 2-VLCC version of example 8, the size of the MILP approximate representation was reduced by making $YK(ik_1,k_5,s_1,t_2) = YK(ik_2,k_1,s_1,t_3) = YK(ik_3,k_5,s_1,t_1) = YK(ik_4,k_4,s_1,t_1) = YK(ik_5,k_7,s_1,t_2) = YK(ik_6,k_2,s_2,t_1) = YK(ik_7,k_3,s_2,t_2) = YK(ik_8,k_4,s_3,t_2) = YK(ik_9,k_6,s_2,t_3) = YK(ik_{10},k_4,s_3,t_1) = 1$. In this way, the best solution for example 8 shown in Figure 8 was discovered in 476.6 s. The CPU time rises to 700.0 s if the computational effort for the two previous steps of the solution scheme amounting to 223.4 s is considered. When compared

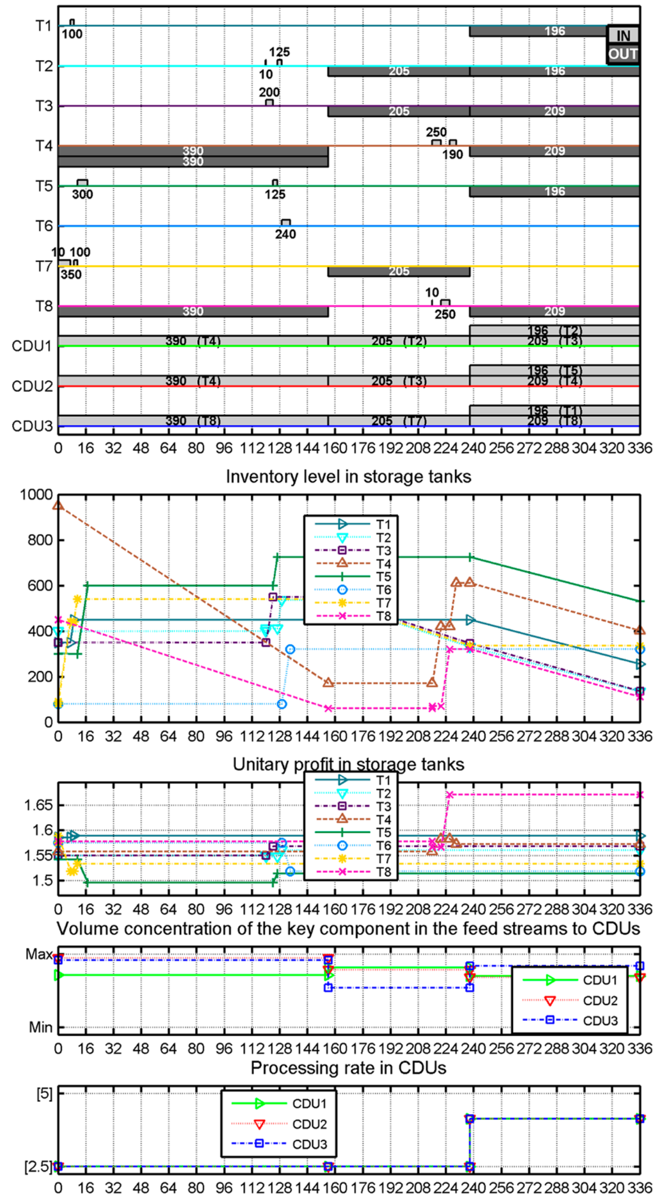

Figure 7. Best solution found for example 4 when considering the three VLCCs.

with the results obtained by Li et al.,[17] it is observed some improvement in the total netback. Similarly, a much better solution was also discovered for example 9 at less computational cost (see Table 5). The best solutions for the other examples with three VLCCs are provided as Supporting Information.

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Table 5. Comparing the Results for Examples 4−11 with Three VLCCs against Previous Works

	our approach		Li et al.[17]		Li et al.[20]	
no.	profit ($)	CPU time (s)	profit ($)	CPU time (s)	profit ($)	CPU time (s)
Examples with Three or More Arriving VLCCs/Vessels						
4	4,645,560	318.1	4,594,030	7511		
5	4,620,920	1065.5	4,642,370	52		
6	4,159,750	102.6	4,146,090	188		
7	4,455,230	295.1	4,429,520	5208		
8	4,081,810	476.6	4,016,650	373		
9	4,571,890	128.7	4,468,430	1958		
10	4,651,940	82.1	4,588,190	999	4,670,100	212
11	4,743,080	546.7	4,716,210	5860		

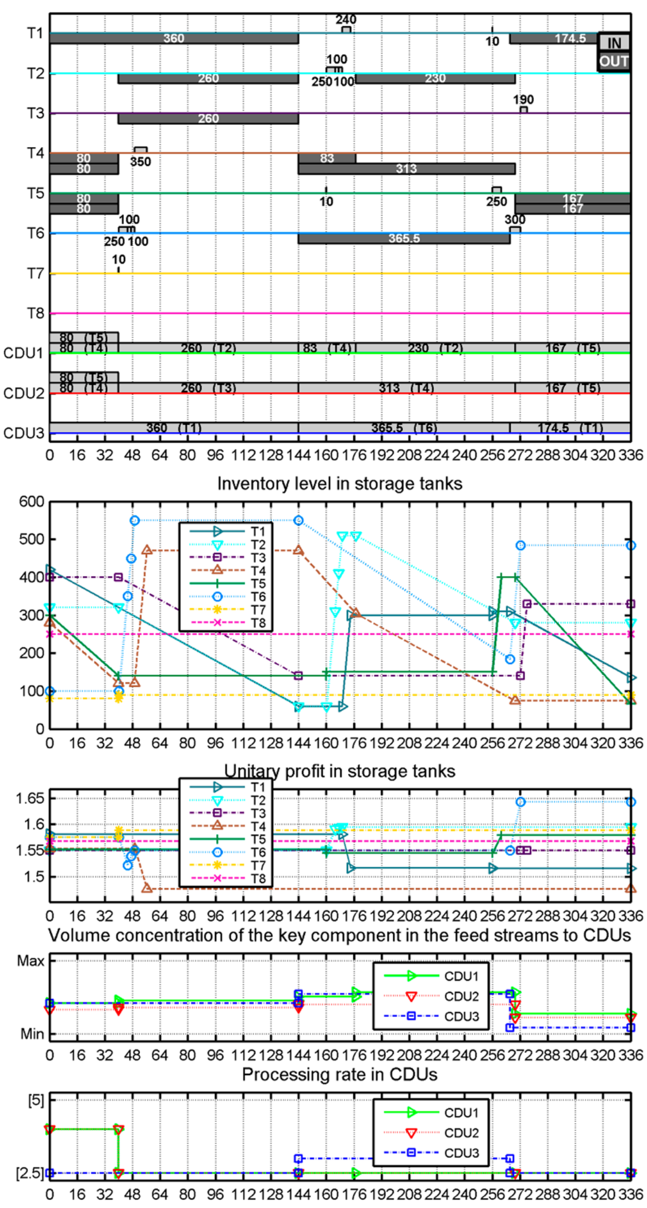

Figure 8. Best solution found for example 8 when considering the three VLCCs.

8.4. Example 11 Involving Three VLCCs and Seven Single-Parcel Vessels. Example 11 is an interesting case study involving three VLCCs and seven one-parcel vessels. While the VLCCs are unloaded offshore at the SBM terminal, the vessels are sequentially docked at a single jetty. The expected departure times of vessels result from assuming a maximum unloading time of 10 h per vessel and the fact that they should be discharged one at a time.

To unload tankers one at a time, they are chronologically ordered in the set **V** by increasing arrival times. A ship cannot start unloading operations before discharging all the previous ones in the set **V**. If two vessels arrive at the same time, the first arising in the set V is unloaded earlier. To facilitate the solution of example 11, it is first solved using a wider range for the CDU feed rate, i.e. [2.0−6.0] instead of [2.5−5.0] kbbl/h. Moreover, the following is adopted: $|\mathbf{S}| = 3$, $|\mathbf{IK}| = 13$. The best solution featuring a total profit of $4,698,740 was discovered in 381.0 s. When the CDU feed rate range is tightened to [2.5−5.0], an additional time slot is required giving rise to a larger MILP model. This size increase is neutralized by using the information provided by the best solution found with [2.0−6.0] as the CDU feed rate limits. The binary variables $YK(ik_1,k_8,s_1,t_3)$, $YK(ik_2,k_4,s_1,t_2)$, $YK(ik_3,k_4,s_1,t_1)$, $YK(ik_4,k_3,s_2,t_1)$, $YK(ik_5,k_5,s_2,t_2)$, $YK(ik_6,k_2,s_2,t_1)$ were prefixed to 1 before solving the MILP. The best MINLP solution for example 11 shown in Figure 9 was discovered in 546.7 s (see Table 4). It features a net profit higher than the one reported by Li et al.[17] (see Table 5).

9. CONCLUSIONS

This work introduces a new continuous-time approach for the scheduling of crude oil operations in marine-access refineries with no separate charging tanks. It relies on an MINLP formulation that accounts for the discharge of different crude oils into multiple storage tanks from ultra large tankers via a single SBM pipeline. It uses global-precedence sequencing variables to establish the ordering of loading and unloading operations in the storage tanks. The computational requirement is reduced by applying an efficient mode of tracking the composition and the inventory of crude mix in the storage tanks. Moreover, synchronized time slots of variable length are used to model the sequence of feedstock supplied to each CDU.

The basic MINLP solution scheme consists of first solving a single MILP approximate formulation obtained from the MINLP by (i) replacing bilinear mass balance equations with linear bounding constraints and (ii) assuming that the marginal profit per barrel of crude blend coming from a storage tank remains equal to the initial value. In the next step, the binary variables are fixed to their current values to convert the original MINLP into an NLP model. By choosing the MILP solution as the starting point, the resulting NLP is easily solved and usually provides near-optimal solutions for the MINLP. This basic solution procedure provides very good solutions at low computational cost for examples involving 1 or 2 arriving VLCCs with multiple compartments and a horizon length of 3−7 days. Almost all the CPU time is consumed to solve the MILP formulation.

An advanced solution methodology has been proposed for large crude scheduling problems involving more carriers, longer time horizons and tighter allowable ranges for the CDU processing rate. For this kind of problems, a higher number of time slots is required and consequently the size of the MILP undergoes a substantial increase. Using the basic solution scheme, a feasible solution for the MILP cannot be found

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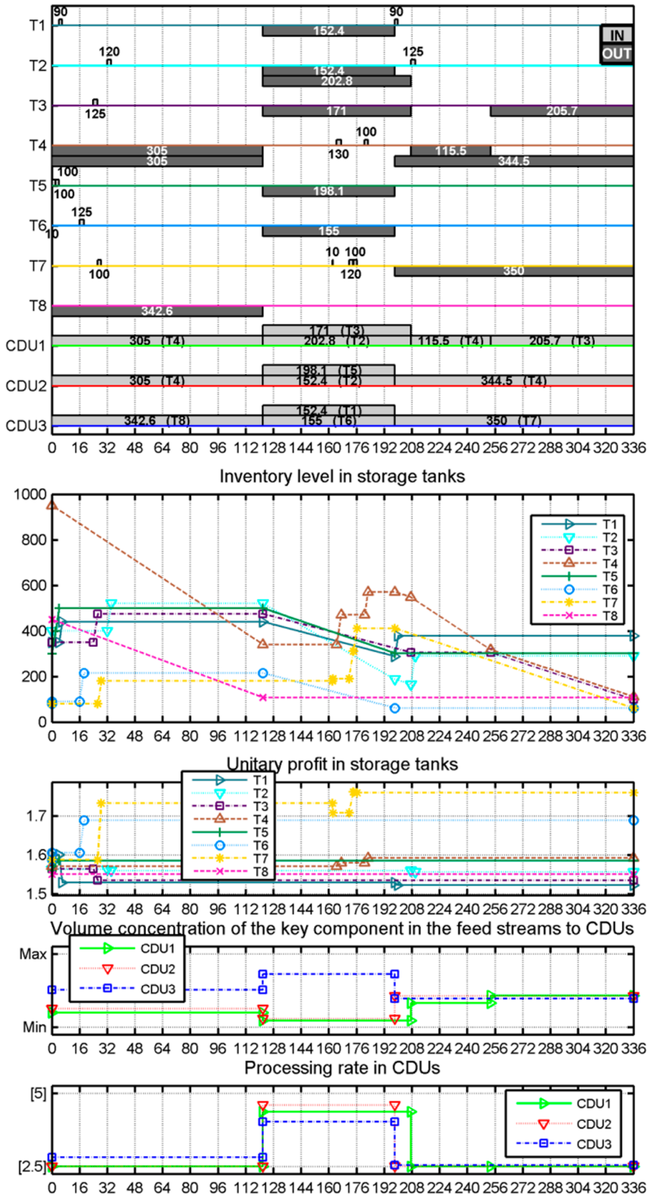

Figure 9. Best solution found for example 11 when considering all the arriving tankers.

within the CPU time limit. The new solution approach views the scheduling horizon as a sequence of vessel-blocks defined by the VLCC arrival times. Moreover, it relies on the idea that tank-CDU assignments for satisfying the demands of CDUs during the vessel-block n have typically a weak influence on similar decisions already made to meet CDU crude requirements during prior vessel-blocks 1, 2, ..., $n - 1$. The advanced solution procedure was successfully applied to a significant number of industrial-size scheduling problems all involving three or more carriers. Results show some improvements in both solution quality and CPU time requirements with regards to previous contributions. Further work is underway to consider more properties to evaluate the quality of the feedstock for the CDUs.

■ ASSOCIATED CONTENT

Supporting Information

The Supporting Information is available free of charge on the ACS Publications website at DOI: 10.1021/acs.iecr.5b01461.

(i) Mathematical expressions for the valid cuts included in the approximate MLP model, (ii) mathematical constraints for tracking the value of the unit margin associated with the crude blend supplied to the CDUs over the time horizon, (iii) tables with all the necessary data for examples 1−11, (iv) model sizes for all the examples, and (v) figures showing the best solutions for all the examples with one, two, and three VLCCs (PDF)

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Notes

The authors declare no competing financial interest.

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■ NOTATION

Sets

CR = types of crude oils
CR_p = type of crude oil in parcel p
IK = lots of crude blends discharged from storage tanks
IV = lots of crude oil from vessels
K = storage tanks
$\mathbf{K}_p$ = storage tanks that can receive crude parcel p
P = arriving parcels of crude oils
$\mathbf{P}_v$ = crude parcels carried by the VLCC/vessel v
S = time slots
T = crude distillation units
V = VLCCs/vessels

Parameters

a_p = size of the arriving parcel p
at_v = arrival time of VLCC/vessel v at the docking station/jetty
$capk_k$ = maximum capacity of the storage tank k
cch = changeover cost per instance
cwv = unit cost for late vessel departure
dem_t = total demand of crude blend for distillation column t
edt_v = expected departure time of VLCC/vessel v
H = length of the scheduling horizon
iik_k^0 = initial amount of crude mix in the storage tank k
$iicr_{cr,k}$ = initial amount of crude type cr in the storage tank k
$invk_{k,\min}$ = minimum inventory of crude mix in the storage tank k
M, MJ = relative large numbers
$rk_{\min}$ = minimum discharge rate of crude mix from a storage tank
$rk_{\max}$ = maximum discharge rate of crude mix from a storage tank
$rt_{\min}$ = minimum CDU processing rate
$rt_{\max}$ = maximum CDU processing rate

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rv_{min} = minimum discharge rate of crude oil from VLCCs/vessels
rv_{max} = maximum discharge rate of crude oil from VLCCs/vessels
sk_{min} = minimum size of a lot of crude blend unloaded from a storage tank
sk_{max} = maximum size of a lot of crude blend unloaded from a storage tank
sst = time to settle and remove brine
sv_{min} = minimum size of a lot of crude oil unloaded from VLCCs/vessels
sv_{max} = maximum size of a lot of crude oil coming from VLCCs/vessels
un_k^0 = initial netback per unit volume of crude mix coming from tank k
$uncr_{cr}$ = unit netback obtained by processing a barrel of crude type cr
$cjcr_{cr}$ = volumetric concentration of the key component in the crude type cr
cjk_k^0 = initial volumetric concentration of the key component in storage tank k
$cjkl_k$ = lower bound on the concentration of the key component in storage tank k
$cjku_k$ = upper bound on the concentration of the key component in storage tank k
cjp_p = volumetric concentration of the key component in the arriving parcel p
$cjtl_t$ = minimum allowed concentration of key component in the feedstock of CDU t
$cjtu_t$ = maximum allowed concentration of key component in the feedstock of CDU t

Binary Variables

$WK_{ik,k}$ = identifies the storage tank k from which the lot ik is unloaded
$WV_{iv,p}$ = denotes that the batch of crude oil iv discharges a portion of parcel p
$XK_{iv,ik,k}$ = denotes that lot iv is loaded into storage tank k before unloading lot ik
$YK_{ik,k,s,t}$ = denotes that lot ik from tank k is processed in CDU t during the time slot s
$YV_{iv,p,k}$ = allocates the batch of crude oil iv discharging parcel p to storage tank k

Continuous Variables

CK_{ik} = end time of the pumping run unloading lot ik from a storage tank
$CT_{s,t}$ = end time of slot s for CDU t
CV_{iv} = end time of the pumping run unloading lot iv from a VLCC/vessel
DT_v = departure time of VLCC/vessel v
$IKK_{k,ik}$ = inventory level of crude mix in the storage tank k at time CK_{ik}
$IKKJ_{k,ik}$ = inventory level of the key component in storage tank k at time CK_{ik}
$IKKN_{k,ik}$ = total potential netback available in tank k at time CK_{ik}
$IKV_{k,iv}$ = inventory level of crude blend in storage tank k at time CV_{iv}
$IKVJ_{k,iv}$ = inventory level of the key component in storage tank k at time CV_{iv}
$IKVN_{k,iv}$ = total potential netback available in tank k at time CV_{iv}
$FK_{ik,k,s,t}$ = amount of crude in lot ik from tank k processed in CDU t during time slot s
$FKJ_{ik,k,s,t}$ = amount of key component in lot ik from tank k supplied to CDU t during slot s
$FV_{iv,p,k}$ = amount of crude in lot iv transferring a portion of parcel p to storage tank k
$FVJ_{iv,k}$ = amount of key component in lot iv sent to storage tank k
LK_{ik} = length of the pumping run unloading lot ik from a storage tank
$LKK_{k,ik}$ = total amount of crude oil loaded into the storage tank k at time CK_{ik}
$LKKJ_{k,ik}$ = total amount of key component loaded into the storage tank k at time CK_{ik}
$LKKN_{k,ik}$ = total potential netback added to tank k up to time CK_{ik}
$LKV_{k,iv}$ = total amount of crude oil loaded into storage tank k at time CV_{iv}
$LKVJ_{k,iv}$ = total amount of key component loaded into storage tank k at time CV_{iv}
$LKVN_{k,iv}$ = total potential netback added to tank k up to time CV_{iv}
$LT_{s,t}$ = length of time slot s associated with the CDU t
LV_{iv} = length of the pumping run discharging lot iv from a VLCC/vessel
$QK_{ik,k}$ = size of batch ik coming from storage tank k
$QKJ_{ik,k}$ = amount of key component in batch ik coming from storage tank k
$QV_{iv,v}$ = size of parcel iv coming from vessel v
$QVJ_{iv,v}$ = amount of key component j in lot iv coming from tanker v
SK_{ik} = starting time of the pumping run unloading lot ik
$ST_{s,t}$ = starting time of slot s for CDU t
SV_{iv} = starting time of the pumping run unloading parcel iv
TD_v = departure tardiness of VLCC/vessel v
$UKK_{k,ik}$ = total amount of crude mix unloaded from storage tank k at time CK_{ik}
$UKKJ_{k,ik}$ = total amount of key component unloaded from storage tank k at time CK_{ik}
$UKKN_{k,ik}$ = total netback withdrawn from tank k up to time CK_{ik}
$UKV_{k,iv}$ = total amount of crude mix unloaded from storage tank k at time CV_{iv}
$UKVJ_{k,iv}$ = total amount of key component unloaded from storage tank k at time CV_{iv}
$UKVN_{k,iv}$ = total netback withdrawn from tank k up to time CV_{iv}
$CJK_{k,ik}$ = concentration of key component in storage tank k while unloading lot ik
CJT_{st} = concentration of key component in the feedstock of CDU t over time slot s

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