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Modelling and uncertainties characterization for robust control

J.L. Figueroa*, S.I. Biagiola

Instituto de Investigaciones en Ingeniería Eléctrica (UNS-CONICET), Avda. Alem 1253, (8000) Bahía Blanca, Argentina

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ABSTRACT

In this work, multi-input multi-output (MIMO) process identification is studied, where the model identification is dedicated to the control design goal. An ad hoc identification procedure is presented which allows estimating not only a nominal parametric process model, but also a bound of the model uncertainty (i.e. modelling errors). The model structure is defined in a way that the identified nominal model and the uncertainties can readily be used for the analysis and design of a robust control system by means of many of the techniques available in the literature. Simulation examples are given to illustrate the method.

1. Introduction

In the past, modelling and control design were conceived as separate problems. Nowadays, this concept has been modified and both processes (i.e. modelling and controller design) are viewed as ineluctably iterative in nature [1].

It is well known that the achievable performance of a control scheme for a particular process is directly related with the quality of the available model of that process. For this reason, many efforts have been dedicated to improve the representation capabilities of the mathematical models, by obtaining more appropriate models at the outset.

A pioneer work due to Boyd and Chua [2] explicitly establishes that any time invariant fading memory systems could be approximated by a nonlinear moving average operator. Based on this result, several model structures have been developed [3–5]. These models share the structure shown in Fig. 1, which consists of a linear dynamic block (in terms of Laguerre or Kautz basis) followed by a static nonlinear block. This kind of models allows to represent any memoryless system as accurately as it is required, and accuracy can be improved by only increasing the model complexity.

Simple models are preferred not just for philosophical but also for practical reasons. Following the principle of parsimony, we attempt to choose the “best” explanation for the process under investigation. The attractiveness of model simplicity appears to be incremented at the time of designing a model-based control system.

This simplification motivated some authors to consider simple structures of controllers. In particular, Hammerstein and Wiener models (see [6] and the references therein) have been used to represent a widely range of systems. The success of these models in control applications is that, under some assumptions, the nonlinearity could be removed from the closed loop and the controller could be designed using any classical linear methodology [7,8].

Now, since that these models are approximations of the real process, the robustness of the designed controllers should be analysed. In order to apply robust control theory, one needs not only a nominal process model, but also a suitable description of the modelling errors which are typically in the form of some bounds of parameter variations [9]. Recently, uncertain description has been obtained not only for Hammerstein and Wiener models [6,10,11] but also for some models of the type described in Fig. 1 [5].

However, most of the robust control analysis and design methodologies are based on linear models (see for example, [12–15]). Classical techniques for robust nonlinear control, usually cover the nonlinearity by an affine convex hull, and perform the analysis on it [16–18]. Only a few algorithms use explicit information about the nonlinearities [19,11] but they are based on simple Wiener models.

* Corresponding author. Tel.: +54 291 4595153; fax: +54 291 4595154.
E-mail addresses: figueroa@uns.edu.ar (J.L. Figueroa), biagiola@uns.edu.ar (S.I. Biagiola).

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In this work, robust identification oriented to control design is accomplished. For this purpose, a suitable model structure is selected to allow the straightforward application of existing robust techniques for controller design [12–15].

As is standard in inversion-based control strategies for the control of Wiener systems, the nonlinearity inverse is required ([20], and citations therein). This is because those control structures embed, in some way, the inverse of the static nonlinearity [21].

However, the approach herein followed, omits the inversion by centering the analysis on the output variables $y(t)$, instead of the intermediate ones ($z(t)$). Therefore, monotonic and slope-restricted nature of the nonlinearities are not necessary conditions to perform the identification procedure as well as the control strategies herein proposed.

The paper is organized as follows. Section 2, deals with the problem of identifying a model structure capable to reflect the observed uncertainty. The identification algorithm is therein developed. In Section 3, two simulation examples are introduced for the purpose of illustrating the implementation of different control schemes (MPC and μ-analysis) that rely on the robust Wiener model previously identified. This article concludes with some final remarks in Section 4.

2. Robust model

The present work addresses a particular and widely used type of block-oriented nonlinear models, the Wiener-like model, and it is assumed it has a parametric representation. The structure of the model herein studied is shown in Fig. 1, with the particularity that we will cover the nonlinear gain with a conic sector.

In this way, the linear block maps an input sequence $\{u(t)\} \in \Re^{N_u}$ to a sequence of intermediate signals $\{z(t)\} \in \Re^{N_z}$. The model output is $\{y(t)\} \in \Re^{N_y}$. The linear map is represented by orthogonal bases [22,10]:

$$z_{i,j,p_{\{i,j\}}}(t) = B_{i,j,p_{\{i,j\}}}(q)u_j(t), \tag{1}$$

for $i = 1, \ldots, N_y$, $j = 1, \ldots, N_u$ and $p_{\{i,j\}} = 1, \ldots, N_{z\{i,j\}}$; where q is the forward time operator, $N_{z\{i,j\}}$ is the number of terms in the orthonormal basis from the jth input to the ith output, u_j is the jth entry on the input vector and the $B_{i,j,p_{\{i,j\}}}(q)$ are the elements that relate the jth input to the ith output via the intermediate variables $z_{i,j,p}$. Therefore, the total number of subsystems is $N_u \cdot N_y$.

The bases are defined as

$$B_{i,j,0}(q) = \frac{(1-\xi_{i,j}^2)^{1/2}}{q-\xi_{i,j}}, \tag{2}$$

and

$$B_{i,j,p_{\{i,j\}}}(q) = B_{i,j,(p_{\{i,j\}}-1)}(q)\ \left(\frac{1-\xi_{i,j}\,q}{q-\xi_{i,j}}\right), p_{\{i,j\}} = 1, \ldots, N_{z\{i,j\}}. \tag{3}$$

Note that from this definition, the total number of internal variables is

$$N_z = \sum_{i=1}^{N_y}\sum_{j=1}^{N_u} N_{z\{i,j\}}. \tag{4}$$

This model allows to use the previous knowledge about the dominant modes of any of the subsystems from each input to each output, including them as parameters $\xi_{i,j}$.

Let us define the components of the vector $z(t)$ that affect the ith output as follows

$$z_i(t) = C_i z(t) \tag{5}$$

where $i = 1, \ldots, N_y$ and C_i is a matrix formed by zeros and a single 1 in each file according with the positions of $z_{i,j,p_{\{i,j\}}}(t)$ for $j = 1, \ldots, N_u$ and $p_{\{i,j\}} = 1, \ldots, N_{z\{i,j\}}$. Then, $C_i \in \Re^{N_{z_i} \times N_z}$ with

$$N_{z_i} = \sum_{j=1}^{N_u} N_{z\{i,j\}}. \tag{6}$$

In this way, a description for the linear part of the Wiener system (Fig. 1) has been provided. Now, it is necessary to define an uncertainty model for the static nonlinearity. For this purpose, the approach followed in [18] is adopted, and the nonlinear static gain is covered by

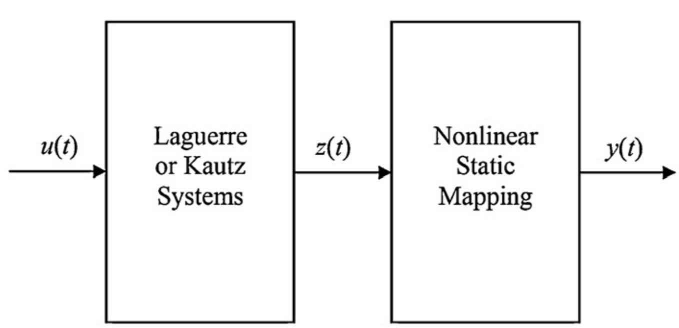

Fig. 1. Wiener-type model.

a conic sector. This means the input–output relation between z and y is confined to certain conic region. The strategy of modelling with conic sectors has been a widely used methodology for nonlinear feedback systems [23,24].

That involves the uncertain behavior of the nonlinearity is assumed to be represented by a polytopic description which is linear in z. Basically, sector bounds of the nonlinear function are determined (which are valid in the operating region) and the following inequalities arise

$$f^{min} \cdot z(t) \le y(t) \le f^{max} \cdot z(t) \tag{7}$$

which involves the uncertain nonlinear functions pass the origin [18]. In other words,

$$y(t) = f(z(t)) \cdot z(t) \tag{8}$$

with

$$f(z(t)) \in \Omega_f = Co\{f^{min}, f^{max}\} \tag{9}$$

where Co stands for *convex hull*.

In this way, the model for the ith output ($i = 1, \ldots, N_y$) must satisfy the following condition

$$y_i(t) \in \mathcal{Y}_i = \left\{ y_i(t) = f_i^T z_i(t), f_i^{lw} \le f_i \le f_i^{up} \right\} \tag{10}$$

where $f_i^{lw} \in \Re^{1 \times N_{z_i}}$ and $f_i^{up} \in \Re^{1 \times N_{z_i}}$ are, respectively, the lower and upper bounds on the parameters that define the conic sector.

In order to compute these bounds, let as consider the set of data $u(t)$ and $y_i(t)$ for $t = 1, 2, \ldots, K$, where K is the number of the available sampled data. Now, taking into account that $z_i(t)$ is a real vector for each input $u(t)$, whose entries could be positive or negative, it is possible to split it by defining $z_i^+(t) = max(z_i(t), 0)$ and $z_i^-(t) = min(z_i(t), 0)$ and to form the vector $Z_i(t) = \left[(z_i^-(t))^T, (z_i^+(t))^T \right]^T$.

At this time, the identification of these bounds must be accomplished. A simple method to solve this problem is to formulate it as an optimization one.

Theorem 1. *The bounds f_i^{lw}, f_i^{up} on the uncertain parameters f_i can be computed by solving the following optimization problem*

$$\min_{f_i^{lw}, f_i^{up}} \|f_i^{up} - f_i^{lw}\|_1 \tag{11}$$

subject to

$$\left[(f_i^{lw})^T, (f_i^{up})^T \right] Z_i(t) \ge y_i(t); \quad t = 1, \ldots, K \tag{12}$$

$$\left[(f_i^{up})^T, (f_i^{lw})^T \right] Z_i(t) \le y_i(t); \quad t = 1, \ldots, K \tag{13}$$

$$(f_i^{up}) - (f_i^{lw}) \ge 0 \tag{14}$$

And the resultant bounds f_i^{lw}, f_i^{up} are the solution to the robust identification problem.[1]

Proof. Note that fulfillment of constraint (12) implies the satisfaction of the upper bound on the data $y_i(t)$ in Eq. (10). In a similar way, inequality (13) involves the satisfaction of the lower bound on the measured process data $y_i(t)$ in Eq. (10). □

It should be remarked that in this model the orthogonal bases could be stated as the matrices A, B in the state space model, while the uncertainty could be concentrated in matrix C, as explain later in Section 3.2.

Last but not less important, it should be taken into account that the measured output could be corrupted with noise. Therefore, this situation, common in practice, should be included in the present identification approach. Data acquisition can involve many different sources of noise, the inclusion of such measurements in the algorithm given by Eqs. (12)–(13) would lead to "artificially" increased uncertainty bounds on the parameters, i.e. model uncertainty would be accounting for noisy measurements. Consequently, conservativeness would be unnecessarily increased. Otherwise, if we consider any output disturbed with noise as follows:

$$y(t) = \tilde{y}(t) + e(t) \quad |e(t)| \le \epsilon \in \Re^+ \tag{15}$$

where y stands for the measured value and $\tilde{y}$ for the "actual" value. Then, it is possible to reduce the conservativeness by changing the constraints (12) and (13) to account for an error margin due to the bound of the measurement noise.

Hence, constraints (12) can be reformulated as follows

$$\left[(f_i^{lw})^T, (f_i^{up})^T \right] Z_i(t) \ge y_i(t) - e(t); \quad t = 1, \ldots, K \tag{16}$$

and taking into account the minimum value for the error bounds (i.e., to consider the less conservative scenario), the following expression is obtained

$$\left[(f_i^{lw})^T, (f_i^{up})^T \right] Z_i(t) \ge y_i(t) - \epsilon; \quad t = 1, \ldots, K \tag{17}$$

In a similar way, constraint (13) can be rewritten as

$$\left[(f_i^{up})^T, (f_i^{lw})^T \right] Z_i(t) \le y_i(t) + \epsilon; \quad t = 1, \ldots, K \tag{18}$$

It must be remarked that this modification allows using the bounds on the measurement error to diminish the bounds on the model parameters. In such way, it could happen some measured output data differ from the model prediction, based on the assumption of noisy

[1] In this theorem the notation $\| \cdot \|_1$ has the typical norm 1 significance, i.e., the sum of the absolute values of the entries of the argument vector.

measurement. However, such difference will never exceed the value of the bound ϵ. Under this hypothesis it is no longer possible to guarantee that the whole measured data will be "justified" by the model.

On the other hand, if a conservative approach is to be adopted, Eqs. (17) and (18) would changed, respectively, to

$$\left[(f_i^{lw})^T, (f_i^{up})^T\right] Z_i(t) \geq y_i(t) + \epsilon; \quad t = 1, \ldots, K \tag{19}$$

and

$$\left[(f_i^{up})^T, (f_i^{lw})^T\right] Z_i(t) \leq y_i(t) - \epsilon; \quad t = 1, \ldots, K \tag{20}$$

This last formulation involves that any measured output would be represented by some model belonging to the identified family of models.

The remaining task is to propose suitable control strategies able to deal with the robust models herein proposed.

3. Robust control schemes

In this section, the model identification methodology presented above will be used to design robust controllers. For illustration purpose, two simulation examples will be performed. In the first one, μ-Theory will be used for robust analysis in the Direct Synthesis Control of a distillation column. In the second one, a robust model predictive control (RMPC) will be applied for controlling a steam generating unit. It is important to remark these examples are merely introduced in order to show the applicability and flexibility of the proposed model.

3.1. Model Based Control

The Model Based Control essence relies on the internal model principle which expresses that control can be achieved only if the control system includes, either implicitly or explicitly, some representation of the process to be controlled.

In this case, a distillation column has been selected as example. This is an appealing application as it is one of the most common unit operations in the chemical industry. Its relevance as well as its complex nature have been the main reasons for being a favorite subject in process systems engineering field. Moreover, in the areas of modelling and control, distillation columns have captured the attraction of many researchers. Such is the case of Skogestad et al. [25,26], whose Column A has been widely diffused. This simulation example is herein selected to illustrate the use of the identification methodology for robust control.

In this case the LV control structure is used. The input $u = [V_B\, L_T]^T$ is a vector formed by the boilup and the reflux flows, respectively. On the other hand, the output $y = [x_B\, x_D]^T$ is a vector formed by the liquid bottom composition and the liquid distillate composition, respectively. Therefore, we deal with a two input-two output process for the identification.

Simulation of the nonlinear model was accomplished in order to collect the required input–output data of this nonlinear process. For such purpose, random signals with uniform distribution around 1% of the nominal steady-state operating point were considered for the inputs (i.e. manipulated variables). A sample time of 10 s was assumed and the input was maintained constant for 100 samples.

The dominant poles in the Laguerre basis were chosen taking into account a preliminary linear identification. In this case, a Laguerre expansion of order 1 was selected with poles $\xi_{1,1} = 0.9581$, $\xi_{1,2} = 0.9385$, $\xi_{2,1} = 0.9645$ and $\xi_{2,2} = 0.9505$. These dominant poles in the Laguerre basis were chosen taking into account a preliminary linear identification.

The results of the robust identification are depicted in Fig. 2. From these plots it is clear that the measurement data are completely represented by the uncertain model obtained. This discrete-time model is transformed into a continuous-time one to design the Direct Synthesis Controller [27]. The resultant state space model is

$$\dot{z}(t) = A^d z(t) + B^d u(t)$$
$$z_i(t) = C_i^d z(t) \quad \text{for } i = 1, 2.$$

and the involved matrices A^d, B^d, C^d are defined in the Appendix A. The uncertain parameters are provided in Table 1. In this case, the nominal parameters are obtained by minimizing a quadratic criterion [22].

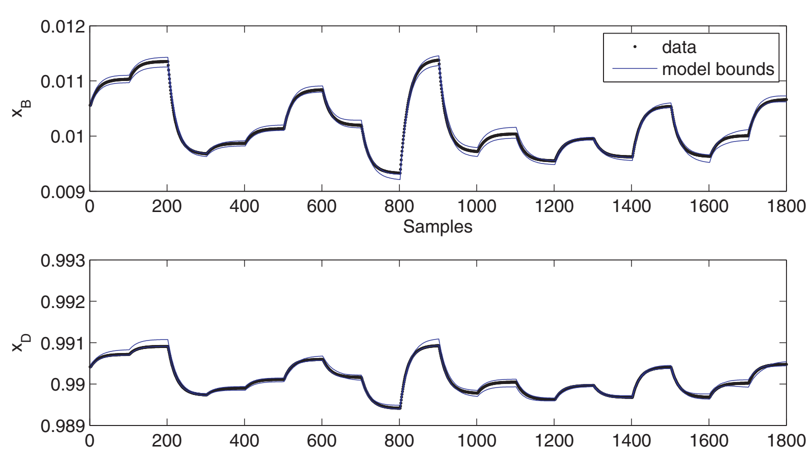

Fig. 2. Lower and upper bounds on the outputs x_B and x_D.

Table 1
Bounds and nominal parameters in the model of the distillation column.

f_1^{lw}	f_1^{nom}	f_1^{up}	f_2^{lw}	f_2^{nom}	f_2^{up}
0.1575	0.1711	0.1763	0.1226	0.1264	0.1411
−0.0102	−0.0056	−0.0056	−0.0189	−0.0181	−0.0181
−0.1826	−0.1731	−0.1606	−0.1315	−0.1249	−0.1219
−0.0291	−0.0291	−0.0246	−0.0180	−0.0043	−0.0043

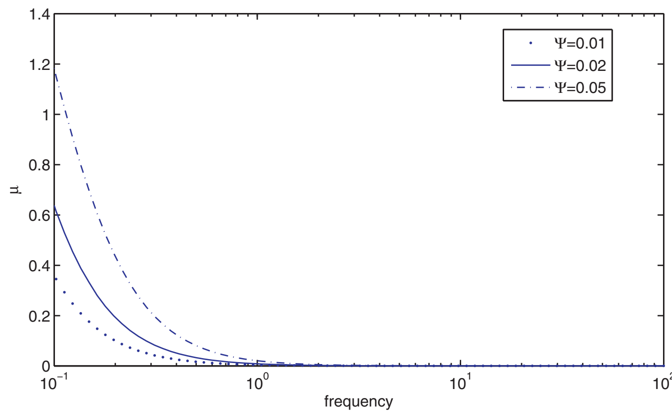

Fig. 3. μ function versus frequency.

The controller is designed based on the nominal model, and it is formed by a cascade of the inverse of the model plus an integral action. In the present work, it is implemented in its space state model as,

$$\dot{x}_c(t) = (A^d - B^d(C^d_{nom} * B^d)^{-1} C^d_{nom} A^d) x_c(t) + \Psi B^d (C^d_{nom} * B^d)^{-1} (y_{ref} - y(t))$$
$$u(t) = -(C^d_{nom} * B^d)^{-1} C^d_{nom} A x_c(t) + \Psi (C^d_{nom} * B^d)^{-1} (y_{ref} - y(t))$$

where y_{ref} is the vector of set points, the parameter Ψ is equal to the inverse of the desired closed loop time constant, and

$$C^d_{nom} = \begin{bmatrix} 0.1711 & -0.0056 & -0.1731 & -0.0291 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.1264 & -0.0181 & -0.1249 & -0.0043 \end{bmatrix}$$

In particular, the parameter Ψ is used to obtain a robust controller. By means of μ function [28] we can select an appropriate value for Ψ [12,13]. Fig. 3 shows the dependence of μ function on frequency for three different values of Ψ. In particular $\Psi = 0.02$ ensures robust stability in the frequency range of analysis.

Figs. 4 and 5 depict the simulated outputs and manipulated variables when the controller parameter is $\Psi = 0.02$. This simulation was performed with the nonlinear model, and the resultant performance is satisfactory.

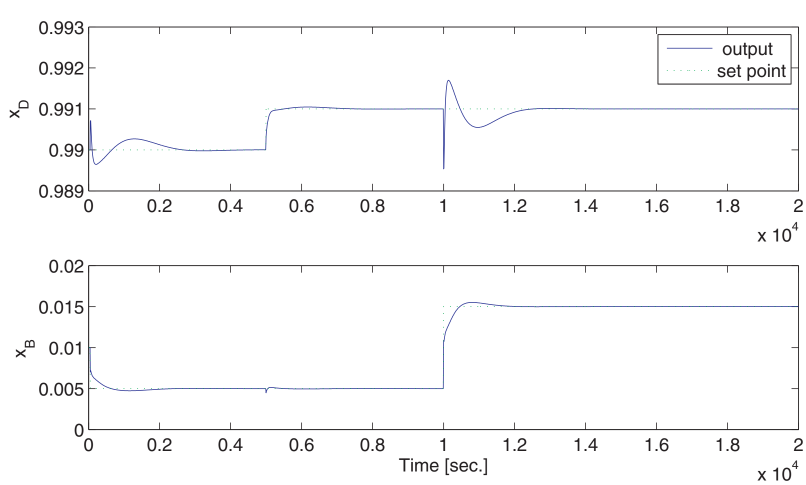

Fig. 4. Controlled variables versus time.

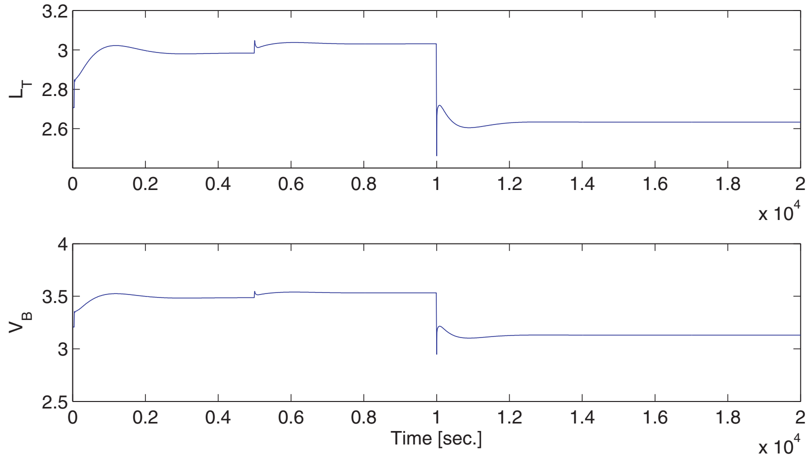

Fig. 5. Manipulated variables versus time.

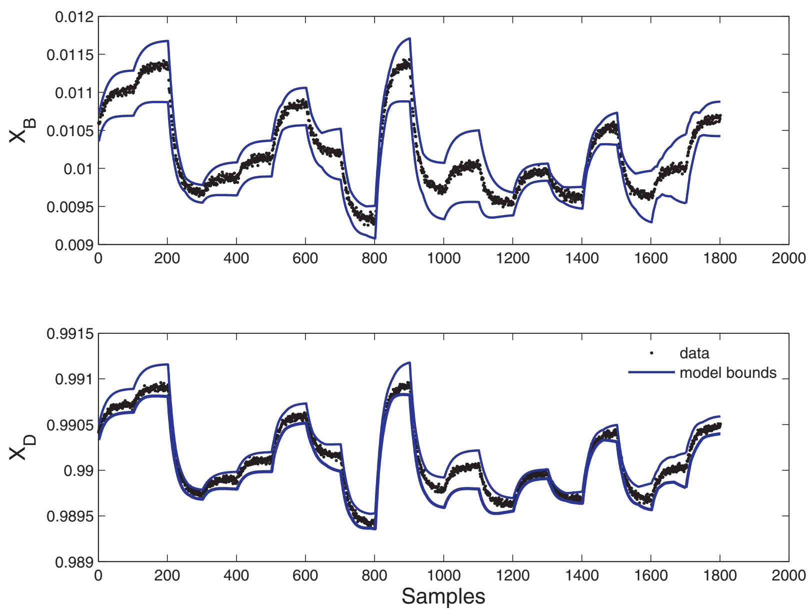

Fig. 6. Bounds on the outputs x_B and x_D based on noisy measurements.

In what follows, further simulation results are shown in order to cope with the more realistic situation in which measured data corrupted with noise are available to accomplish the identification task. For this purpose, measurement corrupted with Gaussian additive white noise with zero mean was assumed and a signal to noise relation of 30 dB. The noisy measurements are depicted in Fig. 6, as well as the new bounds on the model parameters (Table 2). The identification algorithm was performed including the constraints in Eqs. (19)–(20). As observed in Fig. 6, the bounds wrap the whole measured data.

Fig. 7 depicts the new situation of dependence of μ function on frequency, for the same three different values of Ψ. Note that the presence of measurement noise reduces the robustness margin. However, as shown in Fig. 8, the robust controller has a good performance in the setpoint tracking even in the case of noisy measurement. Note that though the measurement noise is equally present in both outputs, the control results show that the controlled variable x_D is much "noisier" than the other one (i.e., x_B). The required manipulated variables to achieve the control action are depicted in Fig. 9.

Table 2
Distillation column: bounds and nominal parameters (noisy measurements).

f_1^{lw}	f_1^{nom}	f_1^{up}	f_2^{lw}	f_2^{nom}	f_2^{up}
0.1309	0.1715	0.2204	0.1156	0.1264	0.1492
−0.0199	−0.0053	0.0157	−0.0338	−0.0182	−0.0120
−0.1973	−0.1724	−0.1470	−0.1495	−0.1251	−0.1218
−0.0642	−0.0298	−0.0037	−0.0093	−0.0040	0.0040

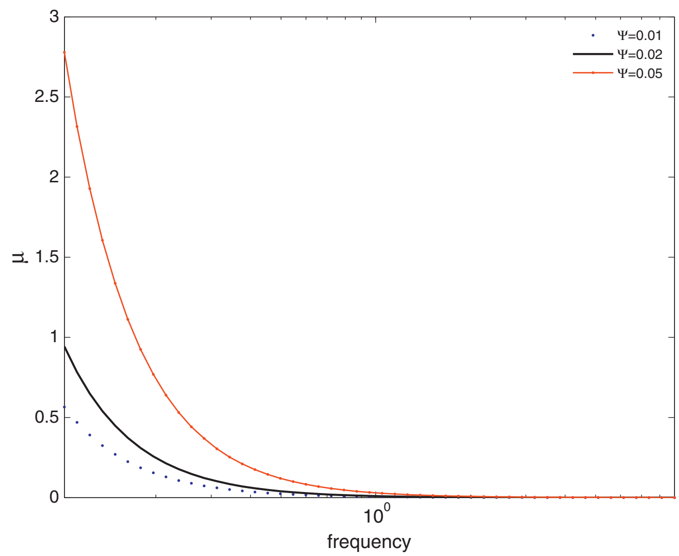

Fig. 7. μ function vs. frequency (for noisy data).

3.2. Model predictive control

In this case, the robust model will be used to design a robust model predictive controller. The control philosophy follows the original work by Khotare et al. [14], modified in [11] to include uncertainties in the matrix C in state space representations. The application case is a steam generating unit (SGU) which can be modelled by the following nonlinear representation [29]:

$$\frac{dP}{dt} = -0.00193SP^{1/8} + 0.000736w_c + 0.014524F - 0.00121L + 0.000176T_e \tag{21}$$

$$\frac{dS}{dt} = 10c_vP^{1/2} - 0.78571S \tag{22}$$

$$\frac{dL}{dt} = 0.00893w_c + 0.002F + 0.463c_v - 6\,10^{-6}P^2 - 0.00914L - 8.2\,10^{-5}L^2 - 0.007328S. \tag{23}$$

The state variables in this nonlinear model are: the pressure (P), the steam flow (S) through the high pressure turbine and the drum level (L). The states P and L are the controlled variables. There are two manipulated variables: the fuel input (F) and the feed water input (w_c), and two disturbances: the feed water temperature (T_e) and the control valve setting (c_v). The steady state values for these variables are shown in Table 3. In the sequel, $y = [P, L]^T$ and $u = [F, w_c]^T$ will be the vectors of controlled and manipulated variables, respectively.

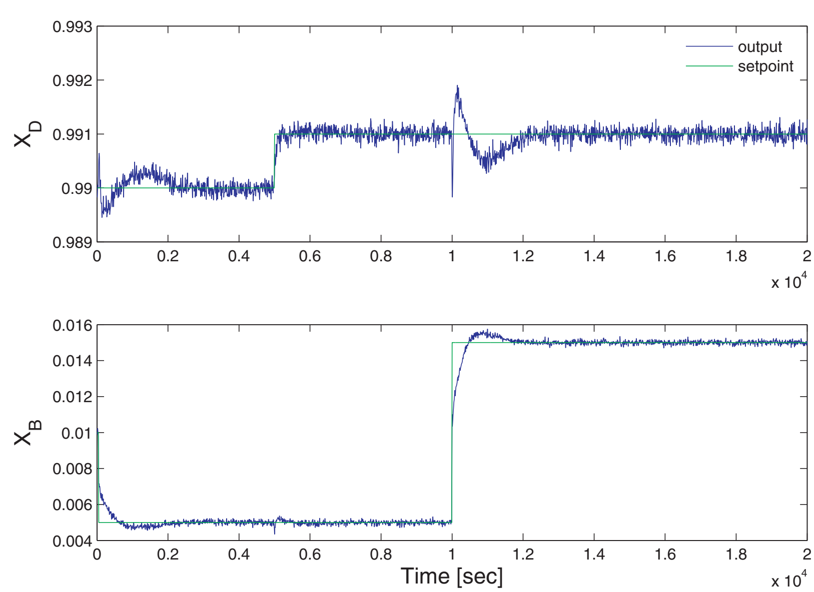

Fig. 8. Controlled variables for the noisy measurement scenario.

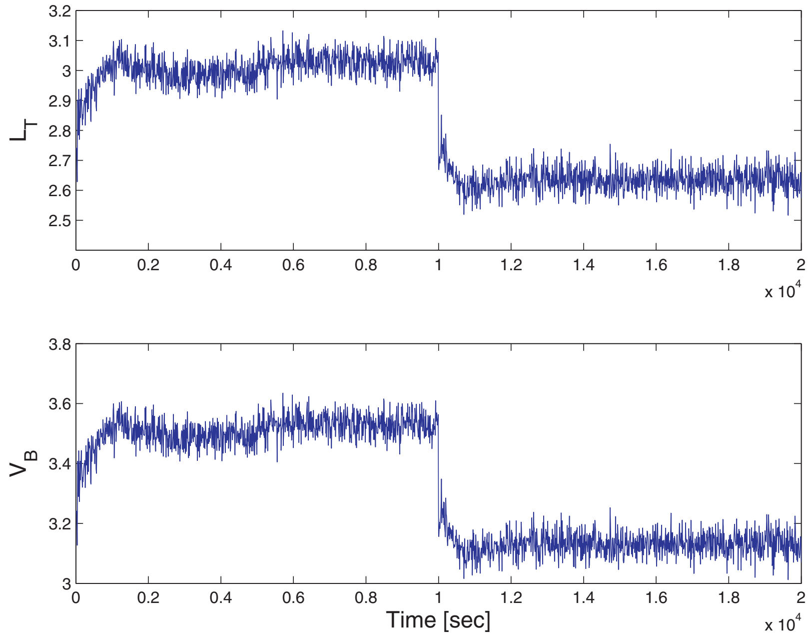

Fig. 9. Manipulated variables for the noisy measurement scenario.

Table 3
SGU variables.

Variable	Value
F (kg/s)	40
w_c (kg/s)	180
T_e (K)	290
c_v	0.8

A set of 1000 data is generated with a sample period of 20 s. The system is excited with uniformly distributed random manipulated variables (F and w_c). It is assumed both signals have a standard deviation of 2% and the variable is maintained constant for 100 samples.

The linear block was modelled as a Laguerre system. Each basis was assumed to be integrated by three terms with dominant poles equal to $\xi_{1,1} = \xi_{2,1} = 0.96$ and $\xi_{1,2} = \xi_{2,2} = 0.8$ (chosen taking into account a preliminary linear identification). In all cases, first order expansion was selected.

The results of the robust identification are depicted in Fig. 10, while Fig. 11 shows, in detail, the lower and upper bounds referred to the data signal (i.e. deviation values are plotted). From these plots it is clear that the measurement data are completely represented by the uncertain model obtained.

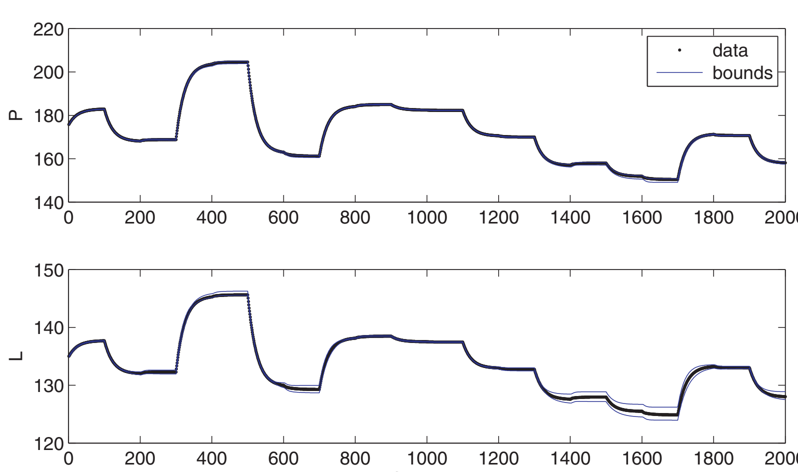

Fig. 10. Lower and upper bounds on the outputs for P and L.

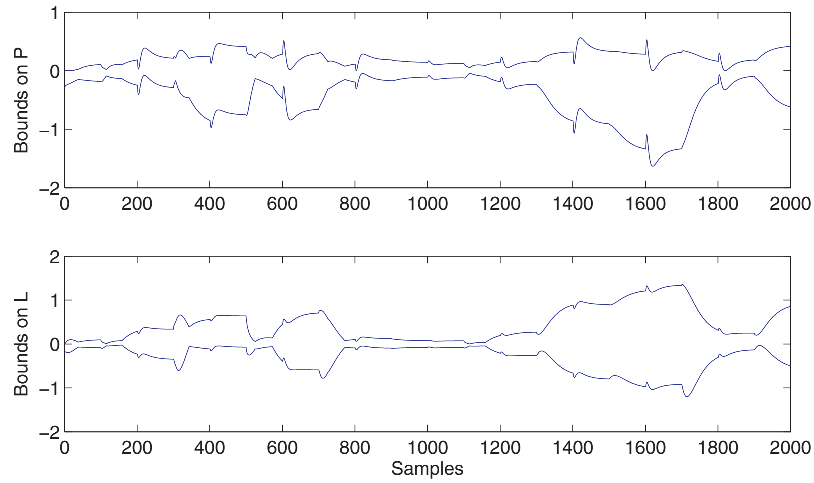

Fig. 11. Lower and upper deviations for outputs P and L.

The resultant discrete time state space model is

$$z(t+1) = A^s z(t) + B^s u(t)$$
$$z_i(t) = C_i^s z(t) \quad \text{for } i = 1, 2.$$

where the involved matrices are defined in the Appendix A. The uncertain parameters are defined in Table 4 (note that the nominal values are not computed because they are not necessary for control proposes). From these bounds on the parameters it is possible to define an uncertain output equation as

$$y^s(t) = C^s z(t)$$

where $C^s \in \Omega_C$, with

$$\Omega_C = Co\{[C^1, C^2, \ldots, C^L]\}. \tag{24}$$

In other words, if $C^s \in \Omega_C$ then, for some $\lambda_i \geq 0$; $i = 1, \ldots, L$ with $\sum \lambda_i = 1$ we have

$$C^s = \sum_{i=1}^{L} \lambda_i C^i. \tag{25}$$

By writing this model in function of deviation variables, and considering a constant set point signal on the horizon (w), the following model is obtained [15],

$$\begin{bmatrix} \Delta z(t+1) \\ y^s(t+1) - w \end{bmatrix} = \begin{bmatrix} A^s & 0 \\ C^s A^s & I \end{bmatrix} \begin{bmatrix} \Delta z(t) \\ y^s(t) - w \end{bmatrix} + \begin{bmatrix} B^s \\ C^s B^s \end{bmatrix} \Delta u(t) \tag{26}$$

$$\begin{bmatrix} y^s(t) - w \end{bmatrix} = \begin{bmatrix} 0 & I \end{bmatrix} \begin{bmatrix} \Delta z(t) \\ y^s(t) - w \end{bmatrix}$$

where $\Delta = 1 - q^{-1}$ and q^{-1} is the delay operator.

Now, defining $A = \begin{bmatrix} A^s & 0 \\ C^s A^s & I \end{bmatrix}, B^s = \begin{bmatrix} B^s \\ C^s B^s \end{bmatrix}, C = \begin{bmatrix} 0 & I \end{bmatrix}, x = \begin{bmatrix} \Delta z(t) \\ y^s(t) - w \end{bmatrix}, y(t) = y^s(t) - w, u(t) = \Delta u(t), A_i = \begin{bmatrix} A^s & 0 \\ C^i A^s & I \end{bmatrix}$ and $B_i = \begin{bmatrix} B^s \\ C^i B^s \end{bmatrix}$ for $i = 1, \ldots, L$, it is possible to solve the RMPC problem with the following objective function [14]

$$J_\infty(t) = \sum_{i=0}^{\infty} \left\{ (y^s(t+i|t) - w)^T Q_1 (y^s(t+i|t) - w) + \Delta u(t+i|t)^T R \Delta u(t+i|t) \right\} \tag{27}$$

Table 4
Parameters in the model of the SGU.

f_1^{lw}	f_1^{up}	f_2^{lw}	f_2^{up}
0.9958	1.0290	0.3926	0.3989
−0.0178	−0.0108	−0.0429	−0.0040
−0.0216	−0.0117	−0.0075	−0.0047
−0.0169	−0.0169	−0.0066	−0.0008

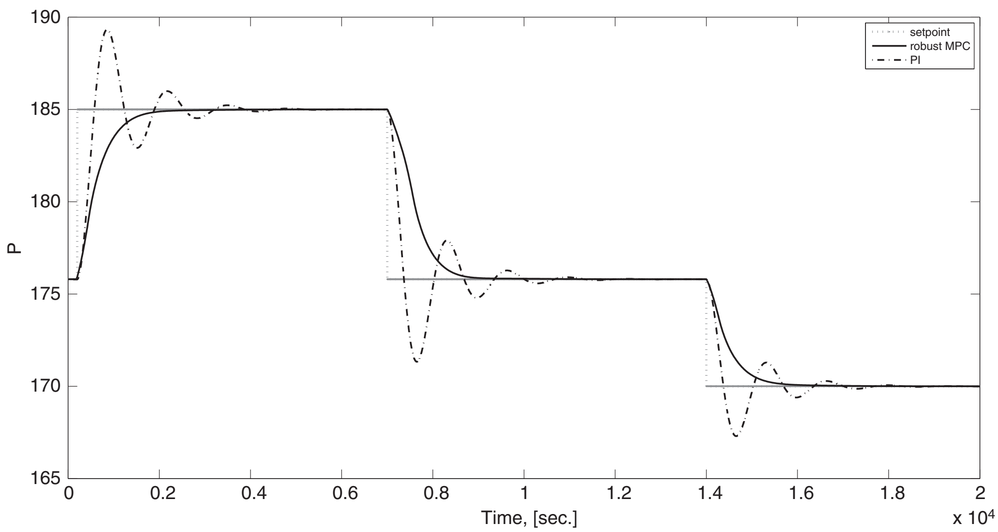

Fig. 12. Controlled variable versus time.

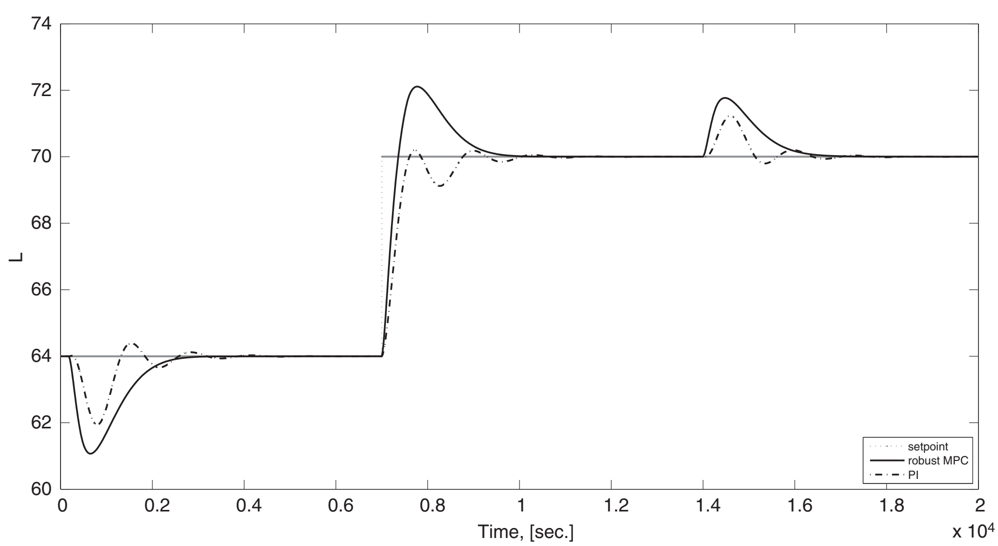

Fig. 13. Controlled variable versus time.

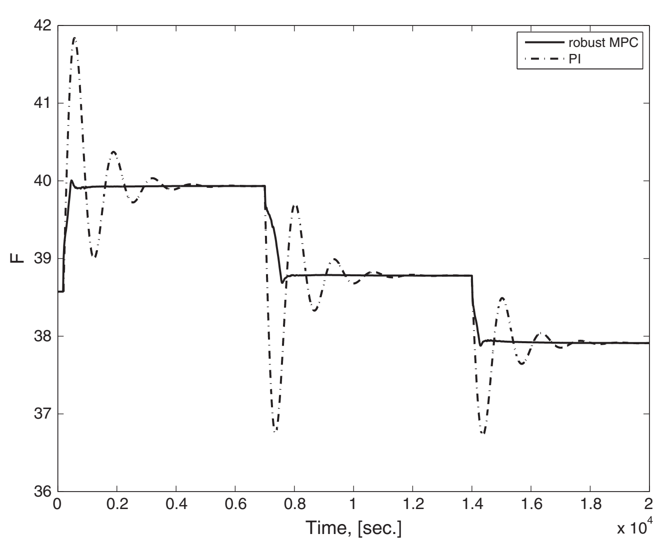

Fig. 14. Manipulated variable versus time.

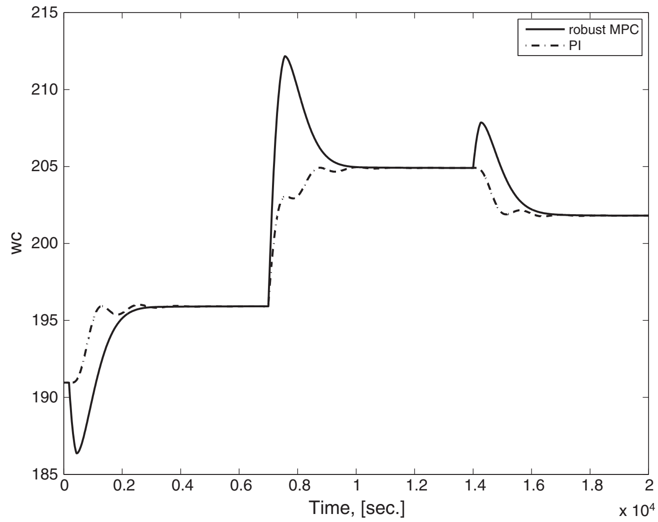

Fig. 15. Manipulated variable versus time.

as

$$u(t) = Fx(t) \tag{28}$$

where

$$F = YQ^{-1}, \tag{29}$$

where $Q > 0$ and Y are obtained from the solution (if it exists) of the following linear objective minimization problem with LMI constraints,

$$\min_{\gamma,Q,Y} \gamma \tag{30}$$

subject to

$$\begin{bmatrix} 1 & x(t)^T \\ x(t) & Q \end{bmatrix} \geq 0 \tag{31}$$

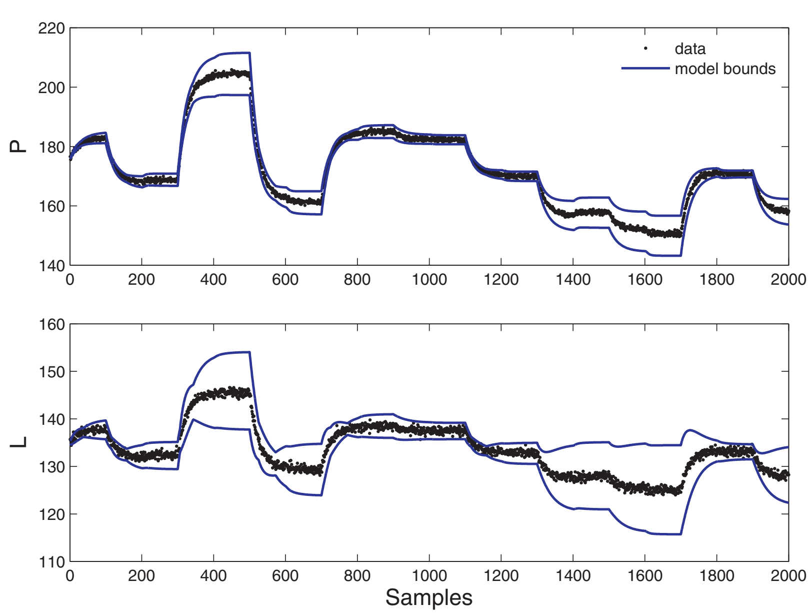

Fig. 16. Bounds on the outputs for the noisy measurement situation.

Table 5
SGU: bounds on the model parameters (noisy measurement).

f_1^{lw}	f_1^{up}	f_2^{lw}	f_2^{up}
0.8181	1.1577	0.1962	0.5258
−0.1583	0.1608	−0.2153	0.1374
−0.0145	−0.0145	−0.0055	−0.0055
−0.0174	−0.0174	−0.0058	−0.0058

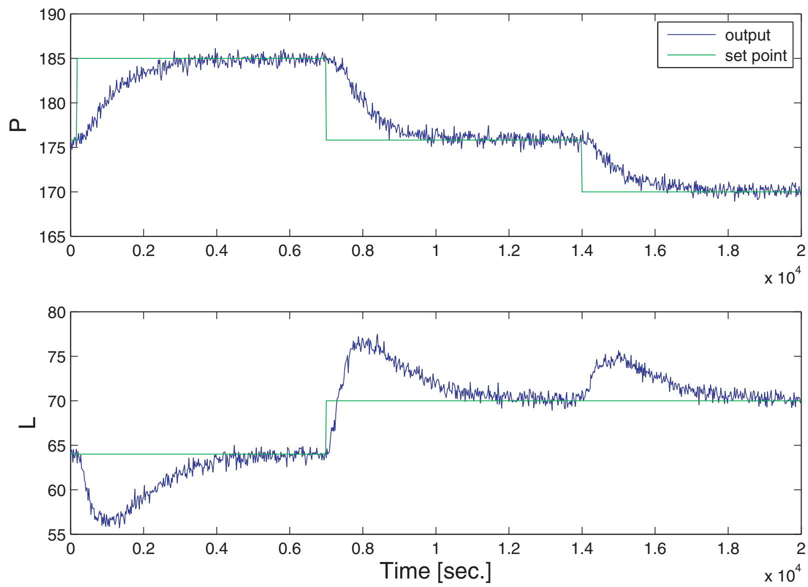

Fig. 17. Controlled variables for the noisy measurement scenario.

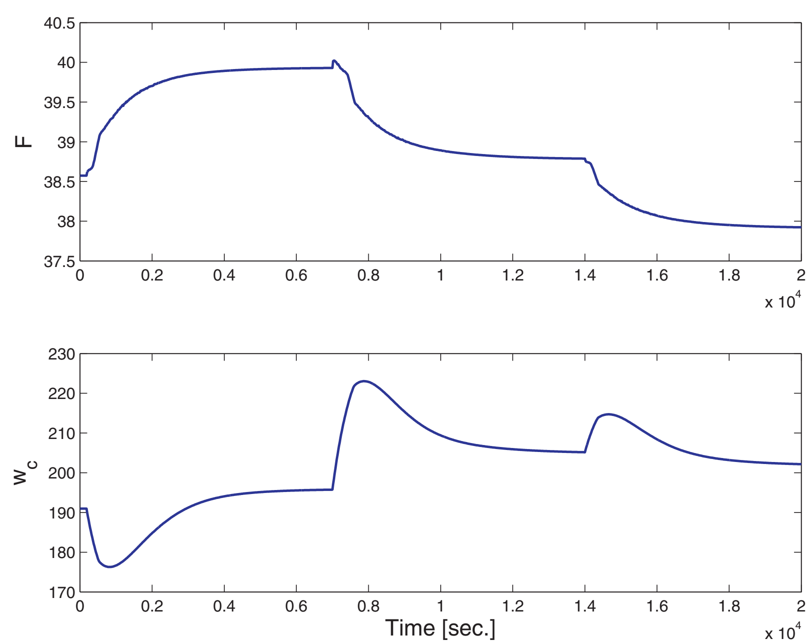

Fig. 18. Manipulated variables for the noisy measurement scenario.

and

$$\begin{bmatrix} Q & QA_j^T + Y^T B_j^T & QQ_1^{1/2} & Y^T R^{1/2} \\ A_j Q + B_j Y & Q & 0 & 0 \\ QQ_1^{1/2} & 0 & \gamma I & 0 \\ R^{1/2} Y & 0 & 0 & \gamma I \end{bmatrix} \geq 0, \quad j = 1, 2, \ldots, L. \tag{32}$$

This approach was used to control the SGU. The selected parameters were $Q_1 = I_2$ and $R = 100 I_2$.

The simulation results are shown in Figs. 12–15, they evidence the good performance of the robust model predictive controller. In order to provide a comparison with a well-known control strategy, two PI controllers were tuned based on input–output data of the process. For the tuning purpose, Ziegler–Nichols method was used. Relative Gain Array (RGA) assessment indicates that the pressure (P) would be best controlled by the fuel flow F; while level L would be best controlled by the feed water flow (wc). The PI-controllers performances are illustrated in Figs. 12 and 13 and the necessary control inputs are shown in Figs. 14 and 15.

Further simulation tests were accomplished to cope with the situation in which measured data corrupted with noise are available. Therefore, measurement corrupted with Gaussian additive white noise with zero mean was assumed, and a signal to noise relation of 30 dB was considered. The noisy outputs measurements as well as the identified upper and lower bounds are depicted in Fig. 16. The estimated parameters bounds are shown in Table 5. As illustrated in Fig. 17, the robust controller evidences good performance in setpoint tracking even for the noisy measurement scenario. Fig. 18 shows the necessary control moves to achieve the control goal.

4. Conclusions

In the present work a dedicated approach for robust identification of uncertain Wiener-like systems was presented. The dynamic linear part is represented by a finite set of discrete Laguerre or Kautz transfer functions, while the non-linear static part can have any possible nonlinearity whenever it is confined to a conic sector. Therefore, it is desirable to choose this sector as smallest as possible to avoid excessive conservativeness.

A parametric identification problem is stated, and it is solved as a convex optimization problem. Note that the identification algorithm is performed in a single step, and no inversion of the static nonlinearity is required. Additionally, no explicit description of such nonlinear function is demanded. This is quite an important advantage as, certainly, the real uncertainty description is hard or impossible to be obtained.

In this way, a family of parametric models are attained, such that the whole set of input–output data can be obtained by such robust model.

As a control oriented identification approach was proposed, therefore, the way this model can be used for controller design was shown.

Simulation results, based on two different systems, have been presented to illustrate the effectiveness of the proposed methodology. For this purpose, two different control methods were selected to show that alternative control strategies can be used based on the robust Wiener model, and very satisfactory performances can be achieved even in the presence of noisy measurements.

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Appendix A.

$$A^d = \begin{bmatrix} -0.0042 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.0082 & -0.0042 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.0062 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.0119 & -0.0062 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.0035 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.0070 & -0.0035 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -0.0049 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0097 & -0.0049 \end{bmatrix}$$

$$B^d = \begin{bmatrix} 0.0286 & 0 \\ -0.0274 & 0 \\ 0 & 0.0345 \\ 0 & -0.0324 \\ 0.0264 & 0 \\ -0.0255 & 0 \\ 0 & 0.0311 \\ 0 & -0.0295 \end{bmatrix}$$

$$C_1^d = \begin{bmatrix} I_4 & 0_4 \end{bmatrix}$$

$$C_2^d = \begin{bmatrix} 0_4 & I_4 \end{bmatrix}$$

where I_4 is the identity matrix of dimension 4 and 0_4 is the zero matrix of dimension 4.

$$A^s = \begin{bmatrix} 0.9600 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.0784 & 0.9600 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.8000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.3600 & 0.8000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.9600 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.0784 & 0.9600 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.8000 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.3600 & 0.8000 \end{bmatrix}$$

$$B^s = \begin{bmatrix} 0.2800 & 0 \\ -0.2688 & 0 \\ 0 & 0.6000 \\ 0 & -0.4800 \\ 0.2800 & 0 \\ -0.2688 & 0 \\ 0 & 0.6000 \\ 0 & -0.4800 \end{bmatrix}$$

$$C_1^s = \begin{bmatrix} I_4 & 0_4 \end{bmatrix}$$

$$C_2^d = \begin{bmatrix} 0_4 & I_4 \end{bmatrix}$$

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