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Analysis of tilt by modulated speckles generated with a double aperture pupil mask

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We present a method based on modulated speckles to detect tilt movement of a diffusing surface. In our proposal a speckle image of the speckle produced by a reflective diffusing surface is formed by a lens having a double aperture. The double aperture yields to an interference process so that the resulting speckle distribution is fringe modulated. The tilting of the diffusing surface is mapped as a shifting of the speckle. Then, the double aperture pupil lens system maps the speckle shifting into a fringes shifting. We study the system performance in terms of the diffuser tilt. Experimental results that confirm our proposal are presented.

1. Introduction

Speckle patterns can be obtained either by using an imaging system or by free propagation [1–3]. Speckles can be considered as carriers of information to be used in optical processing [4,5] and metrology [6]. In particular, speckle photography and speckle interferometry offer an alternative tool for analyzing surface roughness [7], for measuring displacement [8], strain [9], deformation and vibration [10] and slope detection [11].

Note that the characteristics of speckle patterns depend on the pupil aperture. In particular, the speckle formation using diffusers modulated by different single (linear, circular, conical, quadratic, elliptical, modulated graded index and truncated apertures) and multiple apertures is analyzed [12–19]. Although an optical system with a single-aperture pupil is usually employed, some advantage is achieved by use of a double [20] or in general a multiple-aperture pupil [21,22].

Duffy's method for measuring in-plane displacement consists in imaging a diffuser through a pupil mask with a double aperture symmetrically located with respect to the optical axis [20]. As a result of the coherent superposition of the beam emerging from both apertures, the resulting speckle distribution is modulated by Young's fringes. The double or in general the multiple aperture arrangement allows the spectral components of the speckle images to be spread out into the high frequency region of the Fourier transform plane resulting an increased signal to noise ratio. The multiple aperture system in a multiple exposure scheme that changes between exposures allows depicting simultaneously in a single frame several systems of fringes corresponding to different in-plane displacements [23] or in-plane rotations [24,25] between speckled images.

Methods of measuring surface displacements and rotations by the recording of the speckle pattern generated by coherent laser beam have been analyzed in Ref. [26]. Also, surface tilt may be measured by recording a defocused double exposure image and analyzing its optical transform. In Ref. [26], the diffuser surface can be considered as a reflecting mirror system. Under this condition, the diffuser tilting implies that the objective speckle pattern rotates as a whole about an axis belonging to the surface. Then, if the lateral speckle displacement is determined at a given distance of the diffuser, then the surface tilt can be obtained. This property may be used to study the local deformations of a diffusing surface when the deformations produce surface orientation changes. Note that in the case of a rigid body rotating without deformation, the speckle may be observed in the focal plane of a lens [27]. In the direction near the normal to the diffuser, a surface rotation produces a speckle shift in the lens focal plane. Then, a lateral shift of the object produces a translation of the speckle which is not desirable. In Ref. [27] demonstrated that when a collimated laser beam is used and the pattern is stored in the Fourier plane, the speckle displacements produced by the surface tilts about an axis normal to the lens axis are independent of any surface transverse movements in a plane normal to the lens axis. Note that when a

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diverging beam is employed, the pattern is no longer insensitive to transverse surface displacements. Tilt speckle movements and sensitivities are essentially as predicted for collimated illumination beam, but the speckle distribution is not independent of the surface transverse movements. In Ref. [28] a diffuser object lateral shift will produce no change in the speckle pattern and the object rotation results in a translation of the speckle.

In classical interferometrical arrangements, the surface slopes are observed without difficulty by means of the differential method. The wave reflected from the surface is splited by an interferometer into two laterally shifted coherent beams. The reflecting surface slopes may be obtained from the mentioned interference. It is not possible to use the mentioned technique to diffuse objects because of the decorrelation of the same speckle pattern between two arbitrary regions. In Ref. [29] a double aperture interferometer has been used by introducing two plane parallel glass plates in front of each aperture. If four apertures without plates are used, a lateral displacement is produced in two perpendicular directions [29] when the distribution is given a slight defocusing. The detailed methods allow determining two perpendicular components of the slopes variations. An alternative method for recording speckle pattern displacements, and hence diffuser surface movements, is to use a CCD camera and double exposure technique, giving a vector displacement field of speckle pairs.

In our work, a technique to detect tilt movements based on fringe modulated speckles is presented. This technique represents an alternative to Gregory's method [28] allowing a more precise tilt determination but preserving the simplicity of the original proposal. A speckle image of a speckle pattern produced at a given plane (defined as Π plane) by a reflective diffusing surface is formed by a lens having a double aperture mask. This mask yields to an interference process so that the resulting speckle distribution is fringe modulated. The tilting of the diffusing surface is mapped as a shifting of the speckle produced by free propagation at the Π plane. Afterwards, the double aperture pupil lens system maps the speckle shifting into fringes shifting. Our proposal is valid when the speckle correlation is maintained between both states (before and after tilting) of the diffusing surface. In the following in Section 2, the method is discussed and in Section 3, the set-up is described and experimental results that confirm our approach are presented.

2. Discussion of the method

Let us consider that a collimated laser beam is incident, forming an angle $\alpha = 0^o$ onto a reflective diffusing surface plane, which experiences a tilt θ with respect to an axis belonging to the diffuser surface. As it is well known, a speckle is obtained when a collimated laser beam is incident onto a diffuser. Providing that the speckle distribution before and after tilting are fully correlated, the diffusing surface tilt can be mapped as a transversal shifting in a plane perpendicular to the optical axis (Π plane). In this situation, the speckle produced by free propagation at a distance M and observed at an angle β suffers a displacement of

$$\varphi = \left(1 + \frac{\cos\alpha}{\cos\beta}\right)\theta\, M \tag{1}$$

in a plane parallel to the surface before rotation [28,30]. In particular, when the incident plane wave is perpendicular to the surface and the observation is done along the incident direction ($\alpha = \beta = 0^o$) at the speckle Π plane the displacement reduces

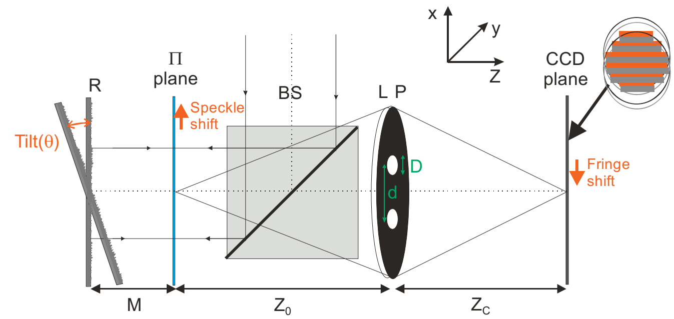

Fig. 1. Scheme to measure diffusing surface tilting by modulated speckle (R: diffuser, L: lens, P: double aperture pupil mask; D: pupil diameter and d: aperture separation; θ: diffuser tilting angle; M: distance from the diffuser to the P plane; Z_0 and Z_C: object and image distance).

to:

$$\varphi = 2\,\theta\, M \tag{2}$$

In summary, this expression allows us to determine the tilting angle θ as the speckle shifting φ in the Π plane. Previous expressions are valid if the diffuser surface can be treated as a reflecting mirror system [30]. Thus, the expression (2) remains valid when the correlation condition is maintained and the speckle displacement is insensitive to the transversal displacement.

The goal of our proposal is to asses tilt angles of the order of several arc minutes. Under these conditions, the displacement at Π plane is of the order of the speckle transversal dimensions. Therefore, the precision of the measurement should be very poor. In order to improve the measurement through the modulation of the speckle, we propose the scheme depicted in Fig. 1. As mentioned above, a speckle pattern is formed onto a Π plane at a distance M from the reflective diffuser surface. Then, this speckle is imaged on a CCD plane by a lens having a double aperture mask. When the pupil mask is used, each point in the image plane receives two contributions, one from each aperture. Then, a speckled image of the speckle at Π plane is produced through each aperture. Besides, the complex amplitude of waves going through each aperture are statistically independent, since different components of the angular spectrum of the scattered light are accepted by them. Thus, the speckle image distribution formed through one aperture is uncorrelated with the distribution obtained through the other one. On the other hand, the resulting speckle pattern appears as the interference of those distributions because they are coherent. Note that the orientation of the fringes that modulate the speckle pattern depends on the orientation of the two apertures. In particular, the fringes are perpendicular to the line joining the apertures. The pitch (spatial period) of the fringe system embedded in each speckle depends on the distance between apertures. Note that as a consequence of the statistical nature of the speckle phenomenon, the fringes spatial period associated to the registered speckle should be treated as an average value. This average spatial period is $\langle p\rangle = \lambda(Z_C/d)$ where d represents the separation between apertures. Then, an image modulated by speckles which are themselves modulated by fringes is obtained in the CCD plane.

As mentioned above, the diffuser tilt is mapped as a speckle shifting φ at Π plane. Then, the resulting modulated speckle pattern at the CCD plane also shifts. This fringe shift is:

$$\varphi' = m\,\varphi \tag{3}$$

where $m = Z_C/Z_0$ is the lateral system magnification.

Then, the diffuser tilt can be determined by measuring the fringe shift. Note that it is not possible to apply the proposed technique to diffusing surface when the speckle pattern decorrelates. In the next section, we verify that the correlation is maintained in our case.

3. Experimental set-up and results analysis

The experimental set-up employed to analyze the reflective diffuser is shown in Fig. 2. A collimated Nd YAG laser beam ($\lambda = 532$ nm) is incident normally onto the diffusing surface under analysis. A speckle pattern is produced by free propagation a distance $M = 50$ mm from the diffuser to the Π plane. A lens L with 200 mm focal length, conjugates the speckle Π plane and the CCD plane. The distances from the Π plane to the lens and from the lens to the CCD plane are 586 mm and 440 mm, respectively. The lens L has attached a pupil mask P with two identical circular apertures. The circular aperture has a diameter $D = 3$ mm and the aperture centers are separated a distance $d = 10$ mm. The average spatial period results $< p > \approx 22$. As it is well known, the average diameter and length of the speckles at the CCD plane are $\langle S_{X-ccd\ plane} \rangle \approx \lambda (Z_C/D) \approx 78\ \mu m$ and $\langle S_{Z-ccd\ plane} \rangle \approx \lambda (Z_C/D)^2 \approx 11.4$ mm, respectively. Note that, if the distance Z_C remains fixed, the parameter D governs the speckle size. Then, in this case the average number of fringes per speckle is approximately 3.3. In the CCD plane, a camera equipped with a zoom microscope is located to capture and afterwards to digitalize the modulated speckle image. In the experimental set-up, we use a PULNIX 6CN CCD camera with 752×582 pixels and 8.6 μm × 8.3 μm pixel area. Our proposal consists in measuring the fringes shift. Therefore, in order to facilitate the measurement of the smaller fringe shifts φ' and smaller tilt angles θ a pattern magnification is required. A zoom-microscope provides a magnification of $4\times$.

In this section is studied the performance of the proposed set-up. When the diffuser surface tilts an angle θ around the y-axis (see Fig. 2) modulated speckles are stored in the CCD camera. In order to make the tilt of the surface, the diffuser object is attached to a precise rotating platform. This device consists of an electronic rotational table with an accuracy of 2×10^{-4} rad. Thus, the rotation of the platform produces the required tilt of the diffuser surface.

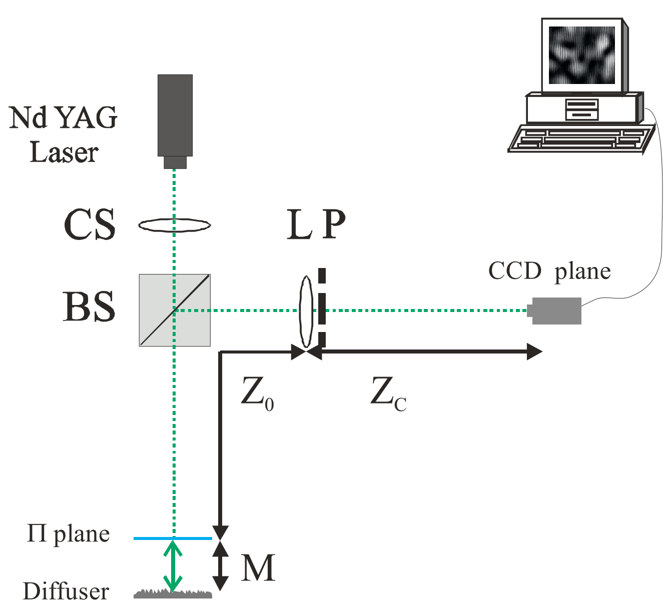

Fig. 2. Experimental set-up designed for diffuser tilt measurements (CS: spatial filter and collimator system; BS: beam splitter; L: lens (focal length $f = 20$ cm), P: double aperture pupil mask with the pupil diameter $D = 3$ mm and apertures separation $d = 10$ mm).

As it is analyzed in the previous section a diffuser tilt produces a displacement φ of the speckles in the image speckle pattern at Π plane and a displacement φ' of the fringes at the CCD plane. Fig. 3 shows the intensity profiles of the modulated speckle patterns corresponding to different diffusing surface tilt angles between -12×10^{-4} rad and 12×10^{-4} rad. The results confirm that a surface tilt is mapped as a transversal shifting of speckles. It is possible to determine from the experimental results that the fringe shift between the reference position (0 rad) and 6×10^{-4} rad corresponds to 20 pixels shift. Then, the tilt angle results 6.2×10^{-4} rad. Let us remark the good agreement bewteen the tilt angle introduced with the platform and the angle obtained by using our model.

Note that the fringe displacement φ' can be expressed as $\varphi' = bpix$ where b is the number of pixel and pix is the pixel width. Also, $\varphi' = \mathrm{m}2\theta\mathrm{M}$. Therefore $\theta = (\varphi'/2\ \mathrm{mM})$ and the relative error of the measures is,

$$\frac{\Delta\theta}{\theta} = \frac{1}{b} + \frac{\Delta M}{M} + \frac{\Delta Z_C}{Z_C} + \frac{\Delta Z_0}{Z_0} \tag{4}$$

and the relative error is of the order of 5%. It should be pointed out that the aperture diameter D and the distance between apertures d control the modulated speckle through the speckle transversal dimension and fringe period. It is clear that to obtain precise measurements the fringe period should contain several pixels.

In earlier experiments [28] note that a two exposures method applied conveniently if the object displacement is greater than the average value of the speckle transversal size. When the spectrum is displayed, this value establishes the size of the diffraction pattern enveloping. Therefore, in order to observe fringes associated to correlated speckles, the fringe spacing should have a pitch to be inside the enveloping of the diffracted light. It means that the displacement in the speckle image plane must be greater than the size of the speckle grain. On the other hand, the object displacement should be short enough otherwise the fringes pitch become very small in the Fourier plane and the measurement is limited by the zero order.

On the contrary in our proposal, the displacement can be smaller than the diameter of the speckle grain in the Π plane. In this case, the whole speckle image at the CCD plane moves with their fringes. Note that despite the displacement magnitude could be shorter than the speckle transversal size in Π plane, a noticeable fringe displacement could be produced. We measure this displacement.

As could be observed from the fringe intensity profile of Fig. 2, in our case the fringes wave vector is parallel to the speckle shift direction. The fringe wave vector depends on the pupil aperture orientation and it could be changed. In general when the wave vector and the shifting direction is not parallel, the fringe shift is given by,

$$p_S = \varphi' \cos\sigma \tag{5}$$

where σ is the angle between the speckle shift direction and the wave vector of the fringes. Note that the fringes wave vector should be parallel to the speckle shift direction ($\sigma = 0$) in order to get the optimum measurement as we are considering in our discussion.

Let us analyze the modulated speckle behavior when a diffuser surface displacement along the z axis is produced. In our experimental conditions to generate the speckle distribution at Π plane, we set the axial distance $M = 50$ mm and the diameter of the illumination spot at the diffuser surface is $T = 1$ mm which gives an average transversal speckle size of $\langle S_{X-\Pi\ plane} \rangle \approx \lambda (M/T) \approx 50\,\lambda$ and $\langle S_{Z-\Pi\ plane} \rangle \approx \lambda (M/T)^2 \approx 2500\,\lambda \approx 1,25$ mm. These averages values established the range of the correlated speckle distributions. Note

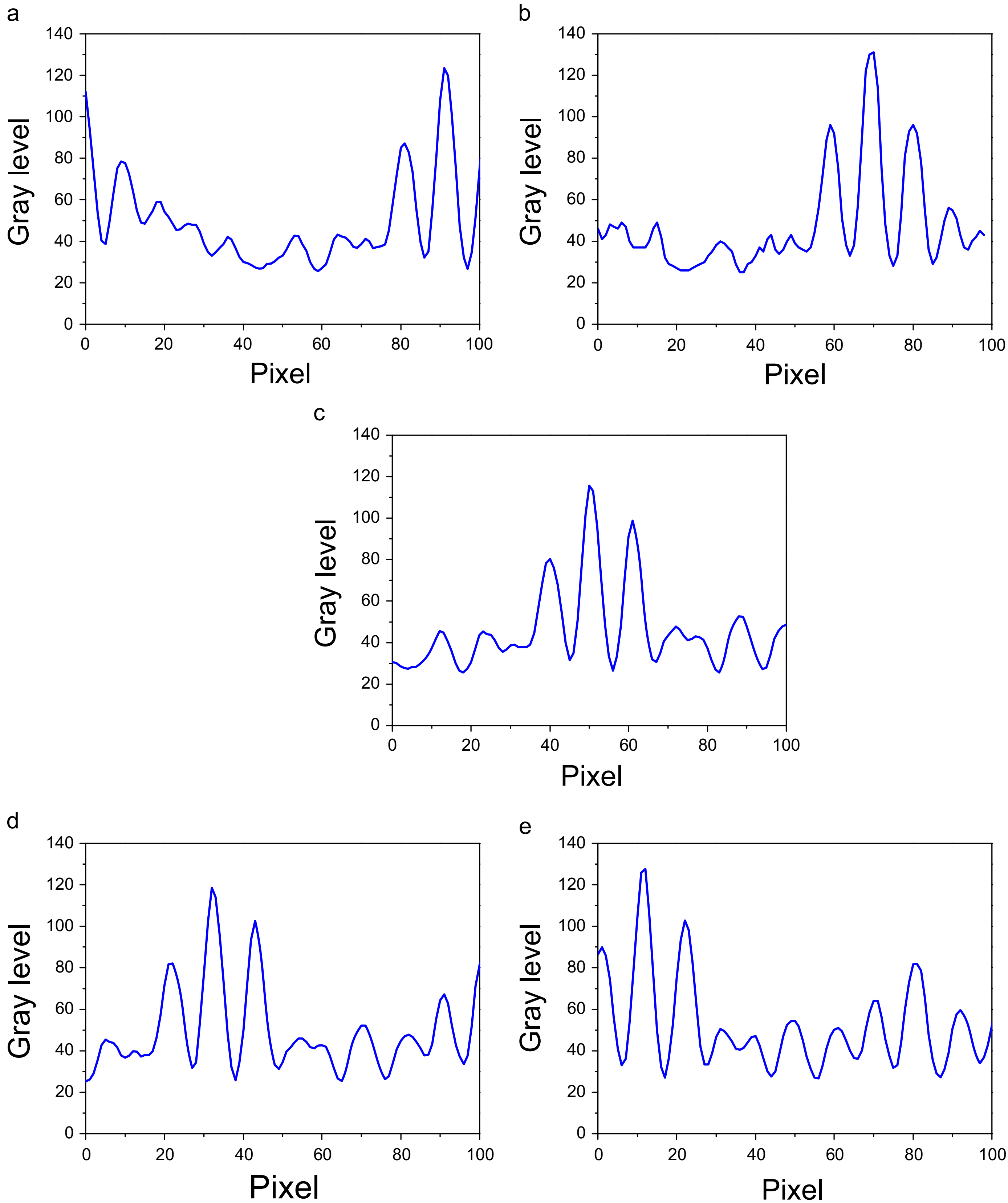

Fig. 3. Modulated speckle profiles corresponding to different tilt angles of the diffuser surface: (a) -12×10^{-4} rad; (b) -6×10^{-4} rad; (c) 0 rad; (d) 6×10^{-4} rad and (e) 12×10^{-4} rad.

that our method is applicable when the correlation condition is maintained. We evaluate the speckle pattern for different surface displacement z=0.2 mm, 0.4 mm, and 0.6 mm, as is illustrated in Fig. 4. Note that the mentioned longitudinal displacements have a negligible influence in the transversal speckle size at Π plane because the z-displacement is shorter than the speckle depth or

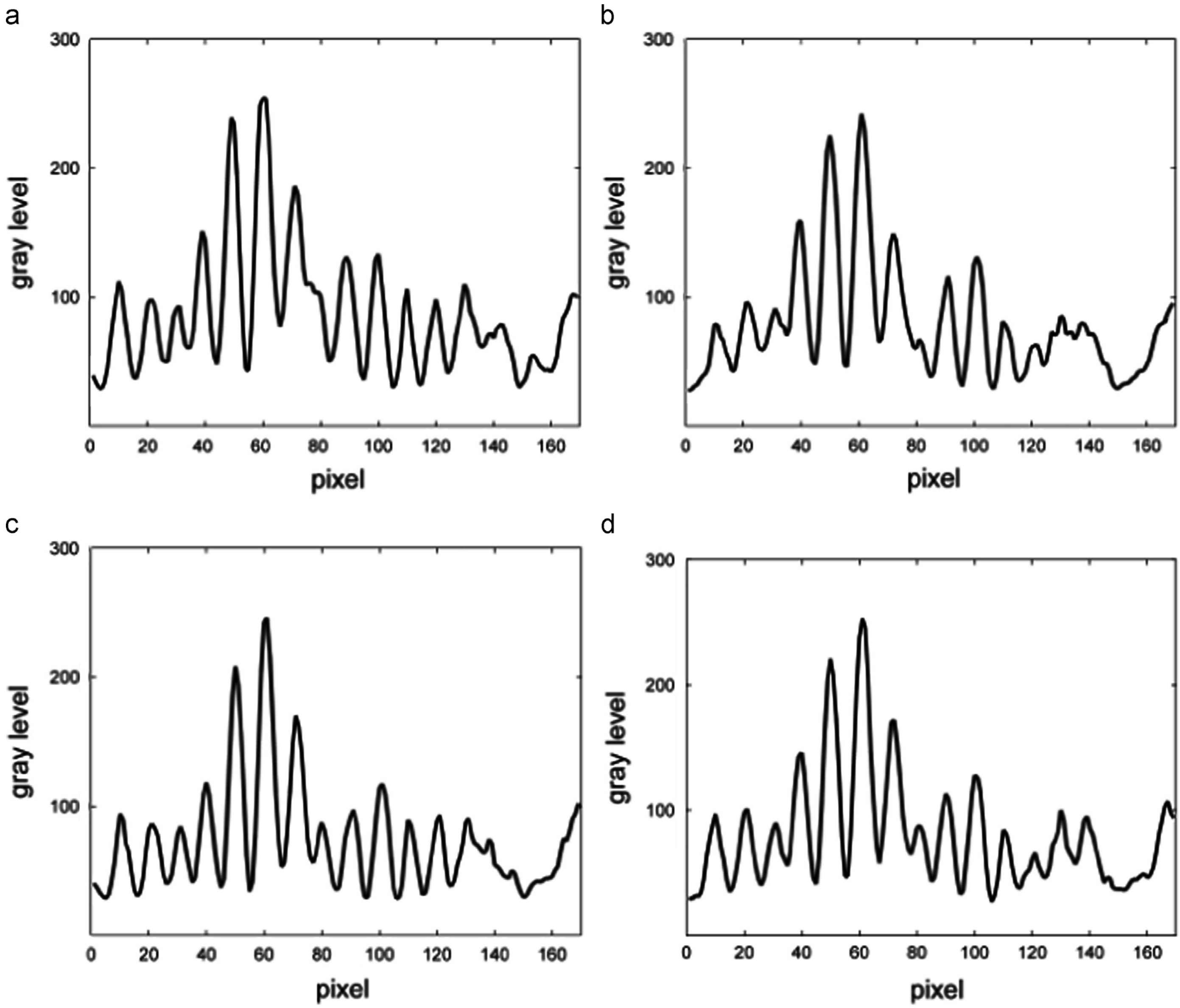

Fig. 4. Modulated speckle profiles corresponding to different diffuser displacements along *z* axis: (a) original diffuser position and (b) 0.2 mm; (c) 0.4 mm; (d) 0.6 mm with respect to the original position.

the longitudinal speckle size. This behavior can be confirmed in Fig. 4 where is observed that the modulated speckle does not change despite the different position the diffuser takes. In fact, the maximum change in the speckle depth size is about $(\Delta S_{Z-\Pi\ \text{plane}}/S_{Z-\Pi\ \text{plane}} -) \approx 2(\Delta M/M) \approx 0.03\ \%$.

The results of Fig. 5 correspond to a 12 mm displacement with respect to the position depicted in Fig. 4(a). Therefore, the object movement with respect to the reference position is greater than the longitudinal speckle size which is confirmed by the decorrelated pattern of Fig. 5. This decorrelation is confirmed by observing Fig. 6(b) that corresponds to the correlation curves obtained between the reference speckle and the speckle displaced 12 mm.

4. Conclusions

In order to detect tilt angles, a speckle image of a speckle pattern produced under free propagation by a reflective diffusing surface is generated in the optical system recording plane. The optical system has a double aperture pupil so that the speckle image is fringe modulated. In this arrangement a diffusing surface tilt change generates in the recording plane a speckle shift which produces a displacement of the modulated fringes. The method could be applied to locally detect the surface slopes of a diffusing object by mapping them as different fringe displacements.

The technique provides sub-speckle measurement precision, because the tilt is mapped into a modulated fringe displacement. In fact, the precision is controlled by the pupil apertures separation and pupil aperture diameter that determines the speckle modulation.

Note that a good agreement is obtained between the tilt produced with the rotating platform and the tilt obtained by using the fringe shift. The technique is particularly well suited when small tilting angles are considered. This condition implies a fully correlated pattern which is guaranteed under the small tilting cases.

In summary, the method senses slope by means of the fringe shifting. It should be emphasized the simplicity of the proposal and the good precision that can be obtained and the possibility to measure in an industrial environment.

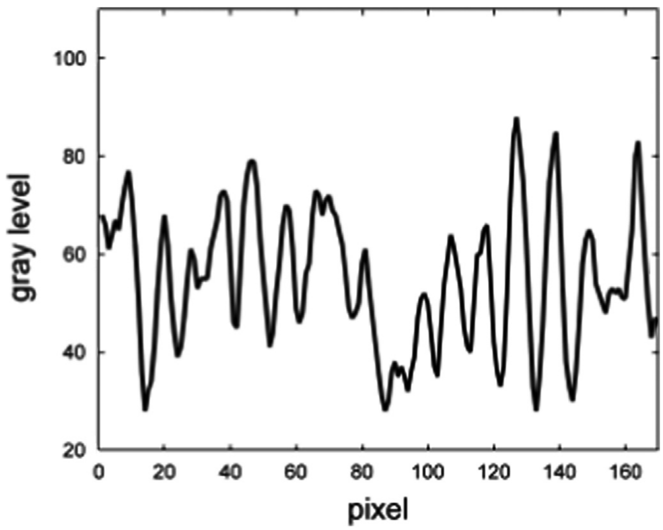

Fig. 5. Modulated speckle profiles corresponding to 12 mm diffuser displacement along z axis. This situation corresponds to a displacement larger than the speckle depth.

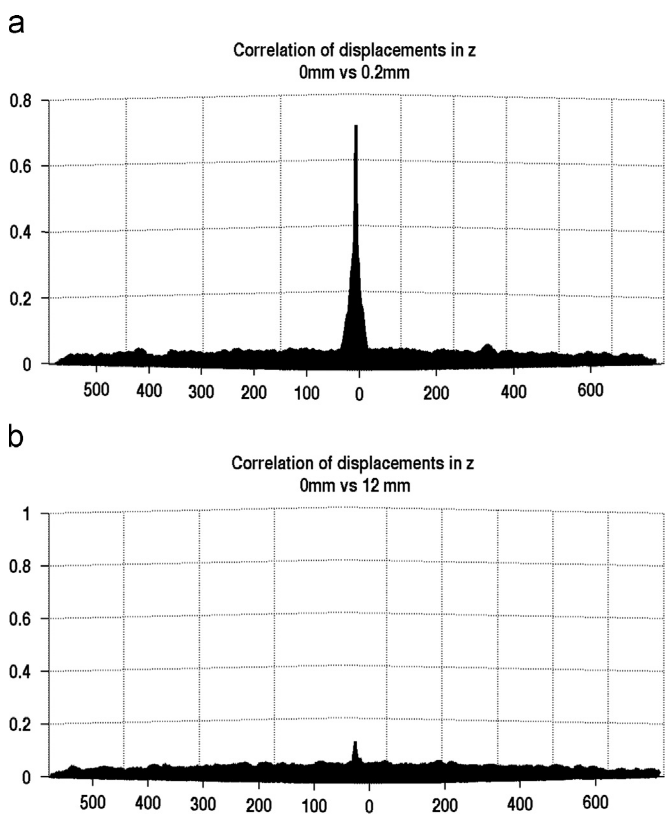

Fig. 6. Correlation curves between original modulated speckle image and the speckle image corresponding to the diffuser displacement along z distances (a) 0.2 mm and (b) 12 mm.

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