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LETTER

Generation and correlation algorithms for ternary complementary pairs of sequences of length $3{\cdot}2^n$

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SUMMARY

Binary complementary pairs of sequences, which are available for certain even length values ($L = 2^n \cdot 10^m \cdot 26^p$), are very interesting for applications with high noise levels and/or highly attenuated signals. To extend the range of useful lengths, some authors have defined ternary complementary pairs of sequences. These sequences have been approached from a theoretical perspective, but further research should be conducted regarding the architectures to process them. This letter describes algorithms for generating and performing the correlation of these sequences using a minimum amount of calculations. These algorithms allow to work with sequences of length $3 \cdot 2^n$, making it possible to attain a wider range of lengths, and, as a consequence, a wider range of noise immunity in some applications. The proposed approach uses a particular delay arrangement that minimizes the memory requirements in hardware applications. The procedure described could also be used in ternary complementary pairs generated from primitives of length different from 3.

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KEY WORDS: ternary complementary pairs of sequences; correlation; efficient algorithms

1. INTRODUCTION

Binary complementary pairs of sequences, also known as Golay sequences [1], have been widely used in the last two decades in applications where highly attenuated signals and/or signals immersed in noise have to be identified. Some examples of these applications are ground-penetrating radars [2], ultrasonic ranging systems [3], and safety sensors for transportation systems [4]. The sum of the autocorrelation functions of these sequences produces a Kronecker delta of amplitude $2L$ (L being the sequences' length), which is easily identifiable under low signal-to-noise ratio (SNR) conditions. After the correlation process, the SNR of the signal is improved by a $\sqrt{2L}$ ratio with respect to the original signal [5]. In this manner, noise immunity level can be incremented using a longer sequence.

Binary complementary pairs of sequences (BCPS) are limited to lengths $L = 2^n \cdot 10^m \cdot 26^p$, where n, m and p are natural numbers [6]. Despite the fact that many lengths are possible, lengths multiple of odd numbers (except 5 and 13) cannot be obtained. To do so, ternary complementary pairs of sequences (TCPS) could be used [7, 8]. TCPS are composed of elements -1, 0, and +1; and the sum of their autocorrelations also provides a Kronecker delta, but of amplitude $2L$-C, where C is the number of zero elements. All mathematical properties of the complementary pairs, except for Kronecker delta amplitude, remain unchanged regardless of the type of elements (binary or ternary). Even though the correlation delta amplitude is lower due to the presence of zero elements in one or both sequences, this zero padding is effective to reduce the inter-pulse interference in applications like Ultra WideBand [10], as it was demonstrated in the work by Cho *et al.* [9].

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If the matter is analysed from a practical viewpoint, there are no algorithms able to generate and perform TCPS correlations in an efficient manner. Di Wu *et al*. [11] proposed two different approaches for constructing mutually orthogonal complementary sets of ternary sequences, yet the author did not provide a generation or correlation algorithm for ternary pairs. Efficient algorithms for BCPS have been developed by different authors, in order to make the generation and correlation processes with a reduced number of calculations. The BCPS efficient generator proposed by Budisin [12] is a recursive algorithm that allows to generate any pair of sequences of length $L = 2^n$, using different permutations of delays. With regard to correlation, a recursive architecture called Efficient Golay Correlator (EGC) was proposed by Budisin [13] and Popovic [14] for different applications. Later on, two optimized correlator algorithms were proposed by the authors of this letter. One of them is the Optimized Golay Correlator (OGC) [15], which improves the Budisin/Popovic's algorithm; the other is the algorithm called O^2GC [16], which performs the simultaneous correlation of two uncorrelated BCPS ('uncorrelated' means that the sum of cross-correlation functions is zero [17]) with a considerable reduction of operations if compared to the straightforward correlation and the other efficient approaches.

This letter proposes algorithms for an efficient generation and correlation of TCPS of length $L = 3 \cdot 2^n$ using the Budisin approach and the OGC concept, respectively. Additionally, in the case of the correlation process, a TCPS correlator which can process two mutually uncorrelated TCPS on a simultaneous basis with a single architecture is developed, with notorious calculations reduction.

2. BINARY COMPLEMENTARY PAIRS OF SEQUENCES GENERATION

The BCPS generation process is based on primitive sets, also called kernels, which are the smallest size sets that fulfil the conditions to be complementary sets. Based on the Budisin approach [12] with the delays ordered in an increasing way, a BCPS of length $L = 2^n$ can be obtained with a recursive concatenation of sequences:

$$\underbrace{\begin{bmatrix} S_{1,n} \\ S_{2,n} \end{bmatrix}}_{\mathbf{S_n}} = \underbrace{\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}}_{\mathbf{H_2}} \cdot \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & w_n \end{bmatrix}}_{\mathbf{W_{2,n}}} \cdot \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & z^{-2^{n-1}} \end{bmatrix}}_{\mathbf{D_{2,n}}} \cdot \underbrace{\begin{bmatrix} S_{1,n-1} \\ S_{2,n-1} \end{bmatrix}}_{\mathbf{S_{n-1}}} \tag{1}$$

where $\boldsymbol{H_2}$ is the Hadamard matrix of order 2, $\boldsymbol{W_{2,n}}$ is the seed matrix with coefficients $w_n = \pm 1$, $\boldsymbol{D_{2,n}}$ is the delay matrix, and $\boldsymbol{S_n}$ is the vector of sequences, respectively. Despite the fact that delays permutation, as proposed in [12], allows to generate different types of BCPS, the use of delays ordered in an increasing manner is more convenient to optimize hardware applications. In this way, delays are arranged in a decreasing order in the correlation process, which allows to reduce the amount of memory bits in the implementation.

The iteration process starts ($n = 1$, $S_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$) with one of the only two existing binary primitives, which can be obtained by using a different seed coefficient:

$$\begin{aligned} w_1 = +1 \rightarrow \begin{bmatrix} S_{1,1} \\ S_{2,1} \end{bmatrix} &= \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & z^{-1} \end{bmatrix} \cdot \begin{bmatrix} S_{1,0} \\ S_{2,0} \end{bmatrix} \\ w_1 = -1 \rightarrow \begin{bmatrix} S_{1,1} \\ S_{2,1} \end{bmatrix} &= \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & z^{-1} \end{bmatrix} \cdot \begin{bmatrix} S_{1,0} \\ S_{2,0} \end{bmatrix} \end{aligned} \tag{2}$$

The successive iterations of algorithm (1) produce larger sequences, all of them generated from these primitives (2). It is worth noting that the sum of the cross-correlation functions between these primitives is null, so it is said that they are mutually uncorrelated [17]. In this manner, given a

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BCPS of any length $L \geq 2$, there is another pair of the same length, which is uncorrelated to the first one, according to the seed coefficient of the first iteration ($n = 1$).

3. EXTENSION TO TERNARY COMPLEMENTARY PAIRS OF SEQUENCES

Based on the BCPS, an analogy with TCPS can be drawn. As stated in the work by Craigen *et al* [8], TCPS are available for a wide range of primitives. Yet this letter analyses the particular case of primitives of length 3. According to Di Wu *et al.* [11], there are two mutually uncorrelated TCPS primitives of length $L = 3$:

$$\begin{bmatrix} S_{1,1}^{\alpha} \\ S_{2,1}^{\alpha} \end{bmatrix} = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 0 & 1 \end{bmatrix} \text{ and } \begin{bmatrix} S_{1,1}^{\beta} \\ S_{2,1}^{\beta} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & -1 & -1 \end{bmatrix} \tag{3}$$

To obtain TCPS of length $L \geq 3$, the same approach as that of BCPS can be applied, that is, a recursive concatenation algorithm (1). In this case, the primitives are different in length with regard to the binary case (2), so every delay element of $\boldsymbol{D_{2,n}}$ is a multiple of 3: $\begin{bmatrix} 1 & 0 \\ 0 & z^{-3\cdot 2^{n-2}} \end{bmatrix}$. The scheme of the complete generator for N iterations is shown in Figure 1.

Once the delays have been defined, the first algorithm iteration should be synthesized according to Eqn (3). Hence, it is necessary to include a generation seed of value ±1, in order to generate two uncorrelated primitives with the same architecture. Transforming (3) to the z-domain, the primitives are as follows:

$$\begin{aligned} \begin{bmatrix} S_{1,1}^{\alpha} \\ S_{2,1}^{\alpha} \end{bmatrix} &= \begin{bmatrix} 1 + z^{-1} - z^{-2} \\ 1 + z^{-2} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 2 + z^{-1} \\ z^{-1} - 2z^{-2} \end{bmatrix} \\ \begin{bmatrix} S_{1,1}^{\beta} \\ S_{2,1}^{\beta} \end{bmatrix} &= \begin{bmatrix} 1 + z^{-2} \\ 1 - z^{-1} - z^{-2} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 2 - z^{-1} \\ -z^{-1} - 2z^{-2} \end{bmatrix} \end{aligned} \tag{4}$$

Based on the mathematical decomposition of [18], both primitives can be synthesized, using the seed w_1 as a parameter, as follows:

$$\begin{bmatrix} S_{1,1} \\ S_{2,1} \end{bmatrix} = \left\{ \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & z^{-1} \end{bmatrix} \right\} \cdot \left\{ \begin{bmatrix} 1 & 0.5 \\ 0.5 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & w_1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & z^{-1} \end{bmatrix} \right\} \cdot S_0 \tag{5}$$

From (5), the primitive $\boldsymbol{S}^{\alpha}$ is obtained with $w_1 = 1$ and the primitive $\boldsymbol{S}^{\beta}$ with $w_1 = -1$. The architecture that implements the algorithm described in Eqn. (5) is displayed in Figure 2:

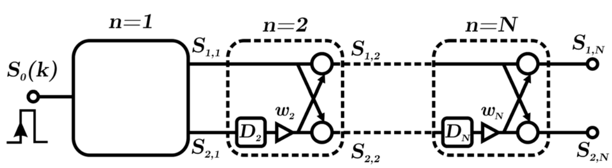

Figure 1. Modular ternary complementary pairs of sequences (TCPS) generation architecture based on (1) with delays multiple of 3.

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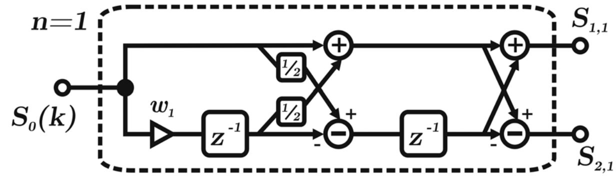

Figure 2. First stage ($n = 1$) of the ternary complementary pairs of sequences (TCPS) generation architecture (Figure 1).

Figure 3 illustrates a time plot of the signals at the output of the different stages of a TCPS generator, using $w_1 = 1$ and $n = 3$. Note that TCPS primitives are generated in the first iteration (top), and, in the following two iterations, the length of the pair is incremented to $L = 6$ (centre) and $L = 12$ (bottom).

4. TERNARY COMPLEMENTARY PAIRS OF SEQUENCES CORRELATION

Ternary complementary pairs of sequences correlation can be addressed using a similar approach to that used in BCPS. Taking into account that BCPS are processed by means of the sum of two correlations, it would be logical to use two straightforward correlators. However, when high values of L have to be used, the number of calculations increases considerably. To avoid the implementation of straightforward correlations and enhance the use of resources in BCPS applications, some efficient correlation architectures have been developed, such as EGC [14] and OGC [15]. Particularly, the OGC approach relies on an inverse generation process, formulated as a recursive algorithm of N stages, which can be employed to obtain the sum of the autocorrelations with substantial calculation reduction.

In the case of TCPS, the OGC approach is valid for all the iterations, excluding the last one ($n = 1$), because the first stage of the generator is the only different one. So, for the first N-1 stages, the correlation algorithm is

$$\underbrace{\begin{bmatrix} C_{S\,1,n-1} \\ C_{S\,2,n-1} \end{bmatrix}}_{\mathbf{C}_{n-1}} = \underbrace{\begin{bmatrix} z^{-3\cdot 2^{n-2}} & 0 \\ 0 & 1 \end{bmatrix}}_{\mathbf{D}'_{2,n}} \cdot \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & W_n \end{bmatrix}}_{\mathbf{W}_{2,n}} \cdot \underbrace{\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}}_{\mathbf{H}_2} \cdot \underbrace{\begin{bmatrix} C_{S\,1,n} \\ C_{S\,2,n} \end{bmatrix}}_{\mathbf{C}_n} \tag{6}$$

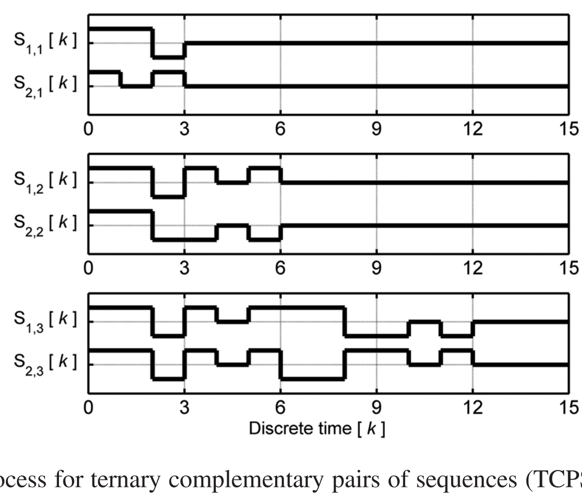

Figure 3. Generation process for ternary complementary pairs of sequences (TCPS) of lengths 3, 6 and 12.

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$\boldsymbol{C_n}$ represents the partial results of the correlation process, where the correlation is obtained at the output $\boldsymbol{C_0}$. Note that the delay matrix, $\boldsymbol{D'_{2,n}}$, is modified with respect to that used in the generation (1), and that the delays are also multiples of 3. Figure 4 represents the architecture corresponding to this correlation algorithm.

To complete the process, the penultimate output ($C_{S1,1}$, $C_{S2,1}$) should be correlated with the corresponding primitive. To complete this step efficiently, it is convenient to start by formulating the sum of the straightforward correlations:

$$Y[k] = \underbrace{\sum_{l=1}^{L} S_{1,1}[l] \cdot C_{S1,1}[k+l]}_{C_{S1,0}} + \underbrace{\sum_{l=1}^{L} S_{2,1}[l] \cdot C_{S2,1}[k+l]}_{C_{S2,0}} \tag{7}$$

Taking into account that the primitive length is $L = 3$, the sum of the autocorrelations for the seed $w_1 = 1$ is

$$Y[k] = \left(C_{S1,1}[k] + C_{S1,1}[k+1] - C_{S1,1}[k+2]\right) + \left(C_{S2,1}[k] + C_{S2,1}[k+2]\right) \tag{8}$$

Transforming into z-domain:

$$Y(z) = \left(C_{S1,1} + C_{S1,1}z^{1} - C_{S1,1}z^{2}\right) + \left(C_{S2,1} + C_{S2,1}z^{2}\right) \tag{9}$$

The equation is reformulated in negative powers of z, in order to obtain a computationally feasible equation represented as time delays:

$$Y(z) = C_{S1,1}\left(-1 + z^{-1} + z^{-2}\right) + C_{S2,1}\left(1 + z^{-2}\right) \tag{10}$$

A similar reasoning can be applied to the other generation seed. Then, the sum of the correlations of the last stage is performed with respect to the value of the generation seed, depending on its sign:

$$\begin{aligned} w_1 &= +1 \rightarrow Y^{\alpha}(z) = \left(-C_{S1,1} + C_{S1,1}z^{-1} + C_{S1,1}z^{-2}\right) + \left(C_{S2,1} + C_{S2,1}z^{-2}\right) \\ w_1 &= -1 \rightarrow Y^{\beta}(z) = \left(C_{S1,1} + C_{S1,1}z^{-2}\right) + \left(-C_{S2,1} - C_{S2,1}z^{-1} + C_{S2,1}z^{-2}\right) \end{aligned} \tag{11}$$

Ordering the terms

$$\begin{aligned} Y^{\alpha}(z) &= \left(-C_{S1,1} + C_{S2,1}\right) + C_{S1,1}z^{-1} + \left(C_{S1,1} + C_{S2,1}\right)z^{-2} \\ Y^{\beta}(z) &= \left(C_{S1,1} - C_{S2,1}\right) - C_{S2,1}z^{-1} + \left(C_{S1,1} + C_{S2,1}\right)z^{-2} \end{aligned} \tag{12}$$

Note that both sums of correlations (12) between the outputs of the penultimate iteration ($n = 2$) and the TCPS primitives (3) are calculated with the same terms, but with some different signs, which are in

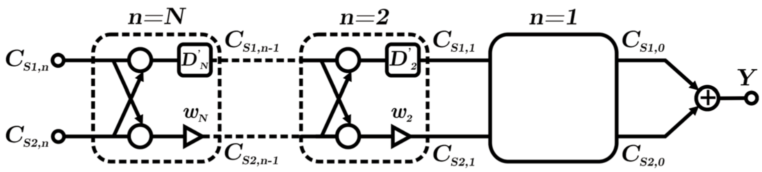

Figure 4. Modular correlation architecture based on (6).

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agreement with seed w_1. In order to make a design in accordance with the generation, Eqn (12) is reformulated using a seed coefficient to discriminate between the two primitives:

$$Y = \left(C_{S2,1} - C_{S1,1}\right)w_1 - \frac{(1-w_1)}{2}C_{S2,1}z^{-1} + \frac{(1+w_1)}{2}C_{S1,1}z^{-1} + \left(C_{S1,1} + C_{S2,1}\right)z^{-2} \tag{13}$$

The architecture that implements (13) is shown in Figure 5.

An easy way to understand the correlation process using the architecture in Figures 4 and 5 is by means of a time plot (Figure 6). A pair of ternary sequences of length $L = 12$ (top) is processed, and pairs of shorter length and more amplitude are obtained. Note that, at the output of the penultimate stage ($C_{S1,1}$; $C_{S2,1}$), the original primitives of the TCPS are recovered, but of amplitude ±4. Finally, a Kronecker delta of amplitude $2L\text{-}C = 20$ is obtained at the output $Y[k]$, delayed L samples.

5. TERNARY COMPLEMENTARY PAIRS OF SEQUENCES SIMULTANEOUS CORRELATION

In the BCPS correlation architecture, the last stage is reformulated using another architecture derived from OGC, known as O^{2}GC [16], which performs the simultaneous correlation of two uncorrelated pairs. In the case of TCPS, the architecture that performs Eqn (13) can be optimized using a similar approach to that of O^{2}GC. Considering that the seed coefficient only assumes values ±1, an efficient architecture that simultaneously performs the correlation of two uncorrelated TCPS is derived (Fig. 7).

With reference to the efficiency of the proposed correlators, the architecture based on Eqns (6) and (13) represents notorious reduction with respect to the straightforward correlation approach. To

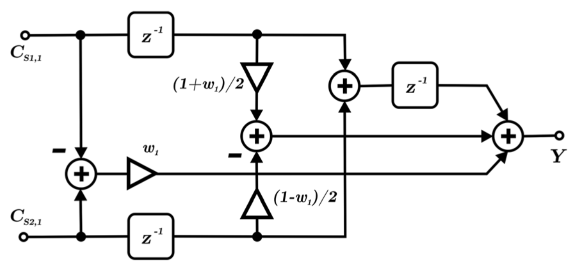

Figure 5. Last stage ($n = 1$) of the ternary complementary pairs of sequences (TCPS) correlator (Figure 4) with the seed coefficient included.

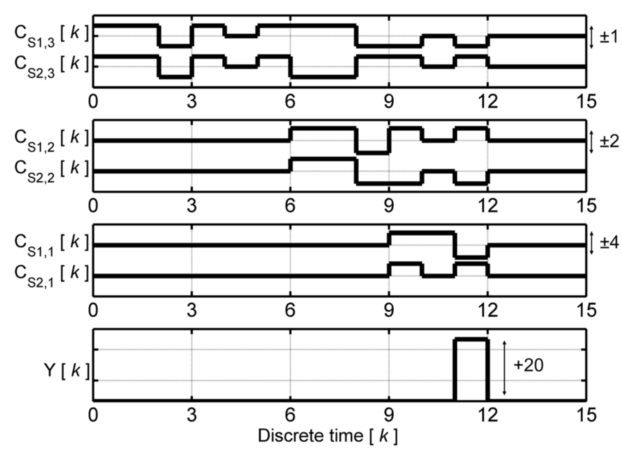

Figure 6. Correlation process for a ternary complementary pairs of sequences (TCPS) of length $L = 12$.

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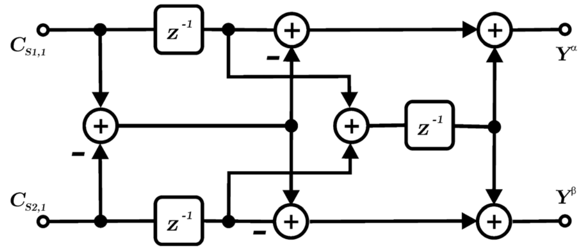

Figure 7. Last iteration ($n = 1$) of the ternary complementary pairs of sequences (TCPS) correlator (Figure 4), which performs the correlation of uncorrelated pairs in a simultaneous way.

Table I. Comparison between the straightforward approach and the algorithms proposed.

	Delays	Additions/subtractions
Straightforward correlation	$4 \cdot (L-1)$	$4 \cdot L + 1$
Correlation based on Figures 4 and 5	$2 \cdot (L-1)$	$2 \cdot log_2(L) + 5$
Correlation based on Figures 4 and 7	$L-1$	$2 \cdot log_2(L) + 6$

correlate with respect to two uncorrelated TCPS, the proposed correlator employs a single architecture with $L-1$ delays and $2 \cdot log_2(L) + 6$ adders/subtractions (the last 6 adders/subtractions are the ones corresponding to the architecture shown in Figure 7). The classical approach, on the other hand, involves four straightforward correlations, two for one of the TCPS and two for the other one, with $4 \cdot (L-1)$ delays and $4 \cdot L + 1$ adders/subtractions. Considering that complementary sequences provide greater immunity to noise with longer lengths, operations reduction is very important. Table I compares the amounts of delays and additions/subtractions necessary to process two uncorrelated TCPS with the classical straightforward approach, and the two algorithms proposed in this letter.

6. CONCLUSIONS

This letter puts forward algorithms to generate and correlate TCPS in an efficient way. The proposed architectures allow to process TCPS of kernel 3 by using recursive algorithms that use less calculations than the classical straightforward approach does. Moreover, a correlator performing a simultaneous correlation of the sequences with respect to the two uncorrelated primitive sets is also developed. This contribution constitutes a significant step towards the practical application of these sequences and extends the application field of complementary sequences from the well-known binary cases to the less used ternary sequences. The proposed algorithms can be extrapolated to different TCPS primitive lengths, allowing to obtain efficient generation/correlation architectures for any TCPS.

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