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## THE ROYAL SOCIETY

# How ants move: individual and collective scaling properties

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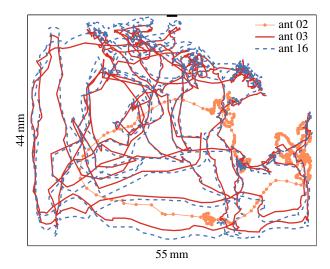
The motion of social insects is often used as a paradigmatic example of complex adaptive dynamics arising from decentralized individual behaviour. In this paper, we revisit the topic of the ruling laws behind the burst of activity in ants. The analysis, done over previously reported data, reconsiders the causation arrows, proposed at individual level, not finding any link between the duration of the ants' activity and their moving speed. Secondly, synthetic trajectories created from steps of different ants demonstrate that a Markov process can explain the previously reported speed shape profile. Finally, we show that as more ants enter the nest, the faster they move, which implies a collective property. Overall, these results provide a mechanistic explanation for the reported behavioural laws, and suggest us a formal way to further study the collective properties in these scenarios.

## 1. Introduction

The behaviour of social insects constitutes beautiful examples of adaptive collective dynamics born out of apparent purposeless individual behaviours [1,2]. The richness of the phenomena offers plenty of opportunities to test the theories dedicated to understand the fascinating aspects of collective organization and behaviour. An emergent issue is to dissect the dynamics that are routed on individual versus those explained by collective forces. (A recent review can be found in [3].) A relevant perspective allowing us to point out the effects of collective behaviour is the analysis of insects' speed. For instance, the walking speed of an ant changes after interaction with other ants [4], where the change depends on the speed of the second ant. Consequently, it has been suggested that a building up in speed, following the interaction with fast returning foragers, might be the basis of an ant's decision to leave the nest for a foraging trip [4,5].

Recent work [6,7] found that ants' bursts of activity moving both inside and outside their colony's nest exhibit a power-law relation between the duration of the activity and its average speed. Successive motion events, defined as the segment of data between two consecutive motionless instances, were found to obey a universal speed shape profile. The authors concluded that this predictability implies that the duration of each ant's movement is 'somehow determined before the movement itself' [7], thus placing important weight into the individual ant's spontaneous behaviour.

In this work, we revisit the topic proceeding to analyse the same datasets as in [6] to disambiguate the individual properties that can be judged as cognitive from the features that can be potentially based on other aspects of the animal's physiology. A null model is introduced allowing us to create synthetic trajectories by adding steps of different ants selected at random. The results of the analysis suggests an alternative explantation simpler than the causation arrows between the ants' duration of activity and their speed proposed in [6]. The origin of the universal shape profile of the speed can be reconnected to a Markov process taking into account the observed ants' speed auto-correlation that is based on the animal's physiology.



**Figure 1.** Examples of typical ants' trajectories: three ants from colony  $C_{55\times44}^2$ , after entering the nest from the point marked with a small bar (middle region of the top side). Notice the movements of two of them (ants 03 and 16) which are manifestly correlated. (Online version in colour.)

Finally, we show a novel and distinctive collective effect by which as more ants enter the nest, the faster they move.

## 2. Methods

#### 2.1. Data

The data re-analysed here were already described in [6], and correspond to Temnothorax albipennis ants, which form small colonies in rock crevices, approximated in the laboratory by a  $100 \times 100 \text{ mm}^2$ Petri dish where food and water are available at will. Ant workers were identified by markers with unique combinations of colour paint dots. Ants' individual trajectories were video-recorded within two nest sizes:  $35 \times 28 \text{ mm}^2$  and  $55 \times 44 \text{ mm}^2$  in a randomized order from each of three colonies (labelled  $C^1$ ,  $C^2$  and  $C^3$ , and indexed according to the nest sizes used). Ants' trajectories were reconstructed manually from the video with a cursor with a time resolution of 0.1 s. To compare with previous work [6,7], we use similar preprocessing steps. The data were coarse-grained to an average sample interval of 0.8s in order to reduce the very small-scale fluctuations. An 'activity event' was defined using a speed threshold  $v_e = 0.001 \,\mathrm{mm \, s^{-1}}$ , below which it indicated that the ant is inactive.

Typical ants' trajectories are plotted in figure 1. Figure 2 shows examples of the speed changes for a group of ants during the first minute recorded. It can be seen that the ants' motion is discontinuous and irregular, eventually stopping and restarting their walk. Indeed, decreases in the ant's speed are related with the so-called marking behaviour in which the ant touches the tip of its gaster to the surface, a known property in various ant species [8–11].

#### 2.2. Model

The causality argument proposed in [6,7] is supported by the comparison with what we can call a *zeroth order* null model, where synthetic realizations of the ants' trajectories were to be made by randomly extracting speeds from the distribution P(v(t)) thus ignoring the possible correlations with the current state of the movement.

Here, we propose an improved *first-order* model taking into account the fact that auto-correlation plays a strong role in animal movement processes. The model is based on the statistical reconstruction of return map for the ants' speeds. This map can be observed in figure 3*a*, where we plot for all time series, all

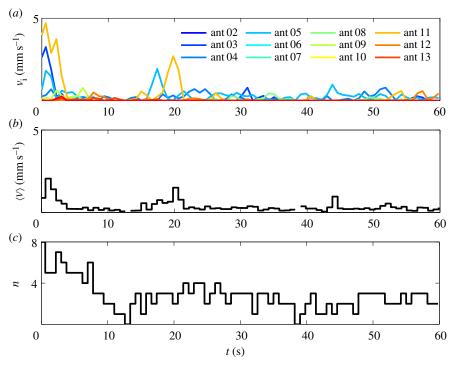
pairs of consecutive values of speeds v(t), v(t+1) (dots in figure 3a). Subsequently, values of v(t) were binned and its corresponding v(t + 1) averaged (circles in figure 3a). Similarly, to construct the synthetic speed trajectories of the null model, we define a conditional probability distribution for the subsequent velocities P(v(t+1)|v(t)) from the data with bins of  $\Delta v =$ 0.1 mm s<sup>-1</sup>. A special bin is used for speeds after rest events (that is, speeds below the threshold  $v_e = 0.001 \,\mathrm{mm \, s^{-1}}$ ). Then, we describe an ant's speed as a Markov process which draws from the binned distribution, using the following iterative procedure: (i) an initial value of speed v(t) is randomly chosen from the bin of speeds following a rest event. (ii) The next speed is determined by randomly choosing a v(t+1) inside the corresponding bin of P(v(t+1)|v(t)). (iii) Such value is then considered as the new v(t)for the new iteration. (iv) If  $v(t+1) < v_e$ , the movement is considered completed. (v) When a movement is finished, a rest time  $\tau$  is extracted at random from the experimental data. (vi) After a time  $\tau$  with no movements, we restart from the first step.

## 3. Results

The results are organized as follows: first we describe the observation that can be derived by analysing the map of discrete consecutive speeds from the raw data. This map is used to define the *first-order* null model, which is relevant to isolate the role of auto-correlation in the individual ant's dynamic. After that, we compare the scaling statistics for both processes: the recorded and null model simulated ants' trajectories, including the event duration, speed, intervals and shape. Finally, we describe a novel dependence of speed as a function of the number of ants moving.

Despite the wide scatter of the data points in figure 3a, the average  $f(v_{(t)})$  (circles and continuous line) make evident the auto-correlation between consecutive steps. Thus, the average dynamics of the ants' speed can be, in part, described by the average map f; i.e. values larger than  $v^*$  (arrow) tend, on average, to decrease (i.e. the average value stays below the identity line). For values of speed smaller than  $v^*$ , there are two tendencies: if the ant is moving, then it will speed up in the next time step. For the case that the ant is not moving, we cannot say anything about when it will start moving (although see below); nevertheless, whenever it does it, it will do it with an average speed of the order of  $v^*$  as indicated by the circled symbol at the origin of figure 3a.

The map f in figure 3a can be used to test the potential origin of the fluctuations in the ants' speed, constructing time series of speeds following a given null hypothesis. Following [6,7], the variables of interest are the individual bouts of motion, defined by the speed fluctuations of a given ant, starting and ending with immobility (see Methods). For instance, figure 3b shows a real event (circles and continues line) lasting 20 s. The other six traces (dashed lines), starting at 4 s, are stochastic realizations constructed using the map of figure 3a. Specifically, time series of speeds were computed iteratively: at each step the next speed value  $v_{(t+1)}$  is calculated as follows: given a speed  $v_{(t)}$ , the next speed is randomly selected from the subset of  $v_{(t+1)}$  values corresponding to the bin  $v(t) - \Delta v/2 < v(t) < v(t) + \Delta v/2$  with  $\Delta v = 0.1 \,\mathrm{mm \, s}^{-1}$ . The computed value became the new current speed and the iteration proceeds. It can be seen, in the example, that the trajectories of speeds constructed this way vary in both size and duration. It is important to note that this first-order null model breaks some, but not all, correlations present in the



**Figure 2.** (a) Speeds of the ants moving in colony  $C_{55\times44}^1$  during the first recorded minute. (b) Whenever at least a single ant is moving we can define the average speed  $\overline{v}(t)$ . (c) The number n of moving ants. (Online version in colour.)

real ants' walks, as shown in the map in figure 3*a* that preserves the average serial correlations in the speed.

As will be discussed later, the stochastic procedure used in our *first-order model* shuffles, in a sense, the 'decisions' of individual ants at each time step. This deletes any predetermined *individual* plan (as argued in [7]) about how long and fast would be any given bout of motion, while maintaining some of the properties of the speeds auto-correlation (see the electronic supplementary material). Very different results are observed (see the electronic supplementary material) if the synthetic realizations were to be made with the *zeroth-order* null model by randomly choosing speeds regardless of their preceding one, as done in [6].

Armed with an appropriate null model, now we can proceed to analyse the ants' data and to compare the statistics with synthetics time series. Of interest here are the 'events', which are defined as the bouts of activity starting and finishing on a quiet instance (recall that quiet was defined as any speed below the  $v_e$  threshold as in [7]).

First, we compute the densities of event's duration T, the event's size S (that is, the total distance covered during the motion event), and their mutual relation, which are plotted in figure 4a-c, respectively. Overall, the results show that the null model agrees very well to replicate more than the 95% of the real events, while fails to replicate the empirical probabilities of the longest lasting events (of the order of  $1 \times 10^{-4} < P < 1 \times 10^{-2}$ ). This disagreement might be a consequence of the higher order correlations which are not taken into consideration in our *first-order* model. Indeed, as we see in the electronic supplementary information, the empirical auto-correlation function decays slower than in our model, which can possibly lead towards longer moves.

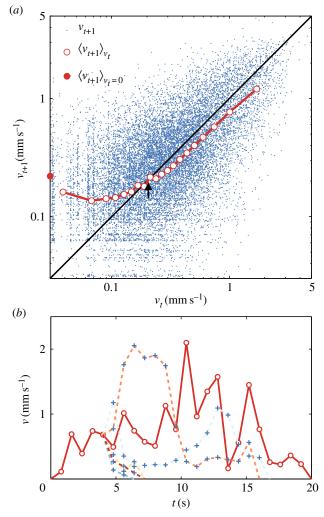
Looking at the scaling relations in figure 4c, is apparent that the exponents could be consistent with a stochastic process where changes in speed are given by an uncorrelated noise  $\Delta v(t+1) = v(t+1) - v(t) = \xi(t)$ , which would yield  $v(T) \propto T^{0.5}$  and therefore  $S(T) \propto T^{1.5}$  [12]. However, the fact

is that *consecutive* speed samples of the raw data are correlated, as demonstrated by the average map of figure 3a. Because of that, it is of less interest to consider the zeroth-order null model in which these correlations are ignored and the time series are built by random shuffling of each raw time series of speeds, where speeds become independent by duration (see the electronic supplementary material and fig. 8 of [6]).

#### 3.1. Shape collapse

The functional relation between the lifetime of an event and its size shown in figure 4c, observed in both real and null model implies that the time series of speed is self-affine [13]. Thus, after appropriate rescaling, an average shape descriptive of the events shall be extracted, as was reported earlier for this data [6], as well as for human activity [14]. We observe in figure 5 that equally satisfactory collapse of the trajectories can be obtained in both cases, the null model (figure 5a) and the raw data (figure 5b). However, there is a small, but relevant, difference. While the shape function of the model is more similar to the expected inverted parabola, the one derived from real walks exhibits a plateau. A less notable feature, but common to both cases, is the presence of a slight asymmetry. These similarities and differences will be discussed later on.

The results so far indicate that the scaling relation and the characteristic shape or the bouts of ant motion are indistinguishable from the results of the null model trajectories in which consecutive speed increments are shuffled at each step among different ants and moments in time. This similarity is one the main results and clearly goes against the interpretation given in [7] when stating that 'ants determine their next move at rest' assigning a cognitive root to the observed universality. To remark the contradiction, note that the null model can be visualized 'as if' at each step each ant uses another randomly chosen ant's decision for its own next step. The observed universal characteristics of ants' movement can be reconstructed using as the only ingredient the

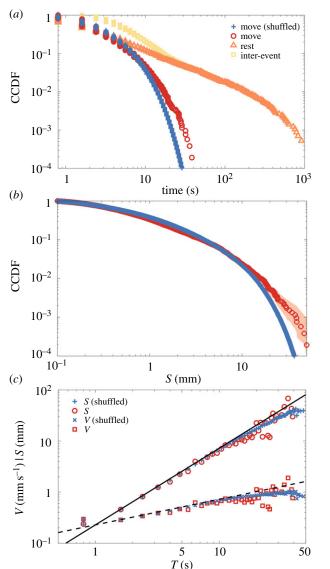


**Figure 3.** Extracting the null model map. (a) Return map extracted from the raw data by plotting consecutive v(t) samples (dots) and its binned average  $f(v_{(t)})$  (circles and continuous line) over-imposed (note the log axis). (b) An example of a real event v(t) (solid line with circles) and six synthetic trajectories (dashed lines). In this example, differently from the implementation in our simulations, the trajectories are the same until t=4 s. Then they evolve independently following our model starting from the real trajectory with initial condition  $v_0=v(4\ s)$  and selecting subsequent steps from the map in panel (a). Note that synthetic trajectories vary in both size and duration. Data from colony  $C_{35\times 28}^2$ , the scaling for other colonies are displayed in the electronic supplementary material. (Online version in colour.)

auto-correlations of speeds, whose origin is probably to be found in the animal's physiology.

## 3.2. Speed dependency with *N*

We observed, in the two colonies with the largest number of ants (but independently from the nest size, see the electronic supplementary material), an additional effect concerning the dependency of speed as a function of the number of ants moving. Figure 6a shows that as this number increases, the average speed also increases. Specifically, at each time t we count the number n of ants in motion, and we compute their average speed  $\bar{v}(t)$ . The quantity  $\langle \bar{v} \rangle_n$  represents the average speed of moving ants in a moment where n ants are moving. To disambiguate the origin of this dependency, we compared the growth in speed observed for the 22 ants tracked in colony  $C_{35\times28}^2$  with the same analysis performed on the same number of artificial time-series obtained from our null model. It is seen (figure 6b) that the speed for the artificial trajectories

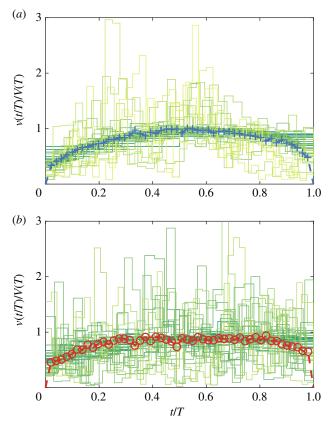


**Figure 4.** Real and null model trajectories exhibit very similar scaling statistics. (a) The complementary cumulative distribution function (CCDF) of the events duration T, the inter-event time and rest time, for shuffled trajectories and real ants. (b) The CCDF of the events size S for real ants and the null model. The orange shaded area indicates the 95% confidence interval for the synthetic CCDF computed via bootstrap. The difference between the two curves is restricted to the very few relatively long events. (c) The scaling of the event size and speed as a function of event duration for the raw data as well as for the null model shows no relevant differences. Continuous and dashed lines correspond to slopes of  $\frac{3}{2}$  and  $\frac{1}{2}$ . Data from colony  $C_{35\times28}^2$ , the scaling for other colonies are displayed in the electronic supplementary material, where we also report a dependence in the numerical value with the nest size already pointed out in [6]. In the simulated data, we generated 1 million synthetic events. (Online version in colour.)

constructed with the null model is independent from the number of ants moving. In this way, we can safely exclude the possibility that it is an artefact related to the larger average speed of longer trajectories; instead the results argue in favour of an effect emergent from the ants' interaction.

#### 4. Discussion

The main findings can be summarized along three points. First, from an individual perspective, the similarity between the

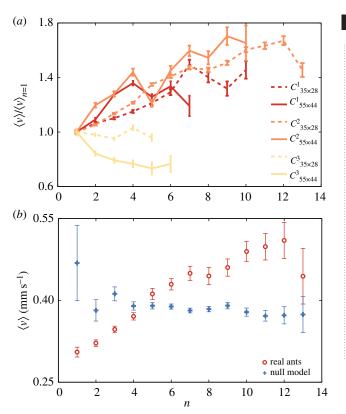


**Figure 5.** The normalized average speed profile for shuffled trajectories (a) and real ants (b) for colony  $C_{35\times28}^2$ , other colonies are displayed in the electronic supplementary material. As observed previously in [6], after rescaling, all the real event (b) profiles collapse to a universal function that presents a plateau close to the average speed at the middle of the trajectory. The same collapse can be observed in the synthetic trajectories in panel (a) that also display a similar skew (see electronic supplementary material) even if the shape of the curve is less flat. (Online version in colour.)

first-order null model and real ants results takes considerable weight from the previous suggestion indicating that the duration of each ant's movement is 'somehow determined before the movement itself' [7]. The proposed causation arrows in previous reports are not found in our results, as the universal shape of the v(t) curve is not a behavioural feature but the necessary consequence in movements characterized by random accelerations integrated over time.

Indeed, at *each* step of the synthetic data, ant *i* future speed is determined by the decision made in another moment by another ant *j*, which completely ignores ant *i*'s past history or determination. It is clear that under such conditions any informed plan or determination taken by ants is hard to sustain.

Second, despite the similarities, it is shown that the average shapes of the individual movement events exhibit a small but relevant difference with respect to the synthetic data. Figure 5b shows that the shape of the empirical events is not completely parabolic as in figure 5a, which is expected from a random walk. It is, instead, rather flat. This type of plateau, according to the analytical results of Baldassarri [12], is expected for a particle executing a random walk on a potential well, in which the restoring 'force' depends on the value of v. In simple words, relatively large increments of v are harder for larger absolute values of v, something that make also sense from a biological standpoint. In passing



**Figure 6.** 'The more the faster'. (*a*) When more ants are in movement, they move faster. We observe that for the two larger colonies,  $C^1$  and  $C^2$ ,  $\langle \bar{\nu} \rangle_n$  grows with n. (*b*) The null model shows that growth of speed with N is not due to single ants' behaviour. (Data from colony  $C_{35\times28}^2$ , other colonies are displayed in the electronic supplementary material.) (Online version in colour.)

notice that the slight asymmetry is not expected for the fully stochastic memoryless linear process in [12].

Third, the analysis unveiled a new observation in this type of data, dubbed here 'the more the faster' effect; as more ants are in motion in the arena, the faster they move. This result is clearly a collective property, which is consistent with a number of theoretical ideas on trail formation, where at a certain density of agents directed motion (and subsequent individual speeds increase) emerges [15,16]. It is our hope that this observation would encourage further experimental studies on the link between speed and colony dimensions, with the particular caution of forming differently sized groups of ants from the same colony, instead of from different colonies, to control for variation among field colonies.

Overall, these results provide a mechanistic explanation for the reported behavioural laws, and suggest a formal way to further study the individual and collective properties of animal trajectories. In particular, the modelling approach we proposed, based on the description of movement patterns as a random process characterizing the evolution of speeds in time, can be enlightening for the open discussion on Lévy flights as models of animal movement [17–19].

The Lévy flight hypothesis, proposing a universal scale-invariant character of animal motion, does not account for the interaction between different animals [20] and for the auto-correlation in consecutive moves [21]. A deeper quantitative understanding of animal movement behaviour can be gained by going beyond the simple characterization of movement patterns in terms of step-length distributions [22]. A model of randomly accelerated walkers, describing the

evolution of speeds as acceleration kicks at random times, already proved successful in showing that the longstanding interpretation of human displacement as Lévy flights was incorrect [23].

The spontaneous movement of animals should necessarily be seen as the product of a continuous decision-making process, adjusting the organism's behaviour in response to both past and present events [18]. This perspective requires accurate analysis of the experimental data, which must be recorded consistently with the spatial and temporal scales of the organism's movement decisions [24,25].

Data accessibility. All data are accessible from the authors upon request. Ants' data have been already shared in [6] and are available at https://data.bris.ac.uk/data/dataset/cmcs6znssfim12zo6zzmur1hq. Authors' contributions. Both authors conceived the study, participated in data analysis, wrote the manuscript and gave final approval for publication.

Competing interests. Authors declare no competing interests.

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