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Letter to the Editor

Comments on "Faber series method for two-dimensional problems of an arbitrarily shaped inclusion in piezoelectric materials" by C.-F. Gao and N. Noda (Acta Mech. 171, 1–13, August 2004)

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The writers wish to congratulate the authors for their rigorous treatment of an important and difficult problem [1]. As stated by Gao and Noda, the inclusion problem is of fundamental importance in engineering sciences and has received considerable attention from both theoreticians and experimentalists.

The authors have developed a successful approach based on the Faber-series method for the anti-plane problem of an arbitrarily shaped piezoelectric inclusion in an infinite matrix and the general solution for an arbitrarily shaped piezoelectric inclusion in an infinite elastic matrix. After that they present numerical results for several special cases of inclusion including elliptical and approximate square and equilateral triangle inclusions.

In the case of the previously mentioned regular polygonal shape the authors have used approximate mapping functions available in [2], which provide reasonable geometries from a rather qualitative continuum mechanics approach. But considerably more accurate results may be obtained if the following straightforward approach is used to obtain the analytic functions which map the infinite ζ-plane with a hole of unit radius onto the infinite z-plane with a hole of regular polygonal shape of apothegm a_p [3], [4].

The mapping function $f(\zeta)$ may be approximated by a truncated polynomial of the form

$$z = a_p \sum_{0}^{N} a_{1-jp} \zeta^{1-jp}, \quad \zeta = re^{i\theta}, \tag{1}$$

where p is the number of axes of symmetry of the configuration.

In order to determine the a_{1-jp}-s one follows the simple procedure of requiring that the real part of Eq. (1) be equal to a_p at a convenient number of points S. In the case of Fig. 1 one has taken $S = 4$.

In the case of point D located at an azimuth π/p, it is in general convenient to require that $\mathrm{Re}\, z = \alpha\, a_p$, $\alpha < 1$, since the corner constitutes a singular point from a mathematical viewpoint on one hand and will introduce high concentration factors on the other, from a physical viewpoint.

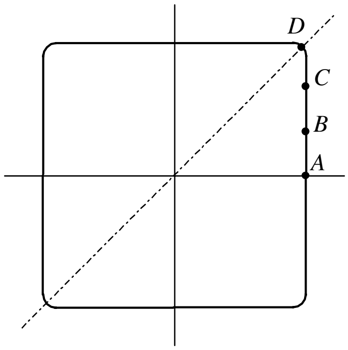

Fig. 1. Point-matching procedure for obtaining the coefficients of Eq. (1); $p=4$, $S=4$

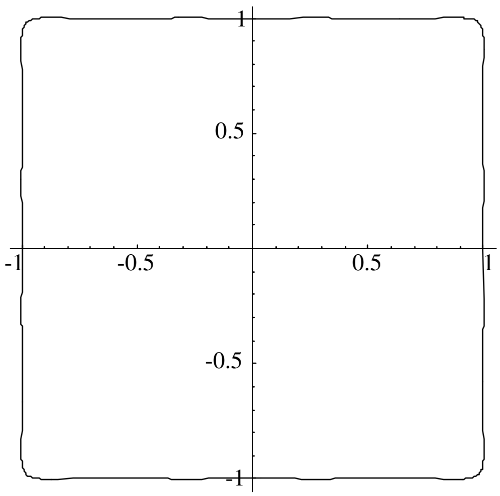

Fig. 2. Square insert with rounded corners ($p=4$, $S=4$)

The procedure leads to a linear system of equations in the a_{1-jp}-s, whose solution is elementary.

It is also interesting to point out that one may include the radius of curvature at the corner point. This is easily performed obtaining the expression for the radius of curvature in terms of parametric relationship but the inclusion of this requirement will make the solution of the system of equations considerably more difficult since the expression of the radius of curvature makes the system nonlinear.

Figures 2 and 3 show configurations of square and hexagonal shapes, respectively, using the first four terms of Eq. (1). The calculated coefficients are $a_1 = 1.17549$, $a_{-3} = -0.189655$, $a_{-7} = 0.0174774$, and $a_{-11} = -0.00330932$ in the case of the square configuration, and $a_1 = 1.06173$, $a_{-5} = -0.0692556$, $a_{-11} = 0.00984779$ and $a_{-17} = -0.00232114$ for the hexagonal configuration. On the other hand $\alpha = 0.98$ and 0.99 for $p = 4$ and 6, respectively.

Another interesting feature is the fact that if one makes $r \ll 1$ in Eq. (1), the first term predominates and one is able to map a finite domain with a hole of rather complicated geometry onto an annulus in the ζ-plane. This circumstance is important from a technological viewpoint since it allows for a simple, approximate solution of certain diffusion and wave propagation problems.

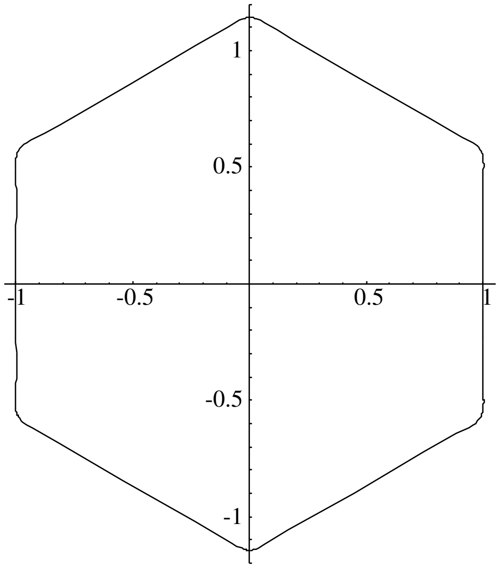

Fig. 3. Hexagonal insert with rounded corners ($p = 6$, $S = 4$)

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Reply to Comments on "Faber series method for two-dimensional problems of an arbitrarily shaped inclusion in piezoelectric materials" by P. A. A. Laura, C. A. Rossit and S. Maíz

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We are grateful to Drs. Laura, Rossit and Maíz for their suggestive comments on our recently published paper [1]. They addressed an important issue: how to determine an approximate mapping function that maps the infinite z-plane with an arbitrarily shaped hole onto the infinite ζ-plane with a hole of unit radius. This is a classical mathematical problem and its solutions have been widely studied. Considering the fact that the readers in the field of solid mechanics are more familiar to the work of Muskhelishvili [2], we adopted Muskhelishvili's result to determine the mapping function. In fact, since Muskhelishvili's masterpiece came out [2], great developments [3] have been made on the problem. As pointed out in the comments, the implementation of these new results is helpful to obtain more accurate solutions for the considered problem, especially for the case where there are some corner points on the boundary. In conclusion, an alternative approach for the determination of the mapping function with high accurate coefficients has been suggested in the comment, and it is believed that the application of the approach will bring more full play to the Faber series method [1] in solving the two-dimensional problems of an arbitrarily shaped inclusion in an infinite elastic material.

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