



# An optimization approach for multiperiod production planning in a sawmill

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## ABSTRACT

Forest industry plays an important role in the national economic and social context, due to the production volume with significant impact in the region where it is located (northeast of Argentina). Sawmill production planning is a key factor for the development of this industry with serious challenges taking into account the involved interrelated tasks. Assuming a multiperiod perspective, several elements can be considered and more interesting results can be attained. The appropriate procurement of logs is an important part of the problem costs and logging can be improved if the sawmill consumption can be foreseen in advance through an integrated approach. In this work, a mixed integer linear programming (MILP) model for the optimal multiperiod planning in sawmills is proposed. An efficient solution is obtained considering a set of cutting patterns (CP) for each type of log that optimize raw material yield. As a result of this optimization, appropriate procurement, distribution policy and stock management of logs can be achieved, as well as suitable production plan along the considered periods in order to fulfil the demand, with significant impact on the sawmill performance and profitability.

## 1. Introduction

The Argentinean forest industry is mainly located in the north-eastern region of the country (85% of the production, approximately 850,000 ha). More than one thousand forest factories (sawmills, plywood mills, pulp mills, and Medium Density Fibreboard factories) are concentrated in this region, where small and medium-sized enterprises represent 98% of these facilities (Broz et al. 2016). For these reasons, forest industry has a very important role in the economic and social development of this region. However, inefficient production and high transport cost have a negative impact on the competitiveness of this sector.

In the particular case of sawmills, efficient production can be achieved through optimal production planning considering raw materials (logs availability), final products (boards) and demands (customers), taking into account the characteristics of primary products (logs diameter and length), and industrial parameters, among others.

Sawmills convert logs of different diameters and lengths into boards using cutting patterns (CP). A CP is an arrangement of rectangles (thickness and width of the boards) within a circle. Each CP can be applied to logs of different lengths, if available. In addition, different CPs can be applied to logs of the same diameter, producing different amounts of diverse boards, as well as residues. Therefore, the

production performance strongly depends on CP selection considering that different yields can be attained and certain product demands must be fulfilled. But also, planning operations at a sawmill involves complex tasks, since several decisions as raw material supply, suitable logs cutting, inventories management and demands satisfaction among others, are simultaneously assessed. Obviously, an efficient cutting operation depends on logs availability, so a proper procurement is a key success factor. Moreover, these decisions lead to various tradeoffs that complicate the problem resolution. Mathematical modeling appropriately allows addressing this problem. Therefore, a mixed integer linear (MILP) programming model for the optimal production planning in a sawmill is presented in this work. The proposed approach addresses all the previous mentioned elements in a multiperiod framework, in such way that all the decisions are simultaneously assessed.

There are many published works dealing with production planning of sawmills. Maturana et al. (2010) present a mathematical formulation for determining the volume and type of logs to be processed assuming that each log is cut according to its optimal CP, i.e. CP selection is not a decision. They consider the production over six weeks, with an ideal scenario of log supply which is then adjusted. Results are compared with a heuristic schedule used by a Chilean company. A similar approach is proposed by Alvarez and Vera (2014) but considering an annual planning period and solving the uncertainties through robust

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Nomenclature			
<i>Subscripts</i>			
<i>a</i>	Providers	$t_{sp}$	Setup time for using CP <i>p</i> (h)
<i>c</i>	Discrete size for trucks	$Tmax_t$	Maximum daily operation time for sawmill (h)
<i>d</i>	Diameters of logs	$\varphi_{dl}$	Weight of a log of diameter <i>d</i> and length <i>l</i> (ton/units)
<i>i</i>	Cross section of boards	$\rho_{dpi}$	Conversion factor: number of boards of cross section <i>i</i> produced when CP <i>p</i> is applied to logs of diameter <i>d</i>
<i>l</i>	Length of logs and boards		
<i>p</i>	Cutting pattern		
<i>t</i>	Time period		
<i>Parameters</i>			
<i>BM</i>	Big constant		
<i>Dis<sub>da</sub></i>	Logs of diameter <i>d</i> and length <i>l</i> available from supplier <i>a</i> (units)		
<i>DM<sub>il</sub></i>	Maximum demand of product with cross section <i>i</i> and length <i>l</i> (units)		
<i>cap<sub>c</sub></i>	Available daily load capacity <i>c</i> (ton)		
<i>Cap<sub>LO</sub></i>	Minimum percentage of allowed load		
<i>Cap<sub>Sil</sub></i>	Stock capacity for board with cross section <i>i</i> and length <i>l</i> (units)		
<i>Cpr<sub>dtp</sub></i>	Production cost for CP <i>p</i> applied to logs of diameter <i>d</i> and length <i>l</i> (\$/unit)		
<i>Crm<sub>da</sub></i>	Unit raw material cost paid for log of diameter <i>d</i> and length <i>l</i> bought to supplier <i>a</i> (\$/unit)		
<i>Crm<sub>pd</sub></i>	Cost for stored old logs of diameter <i>d</i> and length <i>l</i> (\$/unit)		
<i>Cset<sub>p</sub></i>	Unit setup cost for CP <i>p</i> (\$)		
<i>PM<sub>ilt</sub></i>	Minimum production of boards of cross section <i>i</i> and length <i>l</i> required in period <i>t</i> (units)		
<i>PV<sub>il</sub></i>	Selling price of board of cross section <i>i</i> and length <i>l</i> (\$/unit)		
<i>t<sub>dtp</sub></i>	Operating time for processing a log of diameter <i>d</i> and length <i>l</i> using CP <i>p</i> (h)		
		<i>Binary variables</i>	
		$x_{pt}$	Indicates whether the primary CP <i>p</i> is used in period <i>t</i>
		$w_{aclt}$	Indicates whether the provider <i>a</i> sends <i>c</i> tons of logs of length <i>l</i> at day <i>t</i>
		<i>Continues variables</i>	
		<i>CPr</i>	Production cost (\$)
		<i>CR</i>	Raw material cost (\$)
		<i>CSt</i>	Setup cost (\$)
		<i>In</i>	Income for sales (\$)
		<i>Ib<sub>dlt</sub></i>	Logs of diameter <i>d</i> and length <i>l</i> stored in period <i>t</i> (units)
		<i>If<sub>il</sub></i>	Final stock of board of cross section <i>i</i> and length <i>l</i> at the end of planning horizon (units)
		<i>Ip<sub>ilt</sub></i>	Boards of cross section <i>i</i> and length <i>l</i> stored in period <i>t</i> (units)
		<i>It<sub>dlt</sub></i>	Old logs of diameter <i>d</i> and length <i>l</i> stored in period <i>t</i> (units)
		<i>Pi<sub>ilt</sub></i>	Number of boards of cross section <i>i</i> and length <i>l</i> produced in period <i>t</i> (units)
		<i>Qa<sub>dlt</sub></i>	Logs of diameter <i>d</i> and length <i>l</i> used in period <i>t</i> (units)
		<i>Qb<sub>dlat</sub></i>	Logs of diameter <i>d</i> and length <i>l</i> delivered from supplier <i>a</i> in period <i>t</i> (units)
		<i>Qp<sub>dtp</sub></i>	Logs of diameter <i>d</i> and length <i>l</i> processed in period <i>t</i> (units)
		<i>Qt<sub>dlt</sub></i>	Old logs of diameter <i>d</i> and length <i>l</i> processed in period <i>t</i> (units)
		<i>VF<sub>il</sub></i>	Sold boards of cross section <i>i</i> and length <i>l</i> (units)

optimization. In Zanjani et al. (2010), sawmill planning is applied to 30 days horizon time, and the authors also propose robust optimization for solving the problem with random raw material characteristics and yields. They also evaluate the customer service level through backorders, inventory size and costs. In Lobos and Vera (2016), a decomposition algorithm involving two levels is developed: a tactical planning horizon with monthly information and an operational level with detailed weekly production under supply uncertainty are stated. Pradenas et al. (2013) propose an integer programming (IP) model for determining the number of logs to be cut over a period of several days, according to a set of known CPs, in order to fulfil certain demand. They address two different approaches: one based on a metaheuristic algorithm to determine the number of logs and a constructive heuristic to generate the CPs for each log, while the second one solves the exact IP considering CPs generated by heuristic methods.

Dumetz et al. (2015) propose a discrete event simulation model to evaluate and compare different sawing planning strategies, size of the planning horizon, re-planning frequency, and order acceptance criteria. They state that the presented framework represents a tool for choosing a right production planning and ordering management strategies. Wery et al. (2018) integrate a sawing simulator and linear programming optimization for determining which orders should be accepted, what and when to produce, equipment settings and raw material purchase/consumption at each period. These last references give insights for guiding the decision making process in sawing industry with the aim of improving raw material procurement, stock management of logs and boards, and sales policies. These are also very critical issues for production planning.

Although the production yield strongly depends on the CP selection and use, there are few practical tools that support the CPs generation and, to the best of our knowledge, there is no optimization model that involves all the possible CPs for the sawmill production planning. On the other hand, several final products can be obtained from different CPs, and therefore several tradeoffs must be assessed. In this way when production planning is performed, different decisions must be integrated, as raw material purchase, production, inventory, and demand satisfaction. Moreover, sawmills are embedded in a chain that involves complex logistic operations which must be coordinated.

In particular, when a multiperiod approach is adopted, a better procurement can be achieved. Logging can conform to specific production requirements. The northeast region of Argentina is characterized by very favourable conditions for the development of the forest production. However, the climatic conditions affect the quality of the cut logs by the appearance of spots due to fungi. Therefore, these logs must be processed in a relatively short time after logging. Thus, if a specific logs requirement is available, forest harvest can be planned in order to satisfy this demand: logs can be cut in the required sizes and quantities. Hence, it is necessary to address this problem from a comprehensive point of view for attaining a good supply procurement policy along the time horizon, in order to reach the desired production for fulfilling the required demand. Therefore, a multiperiod approach with a time horizon of several days represents a useful tool for the forest industry as a whole.

Taking into account the cited and revised literature, few works have addressed this problem. Undoubtedly, the specific conditions of the northeast region of Argentine encourage this article. However, the

simultaneous consideration of the cutting policy in the forest, the supply of logs, efficient production considering cut patterns, and the satisfaction of demand is a valid problem for any context and has not been deeply addressed until the present.

In order to solve this problem, as previously mentioned, a MILP model for the optimal production planning in a sawmill is formulated in this work. The model embeds all the possible CPs for different log diameters, given by an exhaustive generation algorithm. A multiperiod approach, where all the decisions are simultaneously taken, is adopted. In this way, appropriate log procurement policy, a suitable production plan according to enterprise requirements, an improved stock management, and an accurate demand satisfaction are obtained in order to favour sawmill performance and profitability, assessing all the involved trade-offs. Through the examples, the approach capabilities are highlighted.

The paper is organized as follows: in Section 2 the problem is presented in detail according to a real industrial problem. The proposed mathematical formulation and CPs generation algorithm is described in Section 3. Examples and their corresponding analysis are presented in Section 4, while in the last section the main conclusions are summarized.

## 2. Problem statement

In Fig. 1 the considered process is presented. The raw material (logs) can be provided by different suppliers. Each supplier can send trucks of 30 tons, the usual trucks capacity in the region, which transport logs of the same length, but different diameters. Trucks must be loaded with at least a certain percentage of its capacity. Log price and availability differ according to diameter, length, and suppliers.

When logs arrive at sawmill, they are classified conforming to length and diameter. Then, following the most usual process in the region, they are transferred to the sawing sector where an initial cut is made through a primary saw in which a cant and two flitches are obtained. In the secondary saw, the cant is broken down into dimensional lumber pieces and two flitches. Finally, flitches are processed in a re-sawing unit. This cutting process generates boards of medium and large dimension, with determined cross section area (characterized by width and thickness), and length given by the log length. The log cuts are made according to a selected CP.

Each selected CP can be applied to logs with a determined useful diameter, and is valid for all the different log lengths with that diameter. The useful diameter corresponds to the diameter of the larger cylinder that can be obtained from the log. When a CP is changed, it is necessary to stop the sawmill operation to modify the configuration of the saws. Therefore, the changeover task generates cost and takes time.

The appropriate selection of CPs is critical for sawmill objectives taking into account several results strongly depend on the chosen CPs: operations yield and cost, satisfied demand, stock levels, etc.

At the beginning of the planning horizon, there is an initial logs stock at the sawmill, and daily, logs are bought from different suppliers. Each provider has a limited amount of logs for each length and diameter. Logs are usually cut according to specific requirements of sawmills, and this action can be managed using different criteria, since log prices differ depending on log sizes and the competition among different industries (paper, energy, lumber, etc.) for the use of this raw material.

Every day, a minimum production is required for some types of boards, while a final demand must be fulfilled at the end of the planning horizon. This daily production is settled in order to satisfy specific sales, to guarantee an efficient operation of dryers, etc. The final demand assures an efficient operations management, maintaining appropriate stocks levels for the forecasted sales.

The objective is to determine the detailed production planning of the considered period, i.e. daily decisions about raw material purchases, amount of logs of each type (diameter and length) to be cut with each CP, logs stock level, finished product stock and demands fulfilment in order to maximize the net benefit given by sales income minus sawing cost (raw material, operation, and setups).

## 3. Mathematical modelling

In this section, the mathematical formulation for the optimal planning is posed. Following, the model constraints are stated according to the process stage that they belong.

### 3.1. Raw material procurement and inventory

Each supplier  $a$  can provide a maximum amount of logs of diameter  $d$  and length  $l$ ,  $Dis_{dla}$ , along the planning horizon. Let  $Qb_{dlat}$  represents the amount of logs of diameter  $d$  and length  $l$ , delivered from supplier  $a$  at day  $t$ , then:

$$\sum_t Qb_{dlat} \leq Dis_{dla} \quad \forall d, l, a \tag{1}$$

The log delivery is made by trucks which can transport logs of the same length, but different diameters. As was mentioned in the previous section, the log transportation is made in trucks which allow loads up to 30 tons. This requirement is modelled according to discrete sizes  $c$ , which represent the maximum allowed vehicle load, and at least a certain percentage of the truck,  $Cap_{LO}$ , must be loaded as describe the following equations:

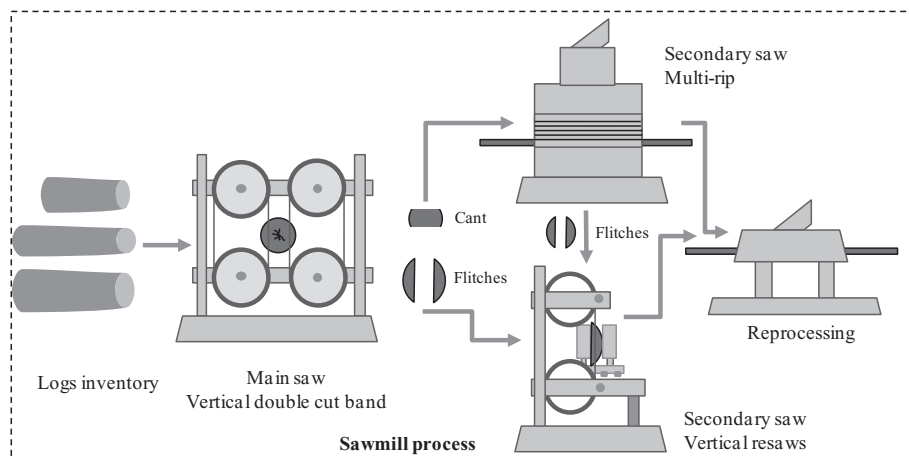


Fig. 1. Stages considered in sawmill process.

$$\sum_d Qb_{dlat} \varphi_{dl} \leq \sum_c cap_c w_{aclt} \quad \forall l, a, t \quad (2)$$

$$\sum_d Qb_{dlat} \varphi_{dl} \geq Cap_{L,O} \sum_c cap_c w_{aclt} \quad \forall l, a, t \quad (3)$$

where  $\varphi_{dl}$  is the weight of a log of diameter  $d$  and length  $l$ ,  $cap_c$  is the available daily load capacity  $c$  taken from a set of discrete values, for example {30, 60, 90, 120, 150, 180} expressed in tons, taking into account the number of vehicles that can be sent from a supplier to a sawmill with the same length.  $w_{aclt}$  is a binary variable that takes value 1 if the supplier  $a$  sends  $c$  tons of logs of length  $l$  at day  $t$ .

Each supplier sends at most one discrete amount of logs for each length every day as state Eq. (4):

$$\sum_c w_{aclt} \leq 1 \quad \forall a, l, t \quad (4)$$

This last constraint means that each supplier can provide 30 t, 60 t, 90 t, etc. each day (or at least  $Cap_{L,O}$ % of these amounts according to Eq. (3)).

The logs used on day  $t$ ,  $Qa_{dlt}$ , can be supplied that day or be in the inventory from the previous day,  $Ib_{d,t-1}$ . After using the required logs, the remaining ones are stored in the inventory,  $Ib_{d,t}$ . Then, the inventory balance is given by the following equation:

$$\sum_a Qb_{dlat} + Ib_{d,t-1} = Qa_{dlt} + Ib_{d,t} \quad \forall d, l, t \quad (5)$$

At the first planning day ( $t = 1$ ), the amount of stored bought logs,  $Ib_{d,0}$ , is equal to zero.

On the other hand, the sawmill not only uses the purchased logs, but also employs the ones it has in inventory from previous periods. Taking into account that the cost may vary according to the date that logs have been cut, the logs from previous periods are managed differently. The sawmill has an initial stock of logs bought in previous periods given by  $I_{d,0}$ . Considering that logs are affected by the appearance of spots due to fungi, the appropriate operation of old logs is very important. Therefore, they can be specifically considered using adequate cost parameters to ease and boost their consumption. In particular, they are employed during the planning horizon following a usual balance for these old logs expressed by:

$$I_{d,t-1} = Qt_{dlt} + It_{dlt} \quad \forall d, l, t \quad (6)$$

where  $Qt_{dlt}$  is the amount of old logs, i.e. raw material not provide by suppliers, of diameter  $d$  and length  $l$  used in the process at day  $t$ .

### 3.2. Log processing

The mathematical model includes, as input data, all the CPs that have been generated according to the available log diameters, demanded boards, and yield imposed by the enterprise. Taking into account a better planning is achieved if it is possible to consider as many CPs as possible, including all the feasible combinations of tables, a CPs generator has been developed. The objective of this CP generator is to systematically arrange rectangles (thickness and width of the boards) within circles. The insertion of the rectangles is done simulating the way in which the sawmill operates. Each log is divided into five sections: a central block, two equal lateral flitches, and upper and lower ones, equal to each other. From these sections, the different admissible boards (products) are obtained. Boards are characterized by its cross section,  $i \in I$ , according to its thickness and width, considering the admissible sizes defined by the sawmill. In this work, it is assumed that the board length adopts the same value that the log length  $l$ .

Let  $P$  be the set of the generated CPs ( $p \in P$ ) and  $\rho_{dpi}$  the conversion factor when CP  $p$  is applied to logs of diameter  $d$ , i.e. the number of products of cross section  $i$  generated when the CP  $p$  has been used in a log of diameter  $d$ . The variable  $Pi_{ilt}$  is defined as the number of boards of cross section  $i$  and length  $l$ , produced in period  $t$ . Eq. (7) determines the

value of  $Pi_{ilt}$  involving the variable  $Qp_{dlpt}$ , which represents the number of logs of diameter  $d$  and length  $l$  that have been cut in period  $t$  using the CP  $p$ :

$$\sum_{p,d} Qp_{dlpt} \rho_{dpi} = Pi_{ilt} \quad \forall l, t, i \quad (7)$$

$$\sum_p Qp_{dlpt} = Qt_{dlt} + Qa_{dlt} \quad \forall d, l, t \quad (8)$$

Eq. (8) states that the number of processed logs of diameter  $d$  and length  $l$  in  $t$ ,  $Qp_{dlpt}$ , is equal to the number of used logs provided by the different suppliers ( $Qa_{dlt}$ ) and those old logs stored in the sawmill ( $Qt_{dlt}$ ).

Let  $x_{pt}$  be the binary variable that takes value 1 if the CP  $p$  is applied on day  $t$ . Then, states that if CP  $p$  is not used at day  $t$ , then no logs are processed through that pattern.  $BM$  is a sufficiently big constant that converts redundant this constraint when  $x_{pt}$  is equal to 1.

$$\sum_{d,l} Qp_{dlpt} \leq BM x_{pt} \quad \forall p, t \quad (9)$$

The operating time for processing a log of diameter  $d$  and length  $l$  using CP  $p$  is defined by  $t_{dlp}$ , while the setup time for using CP  $p$  is given by  $ts_p$ . Therefore, the processing time at each planning day cannot exceed the working day time,  $Tmax_t$ :

$$\sum_{d,l,p} Qp_{dlpt} t_{dlp} + \sum_{p \in P} ts_p x_{pt} \leq Tmax_t \quad \forall t \quad (10)$$

### 3.3. Production management and demand satisfaction

In a multiperiod approach, products requirements are covered throughout the considered periods, accumulating the cut boards every day. Let  $Ip_{ilt}$  be number of boards of cross section  $i$  and length  $l$  stored at day  $t$ . Eq. (11) states the inventory product balance:

$$Ip_{ilt} = Ip_{i,t-1} + Pi_{ilt} \quad \forall i, l, t \quad (11)$$

It is worth noting that the initial product stock,  $Ip_{i,0}$ , is a model parameter, which can be adopted equal to zero if no previous stock is available.

In this approach, it is assumed that the sales of boards of cross section  $i$  and length  $l$  ( $VF_{il}$ ), at the end of the planning horizon,  $t_{final}$ , are less than the maximum demands of each product,  $DM_{il}$ :

$$VF_{il} \leq DM_{il} \quad \forall i, l \quad (12)$$

Let  $If_{il}$  be the variable corresponding to the final stock of product ( $i,l$ ) at the end of planning horizon, after fulfilling the demand. Then, at the end of the planning horizon, the following balance is hold:

$$Ip_{ilt_{final}} = VF_{il} + If_{il} \quad \forall i, l \quad (13)$$

This final stock  $If_{il}$  can be bounded by limited store capacity or enterprise policies,  $CapS_{il}$ :

$$If_{il} \leq CapS_{il} \quad \forall i, l \quad (14)$$

Also, minimum production of boards of cross section  $i$  and length  $l$ ,  $PM_{ilt}$ , can be required at day  $t$ . This condition can represent a specific demand that must be fulfilled, a board arrangement to be dried in the next process stage, or a regular manufacturing policy. This requirement can be satisfied using the production obtained until the current day as is defined in the following equation:

$$\sum_{t' \leq t} Pi_{ilt'} \geq \sum_{t' \leq t} PM_{ilt'} \quad \forall i, l \quad (15)$$

### 3.4. Objective function

As previously mentioned, the objective function considers the net

benefit maximization, given by the difference between the income from sales ( $In$ ) and costs of the raw material ( $CR$ ), production ( $CPr$ ), and setup ( $CSt$ ).

The sales income is calculated as:

$$In = \sum_{i,l} VF_{il} PV_{il} \tag{16}$$

where  $PV_{il}$  represents the selling price of board of cross section  $i$  and length  $l$ .

Raw material cost is given by the unit raw material cost paid for a log of diameter  $d$  and length  $l$  bought to supplier  $a$ ,  $Crm_{dla}$ , and the cost of using stored old logs, where  $Crm_{pdl}$  is its unit cost, as expresses Eq. (17):

$$CR = \sum_{d,l,a,t} Qb_{dlat} Crm_{dla} + \sum_{d,l,t} Qt_{dlt} Crmp_{dl} \tag{17}$$

In Eq. (18) the production cost is shown, where  $Cprd_{dlp}$  is the unit production cost when CP  $p$  is applied to logs of diameter  $d$  and length  $l$ ,

$$CPr = \sum_{d,l,p,t} Qp_{dlpt} Cprd_{dlp} \tag{18}$$

while the setup cost is given by Eq. (19):

$$CSt = \sum_{p,t} Cset_p x_{pt} \tag{19}$$

where the unit setup cost,  $Cset_p$ , depends on the used CP.

Finally, the objective function is:

$$Max In - (CR + CPr + CSt) \tag{20}$$

In this way, the mathematical modelling for the optimal production planning in a sawmill maximizes the net benefit given by Eq. (20), involving constraints Eqs. (1)-(19).

#### 4. Numerical examples

The proposed examples consider logs classified into 5 diameters and 6 lengths. The logs can come from the sawmill stock, which has a total of 385 logs, or can be bought to 3 suppliers: A, B, and C. Each of them has a limited capacity, equal to 13094, 13116, and 12785 units, respectively. Logs availability, according to diameter and length, is shown in Table 1. Each supplier can provide logs using trucks of 30 ton and at least 90% of the vehicle must be loaded in each trip ( $Cap_{LO} = 90\%$ ).

A total of 71 types of products, distributed among 17 cross sections and 6 lengths is considered, while the number of total demanded boards per week is 31646, as shown in Table 2, while the minimum required production for days  $t_1-t_4$  are displayed on Table 3. In this case, at the last day  $t_5$ , no required production is introduced.

The CP generator obtains 1988 CPs, from which 702 are feasible according to the considered diameters, cross section area for different products, and the yield policy applied by the firm. This policy states that more than 50%, 70%, 75%, 80%, and 80% of the logs of diameters  $d_1$  to  $d_5$ , respectively, must be converted into lumber (CP yield) in order to make a better use of the wood. The adopted planning horizon is equal to 5 working days with 8 hours each.

Some model parameters as selling prices, costs, number and types of boards involved in each CP, among others, are not presented due to space reasons, but they are available for interested readers.

The examples are implemented and solved in GAMS (Rosenthal 2017) using CPLEX solver in an Intel(R) Core(TM) i7-3770, 3.40 GHz. The model consists of 111,656 equations, 214,257 continuous and 3,960 discrete variables. The computational time limit is fixed to 1500 s, and the optimality gap is around 5-7% in the presented examples. Some remarks about model performance are discussed at the end of this section.

In the next section, three cases are analyzed. First, the model presented along Eqs. (1)-(20) is solved. In order to show how minimum

production requirements affect the production planning, in the second example Eq. (15) is removed and the resulting formulation is resolved. Finally, in order to arrange trucks arrivals, a limit for them is added and the results are analyzed in Example 3.

#### 4.1. Example 1

After solving the proposed model, the weekly sawmill planning reaches a net benefit equal to \$381,987. In Table 4 the main results of the planning for this case are shown. It can be noted that a total of 2,870 logs are purchased from the different suppliers, distributed in 16 trucks, as details Table 5. Each truck transports logs of different diameters but the same length. Raw material cost includes its transportation; therefore, the supplier selection contemplates raw material availability and proximity. In this case, supplier B is the less convenient since no trucks are delivered along the planning horizon.

A total of 3159 logs are processed, 329 of these corresponds to sawmill initial stock. At the end of the planning horizon, 97 logs are stored, 56 of them are from the initial stock.

Table 6 shows the number of processed logs using different CPs. The same CP is used for logs with the same diameter but different lengths. So, in order to avoid setup times and cost, products with the same cross section  $i$ , but with different lengths, are produced applying the same CP. The CPs are identified with the name “ $p$ ” plus a number, for example “ $p1$ ” is the cutting pattern 1 used for diameter  $d_1$ , and they are numbered from 1 to 1988, the total generated CPs. Along the week, 15 different CPs are used, while only two are repeated (number 1 and 8), i.e. 17 setups are needed.

The number of obtained final boards is equal to 31,710, using the maximum available time (5 working days of 8 hours each). At the end of the time period, 609 units of 12 different types (cross section and length) are stored since they are not included in the demand. Table 7 shows the amount of each type of board in stock at the end of the planning horizon. This inventory is due to CPs include several different boards, and these lumber pieces are not necessarily in the same proportion of the demand or, directly, some types of boards are not required. On the other hand, the unsatisfied demand is equal to 11.5 %.

In the first column of Table 8, the economic results of this example

**Table 1**  
Log availability according to diameter, length, and supplier.

Supplier	Length	Diameter				
		$d_1$	$d_2$	$d_3$	$d_4$	$d_5$
Sawmill	$l_1$	35	24	0	9	4
Sawmill	$l_2$	32	28	13	11	3
Sawmill	$l_3$	7	30	24	3	0
Sawmill	$l_4$	36	33	27	0	1
Sawmill	$l_5$	0	0	0	0	0
Sawmill	$l_6$	36	22	0	6	1
A	$l_1$	560	640	240	80	400
A	$l_2$	484	404	484	323	646
A	$l_3$	571	652	652	816	244
A	$l_4$	412	576	247	659	164
A	$l_5$	665	166	416	249	832
A	$l_6$	168	420	420	252	252
B	$l_1$	240	640	80	800	80
B	$l_2$	646	808	484	323	323
B	$l_3$	244	81	734	408	489
B	$l_4$	576	494	247	576	412
B	$l_5$	416	332	748	249	166
B	$l_6$	504	672	336	252	756
C	$l_1$	640	320	240	320	160
C	$l_2$	404	242	646	242	808
C	$l_3$	571	816	244	816	244
C	$l_4$	82	494	82	82	741
C	$l_5$	665	499	332	665	582
C	$l_6$	252	336	840	252	168

**Table 2**  
Maximum demands on the planning horizon ( $DM_{it}$ ).

Cross section boards																	
Length	$i_1$	$i_2$	$i_3$	$i_4$	$i_5$	$i_6$	$i_7$	$i_8$	$i_9$	$i_{10}$	$i_{11}$	$i_{12}$	$i_{13}$	$i_{14}$	$i_{15}$	$i_{16}$	$i_{17}$
$l_1$	3000	1953	1000	1036	336	106	90	325	101	64	40	41	60	10	200	0	0
$l_2$	1680	2045	419	550	255	150	60	97	400	39	50	13	60	16	200	250	45
$l_3$	527	0	0	568	0	498	0	0	1500	0	0	0	0	0	0	0	0
$l_4$	1820	1805	286	237	248	128	90	165	91	59	28	16	60	28	150	225	46
$l_5$	1300	0	0	200	0	100	0	0	0	0	0	0	0	0	0	0	0
$l_6$	2520	1770	614	750	345	140	0	257	90	70	45	11	60	29	50	0	28

**Table 3**  
Minimum daily required production ( $PM_{it}$ ).

Periods									
Length	$t_1$		$t_2$			$t_3$		$t_4$	
Products	$i_1$	$i_4$	$i_9$	$i_{10}$	$i_{13}$	$i_{15}$	$i_{16}$	$i_5$	$i_8$
$l_1$	1000	800	0	50	50	100	0	200	100
$l_2$	500	400	0	0	0	100	100	100	0
$l_3$	0	0	750	0	0	0	0	0	0
$l_4$	0	0	0	0	0	0	100	0	0
$l_6$	0	300	0	50	50	0	0	100	100

**Table 4**  
Planning results for each day for Example 1.

	Period				
	$t_1$	$t_2$	$t_3$	$t_4$	$t_5$
Purchased logs [unit]	2292	578	0	0	0
Used logs [unit]	703	481	505	774	695
Production time [h]	8	8	8	8	8
Produced boards [unit]	6753	5845	7070	5514	6528

**Table 5**  
Number of trucks (30 ton) of different lengths provided by suppliers.

Lengths							
Supplier	Period	$l_1$	$l_2$	$l_3$	$l_4$	$l_5$	$l_6$
A	$t_1$	1	1	1	1		
A	$t_2$				1	1	1
C	$t_1$	2	2		1		4

**Table 6**  
Processed logs according to CP and time period.

Diameter	CP	Period	Length					
			$l_1$	$l_2$	$l_3$	$l_4$	$l_5$	$l_6$
$d_1$	p1	$t_1$	250					
$d_1$	p1	$t_4$	153			148		81
$d_1$	p4	$t_4$	5	110		104	42	31
$d_2$	p8	$t_1$	222	55				25
$d_2$	p8	$t_5$	5	16	54	50		187
$d_2$	p22	$t_5$	166	69		47		102
$d_3$	p38	$t_2$					50	
$d_3$	p44	$t_2$		6	191			
$d_4$	p181	$t_4$	45	15		40		
$d_4$	p212	$t_1$						90
$d_4$	p213	$t_3$	100					37
$d_4$	p223	$t_3$		33		41		
$d_4$	p433	$t_1$	29	24	3			5
$d_4$	p475	$t_2$		60		64		
$d_4$	p566	$t_3$	124	120		16		34
$d_5$	p731	$t_2$	25	8		14		25
$d_5$	p1494	$t_2$		22		16		

**Table 7**  
Stored boards at the end of the planning horizon.

	$i_6$	$i_9$	$i_{10}$	$i_{11}$	$i_{14}$
$l_1$	153	44		84	40
$l_2$	32		21	70	
$l_4$		5	1		
$l_6$	128	10			21

**Table 8**  
Economical results for studied cases.

	Example 1	Example 2	Example 3
Incomes for sales (\$)	1,286,206	1,225,907	1,289,965
Raw material cost (\$)	510,797	457,163	513,295
Production cost (\$)	329,395	315,014	333,275
Setup cost (\$)	64,027	42,868	70,224
Net Benefit (\$)	381,987	410,862	373,171

are displayed. As can be noted, raw material purchase is the dominant cost, which represents 56.5% of the overall costs. The setup cost represents the time spent for changing knives configuration in the saw. As previously mentioned, in this case, 17 changes are required in the planning horizon. Production cost is proportional to processed logs and, in this case, it represents 36.4% of the total costs.

4.2. Example 2

Suppose that no constraint about minimum daily production is required, i.e. the sawmill has liberty to fulfill the demand at the end of the planning horizon. The optimal solution without considering Eq. (15) which states the daily requirement, improves 7.6% the net benefit of Example 1. Moreover, a very different production plan is attained since a smaller amount of boards are produced, reducing the sales income, but also the overall cost is decreased, as it is shown in the second column of Table 8.

In this case, a total of 2979 logs are purchased (37.5% from A and 62.5% from C), loaded in 16 trucks along the planning horizon, the same amount of trucks of Example 1. However, the raw material cost is 10.5% reduced. The used logs are 3280 units, where 346 log of the initial stock are consumed, i.e. 90% of the total old logs are processed along the week. At the end of the planning horizon, the log stock is equal to 84 units, from which 39 are from the initial sawmill stock. Therefore, the number of stored logs is reduced, which means that a better log procurement policy is applied. Table 9 summarizes the planning information.

From the production point of view, the results are also more convenient. A bigger amount of boards are produced, 31992 lumber pieces, and the unsatisfied demand is reduced to 10.4%. There is also a little idle time, 1.25% of the total time. The number of CPs is reduced to 12, decreasing 33% the setup cost. CPs are scheduled in a better way, reducing the loss due to setup times. Table 10 shows the amount of logs processed with each selected CP each day. The board inventory at the

**Table 9**  
Planning results for each day for Example 2.

	Period				
	$t_1$	$t_2$	$t_3$	$t_4$	$t_5$
Purchased log [unit]	2791	188	0	0	0
Used logs [unit]	520	891	658	682	529
Production time [h]	8	7.9	8	7.6	8
Produced boards [unit]	7082	3806	6134	7514	7456

**Table 10**  
Processed logs according to CP and time periods for Example 2.

Diameter	CP	Period	Length					
			$l_1$	$l_2$	$l_3$	$l_4$	$l_5$	$l_6$
$d_1$	p1	$t_2$	144	321		288		100
$d_1$	p4	$t_3$				104	42	6
$d_2$	p8	$t_4$	239	102		53	50	238
$d_2$	p12	$t_5$	9	92	88	1		8
$d_3$	p22	$t_3$	166	67		45		100
$d_3$	p38	$t_3$		26		11	50	
$d_4$	p44	$t_1$			153			
$d_4$	p181	$t_1$	45			29		
$d_4$	p213	$t_5$	103	48		52		128
$d_4$	p272	$t_2$		14		15		9
$d_4$	P566	$t_1$	124	122		15		32
$d_5$	p731	$t_3$	5	8		14		14

end of the planning horizon is also reduced. In this case, 516 units of 7 different types (cross section and length) are stored. Therefore, compared with the previous example, a better production planning is reached since lower amount of logs are bought and stored, more boards are produced while final inventory is reduced.

This example was presented in order to show how daily production requirements can affect the optimal production planning. Without these constraints, production can be managed more efficiently: unsatisfied demand and final inventory are reduced, and production is increased. These benefits are based on several reasons, but the reduction in the number of used CPs and the consequent shorter setup time greatly improves the solution.

4.3. Example 3

As can be observed from Tables 4 and 9, most log purchases are made in the two first time periods, and none in the rest. As no log inventory cost or other penalty is imposed, it is indistinct the period in which raw material have to be bought. So, the model prefers to buy logs at the beginning of the planning period in order to have available a wide variety of raw material for processing different CPs. It is worth to highlight that logs of different diameters, but equal length, arrive in the same truck. Therefore, when vehicles arrive at the plant in the first days, there is more variety of available diameters for using the selected CP, i.e. a more flexible production.

In order to avoid trucks overlapping and delays in truck unloads, and achieve a better use of the available space for logs storage, a new constraint about the number of trucks that can daily arrive to the sawmill is imposed. In this case, it was stated that no more than 120 tons of logs can arrive per time period, which represents 4 trailers of 30 tons at most:

$$\sum_{d,l,a} Qb_{dlat} \varphi_{dl} \leq 120 \tag{21}$$

Adding this constraint to the model presented for Example 1, the optimal solution gives a net benefit equal to \$373,171. The last column of Table 8 shows the income and costs.

Now, truck arrivals are distributed among all time periods: 4 trucks

**Table 11**  
Planning results for each day restricting the number of truck arrivals per day

	Period				
	$t_1$	$t_2$	$t_3$	$t_4$	$t_5$
Logs purchased [unit]	839	570	555	707	0
Used logs [unit]	715	583	560	581	580
Spent time [h]	8	8	8	8	7.7
Produced boards [unit]	6573	5936	5682	7016	6125

of 30 tons in  $t_1$ , 4 in  $t_2$ , 4 in  $t_3$ , and 4 in  $t_4$  (5 trucks from supplier A and 11 from C). The purchased logs are 2671 units, while the consumed are 3019 units, for which 379 are from the own sawmill initial stock. Therefore, only 6 of these old logs and 31 purchased logs remain in stock, which are significantly lower than the previous cases. The produced boards are 31332 units, while 775 boards of 14 different types remain in the inventory at the end of the planning horizon. Table 11 shows the planning results for each day.

As was previously mentioned, this scenario is less flexible than Example 1. In this case, fewer boards are produced and greater unsatisfied demand is obtained (13%). Also, more boards are stored and 17 CPs are used, while 19 CP setups are required along the planning horizon as shown Table 12.

It is worth mentioning that the purchased logs represent a smaller amount than the first case, but raw material cost is increased due to logs of bigger diameters are bought (see used logs in Table 12), which are proportionally more expensive. Processing bigger logs is also costly. In the same way, setup cost is incremented because 19 pattern changes are applied (2 more than Example 1). From this last table, as well as from Tables 6 and 10 for the previous examples, some insights about the problem can be obtained: types (diameters and lengths) of purchased logs each day, processed logs, used CPs and its changeovers, among others.

Although the economic results is less convenient in this last case (the net benefit is reduced only 3.2%), the planning and logistic features are better coordinated and a better plant operation is achieved.

5. Conclusions and discussion

In this work, a MILP model for the optimal multiperiod production planning in a sawmill was presented. Daily logs procurement, suppliers selection, quantity of processed logs, CPs assignment, log and boards

**Table 12**  
Processed logs according to CP and time period for Example 3.

Diameter	CP	Period	Length					
			$l_1$	$l_2$	$l_3$	$l_4$	$l_5$	$l_6$
$d_1$	p1	$t_1$		250				34
$d_1$	p1	$t_3$	16			126		46
$d_1$	p4	$t_2$	18	80	7	90		63
$d_2$	p8	$t_1$	222	55				10
$d_2$	p8	$t_4$				28	83	175
$d_2$	p12	$t_2$			30			
$d_2$	p22	$t_5$	121	69		47		102
$d_2$	p37	$t_5$		22		23		14
$d_3$	p38	$t_5$		7		25	50	
$d_3$	p44	$t_2$			191			
$d_4$	p181	$t_5$	45	15		40		
$d_4$	p212	$t_1$						78
$d_4$	p213	$t_3$	117					50
$d_4$	p223	$t_3$		33		42		
$d_4$	p433	$t_1$	29	24	3			10
$d_4$	p475	$t_3$		67		63		
$d_4$	p566	$t_4$	124	121		16		34
$d_5$	p731	$t_2$	25	8		14		25
$d_5$	p1494	$t_2$		16		16		

inventories, and operation times are simultaneously obtained in order to satisfy the required demands in the proposed horizon time with maximum net benefit. A complete set of feasible CPs is generated through an exhaustive algorithm, which responds to the firm requirements about types of saws, available log diameters, log yields, and board dimensions. Then, the approach simultaneously selects for each planning day, the proper set of CPs in order to produce the required demand of boards, cutting the suitable amount of logs for reducing raw material, production and setup costs.

The approach also allows determining solutions for other production scenarios according to industry needs as coordinating raw material arrivals. Other topics not addressed in this work, as inventory management of logs and final boards, can be easily incorporated in order to satisfy firm policies.

The multiperiod approach for sawmill production planning, is not only an important issue from the operational point of view for the sawmills, but also allows improving operations management for a set of linked partners: an appropriate procurement and transport of logs and the consequent involved costs, as well as logging tasks.

From the computational point of view, the proposed formulation can present problems due to its combinatorial nature. Since product demands must be fulfilled at the end of the planning horizon but the planning must be performed for each time period (day), there exist many alternative solutions, and therefore, the computational performance worsens. For example, when no daily minimum production is required, the production planning for each period can be exchanged and the same solution from the economical point of view is reached. Although it is a stricter scenario, the same problem occurs when minimum daily production is required, since the remaining production can be carried out in the same way any day. For overcoming this situation, different costs and specific conditions (inventory and procurement requirements, for example) should be added for each time period.

On the other hand, the availability of a complete and exhaustive set of CPs considering the technology existing in the industry is a key

aspect of this proposal to attain efficient solutions.

Even though the large number of generated CPs complicates the model resolution due to a greater number of involved variables, the various tradeoffs among decisions are appropriately assessed. Considering that the same product can be obtained from different CPs and logs, suitable production plans can be proposed that satisfy all the adopted requirements.

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### References

- Alvarez, P.P., Vera, J.R., 2014. Application of robust optimization to the sawmill planning problem. *Ann. Operat. Res.* 219 (1), 457–475.
- Broz, D., Rossit, D.A., Rossit, D.G., Cavallin, A., 2016. Challenge in supply chains management in Argentina: the case of forestry-industrial sector. In: *Proceedings of The 8th International Conference on Production Research – Americas, Santiago de Chile, Chile*.
- Dumetz, L., Gaudreault, J., Thomas, A., Marier, P., Lehoux, N., El-Haouzi, H., 2015. A simulation framework for the evaluation of production planning and order management strategies in the sawmilling industry. *IFAC-Paper-On-line* 48 (3), 622–627.
- Lobos, A., Vera, J.R., 2016. Intertemporal stochastic sawmill planning: Modeling and managerial insights. *Comp. Ind. Eng.* 95 (1), 53–63.
- Maturana, S., Pizani, E., Vera, J., 2010. Scheduling production for a sawmill: A comparison of a mathematical model versus a heuristic. *Comp. Ind. Eng.* 59 (4), 667–674.
- Pradenas, L., Garcés, J., Parada, V., Ferland, J., 2013. Genotype-phenotype heuristic approaches for a cutting stock problem with circular patterns. *Eng. Appl. Artif. Intellig.* 26, 2349–2355.
- Rosenthal, R., 2017. *GAMS A User's Guide*. GAMS Development Corporation, Washington, DC, USA.
- Wery, J., Gaudreault, J., Thomas, A., Marier, P., 2018. Simulation-optimisation based framework for sales and operation planning taking into account new products opportunities in a co-production context. *Comput. Ind.* 94, 41–51.
- Zanjani, M.K., Ait-Kadi, D., Nourelfath, M., 2010. Robust production planning in a manufacturing environment with random yield: A case in sawmill production planning. *Eur. J. Operat. Res.* 201 (3), 882–891.