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Mathematical programming and game theory optimization-based tool for supply chain planning in cooperative/competitive environments

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ABSTRACT

This work proposes to improve the tactical decision-making of a supply chain (SC) under an uncertain competition scenario through the use of different optimization criteria, which allows to manage not only the specific objectives of the SC of interest, but also the way how its clients address their selection between different potential suppliers, identifying best market share for the SC of interest and the strategy to attain it. The resulting multi-objective optimization problem has been solved using the ε-constraint method in order to approximate the Pareto space of non-dominated solutions while a framework based on game theory is used as a reactive decision making support tool to deal with the uncertainty of the competitive scenario. The use of the proposed system is illustrated through its application to a multi-product, multi-echelon supply chain case study, which is intended to cooperate or to compete with another SC of similar characteristics.

Keywords: Supply chain planning; Multi-objective optimization; Game theory; Competitive scenarios

1. Introduction

Given the corporate structure and the market globalization in recent years, industries need to become more competitive trying to obtain more benefits, reduce loses, improve services and products quality, etc. But it is a fact that the efficiency of a company may be constrained by the efficiency of its supply chain management procedures. Many research groups are currently working on developing systematic methods to improve decision making at its different hierarchical levels, including the tactical decision level which has proved to be one of the most important for the day-to-day SC optimization: it intends to produce a master plan that improves the use of the resources of the company (production plants, storage centers, and transport services), with a main objective typically focused to deliver the products to the final consumers in time and with the desired quality and cost.

However, other elements should be also considered during this decision making. For one side, the pressure to manage measures related to the different dimensions of SC sustainability is progressively increasing; this includes directly managed elements affecting the harmful aspects of the different processing and logistic steps to the environment such as: to reduce emissions and discharges of waste, to increase the reuse of natural resources, to recycle, etc., but also should affect decisions about procurement selection and other indirect elements like safety. Another of the most critical issues in the tactical decision making is the need to simultaneously manage comprehensive objectives (overall SC cost, etc.) and specific objectives arising from the different departments of the enterprise (marketing, sales, manufacturing), since each one of these later objectives is related to the different policies that each department considers important (i.e.: just in time policy, customer satisfaction, breakdowns reduction, etc.). Hence, it is necessary to consider the tradeoff among multiple objectives, which in many cases are contradictory, in order to improve the decision making.

For all these reasons, the use of multi-objective optimization (MOO) methods and tools becomes essential to improve the decision making although, as it is well known, they cannot

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Nomenclature

Indexes and sets

g	supply chain ($g = 1,2,\ldots,G$)
h	time periods ($h = 1,2,\ldots,H$)
i	production sites ("sources", $i = 1,2,\ldots,I$)
$I\_G(g)$	production sites (i) belonging to supply chain g.
j	distribution centers($j = 1,2,\ldots,J$)
n	products ($n = 1,2,\ldots,N$)

Parameters

$a(i,n)$	production cost per unit of product n produced at source i ($/unit)
$Bdd(g)$	total budget for supply chain g ($)
$c(i,n)$	inventory – cost per unit of product n at source i ($/unit)
$Djj(n,h,j)$	nominal demand of product n in period h at endpoint j (units)
$Dem(n,h,j)$	demand of product n in period h at endpoint j according to the considered price elasticity of the demand
e_b	escalating factor for (regular production cost, backorder cost, and inventory cost)
Ed	price elasticity of demand (%)
$F(i,h)$	maximum labor level of work at source i in period t (man-hour)
$II(i,n,h)$	initial storage (units)
$k(i,n,j)$	transport cost per unit of product n from the source i to the endpoint j ($/unit)
$l(i,n)$	hour of work per unit of product n produced at source i (man-hour/unit)
$M(i,h)$	maximum machine level available at source i in period h (machine-hour)
$Maxd(i,n)$	maximum acceptable quantity of product n to be distributed from source i in a period (units)
$Maxp(i,n)$	maximum acceptable quantity of product n to be produced at source i in a period (units)
$Mind(i,n)$	minimum acceptable quantity of product n to be distributed from source i in a period (units)
$Minp(i,n)$	minimum acceptable quantity of product n to be produced at source i in a period (units)
$Prate(g)$	discount in the price for supply chain g (%)
$Ps(i,n,j)$	selling Price of product n produced at source i and distributed by endpoint j (100$/unit)
$r(i,n)$	required equipment occupation per unit of product n at source i (machine-hour/unit)
$Rdd(h,i)$	maximum storage space at production plant i in period h (units)
$s(i,n,h,j)$	capacity per truck for product n from source i to endpoint j (units/truck)
$u(i,n,j)$	transport time of product n from source i to end point j (hour/truck)
$vv(n)$	warehouse space required per unit for product n (ft^2/unit)

Decision variables

$Q(i,n,h)$	production of product n in the source i at time h (units)
$T(i,n,h,j)$	quantity delivered from the source i to endpoint j of product n at time h (units)
$W(i,n,h)$	inventor y level at source i of the product n at time h (units)

Binary variables

$X(i,n,h,j)$	binary variable identifying if product n is sent from source i to the endpoint j at time h.
$Y(i,n,h)$	binary variable identifying if the source i produces product n at time h

Objective functions

$CST(g)$	buyers' expenditure at supply chain g ($).
$z1(g)$	total cost of supply chain g. ($)
z2	total delivery time (time to deliver all the products to the consumers, hour)

offer a single final decision, leading to the decision maker the final responsibility to solve the tradeoff among conflicting objectives. Messac et al. (2003) enlisted and analyzed the most effective methods to generate a set of non-dominated solutions, like physical programming, normal boundary intersection, normal constraint, weighted sum, etc. These methods have been successfully applied for supply chain management in several industrial sectors, such as: chemical (Rodera et al., 2002; Sabio et al., 2011), pharmaceutical (Nicolotti et al., 2011), petrochemical (Zhong and You, 2011), food (Kostin et al., 2012), or automotive industries (Cook et al., 2007).

The specific approaches developed to apply these methods are basically classified in two types of procedures: meta-heuristics frameworks (based on genetic algorithms, fuzzy programming, etc.) and mathematical frameworks (like constraint management methods). Both types of approaches have been extensively tested in the literature, exploiting their specific capabilities to manage different objectives for different purposes. Among the first group, a significant number of studies propose the use of genetic algorithms to optimize total cost and service levels during the design step of a supply chain network; for example: Farahani and Elahipanah (2008) propose to minimize both the total cost and the sum of backorders and production excess in all periods, or Altiparmak et al. (2006) suggest to minimize the total cost and to maximize the customer service levels; the design problem has been also addressed by MO fuzzy programming, for example by Chen and Lee (2004), who face uncertain product demands and prices through the incorporation of one objective associated to the robustness of the decision making. Closer to the specific topic of this paper, Li et al. (2012) develop a MO genetic algorithm to solve the integrated planning and scheduling model, considering the different objective functions related to each hierarchical level, and Liang (2008) uses a fuzzy multi-objective approach to optimize the production and distribution planning in a real supply chain case study.

The capacity and robustness of Mathematical Programming based optimization tools justify an increasing number of works proposing the use of these type of procedures to this field: Further to the most conventional applications, like the one proposed by Kostin et al. (2012), where a MILP approach is developed to design a bioethanol SC considering several objective functions (five environmental indicators are simultaneously optimized along with the net present value), Guillén et al. (2005) develop a MOO model to design supply chain networks under uncertain production operation, adding to the usual goals (net present value and demand satisfaction) an objective specifically focused on this new considered element (financial risk); in the same line, Cristobal et al. (2013)

propose to solve the optimal timing of investments in CO_2 capture technologies considering exogenous uncertainty sources (the expected evolution of emission taxes) also in terms of the associated financial risk. More closely related to the hierarchical decision making level focused in this work, Liu and Papageorgiou (2012) present a MILP based approach for the optimization of global supply chains production-distribution planning, where objectives such as total cost, total flow time (summary of the weighted transportation time), and total lost sales in all the time periods are balanced.

In all the cases previously mentioned, the MOO problem is oriented to solve the tradeoff among "local" objectives, and this fact results in a limited representation of the SC working scenario, since the model maintains as underlying assumption that this scenario and the SC of interest are not interacting in both ways. In this sense, although the tradeoff among multiple objectives has been applied to solve different situations, none of the proposed models simultaneously considers objectives from different actors, which would be the bases to obtain a detailed integrated production and inventory planning of the system of interest under cooperation/competition with third parties (other SCs, markets, etc.).

The complexity associated to the multiple objective analysis is additionally complicated by the need to consider external sources of uncertainty into the model formulation: the nature of the planning problem is uncertain (the allocation of production and storage actions is based on the forecasts of the quantity to be demanded, working levels, etc.), so these uncertainties are an absolutely essential element that must be taken into account during its resolution.

The procedures developed to solve decision making problems under uncertainty can be classified as Reactive or Preventive approaches (Sahinidis, 2004). Reactive approaches use the knowledge of the process to obtain the best solution once the uncertainty has been revealed, while preventive approaches explicitly include the potential uncertainties into the model formulation/resolution in order to obtain a solution which optimizes the expected performance over the whole range of possible scenarios, accepting that probably this solution will not be the best suited to address the final situation once the uncertainty is revealed.

The literature in the area of SC planning under uncertainty has been also very active over the last decade, focused on several sources of uncertainties such as: demand (Peidro et al., 2009), product prices (Amaro and Barbosa-Póvoa, 2009), operating and economic conditions (Haitham et al., 2004), etc. In a previous work (Zamarripa et al., 2012), the uncertainty associated to the behavior of other alternative SCs, able to compete for the market demand was analyzed; the game theory (GT) was proposed as a decision technique to determine the best SC operating strategy disregarding further market decisions; the analysis confirmed how the competition behavior of the several coexisting SCs may be described as an uncertainty source, and justified the use of uncertainty models to take into account the eventual decisions of third parties (external SCs).

But the fact is that the uncertain behavior of both, the market and the eventual competitors, will be affected by the policy implemented by the SC of interest, the different customer satisfaction policies (prices, delivery times, etc.) directly impact into the profit of the SC of interest, and the markets are embedded in this competitive scenario, so the decision making policies from third parties, the level of collaboration with other equivalent suppliers (cooperative vs. competitive scenarios), and the way of representing the uncertainty associated to the competitors' behavior, remain unsolved, opening a very promising working line to improve supply chain management strategies in realistic working conditions (Fig. 1).

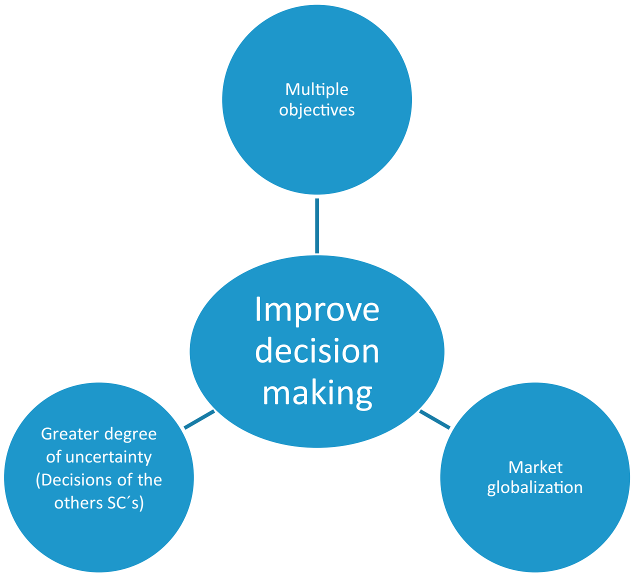

Fig. 1 – Open issues to improve the decision making.

2. Problem statement

The SC planning problem most typically addressed consists on determining the optimal production, storage, backorder, subcontracting and distribution variables associated to a SC network (Fig. 2) which results on the minimization or maximization of a certain performance measure (SC Objective Function) selected according to the specific policy adopted by the enterprise (minimization of the overall total cost, total delivery time, tardiness, or maximization of the benefit, etc.). The main constraints of this problem can be easily derived from physical limitations such as: mass balances, capacities (production, distribution, budget, storage) and time restrictions; these constraints are usually approximated through linear expressions, resulting in a mathematical formulation for these problems typically leading to a mixed integer linear programming (MILP) model, where the binary variables represent the necessity to decide about the use of the SC network resources (allocation to production and distribution elements).

The model originally proposed by Liang (2008) and modified by Zamarripa et al. (2012) has been adopted as a basis for the formulation presented in this paper. This formulation assumes the existence of several supply chains that may work in cooperative or competitive scenarios. In both cases, the mathematical constraints associated to the model will consider the material balances, the budget capacity, the production and storage maximum capacities, and the degree of demand satisfaction. This model has been expanded to consider the different objectives associated to other actors from the whole scenario where the SC of interest is embedded, so it will be used to minimize different Objective Functions (minimize total cost and total delivery time for the cooperative case; minimize the expenses of the buyers and total delivery time for the competitive case) according to the considered scenario.

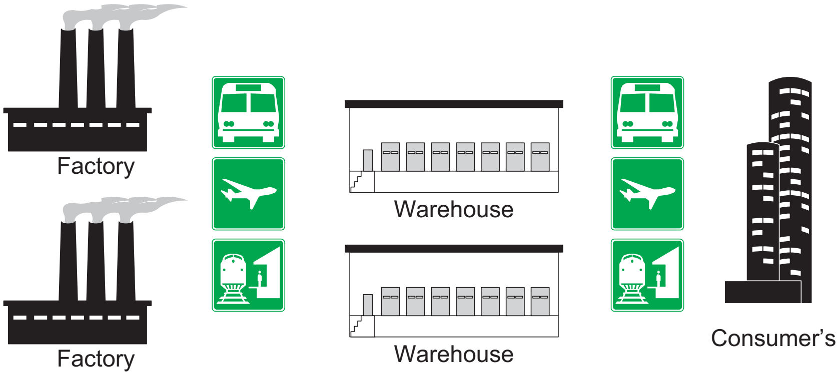

Fig. 2 – Supply chain network configuration.

2.1. *Cooperative problem*

In the cooperative scenario, the problem has been formulated considering the set of different SCs acting as a single coordinated SC ($SC_1 + SC_2 + \cdots + SC_G$). In this scenario, the objective is to obtain the tradeoff between the minimization of the total cost (z1) and the total delivery time (z2). The total cost is based on the economic terms (Eq. (1): production (a_{in}), inventory (c_{in}) and distribution (k_{inj}) costs of each supply chain *g*) and the total delivery time is defined as the time required to distribute the products (*n*) to the distribution centers (*j*) (Eq. (2)).

Minimize the total cost:

$$z1 = \sum_{g}^{G} z1_g = \sum_{g}^{G} \Bigg[\sum_{i \in G(i,g)} \sum_{n} \sum_{h} a_{in} Q_{inh} (1+e_b)^2 + \sum_{i \in I_G(i,g)} \sum_{n} \sum_{h} c_{in} W_{inh} (1+e_b)^2 + \sum_{i \in I_G(i,g)} \sum_{n} \sum_{h} \sum_{j} k_{inj} T_{inhj} (1+e_b)^h \Bigg] \quad (1)$$

Minimize total delivery time:

$$z2 = \sum_{i \in I\_G(i,g)} \sum_{n} \sum_{h} \sum_{j} \left[\frac{u_{inj}}{s_{inhj}} T_{inhj} \right] \quad (2)$$

As it has been previously mentioned, the constraints are associated to the material balances (Eqs. (3) and (4)) and the production levels of the production plants (i), which are constrained by the available labor levels (F_{ih}) and the available machine hours (M_{ih}) at each production plant (Eqs. (5) and (6), respectively). There are also some storage capacities (Rdd_{hi}) associated to each production plant (Eq. (7)); maximum and minimum distribution ($Mind_{in}$, $Maxd_{in}$) and production capacities ($Minp_{in}$, $Maxp_{in}$) are considered by Eqs. (8) and (9), respectively. The total demand satisfaction (Djj_{nhj}) in enforced in Eq. 10, and finally the budget capacity is delimited in Eq. (11).

$$II_{inh} + Q_{inh} - \sum_{k=1}^{J} T_{inhj} = W_{inh} \quad \forall i, n, h = 1 \quad (3)$$

$$W_{inh-1} + Q_{inh} - \sum_{j=1}^{J} T_{inhj} = W_{inh} \quad \forall i, n, h > 1 \quad (4)$$

$$\sum_{n=1}^{N} l_{in} Q_{inh} \leq F_{ih} \quad \forall i, h \quad (5)$$

$$\sum_{n=1}^{N} r_{in} Q_{inh} \leq M_{ih} \quad \forall i, h \quad (6)$$

$$\sum_{n=1}^{N} vv_n W_{inh} \leq RDD_{h,i} \quad \forall h, i \quad (7)$$

$$X_{inhj} Mind_{in} \leq T_{inhj} \leq X_{inhj} Maxd_{in} \quad \forall i, n, h, j \quad (8)$$

$$Y_{inh} Minp_{in} Q_{inh} \leq T_{inh} Maxp_{in} \quad \forall i, n, h \quad (9)$$

$$\sum_{i=1}^{I} T_{inhj \geq Djj_{ihj}} \quad \forall n, h, j \quad (10)$$

$$z1_g \leq BDD_g \quad \forall g \quad (11)$$

Then, the cooperative model consists on Eqs. (3)–(11), as constraints to optimize the production, inventory, distribution and backorder actions under multiple criteria (objective functions Eqs. (1) and (2)).

2.2. *Competitive problem*

In the competitive scenario, the GT is used to identify the different players (SCs), which can consider two types of games: zero-sum and nonzero sum. In this work, the nonzero-sum game is proposed, since the SC of interest will not try to maintain the overall benefit of the system. This strategy is implemented through a payoff matrix, which is made up by the different potential strategies and shows the behavior for each action of the SC against the actions of its competitors.

To play this game (competition behavior), players should deal with the demand share (from the total demand) that customers really offer to each one, and this can be managed basically through their service policy: prices and delivery times. So, in the competitive case, additionally to the already considered objectives, it is necessary to introduce a new objective representing the best deal for the customers (cost for the distribution centers). This has been done through the price rate ($Prate_g$): given the price of the products (Ps_{inj}), an applicable discount has been considered as associated to the source and the destination of the products, and Eq. (12) should be now

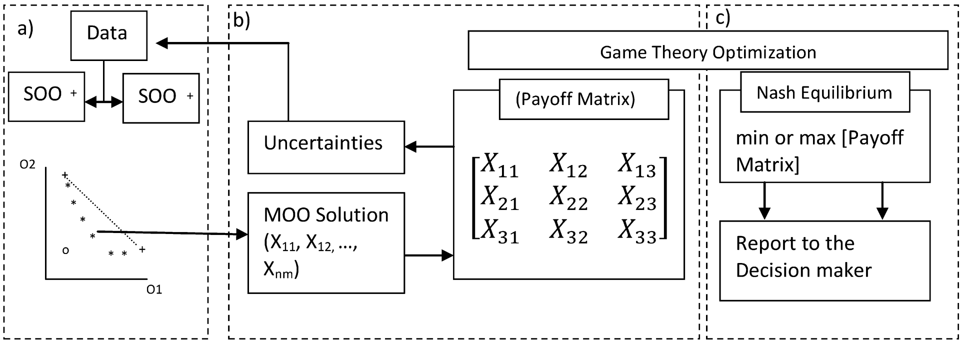

Fig. 3 – Proposed framework.

considered as the new objective function.

$$Min\, CST = \sum_{g}^{G}\left[\left(\sum_{i \in I\_G(i,g)}^{I}\sum_{n}^{N}\sum_{h}^{H}\sum_{j}^{J} Ps_{inj} T_{inhj} Prate_g + z1_g\right)\right] \quad (12)$$

In order to improve the modeling of the demand behavior, a price elasticity of the demand (Ed) has been also considered (Eq. (13)) to indicate the sensitivity of the demanded quantity to the price changes (Varian, 1992), so Eq. (14) indicates the new demand to be satisfied by the set of SCs, based on the original demand satisfaction, the discount rates and prices this elasticity of the demand.

$$Ed = \frac{\Delta D/D}{\Delta P/P} \quad (13)$$

$$Dem_{ihj} = \max_{g}\left[Djj_{nhj} - Ed\, Djj_{nhj}\left(\frac{Prate_g}{100}\right)\right] \quad \forall n, h, j \quad (14)$$

Then, the competitive model aims to optimize the trade-off between the expense of the buyers (Eq. (12)) and the total delivery time (Eq. (2)) for several supply chains, subjected to the constraints modeled in the Eqs. (3)–(9), (11) and (14) under different price policies.

2.3. Multi-objective optimization (MOO)

Among the different decision making methods based on multi-objective optimization proposed in the literature, the ε-constraint method to approximate the Pareto space of non-dominated solutions (based on the formulation proposed by Messac et al., 2003), and the selection of the solution closer to the utopia point, have been the methods used in this work to calculate the values to be included into the payoff matrix (Fig. 3a). These choices are quite common in the literature (Liu and Papageorgiou, 2012; Bojarski et al., 2009, and many others) since not only offer a fast, robust and reliable way of calculation (Sabri and Beamon, 2000) but also ensure a fair and traceable comparison among the different alternatives to be generated.

So the basic MOO procedure can be described as follows (Fig. 3a):

1. Obtain the anchor points
   (i) Solve the single objective optimization (SOO) problem for each objective considered.
   (ii) Identify other main elements of the current problem (utopia point, nadir point, . . .).
2. Divide the Pareto frontier into mk points.
3. Obtain the "ik" Pareto point as the result of a constrained optimization based on the corresponding SOO model.
   (i) Repeat this optimization for all the mk points to approximate the Pareto frontier (. . .. .).
4. Identify the representative solution (for example, the point closer to the utopia point).

2.4. Game theory and equilibrium point

The GT is based on the simulation of the results obtained by a set of players (i = 1, . . ., I) following different strategies (S_n; n = 1, . . ., N). These results are represented through a sort of payments ($P_{i,n}$; i = 1, . . ., I; n = 1, . . ., N) received by each player. In simultaneous games, the feasible strategy for one player is independent from the strategies chosen by each one of the other players. Optimum strategies depend on the risk aversion of the players, so different strategies can be foreseen. Depending on the knowledge about the strategy of the other players, other solutions resulting from the concept of Nash equilibrium point (Nash, 1950), can be devised.

The use of GT as a decision making tool has been extensively proved and, specifically, in a previous work (Zamarripa et al., 2012) it was used to introduce the competition behavior as a source of uncertainty at the tactical decision level of SC Management. In this previous work it was proposed to use the payoff matrix associated to the information of several SCs (players) that compete for the demand of a given market to summarize the solutions of several scenarios considered by each player (SC_1, SC_2, . . ., SC_G), showing how the competitive behavior affects the decision making of the SC of interest. This approach was specifically focused on the presence of competitors so, within the game theory framework, a single objective approach was enough to manage the eventual cooperation or competition of the SC of interest in front of alternative suppliers.

2.5. Proposed framework

In order to deal with the complexity associated to the competition among several SCs, and also to keep looking at different objectives simultaneously, this work proposes the use of an Integrated Optimization Framework composed by a Multi-Objective Mixed Integer Linear Programming model (MO-MILP) embedded in an game theory optimization framework, enhancing the tactical decision making of a SC in competitive/cooperative environments, where decisions are the production, inventory and distribution profiles of the

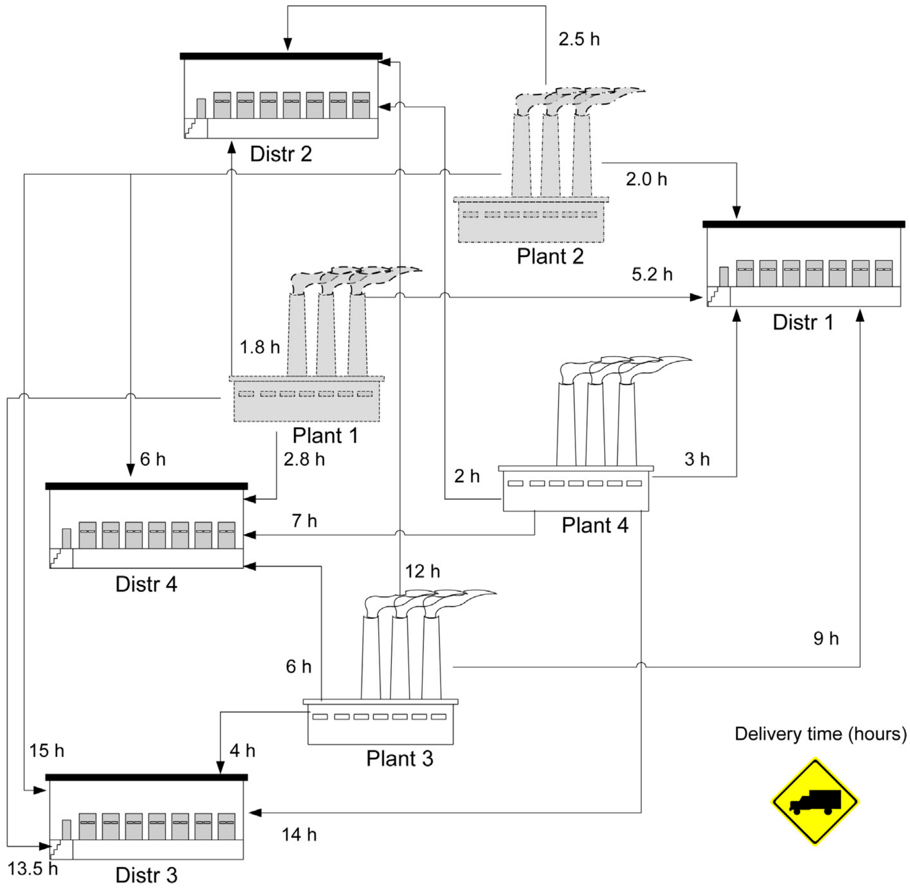

Fig. 4 – Description of the SC network. Plants1–4 serve Distr1–4.

different echelons of the SC of interest over a given time horizon. As previously indicated, its practical implementation is supported by the use of the ε-constraint method to generate the corresponding space of non-dominated solutions, and the systematic selection of the point to be introduced in the payoff matrix to finally to locate the Nash equilibrium point, which represents the best solution for the several scenarios considered.

Specifically, in order to find the best SC policy to face the current competing scenario, it is proposed to use the multi-objective optimization paradigm to model the tradeoff associated to the competition/negotiation among the different interacting elements in the network, namely the several SCs and their markets, through the introduction of external objectives to model this negotiation, which is crucial for the decision making. So the interactions among competitors and markets can be now represented through elements like the customer satisfaction, delivery time, etc.

Fig. 3 schematizes the application of this proposed approach:

1. Read the data of the given scenarios of the payoff matrix (several scenarios of the competition behavior).
2. Solve the MOO (Fig. 3a) for a specific competence scenario.
3. Select the solution which will represent the specific competitive scenario into the payoff matrix among the solutions reported by the MOO.
4. Repeat 1–3 for all the competitive scenarios to be considered in the payoff matrix (Fig. 3b).
5. Report the best value of the payoff matrix (Fig. 3c).

The knowledge obtained from the analysis of the space of the Pareto solutions for the different considered objectives (Fig. 3a) is used to determine the best market share that the SC of interest should attain, assuming that the markets will identify the optimal position for them. Additionally, these Pareto spaces of solutions can be used as a way to assess customers' decisions.

At this point it is worth noting that, since different customers might select in different ways between their different objectives, the way to select the "best solution" from the Pareto frontier should be object of a careful analysis, which would allow to a more flexible application of the procedures proposed in this work.

This optimization framework will really act as a bargaining tool: even when there is a competition and each SC would like to modify its behavior looking for more benefit, the proposed procedure will identify the situation where the tradeoff between objectives will result in a more efficient proposal (less delivery time, cheaper products, etc.). For example: if the SC of interest reduce its costs, the MO Optimization framework will identify not only the expected increased demand from the market, but also the expected increment on the total delivery time associated to this additional demand, which should be also taken into account by the SC managers to find a compromise decision.

Finally, the proposed GT based strategy emulates a sensitivity analysis, characterizing the potential scenarios in order to determine the resulting optimum solutions for each case, using a payoff matrix as basic decision making tool to implement a reactive approach to face market scenario uncertainty. In this way, not only the best solution may be easily implemented when the uncertainty is revealed, but also the best

Table 1 – Comparative results between SC (original data and standalone cases).

	SC1 Liang (2008)	SC1 Standalone	SC2 Standalone
Obj. Funct.	min z1 + z2	min z1 + z2	min z1 + z2
z1($)	788 224	838,652	840,904
z2(hours)	2115	1700	1747
Benefit ($)	–	3,665,347	3,663,095
CST ($)	–	5,342,652	5,344,904

strategies to be used in the different potential situations may be also analyzed and identified in advance, and so used as preventive decision guidelines.

3. Case study

All the concepts presented in section 2 have been applied to a supply chain case study adapted from Wang and Liang (2004, 2005), Liang (2008) and Zamarripa et al. (2012). The network configuration is composed by 2 SCs, SC_1: Plant1/Plant2 and SC_2:Plant3/Plant4, which collaborate or compete (according to the considered situation) to fulfill the global demand from 4 distribution centers. The market demands for Product 1 and 2 at the distribution centers are the ones shown in Table A.3. A discrete time model is used, and in this case-study the time horizon has been considered divided in 3 monthly periods. The information about production capacities and the rest of problem conditions (initial storage levels, transport capacities, and associated costs) can be found in Appendix A (Tables A.1–A.5).

In order to enforce a fair collaboration and/or competition among the 2 considered SCs, the same labor levels, production capacities, production costs (based on the ones originally proposed for Plant1 and Plant2) and initial inventories have been considered also for Plants 3 and 4 respectively (Tables A.1 and A.5). The proposed geographical distribution for Plants 1–2 and distribution centers 1–4 are coherent with the transport times and costs proposed in the original case study. Plants 3 and 4 have been incorporated as represented in Fig. 4, and transport times and costs have been calculated (second term in Table A.2) to be coherent with this representation. Finally, transport costs related to all the distribution tasks (first term in Table A.2) have been modified respect to the original data by a factor of 10 in order to get more significant differences in the obtained results.

Since the overall production capacity of the problem now described (Plants 1–4) is double than the one considered in the original problem (Plants 1 and 2), subcontracting and backorders actions are eliminated. In order to compare cooperative and competitive scenarios under uncertainty, the original demand (Liang, 2008) will be considered.

Different discount rate scenarios have been considered in order to address the capacity of each SC (SC_1 and SC_2) to manage its market share in the competitive problem ($Disc_{sc}$ 0.1–0.4%); a nominal selling price has been also introduced to maintain data integrity (100$ for all products, plants, and distribution centers) and, since changes in the prices are considered, a price elasticity of the demand has been assumed ($Ed = -5$).

4. Results

The proposed framework has been implemented by combining the capabilities of a general purpose numerical computing environment (Matlab®) and a high level mathematical modeling system (GAMS®) with a MILP solver (CPLEX v12), using Matgams (Ferris, 2005) to link both environments. The mathematical model resulting from the proposed case study is relatively small (consists of 384 equations, involving 312 continuous variables and 288 discrete variables) so it requires very low computational effort to be solved (less than 1 CPU second on a Windows XP computer using Intel® Core™ i7 CPU(920) 2.67 GHz processor with 2.99 GB of RAM).

4.1. *Cooperative case*

To better compare the different alternatives considered, the standalone results for each SC (using the aforementioned selection method from the Pareto space of non-dominated solutions) are displayed in Table 1, as well as the results obtained by Liang (2008). The complete sets of non-dominated solutions for both considered SCs, including the ones obtained as solutions of the SOO problems, show how SC_1 is geographically better located than SC_2 for the proposed demand (the one originally demand used as set point by Liang, 2008), although it is worth mentioning that quite different results would be obtained for other demand profiles. The main differences between the results reported by Liang (2008) and the current SC_1 standalone results are associated to the fact that an additional distribution

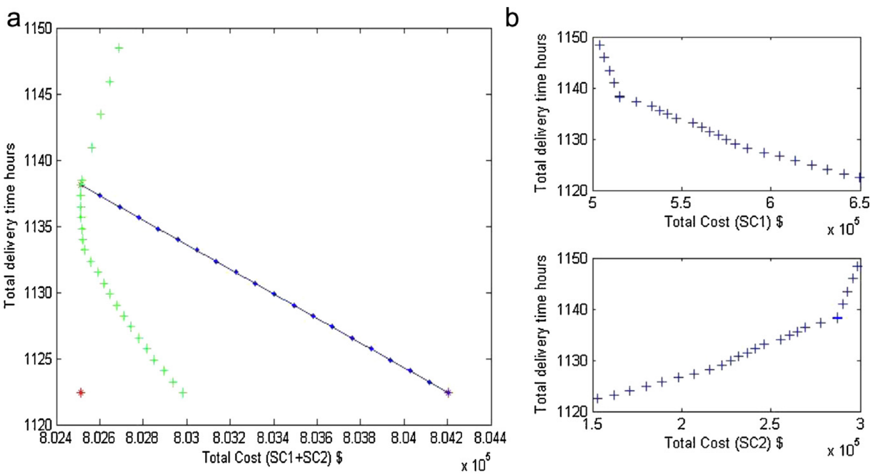

Fig. 5 – Pareto solutions for the cooperative case.

Table 2 – Payoff matrix for the competitive case (discount rate, %).

SC1 policy		SC2 policy 0.0		0.1		0.2	
		SC1	SC2	SC1	SC2	SC1	SC2
0.0	Cost ($)	579,865	222,812	565,800	236,790	532,133	270,379
	Total Cost ($)	802,677		802,590		802,512	
	Delivery time (h.)	1129		1127		1138	
	CST ($)	3,869,465	1,437,212	3,731,500	1,573,800	3,445,333	1,857,998
	Benefit ($)	2,709,734	991,587	2,599,900	1,100,200	2,381,066	1,317,238
0.1	Cost ($)	649,978	153,002	586,993	215,717	565,795	236,793
	Total Cost ($)	802,980		802,710		802,588	
	Delivery time (h.)	1122		1128		1132	
	CST ($)	4,262,938	1,036,802	3,914,662	1,387,544	3,728,330	1,572,416
	Benefit ($)	2,962,980	730,797	2,740,675	956,109	2,596,738	1,098,829
0.2	Cost ($)	642,400	160,544	642,400	160,544	579,865	222,812
	Total Cost ($)	802,944		802,944		802,677	
	Delivery time (h.)	1123		1123		1129	
	CST ($)	4,224,514	1,078,844	4,220,928	1,077,926	3,862,886	1,434,783
	Benefit ($)	2,939,713	757,755	2,936,128	756,836	2,703,154	989,159

cost has been introduced, as justified in the previous section, so now the optimal solution for SC_1 (standalone) is partially driven by the geographical conditions (nearest delivery).

Differences between SC_1 and SC_2 standalone solutions are associated to the different distances from the production sites of SC_2 to the markets. The MO Optimization allows to identify more balanced solutions, like the ones finally presented in Table 1, where the total cost has increased but the total delivery time is much better. Logically, other different policies to solve this tradeoff will lead to different solutions, which finally depend on the specific considered objectives.

The solutions obtained for the cooperative case (when SC_1 and SC_2 work together to meet the overall market demand) are represented in Fig. 5a, which includes the Pareto front for the multi objective problem (total delivery time vs. total cost), the anchor points representing the best optimal solutions for each objective, and the utopia point, which is the unfeasible point resulting from the combination of the best individual values of each objective. The information behind Fig. 5a includes the best market share among the different SCs (SC_1 and SC_2) in each case, since the Pareto solutions represent the balance between both objectives, and one of them is related to the way the markets can be benefited from this cooperation.

In this sense, Fig. 5a may be understood as composed by the individual Pareto frontiers (Fig. 5b), so each solution of the Pareto space for the cooperative case (Fig. 5a) shows how different SC policies lead to different market share situations, which in turn affect to the different considered objectives, e.g., a delivery time of 1122 h would require a total cost of 6.52×10^5 $ for SC_1 and 1.522×10^5 $ for SC_2, which finally lead to the anchor point of 8.042×10^5 $ in the Pareto front (Fig. 5a). Actually, this point has been obtained from the best solution of the SOO problem, as minimizing the total delivery time. It is also worth to note how certain SC policies would lead to globally dominated solutions (upper part of Fig. 5a, where both objectives become worse).

Looking at the tradeoff among objectives, the fact that SC_1 is better located to attend the considered market demand is confirmed, although again the situation should be object of a more careful analysis. For example, it is encountered that, minimizing the total delivery time, in its best solution SC_1 would capture 81.2% of the market, while minimizing the total cost SC_1 would capture only 64% of the market.

4.2. *Competitive case*

When both SCs compete in order to get their best individual results, the model must take into account the markets' preferences based on expected service and cost. Assuming that SC_1 is the "SC of interest", and considering that the main element used by both SCs in order to manage its market share is the price policy (Eq. (12)), the corresponding payoff matrix has been built with the best solutions obtained from the Pareto

Table 3 – Optimum SC1 solutions.

	Cooperative solutions SC1	SC2	Competitive solutions SC1	SC2
Discount	–	–	0.1	0.0
Obj. Funct.	Min *Total cost* and *Total delivery time*		min CST and *Total delivery time*	
z1($)	579,865	222,812	649,978	153,002
Total cost ($)	802,677		802,980	
z2 (hours)	1129		1122.4	
Benefit ($)	3,869,465	1,437,212	4,262,938	1,036,802
CST ($)	2,709,734	991,587	2,962,980	730,797

frontier[1] for several of the considered scenarios (Table 2). A representation of the Pareto solutions for each scenario can be found in Appendix A (Figs. A.1–A.9).

As it is stated in game theory, there could be more than one Nash Equilibrium points in the payoff matrix, but for each case (SC_2 scenario) an optimal solution for SC_1 can be found. As shown in Table 2, when SC_2 offer discounts of 0.0%, 0.1%, and 0.2%, the best SC_1 policies are reached at discount prices of 0.1%, 0.2%, 0.2% respectively, so these solutions represent the higher benefit for SC_1 (2963k$; 2936k$ and 2703k$ respectively). The constraints associated to the production, distribution, and budget capacities act as limitation for the case study presented: e.g., in scenario 1, further discounts in the prices offered by SC_1 will not represent higher benefits.

Table 3 summarizes the solutions of both the cooperative case and the best SC_1 policy for the competitive case assuming that SC_2 plays the scenario 1 with 0.0% of discount rate. As it can be observed, the total cost of the cooperative game is slightly lower ($ 802,677) than the one resulting from the competitive game ($ 802,980). Hence, the cooperative problem represents a global improvement in the overall cost of the SC ($SC_1 + SC_2$). It should be noted that this solution does not correspond to a Nash equilibrium point, which in this case only appears in the scenario with higher discount rates; actually, this solution corresponds to the worst decision for SC_2, even it leads to the most efficient situation in terms of delivery times (market share).

So as a summary, in the cooperative scenario the policy adopted will be to deliver products in the most profitable way to the market; consumers will not be able to decide about the way how SCs will share the market and the proposed solution will be equivalent to the one obtained by a Single Objective analysis: production cost will be globally reduced and customers may benefit from cheapest production policies (unless suppliers adopt oligopoly policies to fix prices) although other customer satisfaction indicators (like delivery times) might be ignored by the suppliers. On the contrary, in a competitive scenario, the entire problem changes and multiple policies to satisfy the market demand should be considered by the suppliers. Specifically in this case: the policy "deliver the cheapest products" should change in order to provide cheaper products that require shorter delivery time, which is the one leading to higher benefits. The knowledge of the way how costumers prioritize these objectives and the way how competitors play this game are essential to maintain SC goals.

5. Conclusions

This work presents an integrated way of using different optimization methodologies to improve supply chain planning decision-making, in order to be able to face the new challenges of present and future market trends (reduction of inventories, challenges associated to new market competition behaviors, production capacity changes, enhanced process/logistic flexibility, etc.). Specifically, a decision making tool has been built by combining the use of the game theory optimization framework and a MOO MILP-based approach to determine the production, inventory and distribution profiles, optimizing the SC planning problem under uncertainty and considering cooperative and competitive multi-objective scenarios.

The proposed approach allows to find more robust decisions, since it emphasizes the role of third parties, which is usually disregarded (or it is assumed as part of the demand uncertainty) to solve the SC planning problem. The uncertainty in the competitors behavior has been independently considered, although the GT allows the identification of the best strategies to be used in different potential situations as a reactive decision making tool. In this sense, the obtained solutions may offer decision guidelines, although further work should be devoted to develop a preventive strategy based on a stochastic programming, considering the multi-objective tradeoffs analyzed in this manuscript.

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Appendix A. Additional case study data and results

The proposed case study, has been developed including information of several examples proposed by Wang and Liang (2004, 2005) and Liang (2008), considers an initial inventory of 400 units of P1 and 200 P2 for both Plant 1 and Plant 3, and 300 P1 and 200 P2 for both Plant 2 and Plant 4. To maintain the competence in the production, distribution and inventory tasks, the maximum and minimum production and distribution parameters have been considered equal for both SC'2 (production min/max 0/10,000 units of products in each time period, and distribution min/max 10/1200 units of products in each travel) (Figs. A.1–A.9 and Tables A.1–A.5).

[1] As previously indicated, the "best" result was defined as the closest solution to the Utopia point, so the selected solutions can be fairly compared with the solutions proposed for the cooperative case – Table 3, but it is worth mentioning that a different selection systematic will lead to different solutions.

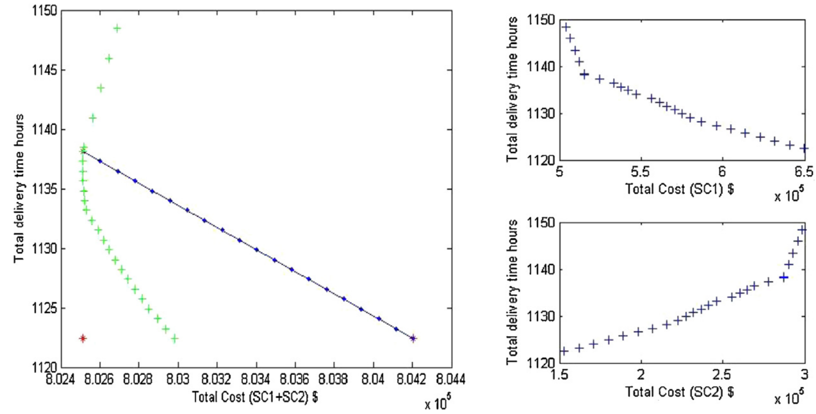

Fig. A.1 – Pareto frontier scenario (% of discount, SC1 0.0, SC2 0.0).

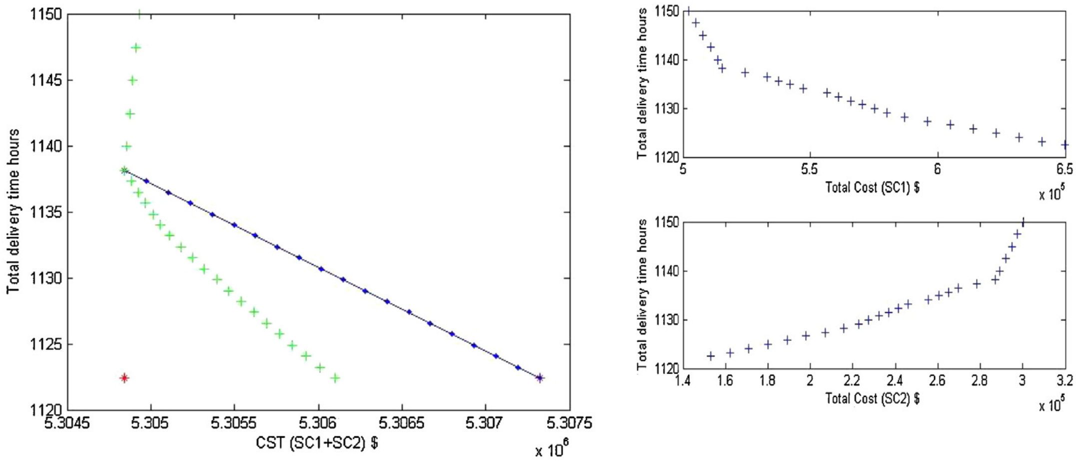

Fig. A.2 – Pareto frontier scenario (% of discount, SC1 0.0, SC2 0.1).

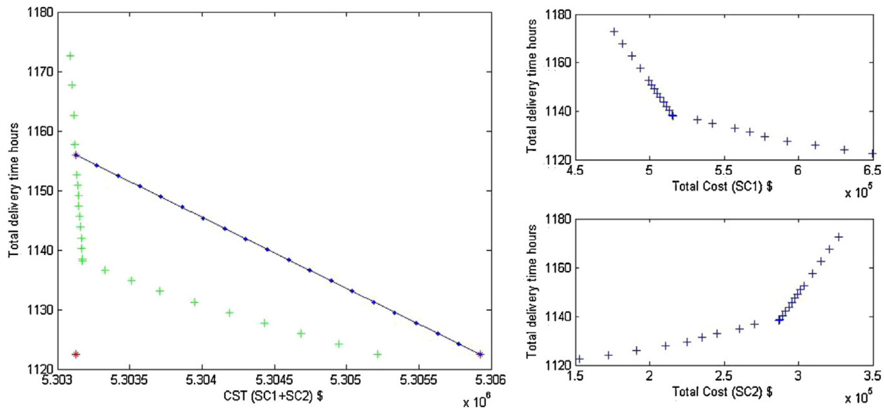

Fig. A.3 – Pareto frontier scenario (% of discount, SC1 0.0, SC2 0.2).

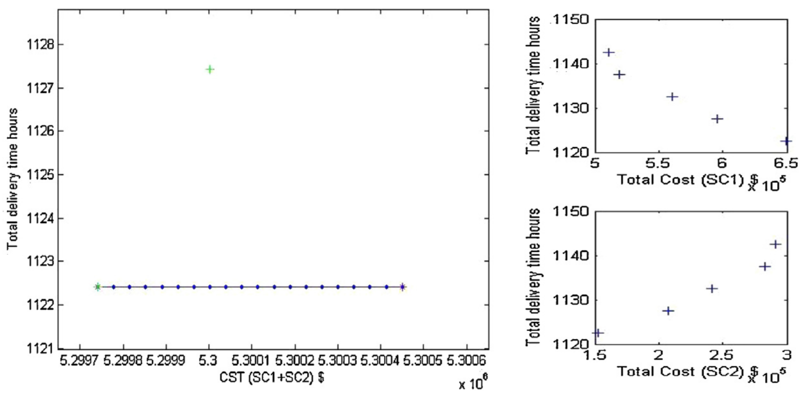

Fig. A.4 – Pareto frontier scenario (% of discount, SC1 0.1, SC2 0.0).

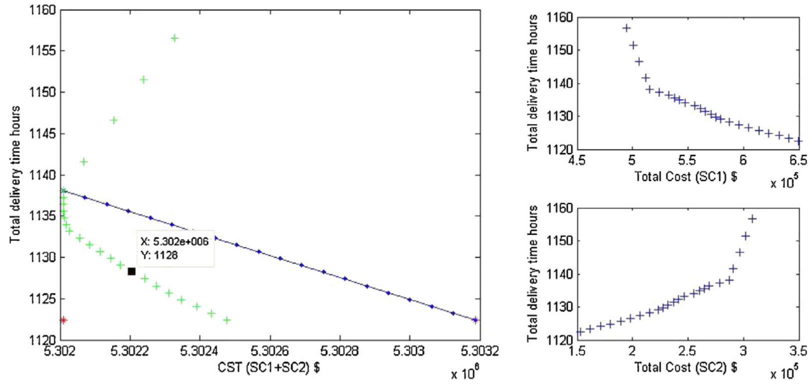

Fig. A.5 – Pareto frontier scenario (% of discount, SC1 0.1, SC2 0.1).

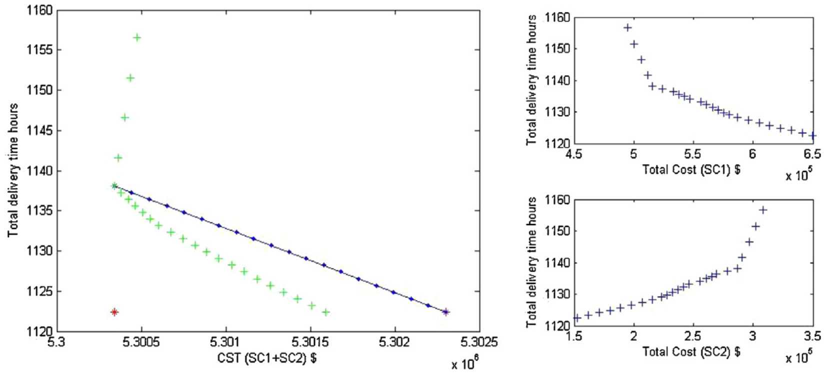

Fig. A.6 – Pareto frontier scenario (% of discount, SC1 0.1, SC2 0.2).

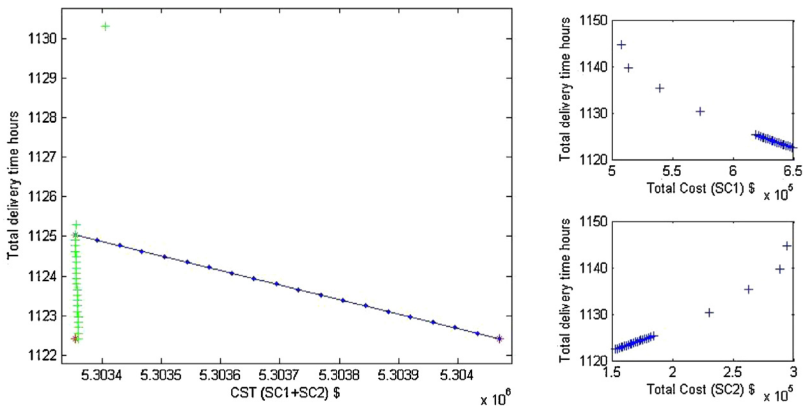

Fig. A.7 – Pareto frontier scenario (% of discount, SC1 0.2, SC2 0.0).

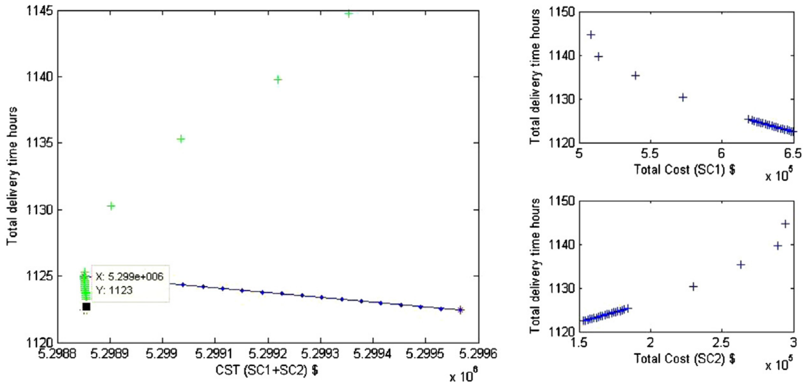

Fig. A.8 – Pareto frontier scenario (% of discount, SC1 0.2, SC2 0.1).

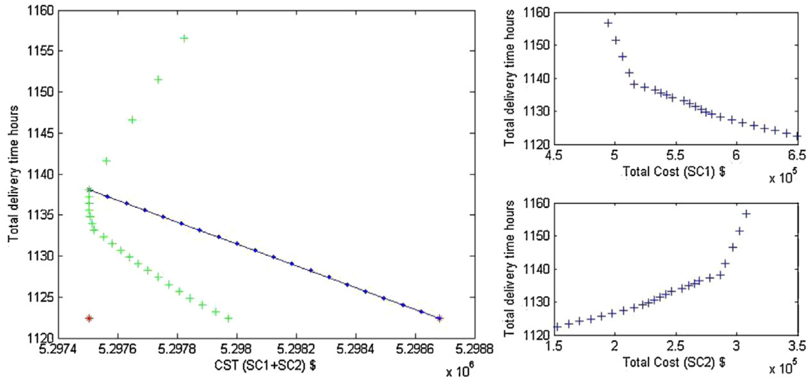

Fig. A.9 – Pareto frontier scenario (% of discount, SC1 0.2, SC2 0.2).

Table A.1 – Problem data.

Source	Time	Product	a_{in}	c_{in}	d_{in}	l_{in}	r_{in}	vv_n	Rdd_i
Plant1	3 months	P1	20	0.3	32	0.05	0.1	2	19500
		P2	10	0.15	18	0.07	0.08	3	
Plant2		P1	20	0.28	20	0.04	0.09	2	16000
		P2	10	0.14	16	0.06	0.07	3	
Plant3		P1	20	0.3	32	0.05	0.1	2	10000
		P2	10	0.15	18	0.07	0.08	3	
Plant4		P1	20	0.28	20	0.04	0.09	2	20000
		P2	10	0.14	16	0.06	0.07	3	

Table A.2 – Network distribution costs/delivery times.

Source	Product	Distribution centers			
		Distr1	Distr2	Distr3	Distr4
Plant1	P1	[a]28/5.2[b]	10/1.8	42/13.5	22/2.8
	P2	25/5.2	9/1.8	40/13.5	20/2.8
Plant2	P1	12/2	15/2.5	50/15	35/6
	P2	11/2	14/2.5	45/15	32/6
Plant3	P1	44/9	59/12	11/4	35/6
	P2	39/9	54/12	10/4	32/6
Plant4	P1	15/3	10/2.0	38/14	41/7
	P2	13/3	9/2	35/14	37/7

Table A.3 – Demand forecast.

Demand product	Distr1			Distr2			Distr3			Distr4		
	t1	t2	t3	t1	t2	t3	t1	t2	t3	t1	t2	t3
P1	1000	3000	5000	820	2300	4000	500	1200	2400	1230	3400	5300
P2	650	910	3000	500	720	2400	300	400	1150	710	1050	3100

Table A.4 – Available labor levels ($F_{i,h}$).

	Time period		
	t1	t2	t3
Plant1	965	1040	1130
Plant2	850	920	990
Plant3	965	1040	1130
Plant4	850	920	990

Table A.5 – Production capacities ($M_{i,h}$).

	Time period		
	t1	t2	t3
Plant1	1550	1710	1870
Plant2	1850	2050	2250
Plant3	1550	1710	1870
Plant4	1850	2050	2250

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