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Influence of negative ions on the humidity effect on the first corona inception

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Abstract
The first corona inception in a divergent electric field is affected by atmospheric parameters. The effect of humidity is a little-debated question and a summary review is first proposed in this paper. Then we present experimental results performed in two different high voltage laboratories. Those two laboratories are located at different latitudes (Germany and northern Argentina) in order to put in evidence the effect of humidity on the inception voltage. At high humidity levels, the standard deviation of the probability distributions is clearly reduced whereas the influence of humidity upon the mean values is made up by quite a large discrepancy in the data. Nevertheless, the critical volume model allows the experimental probability distributions to be calculated with the proviso that the negative ion density directly involved in the primary electron production rate is field dependent and not constant as previously assumed. The calculations show that the total negative ion density is exponentially linked to the humidity.

1. Introduction

The effect of air moisture upon the inception parameters of the first corona, such as the inception voltage level U_i, the time lag t_i and the maximum electric field E_i on the high voltage electrode, is a much debated question. Disagreements arise from the fact that experimental conditions are not always comparable: atmospheric parameters, ion density, gap geometry and the wave shape of the applied voltage may differ. The experimental results which act as 'references' come from the Renardières Group (1977) where voltage impulses are applied to large point to plane gaps. The results of the tests indicate that an increase in humidity leads to an increase of the mean inception voltages. In the same publication, Hahn and Fischer achieve tests in a sealed chamber with controlled humidity levels. They use a rod–plane gap with $d = 20$ cm and $r = 1$ mm and the voltage rate of rise is increased from 0.3 to 10 kV cm^{-1}. There is no strong influence of humidity on the inception voltage except for the somewhat larger standard deviation at low humidity.

In the eighties many studies were conducted on that topic. According to those studies the length of test gaps is usually smaller than 1 m, the tip radius could vary from a few millimetres to tens of centimetres and the variation of humidity is either natural or monitored by means of adequate systems. When the impulse voltage applied to the test gap has a very steep front ($<2\,\mu$s) and a very large tail ($>2000\,\mu$s) the first corona occurs close to or after the time to crest. An increase in the humidity clearly induces an increase the time lag (Allen *et al* 1980, 1981, Berger 1980, Poli 1985). After the time to crest, the applied voltage decreases very slowly (it remains quasi constant) and no clear conclusion can be drawn about the effect of H upon the inception voltage. Otherwise, when the time to crest is long enough to allow the first corona inception to occur during the impulse front, the effect of humidity is made up by the quite large scattering of the t_i and U_i distributions. Nevertheless, Allen *et al* (1981) show that humidity clearly reduces that scattering. Davies *et al* (1988) note that at a low humidity level the observed inception times are widely spread whereas, as the humidity level increases, they become

more grouped and their mean values decrease. For a fixed crest voltage, the mean inception voltage varies linearly with absolute humidity. They notice that the voltage rate of rise influences (i.e. very high crest voltage) the slope of the linear relation between H and U_i. That slope is negative with deep fronts, increases when the crest voltage decreases and becomes positive at the smallest crest voltage. Topalis *et al* (1988) observe that the inception voltage is slightly influenced by humidity, decreasing with an average rate of approximately 0.9% per 1 g m^{-3} increase of humidity. Nevertheless, they add that the scattering of the inception voltage does not allow to confirm that humidity really affects the U_i distribution.

To these experimental tests must be added theoretical studies. It is known that the statistical distribution of the inception voltage, U_i, is linked to the production of a primary electron generated by detachment from the O_2^- ions within a critical volume (Gallimberti 1979, Berger 1980, Poli 1985). How can the humidity affect the U_i distribution? (a) By modifying the negative O_2^- ion density due to clustering processes $[O_2^-(H_2O)_n]$; (b) by modifying their life-time and thus, the production rate of a primary electron; (c) by modifying the effective ionization coefficient $\alpha' = \alpha - \eta$ (difference between ionization and attachment coefficients), which allows the critical volume to be delimited.

The aim of this paper is to discuss how the critical volume model can take the influence of the humidity into account. The analysis is based upon experiments conjointly performed in two different countries where the atmospheric conditions are different enough to record first corona inception parameters at a large range of humidity. It is assumed that, if the clustering process decreases the O_2^- density at a high humidity level, the hydrates act as a 'reservoir' able to re-increase the O_2^- density when the electric field increases within the critical volume.

2. Experimental arrangement

The tests have been performed in the high voltage laboratories of the Technical University of Munich (Germany, temperate climate) and of the National University of Tucumán (Argentina, tropical climate). Particular attention has been paid to calibrating the experimental devices in order to get analogous measurements. In both places the high voltage electrode is a 30° conical tipped brass rod of $R = 0.2$ cm curvature radius, 1 m distant from the ground plate. The generator circuit produces a unipolar oscillating (1-cosine) impulse with 8 μs time to crest. The peak voltage is constant (+250 kV) and the rate of rise, where inception occurs, is about 45–50 kV μs^{-1}. Only the tests done under positive polarity are related here. The recording instrument is a 4-channel digital storage oscilloscope: a voltage signal U from a damped-capacitive divider, a current signal I from a fibre optic system and a photomultiplier signal I_{PM}. The 4th channel is used to monitor the trigger-signal from the impulse-generator's firing gap. The inception voltage of the first corona is detected by means of both the photomultiplier and the current signals. The relative air density δ is always recorded and stays within the 0.95 to 0.98 range. The natural absolute humidity varies from 4 to 12 g m^{-3} in Munich and from 8 to 18 g m^{-3} in Tucumán. The negative and positive ion densities recorded by means of a Gerdien ion counter during the day are in the range of 500 to 1200 ions cm^{-3} and the average ratio of negative to positive ion density is 0.9. It must be noted that the Gerdien ion counter is able to record the density of the small ions only with mobilities greater than 0.5 cm^{-2} V^{-1} s^{-1}. The aim of the ion measurement is to get the same atmospheric composition before each shot and thus to define the time interval between shots.

An ion generator consisting of a sort of metallic brush connected to a dc voltage supply (up to –24 kV) has allowed the negative ion density to be artificially increased in the laboratories.

In Munich, the HV generator is located in a separate laboratory, the high voltage being brought into the test chamber (133 m^3) by a HV bushing. In Tucumán, the test gap and the high voltage generator stand in one single large steel hall ($16 \times 14 \times 12$ m^3).

Table 1. Description of the 17 series: atmospheric parameters, mean inception voltage and their standard deviations.

	No of shots	P (mb)	T (°C)	H (g m^{-3})	H (%)	$U_{i,50}$ (kV)	SD (kV)	SD (%)
Munich	19	961	24	11.8	55	147	22	15
	22	970	21	7.3	42	187	34	18
	20	960	21	3.8	21	179	29	16
	20	960	21	3.8	21	181	15	8
	20	960	21	3.8	21	173	23	13
	100	957	17	3.6	25	175	24	14
	10	975	18	5.7	36	150	32	21
	20	975	18	6.0	41	158	21	13
	10	975	18	6.5	39	163	20	12
Tucumán	20	967	25	17.0	72	132	9	7
	10	967	24	13.0	58	108	14	13
	12	962	28	18.0	68	113	6	5
	5	965	18	10.5	45	158	17	11
	5	965	18	11.3	45	138	30	21
	5	965	19	12.5	46	128	8	6
	10	975	17	8.0	55	116	29	25
	15	994	17	8.8	63	132	21	16

3. Experimental results

Seventeen series, 9 from Munich and 8 from Tucumán, have been selected to evaluate the influence of the humidity upon the inception voltage U_i. The stochastic nature of the inception voltage of the first impulse corona in an air gap, even under controlled atmospheric conditions, is well known. Although the probability distribution does not always follow a normal law, each experimental series can be characterized by its mean value, $U_{i,50}$, and its standard deviation, SD. They are summarized in table 1 with the associated atmospheric conditions. From the total data, statistical tests show that the $U_{i,50}$ values are correlated with H. The global tendency can be observed in figure 1 but the Munich and Tucumán test trends are shown separately. As regards the data from Munich, the effect is statistically significant with a slope of about 2% per g m^{-3}, while no slope is obtained as regards the data from Tucumán. Nevertheless, as already observed by many authors, the series present a large scattering of the results, especially at low humidity levels. In order to underline the effect of humidity upon the spread in figure 1, the $U_{i,50}$ values have been plotted with the extreme values of each

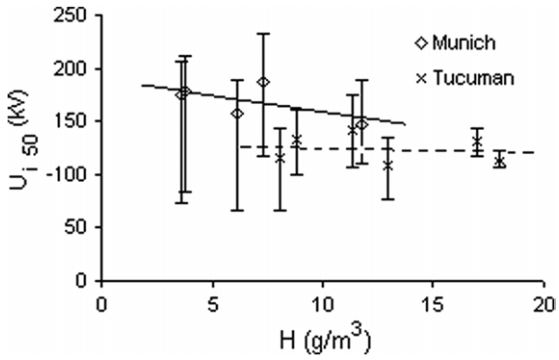

Figure 1. Mean inception voltages $U_{i,50}$ with their extreme values as a function of the absolute humidity H for both laboratories. The lines are the linear regression from Munich data (solid line) and from Tucuman data (dotted line).

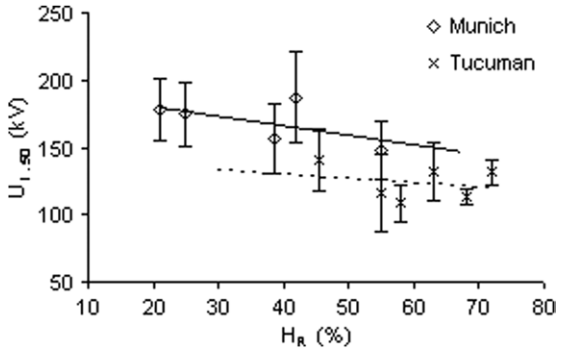

Figure 2. Mean inception voltages $U_{i,50}$ with SD as a function of the relative humidity H_r for both laboratories. The lines are the linear regression from Munich data (solid line) and from Tucuman data (dotted line).

series. We can see that for $H > 15\,\mathrm{g\,m^{-3}}$, the spread is significantly reduced. At lower humidity levels, the spread is large and we are thus entitled to doubt the real effect of H upon the mean inception voltage. The $U_{i,50}$ values can also be plotted as a function of the relative humidity H_r (figure 2). The correspondence between the two laboratories looks more coherent.

On the other hand, more than the mean values of the inception voltage distributions, their minimum values, $U_{i,0}$, are also a crucial parameter. $U_{i,0}$ represents the maximum voltage at which no corona occurs. Experimentally, it is hardly detectable under natural conditions since the probability of getting such a low inception voltage is close to 0. Each series should include several hundred of shots to attain the registration of such low inception voltage. Its value can be estimated approximately as being below the $U_{i,50}$ voltage by three times the standard deviation SD. Since an artificial increase of the negative ion density in the air, close to the HV electrode, considerably lowers the inception voltage (Ortega *et al* 2005), an ion generator has been used in additional tests. From those tests, a minimum inception voltage of $U_{i,0} \approx 40\,\mathrm{kV}$ has been measured. The effect of the humidity could not be put in evidence. It seems that the effect is made up by the uncertainty of the measurements. If the influence exists, it must be very slight.

4. The critical volume model

First of all, the spatial distribution of the electric field in the vicinity of the HV electrode and the voltage–time relationship must be accurately known to solve the equations involved in the application of the model. In this work, the electric field is calculated by the charge simulation method (Singer *et al* 1974). The program allows the field lines to be identified and, by regression techniques, each line is located by means of a 3rd order polynomial, $z = p_i(r)$, i being the number of the field line and (r, z) the cylindrical co-ordinates. Along each field line, the reduced electric field $E_i(r, z)/U_p$, U_p being the HV potential, is also expressed by means of a 3rd order polynomial $E_i = f_i(r, z)$.

The critical volume model has been described by various authors. In non-homogeneous electric fields, the positive first corona can be described as a sheath of positive streamers the origin of which is an electronic avalanche triggered by a primary electron. The critical volume model admits that the primary electron comes from a O_2^- ion. The principle of the model is to express the probability that such an ion loses its electron by collisional detachment within a critical volume. Therefore, the presence of such an ion within the critical volume and its life-time τ_d are taken into account. The probability of corona inception between the time $t = 0$ and t_i is given by

$$P(t) = 1 - \exp\left(-\int_0^t \frac{n_-}{\tau_d(t')} V_c(t')\,\mathrm{d}t'\right). \qquad (1)$$

The equivalent time-constant, τ_d, and the critical volume V_c are field (and time) and humidity dependent whereas n_- is usually assumed to be a constant. The dependence of those three parameters with the humidity is now discussed.

The ratio n_-/τ_d is the production rate of a primary electron. In dry air, this ratio is equal to n_0/τ_0 where n_0 is the O_2^- ion density and τ_0 the life-time of their collisional detachment. τ_0 is strongly dependent on the electric field E and an exponential relationship represents that variation rather satisfactorily:

$$\tau_0 = A \cdot \mathrm{e}^{B/E}, \qquad (2)$$

where A and B are constants experimentally optimized by Frommhold (1964). In moist air, the O_2^- ion density is reduced due to clustering processes with H_2O molecules as follows:

$$O_2^-(H_2O)_n + H_2O + M \underset{k_{-(n+1)}}{\overset{k_{(n+1)}}{\leftrightarrows}} O_2^-(H_2O)_{n+1} + M \qquad (3)$$

with $n = 0, 1, 2, 3, \ldots$ Oblique effects of the water molecules on, for instance, CO_2 molecules, modify the O_2^- ion density (Bastien *et al* 1975, Poli 1985). Those effects may or may not be taken into account according to the different authors.

The τ_0 parameter is thus superseded by an equivalent life-time. Allen *et al* (1981) have preserved the shape of the relationship of equation (2) but proposed two expressions that are humidity dependent for the two coefficients $A(H)$ and $B(H)$. On the other hand, the thermo-kinetic constants being known, the study of the equilibrium of the different ion species involved in equation (3) shows that an increase in the electric field stimulates the declustering process

(Gallimberti 1979, Berger 1980, Poli 1985). It is also shown that the electron release from a cluster ion has a very low probability and that the declustering process caused by the field increase is essential to compensate the decrease of the O_2^- ion density due to humidity. As a consequence, τ_d is the equivalent life-time of the cascade declustering and of the detachment process from O_2^-. It is defined as follows:

$$\tau_d = N_r(E, H) \cdot \tau_0(E), \tag{4}$$

where $N_r(E, H)$ is the ratio of the total negative ion density formed by O_2^- and hydrates, n_-, on the O_2^- ion density $n_{O_2^-}$ (Berger 1980). The $N_r(E, H)$ function can be calculated as a function of the electric field and of the humidity from the system of rate equations for the chain of reactions of equation (3) (Gallimberti 1979, Berger 1980, Poli 1985).

In equation (1), the n_- density is the global density of the negative ions. Usually n_- is assimilated to the density commonly measured for a natural background which is some hundreds of ion cm^{-3}. Allen *et al* (1981) have validated the model with their own experimental results assuming a much smaller n_- value. That reduction is attributed to the large positive space charge produced by the successive first coronas in the vicinity of the HV electrode. Anyway, in all previous works, that parameter has been assumed to be constant as regards the electric field and humidity.

The critical volume, V_c, in equation (1) is defined to allow, firstly, an avalanche to grow and, secondly, that avalanche to reach a critical size. The first condition requires the effective ionization coefficient $\alpha' = \alpha - \eta$ to be positive and the second one, thanks to Meek's criterion, is expressed in the following form

$$\int_{x(\alpha' \geqslant 0)}^{x\,(\text{anode})} \alpha' \cdot \mathrm{d}\ell \geqslant k. \tag{5}$$

The coefficient α' is field, pressure and humidity dependent. Semi-empirical expressions have been proposed by many authors. The following expression deduced by Hartmann (1980) has been used in our calculation:

$$\frac{(\alpha - \eta)}{p} = M\left[A\left(1 + \frac{C}{N(E:p)^3}\right)\mathrm{e}^{B \cdot p/E} - O \cdot E_0\right] \tag{6}$$

where $M = 1 + 0.01H$, $A = 1750$, $B = 40\,000$, $C = 1.15 \times 10^{12}$, $N = 1 + 0.032H$, $O = 1 + 0.175H^{0.1}$ and $E_0 = \frac{0.9}{1.49+\exp(-p/587)}$. The coefficients are expressed in SI units, H in g m^{-3} and the pressure in torr.

At atmospheric pressure, when the field $E < 76$ kV cm^{-1}, that coefficient is higher in wet air whereas for fields $E > 76$ kV cm^{-1} the effect of H is reversed. Allen (1985) has completed this work taking into account the influence of H on the coefficient of free diffusion of the electron. He concludes that the length of avalanche required to obtain the Meek's criterion increases with increasing humidity. That effect diminishes with high values of the field E. Hartmann (1980) adds that the onset field of streamer inception increases with H with small tip radii and this tendency is reversed with large tip radii. The critical radius is about 1 cm. Our electrode configuration belongs to the first group and an increase of the minimum inception voltage could be expected. As aforementioned, that influence could not be put in evidence with our measurement.

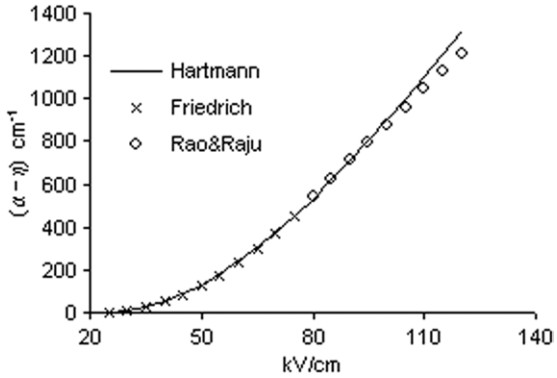

Figure 3. Effective ionization coefficient calculated from Friedrich, Rao and Raju and Hartmann as a function of the electric field at $H = 0$ g m^{-3} and $p = 1000$ mb.

The value of the k coefficient, also called the streamer constant, is a debated question. This coefficient is fundamental to determine the streamer breakdown criterion. In the critical volume model, the magnitude of k has been believed to be equal to 18. Nevertheless, various changes have been brought to both the interpretation of the streamer breakdown criterion and the magnitude of k. Petcharaks (1995, 1999) shows that, using the α' function from Friedrich (1992) when the electric field is in the range 25.9–79.4 kV cm^{-1} and from Rao and Raju (1971) when the electric field is in the range 79.4–140 kV cm^{-1}, the calculated inception voltages coincide with the experimental results in dry air for $k = 9.15$. According to the author, the high value of k (= 18) adopted in the past could be essentially attributed to erroneous effective ionization coefficients. Figure 3 shows that the α' expression from Hartmann is consistent with the last two expressions in the range 20–120 kV cm^{-1}.

5. Application of the critical volume model and results

The first step is to calculate the critical volume, that is to say to solve equation (5). The integral is numerically calculated along each line and the critical volume is thus defined by checking the two conditions, $\alpha' \geqslant 0$ and equation (2). The k coefficient value, ranging from 10 to 20 in the literature, can be chosen by conserving the value leading to a minimum inception voltage, $U^*_{i,0}$, comparable to the experimental threshold, $U_{i,0}$. When H varies from 1 to 20 g m^{-3}, at atmospheric pressure, $k = 10$ and 18 lead to $U^*_{i,0}$ calculated values in the range 39.5 to 40.5 kV and 46.7 to 47.1 kV, respectively (figure 4). Therefore, the value $k = 10$ is more satisfying since the experimental threshold is about 40 kV. This value is in accordance with the conclusions reached by Petcharaks with equivalent effective ionization coefficients (figure 3). It can be noted in figure 4 that the humidity has a weak influence on the minimum inception voltages (less than 0.15% per g m^{-3}).

Then, the equivalent life-time τ_d is calculated either from equation (4) or from the Frommhold expression adapted by Allen *et al* (1981) for the humidity dependence. In equation (4), the assumption that the ratio $N_r(E, H)$ can be calculated as a function of time supposes that the equilibrium of the equation system defined by equation (3) is instantaneous.

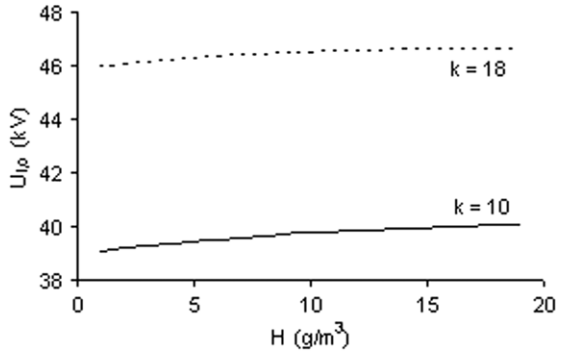

Figure 4. Calculated minimum inception voltage as a function of the absolute humidity versus two k values (equation (5)).

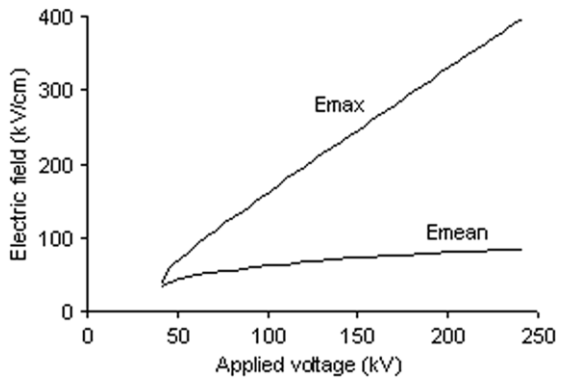

Figure 5. Mean and maximum electric values along the z axis within the critical volume as a function of the applied voltage.

We agree with this assumption since the equilibrium time is about 0.1 μs (Berger 1980) and remains relatively small regarding the rate of rise of the applied voltage.

On the other hand, in this work, the field is strongly non-homogeneous. The use of the mean electric field within the critical volume at any time to calculate τ_d can introduce substantial errors. The analytic formulation of the electric field distribution allows the critical volume to be divided into sub-volumes. Using the mean field, the τ_d value can be underestimated by a factor close to 100 at worst. As an illustration, figure 5 compares the mean and the maximum electric field values along the z axis calculated within the critical volume.

As far as the calculated statistical distribution is compared with the experimental one, the model can give satisfactory results only if the negative ion density, n_-, in equation (1), is adequately adjusted as a function of the humidity. Nevertheless, the fitting to all of the experimental data is better achieved if n_- is also field dependent. That result has led us to assume that the ion density, n_-, in equation (1) is restricted to the O_2^- density. Indeed, that density depends both upon the total ion density which is humidity dependent and upon the electric field because of the declustering process. Equation (1) becomes:

$$P(t) = 1 - \exp\left(-\int_0^{t_i} \frac{n_{O_2^-}(E,H)}{\tau_d(E,H)} V_c(E,H)\,dt\right), \qquad (7)$$

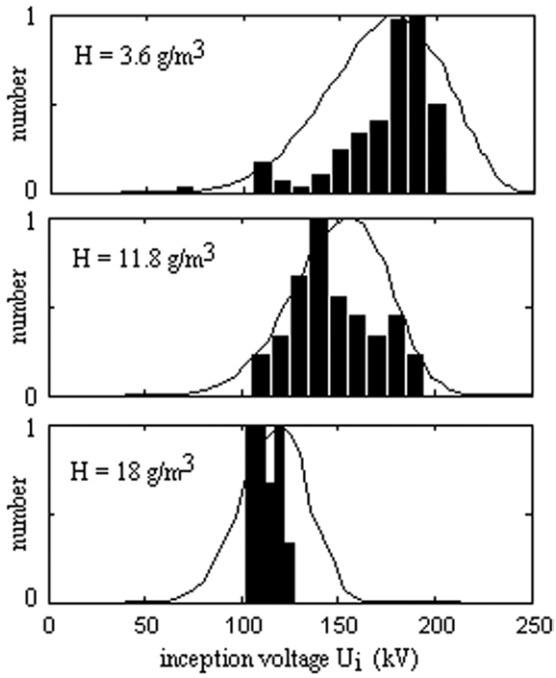

Figure 6. Experimental and calculated (solid line) normalized probability distributions of the inception voltage at three humidity levels.

where E is time dependent and the O_2^- density expressed as follows:

$$n_{O_2^-}(E,H) = \frac{n_-(H)}{N_r(E,H)}. \qquad (8)$$

Now, the statistical distributions are calculated using equations (7) and (8) and the $n_-(H)$ values are deduced from the comparison with the whole series described in table 1 using a least square method. Figure 6 shows three comparisons of experimental and calculated distributions. In the calculation the temperature and the pressure have a weak effect on the results.

The $n_-(H)$ values are plotted on a semi-logarithmic graph as a function of H in figure 7. The graph shows a significant exponential relationship between the ion density and the absolute humidity. That relationship can be expressed as follows:

$$n_-(H) = N_0 \exp(H/H_s) \qquad (9)$$

with $N_0 = 83$ ions cm^{-3} and $H_s = 2.4$ g m^{-3}. N_0 represents the ion density in dry air. Its order of magnitude is comparable to the ion density measured with Gerdien tubes. The departure of the data in relation to the linear regression can be attributed to a natural variation of the ion density. Indeed, the linear regression plotted in figure 7 gives a 95% confidence interval of the N_0 parameter of 19–370 ions cm^{-3}. Those values are in the range of the usual density measured. In figure 7, the data are surrounded by the two lines calculated from equation (9) with the two N_0 values, 19 and 370 ions cm^{-3}.

A similar relation can be obtained if $n_-(H)$ is plotted as a function of the relative humidity:

$$n_-(H) = N_{0,r} \exp(H_r/H_{s,r}) \qquad (10)$$

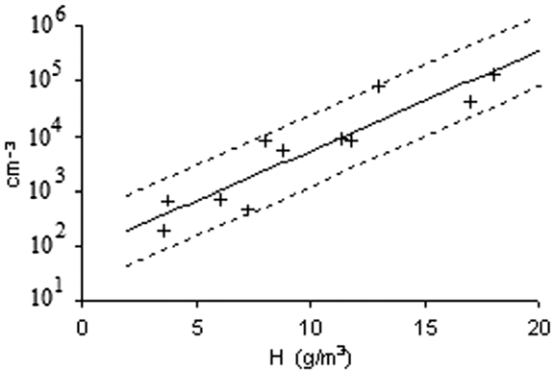

Figure 7. Total ion density n_- deduced from the model as a function of H. Equation (9) is plotted for $N_0 = 83$ ions cm^{-3} (solid line) and $N_0 = 19$ and 370 ions cm^{-3} (dotted lines).

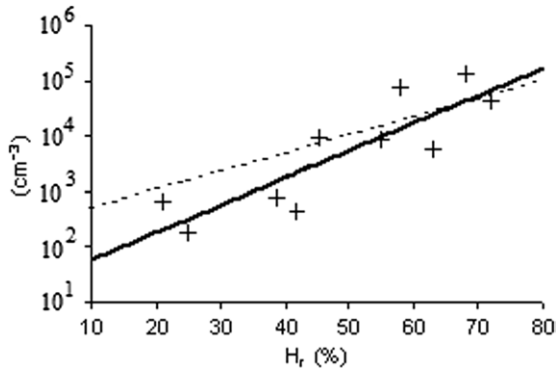

Figure 8. Total ion density n_- deduced from the model as a function of H_r (+) with the least square regression (solid line) and compared with Diaz's relationship (dotted line).

where $N_{0,r} = 18.7$ ions cm^{-3} and $H_{s,r} = 8.8\%$. The range within a 95% confidence interval of $N_{0,r}$ is from 1 to 262 ions cm^{-3} which is consistent with the N_0 range. The data from figure 7 exhibit a neat increase over approximately 17 g m^{-3} and 60%, respectively.

In return, the critical volume size is only slightly affected by the humidity due to the reverse influence of H upon α' as the electric field range within the volume is both greater and smaller than the critical value ($E = 76$ kV cm^{-1}) put in evidence by Hartmann (1980). Thus, the effect of humidity upon the inception parameters is mainly based on the competition between the increases of the equivalent lifetime of the O_2^- ions and of $n_-(H)$ density with H.

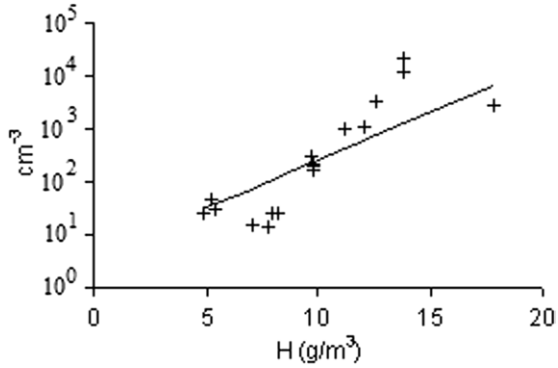

Figure 9. Total ion density n_- as a function of H_r (+) deduced from previous data (Allen *et al* 1981). The solid line is equation (9) with $N_0 = 4$ ions cm^{-3}.

6. Discussion

The new assumption introduced in this work concerns, first, the dependence of the total ion density on humidity and, second, the fact that the electron production rate in equation (1) must be expressed as a function of the O_2^- ion density which is humidity and field dependent. The dependence of an n_- density humidity can be justified as follows. The usual assumption that n_- is constant is based on measurements of negative atmospheric ions. For instance, in our laboratories, the negative ion density has been measured using a coaxial Gerdien counter already described (Diaz *et al* 1997) and a density ranging from 200 to 1000 cm^{-3} has been recorded. The Gerdien counter is usually designed to collect ions the mobility of which is greater than about $\mu_s \approx 1$ cm^2 V^{-1} s^{-1}. According to the Langevin theory, the mobility of ions in air depends upon their sizes more than their masses. Ions with a mobility greater than μ_s are thus called 'small ions'. Recently, Diaz *et al* (2003) have achieved ion density measurements with ion counters adapted to the ion mobility. More than the classical Gerdien tube, an special ion counter has been designed in order to measure ions with a mobility below μ_s down to 4×10^{-4} cm^2 V^{-1} s^{-1}. Such ions are called 'intermediate ions' and 'large ions'. The authors have recorded densities varying from 120 to 780 small ions cm^{-3} and from 5000 up to 140 000 intermediate and large ions cm^{-3}. Those measurements have been correlated with a natural variation of humidity. Whereas the small ion density (negative or positive) does not exhibit any correlation with humidity, the intermediate and large ion densities, N_-, vary exponentially with the relative humidity. The authors have experimentally deduced an exponential relationship between N_- and relative humidity, H_r:

$$N_-(H_r) = N_{0,r} \cdot e^{H_r/H_{s,r}} \qquad (11)$$

with $N_{0,r} = 251$ cm^{-3} and $H_{s,r} = 13.3\%$. It is obvious that the intermediate or large ions measured are not reduced to the O_2^- hydrates since many other cluster families are present in air (Bastien *et al* 1975). Nevertheless, the similitude between equations (10) and (11) is interesting. The two regressions are plotted with the calculated values in figure 8.

Furthermore, a similar relation can be observed using the data published by Allen *et al* (1981). The gap length was 0.6 m and the tip radius 0.01 m. The model was applied with an ion density (n_- in equation (1)) chosen as a parameter depending upon atmospheric conditions but constant as regards the electric field. It was deduced from comparison between measured and calculated probability distributions of the inception voltage at different humidity levels. Despite an order of magnitude approximately 20 times smaller, figure 9 shows that a large number of data follow an exponential variation with H.

A simplified analysis of an analytic expression of the ion mobilities can reinforce the idea of the dependence between n_- and H. Since the motion of the ions is governed by the

dynamics of the force system (electrostatic and viscous drag forces), the ion mobility in air can be expressed as a function of the molecule diameter, D, as follows:

$$\omega = \frac{eC}{3000 \cdot \pi \cdot \mu \cdot D} \ (\mathrm{m^2\,V^{-1}\,s^{-1}}), \qquad (12)$$

where μ is the kinematic viscosity of air ($1.45 \times 10^{-5}\,\mathrm{m^2\,s^{-1}}$), e the charge electron ($1.6 \times 10^{-19}\,\mathrm{C}$) and C the Cunningham correction:

$$C = 1 + 2.468\frac{\lambda}{D} + 0.828\frac{\lambda}{D}\exp\left(-\frac{0.438 \cdot D}{\lambda}\right), \qquad (13)$$

where λ is the mean free path of molecules in air ($\approx 6.5 \times 10^{-8}\,\mathrm{m}$).

The diameters of the oxygen molecules is about 0.4 nm. With such a value, equation (12) gives a mobility of about $1.6\,\mathrm{cm^2\,V^{-1}\,s^{-1}}$. The mobility decreases below $0.1\,\mathrm{cm^2\,V^{-1}\,s^{-1}}$ when the radius reaches 16 nm. The size of the O_2^- hydrates has not been measured but it may be presumed that the clusters belong to the intermediate ion family and, consequently, their density can be H dependent as shown by Diaz *et al*.

The second assumption supposes that the electron production rate depends on the O_2^- ion density. This assumption is first justified by the good correlation between calculated and experimental probability distributions. Secondly it also allows the decrease of the spread to be explained. Figure 10 shows the variation of the ratio $n_{O_2^-}/\tau_d$ (from equation (8)) with the electric field at two humidity levels. It can be seen that, in moist air, the ratio is smaller than in dry air as long as the field is smaller than a field threshold which is approximately $40\,\mathrm{kV\,cm^{-1}}$. Over the threshold, the ratio is greater in moist air. That means that, in moist air, the probability of very low and very high inception voltages is reduced in comparison with dry air conditions. Indeed, at a low humidity level, the clustering process is not dominant, $n_{O_2^-}$ is not considerably affected, and when the field increases in the gap, the declustering process does not produce a consequent increase of the O_2^- density. A large discrepancy of the inception voltage can be expected. In return, at a high humidity level, the clustering process significantly reduces $n_{O_2^-}$ at low field condition but an increase of the electric field considerably increases the O_2^- density, together with the inception probability which rapidly tends to unity. Those cumulated effects at low and high field (or voltage) induce a reduction the discrepancy of the distribution.

In equation (9), N_0 approximately coincides with the ion density commonly observed which ranges from 100 to $1000\,\mathrm{ions\,cm^{-3}}$ at low humidity levels. Thus, natural variation (diurnal, seasonal, etc) can be the reason for the large discrepancy in the inception voltage. For instance, for $H = 4\,\mathrm{g\,m^{-3}}$, if N_0 is varied from 30 to $300\,\mathrm{ions\,cm^{-3}}$ in equation (9), the variation of the most probable inception voltage provided by the model is 32 kV (20%). That variation is comparable to the departure given in table 1 at low humidity levels. Furthermore, the efficiency of the model must be highlighted since despite an exponential increase of n_-, that variation is reduced at high humidity levels since for $N_0 = 30$ and $300\,\mathrm{ions\,cm^{-3}}$ $U_{i,50}$ sustains an increase of 19 kV (16%).

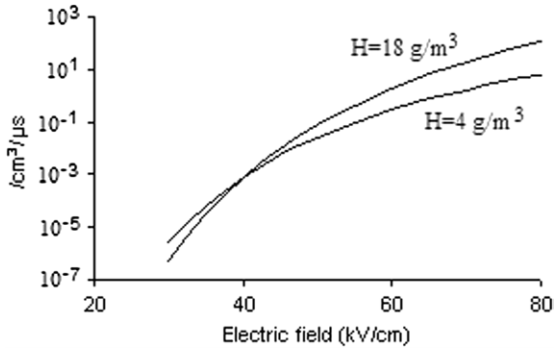

Figure 10. Electron production rate $n_{O_2^-}/\tau_d$ (equation (8)) as a function of the electric field at two humidity levels.

7. Conclusion

The experiments carried out in both laboratories have shown that the effect of humidity upon the probability distribution of the inception voltage of the first corona is made up by a large discrepancy in the data. The spread seems to be sensitively reduced when the moisture of the air exceeds a limit of about $15\,\mathrm{g\,m^{-3}}$ or 60% when relative humidity is concerned. That reduction can be justified by the critical volume model assuming that the electron production rate depends on the O_2^- density which is a function of humidity and of electric field. An exponential relationship between n_- and H allows the experimental statistical distributions to be satisfactorily calculated. That variation coincides with recent measurements achieved with ion counters especially devised for intermediate and large ions. Nevertheless, an accurate measurement of the hydrate ion densities and their mobility, particularly the intermediate ions, is essential to confirm that dependence. Indeed, the regressions of equations (9) or (10) have a large residue especially at high humidity levels. If the exponential relation between ion density and humidity proves to be exact with further measurements, the natural variation of small ion density can explain the discrepancy experimentally observed in the inception voltage. Paradoxically, the discrepancy is more important at a low humidity level. That paradox can be physically explained by the theory of the critical volume model. Also, the reason for the threshold of $15\,\mathrm{g\,m^{-3}}$ or 60% corresponding to a deep increase of the ion density requires further investigation and cannot be clearly explained for the time being. Lastly, the critical volume model still allows the minimum inception voltage to be calculated with the assumption that the k parameter is about 10 and not 18, contrary to what has been often been suggested.

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References

Allen N L, Dring D and Burger S 1980 The influence of humidity on positive impulse corona in a rod/plane gap *6th Int. Conf. on Gas Discharge and Their Applications (Edinburgh) IEE Conference Publication* **189** pp 122–5

Allen N L, Berger G, Dring D and Hahn D 1981 Effects on humidity on corona inception in a divergent electric field *IEE Proc.* **128** 565–70

Allen N L 1985 The effect of humidity on avalanche growth and streamer initiation *J. Phys. D: Appl. Phys.* **18** 47–52

Bastien F, Haug R and Leguiller M 1975 Simulation sur ordinateur de l'évolution temporelle des ions négatifs de l'air. Application au cas de la décharge couronne négative *J. Chim. Physique* **72** 105–12

Berger G 1980 Delay in the formation of the positive corona discharge in air *PhD Thesis* Université Paris-Sud Orsay pp 109–62

Davies A J, Matallah M, Turri R and Waters R T 1988 The effect of humidity and pressure on corona inception in a short air gap at breakdown voltage levels *9th Int. Conf. on Gas Discharge and Their Applications (Venezia), IEE Conference Publication* pp 185–9

Diaz R, Tagashira R, Rühling F and Heilbronner F 1997 The impulse corona inception in inhomogeneous fields: a model *10th Int. Symp. on Highvoltage Engineering (Montreal)* vol 5 pp 363–6

Diaz R, Fernandez F M and Silva J 2003 Humidity-ion density relationships *13th Int. Symp. on Highvoltage Engineering (Delft)* pp 222.1–222.4

Friedrich G 1992 Die physicalischen Entladungsparameter der synthetischen Luft unter dem Einfluss der Feuchte und der Temperatur *PhD Thesis* ETH 9992

Frommhold L 1964 On delayed electrons in electron avalanches, especially in oxygen and air, due to growth of negative ions O^- *Fortschr. Phys.* **12** 597

Gallimberti I 1979 The mechanism of the long spark formation *J. Physique Coll. (Suppl.)* **7** 194–9

Hartmann G 1980 Theoretical evaluation of Peek's law *Proc. IEEE* **IAS-15** 1113–7

Les Renardières Group 1977 Positive discharge in long airgaps at Les Renardières: 1975 results and conclusions *Electra* **53** 31–128

Ortéga P, Rühling F, Heilbronner F and Diaz R 2005 Charge–voltage relationship of the first impulse corona in long airgaps 2005 *J. Phys. D: Appl. Phys.* **38** 2215–26

Petcharaks K 1995 Applicability of the streamer breakdown criterion to inhomogeneous gas gaps *PhD Thesis* ETH 11192

Petcharaks K 1999 XI A contribution to the streamer breakdown criterion *Int. Symp. on Highvoltage Engineering (London)* vol 3 pp 19–22

Poli E 1985 Positive corona inception under variable humidity conditions *8th Int. Conf. on Gas Discharge and Their Applications (Oxford) IEE Conference Publication* pp 593–6

Rao C and Raju G 1971 Growth of ionization currents in dry air at high value of *E/N J. Phys. D: Appl. Phys.* **4** 494–503

Singer H, Steinbigler H and Weiss P 1974 A charge simulation method for the calculation of high voltage fields *IEEE Trans. Power Appl. Syst.* **93** 1660–8

Topalis F V, Katsabekis T, Bourkas P D and Stathopoulos I A 1988 Corona inception and breakdown in medium air under switching impulse voltage *9th Int. Conf. on Gas Discharge and Their Applications (Venezia) IEE Conference Publication* pp 467–70