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Enterprise optimization for solving an assignment and trim-loss non-convex problem

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Abstract

In the present work, the planning and cutting problem for the corrugated board boxes industries is presented. This problem belongs to the category of the trim-loss problem, which is essential in the paper-converting supply chain management. Bilinear terms in demand and stock constraints, for instance, lead to a non-convex formulation. Two global convex models are formulated and tested. Results obtained in the problem solution are shown. The most efficient model is implemented by means of Java programs and GAMS, a mathematical optimization program. The system is linked to the company ERP (enterprise resource planning) system. Several issues are optimized and improved: waste generation, energy demand, environmental impact and production costs. Paper reel stock management is improved due to more accurate and statistical information obtained by the system. The planning system linked to the ERP connection allows the integration of customers and suppliers increasing the company competitiveness.

Keywords: Cutting stock; Trim-loss; Enterprise optimization; Corrugated board; MINLP; MILP

1. Introduction

The scope of manufacturing logistics begins at the point where end-item customer demands are determined, and extends to the point where they are fulfilled (Wu & Golbasi, 2004). According to these authors they distinguish narrow and a broader view of manufacturing logistics. The first one includes the planning, scheduling and control of all activities resulting in the acquisition, processing, movement and storage of inventory. These activities include order acceptance, production planning and scheduling, inventory control, inventory distribution, and the design of the corresponding decision processes and decision support systems. The second one considers the flow of material, information, and services across enterprise, industry and national boundaries.

In this work, it will be described the planning models considered for the carton corrugated industry, the implementation of the selected model as a business intelligence module and its link to the company's ERP in order to share the information with customers and suppliers of the supply chain to increase the services across the company limits.

Competition between single companies changes into competition between supply chains. Therefore, it can be expected that the importance of methods for management and production process optimization will increase with companies seeking global instead of local optimums (Trkman, Indihar Štemberger, & Jaklic, 2005). Particularly, waste generation is one important issue in today's supply chain management. In first place, waste means increasing costs, with no positive effects in customer satisfaction; and environmental impact, also an objective of increasing interest. Consequently, in the cutting process waste should be minimized to be competitive in the global market and friendly to the environment. There are some industries that generate waste during its cutting process such as the wood industry (Venkateswarlu, 2001), the paper-converting industry (Westerlund & Isaksson, 1998) and the steel industry (Vasko, Newhart, & Stott, 1999).

Grossmann and Westerberg (2000) pointed out that chemical engineering in the context of process system engineering (PSE) has evolved in the past decades from being rooted in the concept of unit operations to one based on engineering sci-

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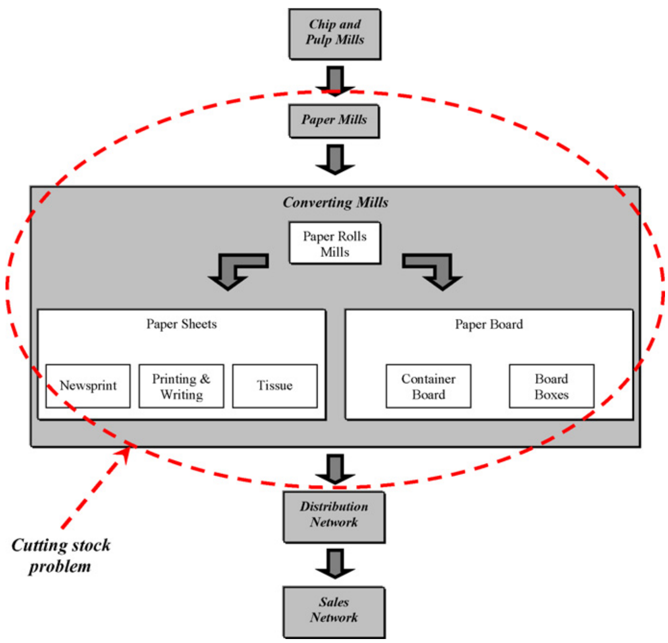

Fig. 1. The pulp and paper supply chain.

ence and mathematics. They have proposed a new definition of PSE where the discipline is concerned with the improvement of decision-making processes for the creation and operation of the chemical supply chain. It deals with the discovery, design, manufacture, and distribution of chemical products in the context of many conflicting goals. Although this work does not deal with a chemical process, the theoretical issues and the goal pursued corresponds to models generation for the prediction of performance, and decision making for an engineered system, which is also a main concern for PSE area.

In this work a real-world industrial problem of production planning and cutting optimization for corrugated board boxes is presented. However, as shown in Fig. 1 (adapted from Carlsson, D'Amours, Martel, & Rönnqvist, 2006), the cutting stock is a common problem in almost every link on pulp and paper supply chain. Additionally, packaging production industry is also involved in almost all manufacturing supply chains. As a consequence, the industry studied performs their activities in a very competitive market. An efficient production plan improves company competitiveness providing convenient product prices and just in time order deliveries.

In many articles, the cutting stock problem has been studied with different goals such as minimal trim-loss (Harjunkoski, Westerlund, Isaksson, & Skrifvars, 1996; Harjunkoski, Westerlund, & Pörn, 1999; Trkman & Gradisar, 2007), minimal production costs (Harjunkoski, Westerlund, Pörn, & Skrifvars, 1998; Harjunkoski et al., 1999), minimal number of patterns (Johnston & Sadinlija, 2004), minimal total length and overproduction (Correia, Oliveira, & Ferreira, 2004), etc. In some cases, mathematical optimization cannot achieve to optimal solution in reasonable execution time and heuristic techniques are also approached (Beasley, 2004; Riehme, Scheithauer, & Terno, 1996). Often rough simplifications are made when formulating the trim-loss problem. These simplifications have a great influence on the result and many times heuristic procedure may give better results in practice than an optimized solution (Westerlund & Isaksson, 1998). Those simplifications refer to some important practical issues connected to the problem. In fact, stock constraint is usually disregarded, formulations assume a unique reel length and width and different paper types are not considered. In this case, simplifications are avoided in order to find a proper solution for a real industrial problem.

Although the trim-loss costs objective is here considered the most representative function to define a "real-world" optimal solution, some other objective functions are also used in order to compare models behavior and solution strategies. The first model developed is a non-convex MINLP which is transformed by two different methods to obtain a global solution. Besides global optimization, in this work some other targets are pursued such as computational efficiency, detailed problem representation, planner intervention in the problem inputs and constraints and integration to the company information system.

In the following section, some issues related to the production process and product characteristics are considered to understand the problem formulation. The third section contains the problem statements and background. The next section refers to models formulation and convexification techniques. In the following section, different objective functions are presented. Some examples were analyzed in section six to illustrate models computational performance for the proposed objective functions and compare models results. Critical considerations concerning implementation features, as models integration to ERP systems and its influence on paper supply chain are considered in section seven. In the last section, conclusions refer to final discussion and outlines future research goals.

2. Process description

The raw material to produce board boxes is usually a set of paper reels which correspond to different paper types depending on the board characteristics needed on the final product. Paper reels of different width and lengths provided by different suppliers can be used to produce the corrugated board and cut the sheets. The assignment of paper width to cut patterns according to the paper stock available is an important consideration in problem formulation.

The corrugated board is produced using several paper layers. There are two main kinds of layers: liner layer and fluted layer. The board structure is mainly influenced by the number of layers assigned and the paper type in each layer. The most used boards in the industry are the *single wall*, that it is a rigid structure, which has two external liner layers and one middle fluted one and the *double wall board*, which is also a rigid structure, formed by three liner layers, two external and one central, and two fluting layers located between the central liner and one of the externals, respectively.

The production of corrugated sheets is a continuous process. The first step is to place the paper reels in the corrugator. Next, some layers are corrugated to form the required flute where adhesive is then added to glue the liner layers. Once corrugated board

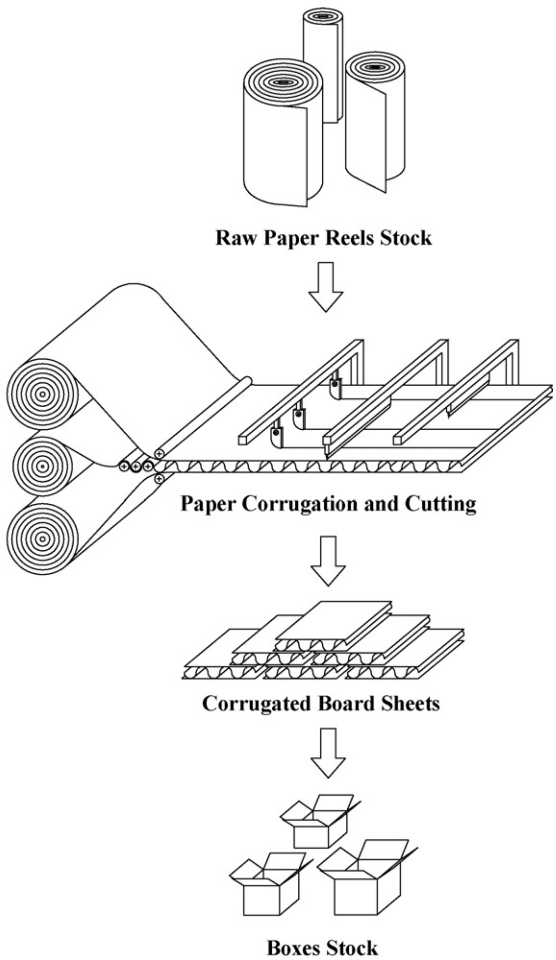

Fig. 2. Board boxes production process.

is formed, it goes through the cutting section where board sheets are finally obtained. Fig. 2 shows the process main stages.

The cutting machine has *Nlong* knives to cut the boards lengthwise and *Ntrans* knives for the transversal cuts. Those characteristics limit the number of different sheets to cut per time. Using *Nlong* knives *Nlong* + 1 board parts could be obtained; the two external ones must be discarded because the

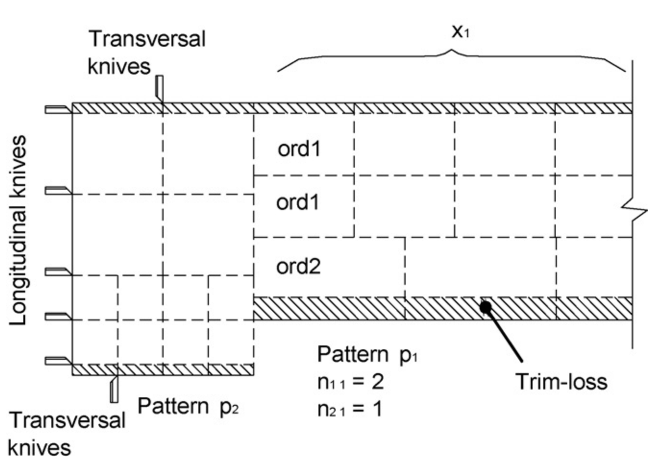

Fig. 3. Cutting process.

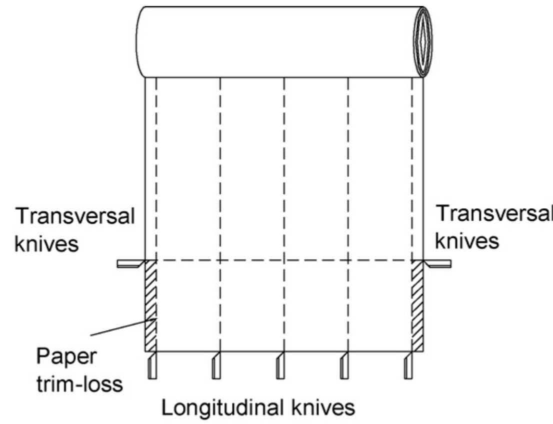

Fig. 4. Patterns with same order sheets.

layers are not perfectly glued. This allows the cutting of at most *Nlong* − 1 sheets per board wide and gives a minimum trim-loss *Permin* (see Fig. 3). The *Ntrans* knives limit to *Ntrans* the different lengths to cut.

In Section 4, a general problem formulation is presented to solve the trim-loss problem in the corrugated board boxes industry independently of the corrugating and cutting machine to be used. However, a machine with five longitudinal knives and two transversal ones is considered as an example, in order to illustrate the different type of cutting patterns, which are as follows:

(1) Patterns corresponding to a single sheet order: the possible number of sheets to cut varies from one to four; the final value depends on the order width and the paper reels width used in each layer. A sketch of this pattern type can be observed in Fig. 4.
(2) Pattern 1-1: corresponds to two sheet orders with different width and length. When combining sheets of different orders both of them must have the same board and paper types in each layer. In this case, only three of the slitting knives and two transversal ones are used. A scheme of this pattern type is illustrated in Fig. 5.
(3) Pattern 2-2: these patterns have two sheets of one order and other two of a different one; it is shown in Fig. 6.

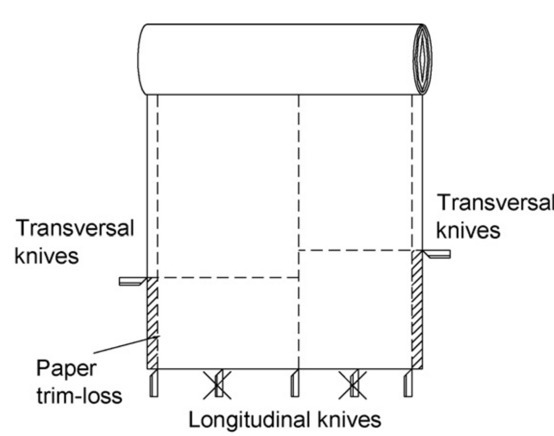

Fig. 5. Pattern type 1-1.

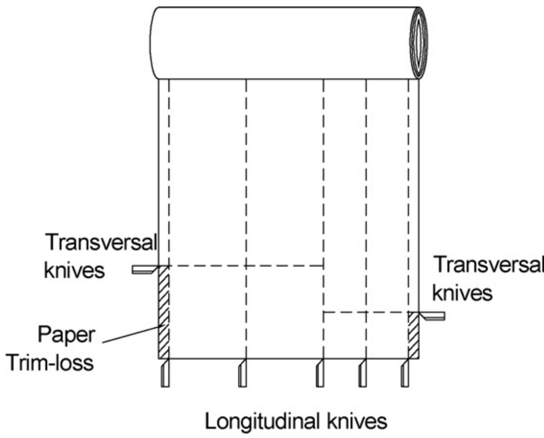

Fig. 6. Pattern type 2-2.

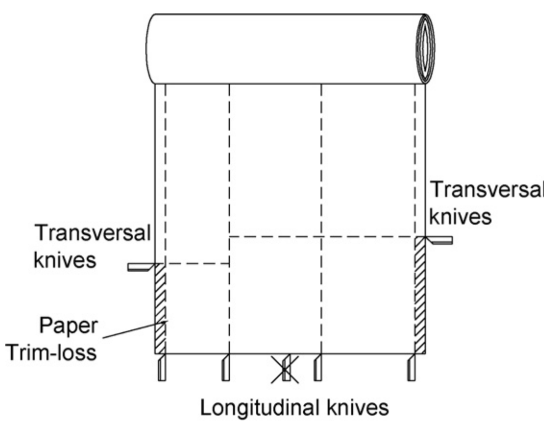

Fig. 7. Pattern type 1-2.

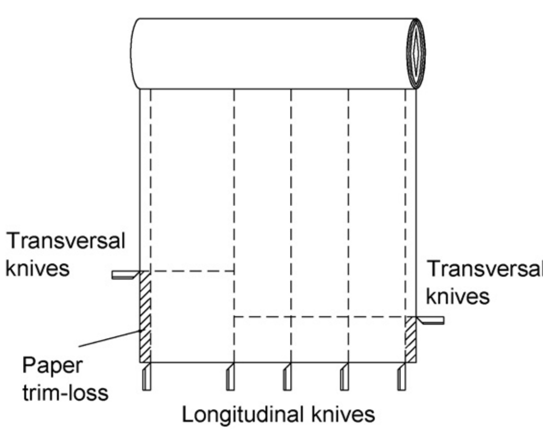

Fig. 8. Pattern type 1-3.

(4) Pattern 1-2: these patterns cut a sheet of one order and two of another one, which is illustrated in Fig. 7.
(5) Pattern 1-3: these patterns cut a sheet of one order and three of another one. A representative scheme of this pattern type is shown in Fig. 8.

3. Problem statement, background

The board boxes process includes two main steps. First, the board must be produced from single layers involving an assignment problem and then the board is cut into smaller pieces to produce the sheets that form the boxes. Assignment decisions usually implicate binary variables in problem formulation leaving more difficult models and time-consuming solutions. The second step is related to the cutting stock problem. There are many articles dealing with the cutting stock problem where the raw paper reels are cut into smaller ones (Sweeney & Haessler, 1990; Valério de Carvalho, 2002; Westerlund, Harjunkoski, & Isaksson, 1998). Generally, the solution strategy defines optimal cutting patterns, and then determines the number of patterns to produce in order to minimize a cost function. The cutting process generates the trim-loss because raw paper widths are not exact multiples of customer orders widths, and so larger paper reels are needed to form feasible patterns. However, the production of corrugated board boxes problem has not been considered yet. In this case, not only trim-loss problem is crucial but also board structural definition (the number of layers, paper weight and color), the assignment of the paper width for each layer and also the combination of orders according to their board type, among others. It must be also taken into account that reels of different widths and lengths are available in stock and as a result, the problem complexity is very high due to the huge number of product combinations and variables to handle. Discrete decisions and non-convex relationships are also involved. In fact, due to its NP-hard and non-convex nature, solution strategies and problem representations have an important influence on computational efficiency. Fig. 9 shows some orders combination constraints and paper assignment characteristics.

4. Model formulation

4.1. The original model

The problem formulation produces a non-convex MINLP model where global optimality cannot be guaranteed. A group of orders i must be satisfied considering an accepted overproduction η_i. Stock constraint is also considered. The cutting machine has *Nlong* longitudinal knives and *Ntrans* transversal ones. Maximum and minimum trim-loss are given and must be satisfied when assigning orders i and paper widths ap to pattern p. Each customer order has a board type which is defined by the number of layers k and the paper type tp corresponding to each one. Another constraint is that orders assigned to pattern p cannot have different board type. The $Not\_comb_{ii'}$ set includes all pair of orders that cannot be assigned to the same pattern. Fig. 9 further clarifies these concepts. The run length x_p for each pattern p has upper and lower bounds, both related to the feasible production time. Thus, the problem of minimizing the total trim-loss cost can be formulated follows:

$$\text{Min } Z = \sum_{\forall p \in P} \sum_{\forall k \in K} cp_{pk} \tag{1}$$

s.t.

$$\sum_{\forall i \in I} n_{i\,p} \leq Nlong - 1 \quad \forall p \in P \tag{2}$$

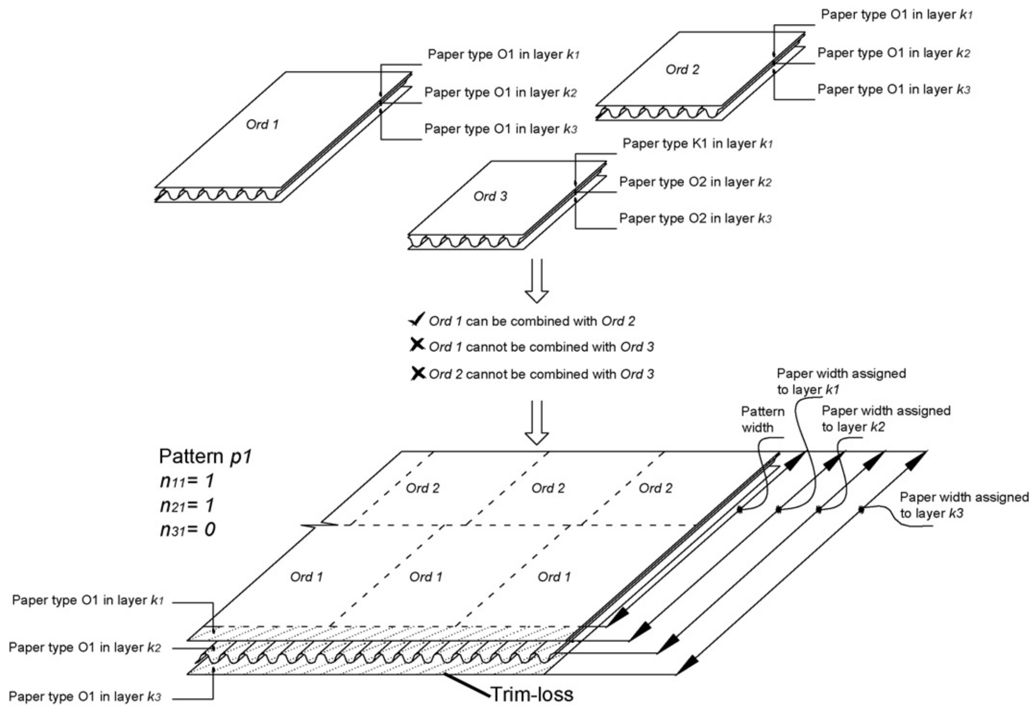

Fig. 9. Orders combination and pattern structure.

$$\sum_{\forall i \in I} y_{i\,p} \le Ntrans \quad \forall p \in P \tag{3}$$

$$ta_{pk} \ge \sum_{\forall ap \in AP} w_{ap\,k\,p} \cdot Wap_{ap} \cdot x_p \qquad \forall p \in P, \quad \forall k \in K \tag{4}$$

$$\sum_{\forall ap \in AP} w_{ap\,k\,p} \le 1 \qquad \forall p \in P, \quad \forall k \in K \tag{5}$$

$$ua_{pk} = \sum_{\forall i \in I} n_{i\,p} \cdot Wi_i . x_p \qquad \forall p \in P, \quad \forall k \in K \tag{6}$$

$$cp_{p\,k} \ge Co_{p\,k} \cdot (ta_{p\,k} - ua_{p\,k}) \qquad \forall p \in P, \quad \forall k \in K \tag{7}$$

$$y_{i\,p} + y_{i'\,p} \le 1 \qquad \forall i \ne i' | (i, i') \in Not\_comb_{i\,i'}, \quad \forall p \in P \tag{8}$$

$$\left(\sum_{\forall ap \in AP} w_{ap\,k\,p} \cdot Wap_{ap} \right) - \sum_i n_{i\,p} \cdot Wi_i \ge Per\,min \cdot yr_p \qquad \forall p \in P \tag{9}$$

$$\left(\sum_{\forall ap \in AP} w_{ap\,k\,p} \cdot Wap_{ap} \right) - \sum_i n_{i\,p} \cdot Wi_i \le Per\,max \cdot yr_p \qquad \forall p \in P \tag{10}$$

$$\sum_{\forall ap \in AP} w_{ap\,k\,p} \ge yr_p \qquad \forall p \in P, \quad \forall k \in K \tag{11}$$

$$\sum_{\forall p \in R_{p\,k\,tp}} \sum_{\forall k \in R_{p\,k\,tp}} x_p \cdot \alpha_k \cdot w_{ap\,k\,p} \le S_{tp\,ap} \qquad \forall tp \in TP, \quad \forall ap \in AP \tag{12}$$

$$\sum_{\forall p \in PO_{p\,i}} n_{i\,p} \cdot x_p / Li_i \ge D_i \quad \forall i \in I \tag{13}$$

$$\sum_{\forall p \in PO_{p\,i}} n_{i\,p} \cdot x_p / Li_i \le D_i \cdot (1 + \eta_i) \quad \forall i \in I \tag{14}$$

$$x_p \ge CRmin_p \cdot yr_p \quad \forall p \in P \tag{15}$$

$$x_p \le CRmax_p \cdot yr_p \quad \forall p \in P \tag{16}$$

$$\sum_{\forall p \in P} y_{i\,p} \ge 1 \quad \forall i \in I \tag{17}$$

$$y_{i\,p} - n_{i\,p} \le 0 \qquad \forall i \in I, \quad \forall p \in P \tag{18}$$

$$n_{i\,p} - Nlong \cdot y_{i\,p} \le 0 \qquad \forall i \in I, \quad \forall p \in P \tag{19}$$

$$yr_{i\,p} \ge y_{i\,p} \qquad \forall i \in I, \quad \forall p \in P \tag{20}$$

$$yr_p \le 1 \quad \forall p \in P \tag{21}$$

$$yr_p \le \sum_{\forall i \in I} y_{i\,p} \quad \forall p \in P \tag{22}$$

$$yr_p, \quad x_p, \quad cp_{pk}, ta_{pk}, ua_{pk} \in R^+$$

$$n_{i\,p} \in N$$

$$y_{i\,p}, w_{ap\,k\,p} \in \{0; 1\}$$

The objective function presented in Eq. (1) is the cost of the trim-loss, represented by the sum of cp_{pk} which is the cost of the trim-loss for pattern p on layer k. Note that cp_{pk} is a positive variable that depends on the conformation of the pattern in each layer and the length x_p.

Eq. (2) defines that the number of sheet to be cut per pattern p could be at most $Nlong - 1$, where $Nlong$ represents the number of longitudinal knives in the cutting machine, and n_{ip} is an integer variable that indicates the number of units of the order i assigned to pattern p.

Eq. (3) defines that the number of different orders assigned to each pattern could be at most $Ntrans$, where y_{ip} is a binary variable representing the fact that order i is assigned to pattern p or not.

Eq. (4) defines the total area assigned to each layer k of pattern p, ta_{pk}, where $w_{ap\,k\,p}$ is a binary variable that is one if the paper width ap is used in the layer k of the pattern p and zero otherwise (see Fig. 9), $Wap_{\rm ap}$ is a parameter that represents the paper width and x_p is the variable that determines the pattern length. Eq. (5) constrains the number of widths ap assigned to each layer k of pattern p to one.

In Eq. (6), the used area in each layer k of pattern p is defined as ua_{pk}, which depends on the number of orders i assigned to p, n_{ip}, their widths Wi_i and the pattern length x_p.

Eq. (7) defines the pattern trim-loss cost for each layer k, cp_{pk}, where Co_{pk} is a parameter that indicates the paper cost in layer k of pattern p. The trim-loss area in each layer k of pattern p is calculated as the difference between variables ta_{pk} and ua_{pk} defined in Eqs. (4) and (6), respectively.

Eq. (8) establishes that orders i cannot be combined in the same pattern with order i' because they have different type of board, as was set-up in set $Not\_comb_{ii'}$.

Eqs. (9) and (10) define a minimum and maximum trim-loss per width of pattern p, respectively. Eq. (9) establishes that the difference between the width of the paper used in each layer k of pattern p and the pattern width, which depends on the number of orders i assigned to p and their widths, must be greater than $Permin$. Note that only one width ap can be selected for each layer k of pattern p[1], so $\sum_{\forall ap \in AP} w_{ap\,k\,p}$ is at most one. In fact, if no width ap is assigned to pattern p, it means that the pattern p is not used and consequently, the pattern width will be also zero. Similarly, Eq. (10) determines that the width of each pattern p could have at most a maximum waste, $Permax$, corresponding to business rules reasons.

Eq. (11) establishes that if pattern p exists ($yr_p = 1$), then some paper width ap must be assigned to each layer k of the pattern p. Note that yr_p is not a binary variable, it only takes the values of 0 or 1 constrained by Eqs. (20)–(22).

In Eq. (12) the parameter $S_{tp\,ap}$ represents the stock of the paper (m) of type tp and width ap. This constraint establishes that if the width ap was assigned to the layer k of pattern p, the length used in all layers of all patterns can be at most $S_{tp\,ap}$. Parameter α_k is a coefficient for the paper consumption in layer k (greater than 1 for flute papers). The set $R_{pk\,tp}$ determines the paper type tp associated to each layer k of pattern p.

Eqs. (13) and (14) are the demand constraints, where PO_{pi} associates orders i to the same type of board patterns p where they can be assigned. Eq. (13) defines that the number of sheets produced for one order i in the patterns p must be greater than the demand D_i. The Eq. (14) establishes an over-production upper bound η_i which gives flexibility to the cutting plan.

[1] See Eq. (5).

Eqs. (15) and (16) give a minimal and maximum run length for pattern p $CRmin_p$ and $CRmax_p$, respectively. Both constrains are related to the production time limits.

Eq. (17) determines that every order i must be assigned to some pattern p.

Eqs. (18) and (19) are logical constraints relating variables n_{ip} and y_{ip}. The first one determines that if n_{ip} is zero, y_{ip} must be zero too. The second one defines that if n_{ip} is greater than zero, y_{ip} must be one.

Eqs. (20)–(22) represent lower and upper bounds to the positive variable yr_p. Eq. (20) is a lower bound and establishes that if some order i is assigned to pattern p, then yr_p must be at least one. In Eq. (21) yr_p is limited to one. By Eq. (22) yr_p must be zero if no order i is assigned to pattern p.

To assure a global solution, this first model must be reformulated. For that purpose, two different strategies are selected. In the first one, some transformation techniques are applied in order to eliminate bilinearities. The resulting MILP model is bigger than the original one in terms of constraints and variables. The second strategy is a two-step procedure. The initial step generates feasible cutting patterns for a set of orders. The algorithm generating the patterns of different orders must combine only corrugated sheets having the same characteristics: same number of layers, same flute type and paper class for each layer. Then a MILP optimization model is solved selecting a subset of the patterns and the length to cut to satisfy the demand and stock constraints.

4.2. Convexification techniques

The non-convexities in problem (1)–(22) arise from Eqs. (4), (5) and (12)–(14), where some bilinear term appears. Traditionally, in cutting stock problems, bilinearity comes from the demand constraints as presented in Eqs. (13) and (14), by the product $n_{ip} \cdot x_p$. However, in the formulation considered this term also appears in Eq. (5) to calculate the used area in pattern p. Another bilinear term corresponds to the paper width assignment in each layer k of pattern p. The product $w_{ap\,k\,p} . x_p$ is used to calculate the total area of pattern p in Eq. (4) as well as the paper consumption in the stock constraint (12), which is usually disregarded.

A number of transformation techniques to overcome bilinear terms have been studied in Harjunkoski et al. (1999) and Pörn, Harjunkoski, and Westerlund (1999). Any method supposes an expansion in terms of number of variables and constraints. In this work a well known linear transformation is considered. The strategy redefines variables and constraints of the initial model. The first transformation adds a binary variable β_{ipj} to define n_{ip} as follows:

$$n_{i\,p} = \sum_{\forall j \in J} j \cdot \beta_{i\,p\,j} \quad \forall i \in I \tag{23}$$

$$\sum_{\forall j \in J} \beta_{i\,p\,j} \le 1 \qquad \forall i \in I, \quad \forall p \in P \tag{24}$$

$$n_{i\,p} \in R^+, \quad \beta_{i\,p\,j} \in \{0, 1\}$$

Then, a slack variable s_{ipj} is also introduced, and the following transformation constraints must be added:

$$s_{i\,p\,j} - x_p \le 0 \qquad \forall i \in I, \quad \forall p \in P, \quad \forall j \in J \tag{25}$$

$$-s_{i\,p\,j} + x_p - CRmax_p(1 - \beta_{i\,p\,j}) \le 0 \quad \forall i \in I, \quad \forall p \in P, \quad \forall j \in J \tag{26}$$

$$s_{i\,p\,j} - CRmax_p \cdot \beta_{i\,p\,j} \le 0 \qquad \forall i \in I, \quad \forall p \in P, \quad \forall j \in J \tag{27}$$

$$s_{i\,p\,j} \in R^+$$

Note that with this transformation, Eqs. (5), (13) and (14) can be rewritten as follows, respectively:

$$ua_{p\,k} = \sum_{\forall j \in J} \sum_{\forall i \in I} s_{i\,p\,k} \cdot Wi_i \qquad \forall p \in P, \quad \forall k \in K \tag{28}$$

$$\sum_{\forall j \in J} \sum_{\forall p \in P} j \cdot s_{i\,p\,j}/Li_i \ge D_i \quad \forall_i \in I \tag{29}$$

$$\sum_{\forall j \in J} \sum_{\forall p \in P} j \cdot s_{i\,p\,j}/Li_i \le D_i \ (1 + \eta_i) \quad \forall i \in I \tag{30}$$

Considering the bilinear term $w_{ap\,k\,p} \cdot x_p$ another similar transformation is defined introducing a positive slack variable $l_{ap\,k\,p}$ and using $w_{ap\,k\,p}$:

$$l_{ap\,k\,p} - x_p \le 0 \qquad \forall ap \in AP, \quad \forall p \in P, \quad \forall j \in J \tag{31}$$

$$-l_{ap\,k\,p} + x_p - CRmax_p(1 - w_{ap\,k\,p}) \le 0 \quad \forall ap \in AP, \quad \forall p \in P, \quad \forall j \in J \tag{32}$$

$$l_{ap\,k\,p} - CRmax_p \cdot w_{ap\,k\,p} \le 0 \qquad \forall ap \in AP, \quad \forall p \in P, \quad \forall j \in J \tag{33}$$

$$l_{ap\,k\,p} \in R^+$$

The constraints (4) and (12) can now be written as shown in Eqs. (34) and (35), respectively:

$$ta_{p\,k} \ge \sum_{\forall ap \in AP} l_{ap\,k\,p}.Wap_{ap} \qquad \forall p \in P, \quad \forall k \in K \tag{34}$$

$$\sum_{\forall p \in R_{p\,k\,tp}} \sum_{\forall k \in R_{p\,k\,tp}} l_{ap\,k\,p} \cdot \alpha_k \le S_{tp\,ap} \qquad \forall tp \in TP, \quad \forall ap \in AP \tag{35}$$

The solution procedure is mainly improved if binary variable y_{ip} is eliminated from the initial model (Eqs. (1)–(22)) and replaced by $\sum_{\forall j \in J} \beta_{i\,p\,j}$. Instead of using Eqs. (7), (17), (20) and (22); Eqs. (36)–(39) are introduced.

$$\sum_{\forall j \in J} \beta_{i\,p\,j} + \beta_{i'\,p\,j} \le 1 \qquad \forall i \ne i' | (i, i') \in Non\_comb_{ii'} \forall P \tag{36}$$

$$\sum_{\forall p \in P} \sum_{\forall j \in J} \beta_{i\,p\,j} \ge 1 \quad \forall i \in I \tag{37}$$

$$yr_p \ge \sum_{\forall j \in J} \beta_{i\,p\,j} \qquad \forall i \in I, \quad \forall p \in P \tag{38}$$

$$yr_p \le \sum_{\forall i \in I} \sum_{\forall j \in J} \beta_{i\,p\,j} \quad \forall p \in P \tag{39}$$

Eqs. (18) and (19) are not needed in this formulation because y_{ip} has been eliminated and n_{ip} is now defined by Eq. (23).

Consequently, the final model is now redefined by Eqs. (1)–(3), (6), (8)–(11), (15), (16) and (28)–(39).

4.3. Two steps formulation

Another strategy to transform the problem formulation into a linear one is to separate decisions into two stages. This method is generally used because it is simple and efficient (Westerlund et al., 1998; Westerlund & Isaksson, 1998). First, a pre-generation model defines all feasible cutting patterns which are then part of the input data in a MILP optimization model. Neither additional constraints nor variables are introduced in this formulation. As a result not only optimal solution is guaranteed but also better solution performance.

The equations presented in this section consider that customer's pending orders, paper reels stock and its cost are known in order to define a set of feasible patterns. If the following constraints are satisfied then a feasible cutting pattern is generated. The procedure is recursively repeated until all pending orders i are analyzed and feasible patterns p defined.

$$Wf_p = \sum_{\forall i \in I} N_{i\,p} W_i \qquad \forall p \in P, \quad \forall k \in K \tag{40}$$

$$Per\,max \ge Per\,min \tag{41}$$

$$WTP_{p\,k} - Per\,max \le Wf_p \le WTP_{p\,k} - Per\,min \quad \forall p \in P, \quad \forall k \in K \tag{42}$$

$$Cm_{p\,k} = Co_k \cdot (WTP_{p\,k} - Wf_{p\,k}) \qquad \forall p \in P, \quad \forall k \in K \tag{43}$$

$$Y_{i\,p} + Y_{i'\,p} \le 1 \qquad \forall i \ne i' | (i, i') \in Not\_comb_{i\,i'}, \quad \forall p \in P \tag{44}$$

$$\sum_{\forall i \in I} N_{i\,p} \le Nlong - 1 \quad \forall p \in P \tag{45}$$

$$\sum_{\forall i \in I} Y_{i\,p} \le Ntrans \qquad \forall p \in P, \quad \forall i \in I \tag{46}$$

The generation process is defined by Eqs. (40)–(46), where Wf_p is the pattern width, calculated in Eq. (40) as the sum of the number of sheets N_{ip} of order i in the pattern p per W_i that represents the sheet width of order i. Note that N_{ip} is used with the same purpose as n_{ip} in the previous MINLP model (Eqs. (1)–(22)). However, in the present formulation this value is calculated by the pre-generation model trying all the possible conformations of the patterns. The parameter *Permax* is defined by the planner corresponding to the maximum trim-loss allowed for the pattern p. The parameter WTP_{pk} corresponds to the paper width used for layer k. Eq. (42) assures that the width of each pattern p has at most a maximum waste of *Permax* and at least a minimum *Permin* in each layer k.

In Eq. (43), the parameter Cm_{pk}, which corresponds to the cost of the trim-loss per meter of pattern p, is calculated multiplying the paper cost in each layer k, Co_k, and the trim-loss per meter in layer k, denoted by $(WTP_k - Wf_p)$.

When combining different orders they must have the same board which means that the paper type must be the same on each layer k, and also the number of layers. Eq. (44) means that if two different orders i and i' are assigned to pattern p they cannot belong to set $Not\_comb_{ii'}$.

Eq. (45) establishes that the number of sheets cut per pattern p could be at most $Nlong - 1$. The sum of Y_{ip} determines the number of different orders assigned to pattern p, which is limited to *Ntrans* by Eq. (46). Note that Y_{ip} is a parameter but plays the same function of the variable y_{ip} in the initial MINLP.

The MILP optimization model is formulated as follows:

$$\text{Min } Z = \sum_{\forall p \in P} \sum_{\forall k \in K} cp_{p\,k} \tag{47}$$

s.t.

$$cp_{p\,k} = Cm_{p\,k} \cdot x_p \qquad \forall p \in P, \quad \forall k \in K \tag{48}$$

$$\sum_{\forall p \in R_{p\,k\,tp}} \sum_{\forall k \in R_{p\,k\,tp}} x_p \cdot \alpha_k \leq S_{tp\,ap} \qquad \forall tp \in TP, \quad \forall ap \in AP \tag{49}$$

$$\sum_{\forall p \in PO_{p\,i}} N_{i\,p} \cdot x_p / Li_i \geq D_i \quad \forall i \in I \tag{50}$$

$$\sum_{\forall p \in PO_{p\,i}} N_{i\,p} \cdot x_p / Li_i \leq D_i \cdot (1 + \eta_i) \quad \forall i \in I \tag{51}$$

$$x_p \geq CRmin_p \cdot yr_p \quad \forall p \in P \tag{52}$$

$$x_p \leq CRmax_p \cdot yr_p \quad \forall p \in P \tag{53}$$

$$x_p, \;\; cp_{p\,k} \in R^+$$

$$yr_p \in \{0; 1\}$$

In Eq. (47) the objective function is defined, representing the paper trim-loss cost, where the variable cp_{pk} is the cost of the trim-loss of pattern p. This variable is calculated in Eq. (48) multiplying the parameter cost Cm_{pk} defined in the pre-generation model and the pattern length x_p. Eq. (49) determines that sum of the length of each paper layer k in all patterns p must not exceed the paper length in stock, $S_{tp\,ap}$. Eqs. (50) and (51) are the demand constraints.

In Eqs. (52) and (53) if a pattern p is executed ($yr_p = 1$), it must be longer than or equal to a minimal and maximum run length $CRmin_p$ and $CRmax_p$, respectively.

Note that the objective function, Eq. (47), is equal to Eq. (1) in the first formulation. However, it is repeated to facilitate the model reading at this point. The same criterion was used with Eqs. (52) and (53) which are the same than Eqs. (15) and (16) with the difference that in the MINLP model yr_p is a positive variable not binary like in this last model (Eqs. (47)–(53)).

The initial model (Eqs. (1)–(22)) is really simplified by this strategy with no additional variables nor constraints. The main difference in this model is that the cutting patterns pre-generation process avoids assignment decisions in the optimization model which strongly affect computational performance.

5. Different objective functions

Besides solution strategies, the objective function may also have influence on the solution quality and efficiency. Choosing a suitable objective function should not disregard economical or environmental considerations. In real cases, usually the trim-loss costs (Eq. (1)) results one of the most appropriate, however others can be used depending on the goals pursued.

The first alternative objective considered is given by the following equation:

$$\text{Min } \; TPC = \sum_{\forall p \in P} \sum_{\forall k \in K} cp_{p\,k} + \sum_{\forall p \in P} CY \cdot yr_p \tag{54}$$

Eq. (54) adds to the trim-loss cost the changing pattern cost. The parameter *CY* indicates the cost of changing a pattern. Usually, the pattern change results in some paper trim-loss because of the paper reels remotion, parts of the machine changed and the set-up time to place the knives.

Another goal is the total production cost defined by Eq. (55).

$$\text{Min } \; PC = \sum_{\forall p \in P} \sum_{\forall k \in K} Co_{p\,k} \cdot ta_{p\,k} \tag{55}$$

This objective could be useful when global enterprise costs must be minimized.

When environmental issues are pursued the weight of the trim-loss may be considered as an objective function.

$$\text{Min } \; TW = \sum_{\forall p \in P} \sum_{\forall k \in K} V_{p\,k} \cdot (ta_{p\,k} - ua_{p\,k}) \tag{56}$$

The parameter V_{pk} represents the paper grammage in each layer k of pattern p.

6. Results

In order to illustrate models performance three examples are solved. Models have also been executed in real productive scenarios. The objective functions discussed are considered in examples one and two.

Table 1
Models main characteristics and results, example 1

Strategy	Single equations	Discrete variables	Total variables	*f*(*x*): *Z* (Eq. (1))		*f*(*x*): *TPC* (Eq. (54))		*f*(*x*): *PC* (Eq. (55))		*f*(*x*): *TW* (Eq. (56))	
				OV ($)	ST (s)	OV ($)	ST (s)	OV ($)	ST (s)	OV (kg)	ST (s)
Applying linearization techniques: Eqs. (2), (3), (6), (8)–(11), (15), (16) and (28)–(39)	2064	380	931	107.9	33.52	157.9	10.84	2631	7.83	90.22	13.81
Two-steps formulation: Eqs. (40)–(46), (48)–(53)	49	13	27	107.9	0.187	157.9	0.234	2631	0.046	90.22	0.109

f(*x*): objective function; OV: objective value; ST: solution time (s) (GAP 0%).

Table 2
Results obtained with five orders example, minimizing *Z*

p	i	$n_{i,p}$	i'	$n_{i',p}$	k_1		k_2		k_3		$\sum_{\forall i \in I} n_{i\,p} \cdot Wi_i$ (mm)	x_p (m)	$\sum_{\forall k \in K} cp_{pk}$ ($/pattern)
					tp	ap (mm)	tp	ap (mm)	tp	ap (mm)			
1	Ord 1	2	Ord 2	2	O1	1200	O1	1200	O1	1200	1180	500	4.50
2	Ord 1	3	Ord 2	1	O1	1300	O1	1300	O1	1300	1250	500	11.25
3	Ord 3	3	–	–	K1	1000	O2	1000	O2	1000	960	787.5	17.40
4	Ord 4	2	Ord 3	1	K1	1400	O2	1400	O2	1400	1360	755	16.69
5	Ord 5	2	–	–	O2	1500	O1	1500	O2	1500	1422	1496.25	58.05

Table 3
Models main characteristics and results, example 2

Strategy	Single equations	Discrete variables	Total variables	*f*(*x*): *Z* (Eq. (1))		*f*(*x*): *TPC* (Eq. (54))		*f*(*x*): *PC* (Eq. (55))		*f*(*x*): *TW* (Eq. (56))	
				OV ($)	ST (s)	OV ($)	ST (s)	OV ($)	ST (s)	OV (kg)	ST (s)
Applying linearization techniques: Eqs. (2), (3), (6), (8)–(11), (15), (16) and (28)–(39)	1944	368	889	131.7	2585	191.7	1924	3351	4164	110.6	1841
Two-steps formulation: Eqs. (40)–(46), (48)–(53)	85	28	57	131.7	0.39	191.7	0.140	3351	0.406	110.6	0.312

f(*x*): objective function; OV: objective value; ST: solution time (s) (GAP 0%).

Because of the problem complexity, the approach used to solve the corrugation and cutting problem has key influence on the execution time. The strategy selected for the implementation must guarantee not only an optimal solution but also reasonable resource consumption.

The models were posed in GAMS system. CPLEX 9.0 was used to solve the MILP formulations, they have been executed over a PC having an Intel Pentium D 2.8 GHz processor.

6.1. Example 1

The first example is small, with five customer orders and ten patterns available to use. In Table 1, models main characteristics and results are presented.

As shown in Table 1, although the first model[2] has short execution time for every objective function considered, two-steps formulation runs are much faster because of the reduced number of equations and variables.

When minimizing the trim-loss cost (*f*(*x*): *Z*), patterns configuration in the final solution is shown in Table 2.

6.2. Example 2

The second case considered has seven customer orders to be satisfied in seven available patterns p. Models characteristics and major results are shown in Table 3.

In this example, there is a great difference between models performances. This still small case shows how the second alternative is much better in terms of computational efficiency. Consequently, the first alternative will unlikely solve a real productive problem as presented in example 3.

6.3. Example 3

This last example represents an industrial case with 26 orders which was run using Eq. (54) as the objective function. Results and models configuration are shown in Table 4.

[2] Eqs. (1)–(3), (6), (8)–(11), (15), (16) and (28)–(39).

Table 4
Models main characteristics and results, example 3

Strategy[a]	Single equations	Discrete variables	Total variables	$f(x)$: TPC – (Eq. (54))		
				OV ($)	GAP	ST (s)
Applying linearization techniques: Eqs. (2), (3), (6), (8)–(11), (15), (16), (28)–(39) and (54)	14,697	2440	5641	–	–	>75,000
Two-steps formulation: Eqs. (40)–(46), (48)–(53)	579	254	2155	473.67	0%	0.703

$f(x)$: objective function; OV: objective value; ST: solution time (s).
[a] These characteristics are also valid if instead of trim-loss cost any other objective function is used, such as Eqs. (1), (55) or (56).

Reaching a solution in one step is a very difficult task because the techniques applied to get a linear formulation from the original model expands the integer space. The combinatorial complexity resultant, as shown in Table 4, makes the problem impossible to solve by this strategy.

On the contrary, the two-steps method reduces the number of variables and constraints in comparison to the original formulation which leads to extremely short execution time even in industrial scenarios.

Patterns configuration in the final solution is shown in Table 5. The pattern generation procedure creates 254 feasible patterns. These patterns are input data of the MILP model which chooses to use 20 of them in the final solution. Variable yr_p are set to 1 when pattern p is selected, to satisfy customer orders and minimize trim-loss and changing pattern costs (TPC).

7. Model implementation

In the previous section, some examples were solved to compare the computational efficiency of the models. The two-steps algorithm was chosen to implement in the company. Besides the execution time, this model has the advantage that the planner can handle some problem parameters allowing the analysis of several scenarios. The planner can manipulate the following parameters: maximum waste allowed *Permax*, maximum and minimum number of patterns per order, the number of longitudinal and transversal knives in the cutting machine, minimum run length $CRmin_P$ and the mandatory and optional orders to consider, while the plan for mandatory orders must be solved, optional are used to combine and produce a best set of patterns. Smithin and Harrison (1982) suggested that there is an equally important dimension concerned with practical and interpersonal issues which is often overlooked in tackling cutting stock problems. Human expertise should not be disregarded because it can provide a competitive advantage over the system solution.

An interface written in Java has been implemented such that the planning and cutting system can be linked to the company ERP (an Oracle E-Business Solution). The model takes the input values from the ERP: customer orders and paper reels stock information. Then the planner selects the orders to produce and the plan parameters, executes the optimization MILP problem, an automatic data transference is performed from the ERP to a GAMS model, then an intermediate HTML results file is generated so that the planner can analyze it. He can change the input if needed and run again the optimization problem, this process can

Table 5
Results obtained with 26 orders example, minimizing TPC

p	i	$n_{i,p}$	i'	$n_{i',p}$	k_1		k_2		k_3		$\sum_{\forall i \in I} n_{i\,p} \cdot Wi_i$ (mm)	x_p (m)	$\sum_{\forall k \in K} cp_{p\,k}$ ($/pattern)
					tp	ap (mm)	tp	ap (mm)	tp	ap (mm)			
1	Ord 14	2			K1	1100	O2	1100	O2	1100	1066	705.5	13.26
2	Ord 1	2	Ord 2	2	O1	1200	O1	1200	O1	1200	1180	562.5	5.06
3	Ord 4	2	Ord 24	1	K1	1400	O2	1400	O2	1400	1380	1066.8	11.84
4	Ord 5	1	Ord 10	1	O2	1100	O1	1100	O2	1100	1061	1092.5	21.19
5	Ord 5	1	Ord 11	1	O2	1100	O1	1100	O2	1100	1080	1400	13.86
6	Ord 1	1	Ord 6	3	O1	1400	O1	1400	O1	1400	1380	900	8.1
7	Ord 7	3	Ord 18	1	O1	1200	O1	1200	O1	1200	1170	708.33	9.56
8	Ord 3	1	Ord 8	2	K1	1400	O2	1400	O2	1400	1380	1400	15.54
9	Ord 8	1	Ord 25	1	K1	1200	O2	1200	O2	1200	1154	800	20.32
10	Ord 3	1	Ord 9	2	K1	1400	O2	1400	O2	1400	1380	930	10.32
11	Ord 10	2	Ord 17	2	O2	1300	O1	1300	O2	1300	1280	753.75	7.46
12	Ord 5	1	Ord 11	2	O2	1500	O1	1500	O2	1500	1449	500	12.7
13	Ord 12	2	Ord 21	2	O1	1500	O1	1500	O1	1500	1450	862.5	19.41
14	Ord 13	1	Ord 15	1	K1	1200	O2	1200	O2	1200	1178	1345	16.41
15	Ord 13	1	Ord 23	1	K1	1200	O2	1200	O2	1200	1167	1366	24.86
16	Ord 13	1	Ord 25	1	K1	1300	O2	1300	O2	1300	1267	622.4	11.33
17	Ord 15	1	Ord 23	1	K1	1300	O2	1300	O2	1300	1219	500	22.4
18	Ord 16	2	Ord 19	2	O2	1200	O1	1200	O2	1200	1170	985	14.68
19	Ord 20	2	Ord 26	1	O1	1300	O1	1300	O1	1300	1276	889	9.60
20	Ord 18	1	Ord 22	2	O1	1500	O1	1500	O1	1500	1478	577.85	5.72

be repeated until the plan is accepted. At this point, the results generated are automatically transferred to the ERP system. This information includes patterns to produce and theoretical paper consumptions (paper stock reservation). This system has been a great evolution for the company-supply chain integration. The previous version was a Visual-Basic ad hoc program used by the company planner, inputs and outputs were made by means of spreadsheets not integrated to the company system. Plan results were loaded by hand, control of the production plan (theoretical versus real) was not followed, people in the company did not trust in the information loaded. By this system, the production plan control is made by the ERP system, all the data is automatically read and loaded, several terminals were distributed on the plant floor so that an automatic caption of the plan progress is made. Information system data are now reliable, statistics data can be generated, and it is possible to integrate customers and suppliers to exchange information, company managers are now involved in this process to be more competitive in the market.

8. Discussion and conclusions

In this work, enterprise optimization related to an assignment and trim-loss problem was presented. The cutting stock problem is one of the major issues in paper converting supply chain. Additionally, lower costs in boxes production will improve competitiveness in almost every supply chain because packaging industry has a great influence in transportation and logistic cost. The first model developed is a MINLP formulation which presents two different bilinearities in some equations. To obtain global solution, two transformation techniques were applied. One of the methods corresponds to linearization techniques, as presented by Harjunkoski et al. (1999). This strategy solves the problem in one step. However, some additional integer and positive variables and constraints are introduced increasing the combinatorial complexity. Consequently, it cannot be applied to real enterprise optimization problems due to long computational time.

A two-stages model has also been developed to avoid original non-convexities. A similar procedure has also been applied by Westerlund et al. (1998) and by Westerlund and Isaksson (1998). The formulation developed is much simpler than the first method, eliminating some decisions in the optimization model due to a pattern pre-generation procedure. It also provides a more robust and faster problem solution. One important feature of this mathematical model is the reduced time spent to reach the solution which allows the planner to evaluate in a few minutes several scenarios by manipulating the parameter values, comparing with the old system where the planner spent around 4 h to generate the first valid solution.

Comparing plans generated by the old and new system, the latter approach reduces up to 30% the trim-loss cost. No simplifications were done to represent the real productive context. The system is now in production and has been used for several of months giving very good results. Having linked the system to the company ERP besides of the advantages of the integration, via the ERP the control of the production plan generated by the model is facilitated giving an extra feature to the whole system.

Some discrete decisions connected to this problem could be treated as disjunctions. Disjunctive formulation usually facilitates the problem understanding. Future work includes the generation of a disjunctive formulation of this problem. Some of this work is already in progress.

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