

High-quality optical vortex-beam generation by using a multilevel vortex-producing lens

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In the present work, we propose a method to generate high-quality optical vortices with a reduced number of phase levels by using multilevel vortex-producing lenses (VPLs). The VPL is implemented in a liquid-crystal spatial light modulator with limited capacity to project phase levels. The proposed method significantly improves the quality of the optical vortex obtained by employing spiral phase plates with the same number of phase levels. Simulations and experimental results confirming the effectiveness of the method are presented. © 2013 Optical Society of America

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An optical vortex (OV) of an electromagnetic beam corresponds to a point in space where its intensity vanishes, its phase is undetermined, and the total orbital angular momentum (OAM) around the vortex is nonzero. The most common form of OV beams is the Laguerre–Gaussian, which is featured by a doughnut-shaped intensity pattern and a phase with an azimuthal dependency $e^{im\theta}$ where m corresponds to the topological charge or number of 2π phase discontinuities of the wavefront [1].

Although OAM was recognized since the 1950s in the higher-order transitions of atoms [2], it was only in 1992 when Allen *et al.* [3] demonstrated that Laguerre–Gaussian beams have nonzero OAM. This characteristic allowed the use of this type of laser beam in a wide range of engineering and scientific applications. Nowadays, the properties of the OVs are present in telecommunications systems [4], imaging [5], image processing [6,7], metrology [8], optical tweezers [9,10], integrated optics [11], and nanophotonics [12], among others. Moreover, technological applicability of optical singularities has made the problem of constructing high-quality OV beams become one of great relevance and interest [11,13].

Two common techniques for the generation of OVs are based on the diffraction of laser beams by a computer-generated hologram [14,15] or by a spiral phase plate (SPP) [7,16,17]. In the case of SPPs, a fundamental TEM₀₀ laser beam goes through an SPP of transmittance $t(\theta) = e^{im\theta}$, thus producing in the far field a Laguerre–Gaussian beam of topological charge m .

A convenient way to generate the SPP is by using a liquid-crystal spatial light modulator (LC-SLM). In this case, the ideal condition is fulfilled if all phase values of the transmittance can be addressed onto the LC-SLM. However, there are some LC-SLMs that only achieve a total phase modulation $\phi_{\max} < 2\pi$ rad being unable to have a continuous modulation from 0 to 2π . Therefore, the wrapped SPP must be built by a multilevel phase with $N \leq 2\pi/2\pi - \phi_{\max}$ steps, N being the number of phase levels addressed in the LC-SLM. In this sense, each phase step accounts for a relative phase difference regarding its neighbors of $2\pi/N$ rad. Note that, although the LC-SLM

does not reach a 2π phase shift, it is possible to demonstrate that, with this discretization, the Burgers vector modulus around the dislocation is an integer multiple of 2π [18].

If the quality of a Laguerre–Gaussian beam is evaluated in terms of its intensity symmetry and its phase distribution, it is possible to show that the quantized phase levels degrade the quality of the OV. Guo *et al.* [19] studied the beam resulting from a multilevel SPP and obtained expressions for the intensities of the vortex components as a function of the number of phase steps. Kotlyar and Kovalev [20] approximated the diffraction efficiency of the multilevel SPP using polygon regions and derived somewhat similar results using Fraunhofer diffraction theory. Later, Zhang *et al.* [17] experimentally corroborated the validity of the theoretical models for an OV with a topological charged one, built by a multilevel SPP with $N < 6$.

It is known that for a diffractive optical element (DOE) with no resolution limitations, the quality obtained from the complex amplitude pattern depends on the DOE efficiency [21], which is given by

$$\eta = \left[\frac{\sin(\frac{\pi}{N})}{\frac{\pi}{N}} \right]^2 \quad (1)$$

As shown by Zhang *et al.* [17], SPPs with diffraction efficiencies below 0.9 ($N < 6$) produce poor-quality OV beams. Aiming at comparing their experimental results with our simulations, Fig. 1 shows the intensity and phase of OV beams simulated for different values of N (the ideal situation corresponds to $N = 256$).

As a quality estimator of the OVs, we calculate the mean square error (MSE) between the OV computed with $N = 256$ and the OVs produced for different values of N and normalized by using the value for $N = 1$. The MSE can be written as

$$\text{MSE}(N) = \frac{\sum_{i,j} (I_{256}(i,j) - I_N(i,j))^2}{\sum_{i,j} (I_{256}(i,j) - I_1(i,j))^2}, \quad (2)$$

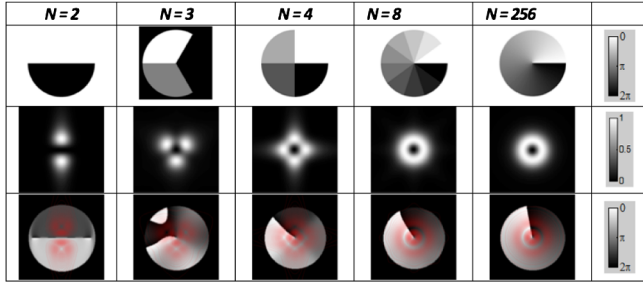


Fig. 1. Simulation of a Gaussian beam diffracted by a multilevel SPP with $m = 1$. First row: number of phase levels used to generate the SPP; second row: multilevel SPP; third row: intensity pattern in the physical lens back focal plane; fourth row: phase pattern in the physical lens back focal plane. SPP diameter $D_{\text{SPP}} = 0.4$ mm, Gaussian beam waist $w = 0.13$ mm, $f = 12$ mm, $\lambda = 532$ nm. Red lines correspond to intensity contours.

where (i, j) correspond to each image pixel and I_N is the intensity of the OV generated with a phase mask of N levels.

A graph comparing the MSE for different values of N is displayed in Fig. 2. The poor quality observed in the diffraction patterns for SPPs with $N < 6$ can be understood in terms of the contributions of OVs with topological charges different to the desired one [17,19]. For example, for $m = 1, 2$, and 3 and $N = 4$, there are contributions of about 19%, 60%, and 91% from other vortices, respectively. This fact results in a huge reduction of the OV quality, as can be seen in Fig. 3. However, it should be stressed that the radial contribution of each vortex depends on the intensity and phase distribution of the incident beam. In that sense, if the phase of the incident beam is modified adequately, or alternatively, another appropriate phase is encoded in the LC-SLM, the radial distribution of undesired vortices can be changed, diminishing their corresponding detrimental effect on efficiency of generating the desired one.

Pointing to the direction of previous ideas, Heckenberg *et al.* [16] produced OVs by using a spiral Fresnel zone plate. However, they pointed out that they went through several problems common to in-line holography of

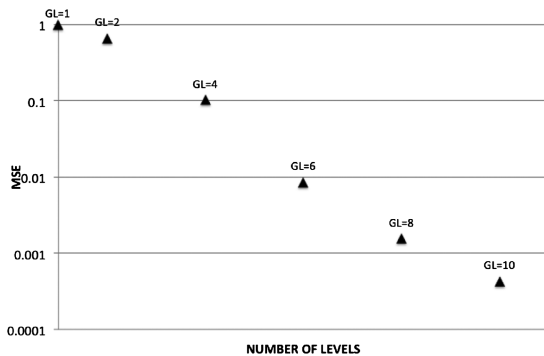


Fig. 2. MSE (measured with respect to the ideal case, $N = 256$) of the intensity patterns obtained from the simulation of a Gaussian beam diffracted by a multilevel SPP with $m = 1$. The intensity pattern of a multilevel SPP with $N = 1$ corresponds to the MSE normalizing value. Simulation parameters are the same as in Fig. 1.

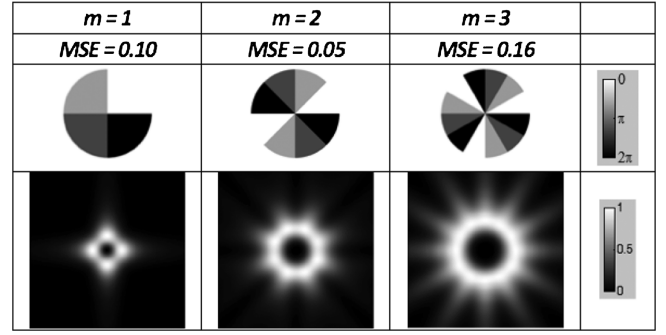


Fig. 3. Intensity patterns obtained from SPPs for $N = 4$: (first row) vortex topological charge; (second row) MSE with respect to the ideal case of $N = 256$; (third row) SPP; (fourth row) intensity pattern in the physical lens back focal plane. Simulation parameters are the same as in Fig. 1.

the Gabor type. Furthermore, Crabtree *et al.* [7] used vortex-producing lenses (VPLs) to create a more compact space-invariant optical Fourier image-processing system. In both cases, the ideal continuous phase-mask transmittance is expressed as [7,16]

$$t(r, \theta) = e^{im\theta} e^{-i\frac{\pi r^2}{f_{\text{FR}}}}, \quad (3)$$

where (r, θ) are the polar coordinates, λ is the wavelength, and f_{FR} is the Fresnel lens focal distance.

Unlike in previous reports, in this work we demonstrate that high-quality OV beams can be generated by using multilevel VPLs projected on LC-SLMs with a reduced number of phase levels. VPLs result from the combination of an SPP with a Fresnel lens [7] (see Fig. 4). When a VPL is discretized, it presents a number of radial periods that depend inversely to its focal distance f_{FR} and directly to its diameter D . These periods have a strong effect on the reduction of the contribution of non-desired topological charges. As explained below, this can be achieved by including an appropriate Fresnel lens as in Eq. (3) and observing the outcome at the corresponding focal distance of the compound optical system. It is important to highlight that an OV generated by a Gaussian TEM₀₀ beam impinging into an SPP is observed at the Fourier plane of the physical lens of the system. In the case of a VPL, however, the OV is observed at an output plane located at a distance z from the physical lens' focal plane, (see Fig. 5), and this work demonstrates there is an optimum distance z_{opt} where a high-quality OV beam can be obtained.

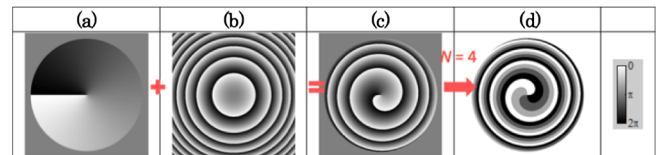


Fig. 4. VPL building procedure: (a) SPP with $N = 256$ is computed. (b) Fresnel lens with $N = 256$ is computed. (c) The SPP and the Fresnel lens are superimpose to generate a VPL with $N = 256$. (d) The VPL is transformed into a multilevel VPL with $N = 4$ and displayed on the LC-SLM. This example is for a VPL with $m = 1$.

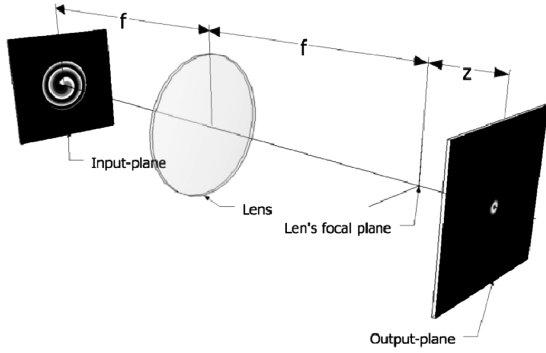


Fig. 5. Basic optical setup for simulations and experiments.

Because the Fresnel lens and the physical lens form a compound lens, the back focal plane of this system does not match with the corresponding one of the physical lens. Taking into account that the Fresnel lens is at the front focal plane of the physical lens, the new back focal plane is at a distance z_{opt} from the focal length of the physical lens and is given by the expression [22]

$$z_{\text{opt}} = \frac{f^2}{f_{\text{FR}}}, \quad (4)$$

z_{opt} being measured from the physical lens focal plane (see Fig. 5). Then, by observing the optical field on this optimum plane, a high-quality OV beam can be detected, even for DOEs with few phase levels.

Figure 6 presents results for simulations of three OV beams of topological charges $m = 1, 2$, and 3, generated with multilevel VPLs with $N = 4$. It must be highlighted that there is an astonishing improvement in the OVs quality with only the inclusion of a Fresnel lens and the detection at the optimum output plane. To quantify this improvement, the MSE is calculated (in the same way as in Fig. 1) and displayed in Fig. 6. As it is apparent, the MSE is diminished several orders of magnitude with respect to the SPP cases shown in Fig. 3. It should be stressed that, to reach a $\text{MSE} = 0.002$ by using a multilevel SPP, it would be necessary to use at least eight

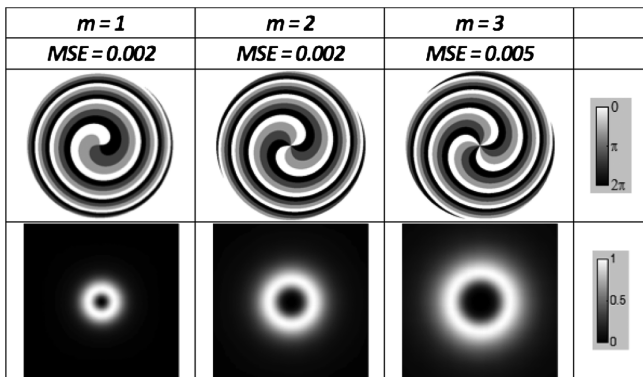


Fig. 6. Optical vortices generated by using multilevel VPLs with $N = 4$: (first row) vortex topological charge; (second row) MSE with respect to the ideal case of $N = 256$; (third row) VPL; (fourth row) intensity pattern at z_{opt} . VPL diameter $D_{\text{VPL}} = 0.4$ mm; $w = 0.13$ mm; $f = 12$ mm; $f_{\text{FR}} = 12$ mm; $\lambda = 532$ nm.

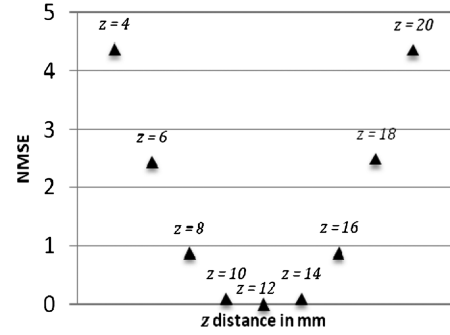


Fig. 7. OV quality (MSE) dependence with respect to the observation plane. Simulation parameters are the same as in Fig. 6. $\text{MSE} > 1$ are due to the scaling phenomenon of the OV beam with respect to the ideal case. This example is for a VPL with $m = 1$.

levels, which corresponds to having more than 95% of the light in the desired topological charge OV. Then these simulations confirm that it is possible to obtain high-quality OV beams by using a multilevel VPL with a reduced number of levels.

Figure 7 shows the OV quality dependence with respect to the observation plane. These results confirm that there is a plane where the OV beam reaches a maximum quality, and the plane is given by the z_{opt} . Note that the increase in MSE as the z distance changes away from the optimal position z_{opt} is mainly due to a defocusing effect. This happens because the MSE is an indicator of similarity calculated point to point, and it is sensible to defocusing.

The dependence of the OV quality on f_{FR} at z_{opt} , for fixed values of $N = 4$, $D_{\text{VPL}} = 0.4$ mm, and $w = 0.13$ mm, is depicted in Fig. 8. It can be noticed that as the parameter f_{FR}/w grows, the MSE of the OV beam generated with the multilevel VPL tends to the MSE value of the OV beam generated with a multilevel SPP (see Fig. 2). This behavior is expected because as long as f_{FR} grows, the similarity between the VPL and the SPP at their centers increases. Then, for larger values of f_{FR}/w , only a tiny region around the center of the VPL is illuminated by the Gaussian beam, and this situation corresponds to the case of Fresnel lens absence. The above results show that inclusion of a Fresnel lens in the LC-SLM strongly contributes to obtain a high-quality OV, provided it is observed at the correct plane.

To validate the previous analysis, experimental results are presented. Our experimental setup consisted of a

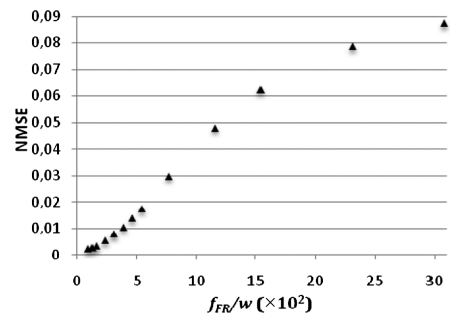


Fig. 8. OV quality dependence with f_{FR}/w . Simulation parameters are the same as in Fig. 6, except for f_{FR} , which is varied.

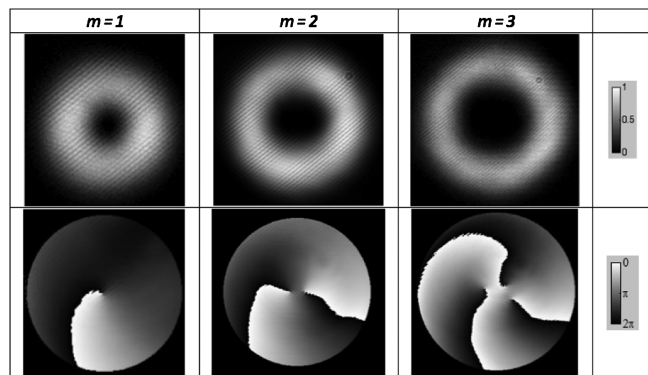


Fig. 9. Experimental results for OV beams generated by projecting VPLs in a four-phase levels transmission LC-SLM.

TEM₀₀ doubled Nd:YAG laser ($\lambda = 532$ nm), which lit a Holoeye LC2002 transmission LC-SLM with a maximum phase shift of $\phi_{\max} = 1.5\pi$. Then our VPL had $N = 4$ levels. The beam waist was $w = 6$ mm, the physical lens' focal distance was $f = 300$ mm, and the Fresnel lens focal distance was $f_{\text{FR}} = 10$ m. The OV was observed nearly at $z = 9$ mm. The intensity of the OV beam was recorded with a CCD camera coupled to a microscope system with a 20X objective. The phase was obtained by using a five-step phase-shifting technique [23,24]. The results are presented in Fig. 9, for OV beams of charges $m = 1, 2$, and 3. As evidenced, the experimental results are in agreement with the simulations. The breakdown of OVs with $m > 1$ in m single-charged OVs may be explained by the presence of a background field [25] and may also be due to the presence of aberrations in the optical system [26]. More research in this direction has to be performed.

In conclusion, this work has demonstrated that, for a reduced number of phase levels, optical vortices reached by including an appropriate Fresnel lens, i.e., obtained by multilevel VPLs, are much better than those obtained by using multilevel SPPs, provided they are observed at the right plane. Therefore, the present work opens a real possibility to generate high-quality OV with low-cost LC-SLM. Moreover, the results of the present manuscript can be extended to the fabrication of fixed phase masks of a reduced number of levels.

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