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Multicriteria Optimization Model for Supply Process Problem under Provision and Demand Uncertainty

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ABSTRACT: Supply processes play an important role in customer satisfaction and company costs. The main characteristics of this problem are given by several decisions that follow a hierarchical structure and a very uncertain context, conditioning the success of the solutions proposed. Two significant sources of uncertainty are considered in this work, namely, provision and demand, both modeled as exogenous variables with random behavior. An optimization model is formulated to reduce the effects of the uncertainty in the company supply process. Because of the problem complexity, a multicriteria model is required to bring a comprehensive solution. Several Pareto-optimal solutions are obtained through application of the ε-constraint technique. The original formulation is a nonconvex one that is then transformed to obtain a disjunctive linear model that guarantees a global result.

1. INTRODUCTION

Competitiveness in global markets is a challenging goal in today's economy. Production companies show a permanent interest in improving all of their business activities with the aim of diminishing costs, growing profits, and increasing customer satisfaction. In this context, the supply process is one of the most important activities in the company operation because it involves planning material requirements; selecting suppliers; and defining inventory levels, delivery alternatives, and commercial agreements to satisfy product demand. These activities are greatly affected by business uncertainty. For that reason, company managers and researchers have focused their work on modeling risk factors in decision making.[1−3]

Provision uncertainty is one critical point for most companies given their reliance on their suppliers to manage their inventories and satisfy demand. The evaluation and selection of sources are then essential to improve a company's performance.[4] Hence, in addition to prices, discounts, and other commercial characteristics, suppliers' provision failures are taken into account in this work.

On the other hand, demand forecast accuracy is also an issue to address. Several demand uncertainty sources can be identified in the business environment. According to the horizon length, those sources are classified as short-term and long-term uncertainties.[5] The former includes orders canceled at the last minute, urgent requests from clients, changes in the priority of an order, and so on. Long-term uncertainty involves price fluctuations in final products, seasonal variations in demand levels, changes in consumer preferences, and increases in clients' expectations, among other.

It is noteworthy that, if demand uncertainty is underestimated, planning decisions will not prepare a company against risks in addition to not allowing it to take advantage of market opportunities. In this context, there are two main alternatives to consider: adjusting product stock levels, which increases the probability of stock-out situations, and raising stock levels to ensure customer satisfaction, thereby increasing inventory costs and capital invested.

The inclusion of demand variability in planning problems aims to mitigate the negative effects of uncertainty. Doing so provides better information to the company, which can definitely improve its planning process accuracy. Demand is not only one of the main uncertainty sources, but also one of the major influences over economic results. Because demand is an exogenous variable, companies can influence it by applying marketing tools such as promotions and publicity, but they can never control it.

A growing interest in modeling demand uncertainty is observed in many scientific works. Gupta and Maranas[6] developed a model of two-stage stochastic programming with uncertain demand. This parameter was modeled using a normal distribution with a known mean and a known standard deviation that was estimated in the second stage through the analytical integration of the distribution curve. As an advantage, this method does not increase the model size, but it does introduce nonlinearities into the formulation.

A normal probability distribution was also applied by You and Grossmann[7] to represent uncertainty in demand behavior. The model designed a three-level supply chain to minimize investment costs, transportation costs, fixed costs in order satisfaction, inventory holding costs in distribution centers, and safety stock costs to ensure certain service levels in demand. The service level is defined as the probability that the inventory level is greater than the demand. The original proposed model is an integer nonlinear program (INLP) because of inventory costs in the objective function.

The objective function is also a notable issue in this article. As a general practice, cost minimization and profit maximization are the most common objective functions in planning problems regarding purchases, inventory, and sales. However, sometimes, there are other targets in the company planning area that are difficult to measure as unique criteria. This is especially true as problems increase in terms of the numbers and types of decisions considered. Indeed, in many cases, it is certainly convenient to define several objective functions to optimize.

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Multicriteria programming includes problems with two or more objectives in conflict. In this area, if a problem is well-formulated, no solution can optimize all targets at the same time. In contrast to single-objective optimization, a solution to a multi-objective problem is more a concept than a definition.[8] Typically, there is no single global solution, and the predominant concept to calculate an optimal point is given by Pareto optimality.[9] One important goal in this kind of problem is to find these solutions and quantify how good each solution is in relation to the others.

One method extensively used to model this context is given by the ε-constraint technique, proposed by Haimes et al.[10] This approach has been mainly applied for supply chain design or enterprise-wide optimization, both characterized by a wide decision scope and a large number of goals to satisfy.[7,11−13] This procedure presents multiple Pareto solutions that are analyzed to estimate the best result for the problem. At this point, manager expertise is crucial to determine the selected solution and understand the tradeoff between the conflictive targets.

This work integrates and completes two problems presented by Rodriguez and Vecchietti.[14,15] In this case, a procurement problem is tackled considering that demand follows an uncertain behavior. This context leads to a much more realistic representation that is also more difficult to solve. In the present article, the formulation of the procurement optimization problem includes selection of contracts offered by suppliers, raw material purchases under provision uncertainty, inventory levels, and delivery decisions. Uncertain demand is now a random parameter modeled by a probabilistic approach which introduces nonconvexities in the formulation. A binary reformulation is applied to linearize the original model and guarantee a global optimum. Because of the complexity of this new problem, several objectives are selected to find an optimal solution. As mentioned, the ε-constraint technique is applied to obtain several Pareto solutions.

Finally, the problem presents several discrete decisions that follow a hierarchical structure in which many constraints must be satisfied subordinate to the decisions already made. Then, a natural representation of this process is approached by the use of generalized disjunctive programming (GDP).[16] We propose a comprehensive disjunctive formulation that integrates supplier selection, material purchases, inventory levels, demand satisfaction, and purchase contract decisions under provision and demand uncertainty on a medium-term planning horizon.

2. PROBLEM DESCRIPTION

The problem approached is centered in a company that aims to define purchases of raw materials in a multiperiod horizon plan. Products are manufactured using several materials that are grouped into families. This provides flexibility in purchase decisions because different material types can be used to satisfy a final product, provided that they belong to the same family.

Another important decision in the problem is given by the selection of the suppliers. In this work, a potential group of providers offers a variety of purchase contracts to sign with them. In general terms, those contracts are classified as quantity-flexibility type, where the buyer commits to purchase a minimum amount with a discounted price. Special characteristics, such as longer relationship terms constraints and payment flexibility, are also included. Additionally, contracts include a maximum quantity to restrict the order size.

Uncertainty in the provision process exists because the suppliers could send just part of the requested amount. Applying a discrete probability distribution, each possible failure range is associated with an occurrence probability, as shown in Figure 1. Then, the expected received quantity of each material depends on the amounts ordered and the suppliers selected.

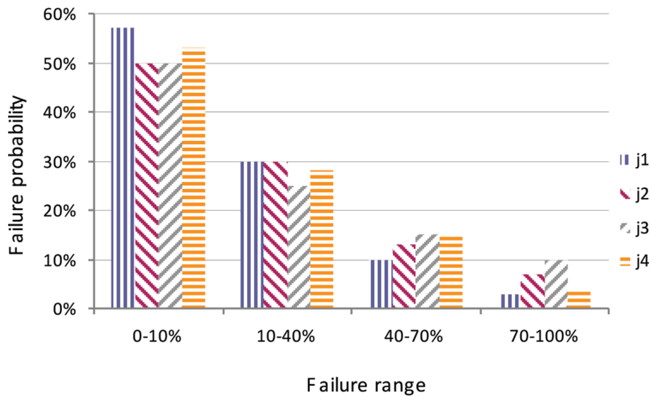

Figure 1. Supplier uncertainty with a discrete probability distribution.

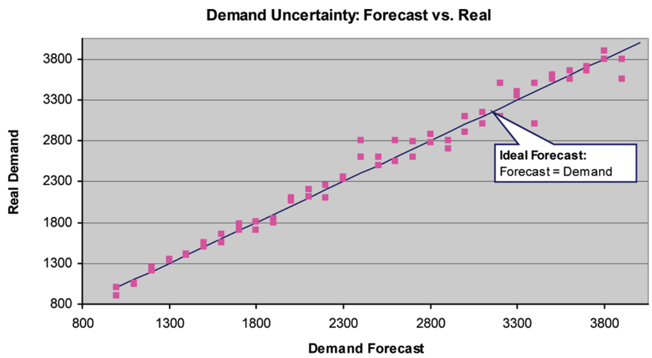

Figure 2. Demand uncertainty: Forecast versus actual.

It is assumed that partial deliveries are allowed to complete a purchase order and that transportation cost is paid by the company. It is also considered that different transportation sizes have different fixed cost. This situations leads to nonlinear terms, as explained in the Problem Formulation.

As mentioned in the previous section, considering deterministic demand is a sheer simplification of a real context. Especially when the planning horizon exceeds the short term, the demand value is, in fact, a data point from a demand forecast. Actual demand, in contrast, is historic data given by the client's requests whether they were satisfied or not. In general, a comparison between demand forecast and historic demand shows some differences as presented in Figure 2. In some cases, several demand forecasts are considered to evaluate different optimistic and pessimistic scenarios.[17]

The normal distribution is widely used in the literature to model demand uncertainty.[18−21] Clients' behavior in general corresponds to this type of function.[22,23] Additionally, the central limit theorem supports the idea of using a normal distribution to represent demand under uncertainty.[24] For those reasons, this distribution was assumed in this article to model the demand parameter as a random exogenous variable. However, it is remarkable that the proposed approach could also be used assuming a different distribution function, simply changing the corresponding functional form in the formulation.

The objectives pursued include minimizing material purchase costs, inventory holding costs, and delivery costs and maximizing demand satisfaction. The first three objectives are considered in the

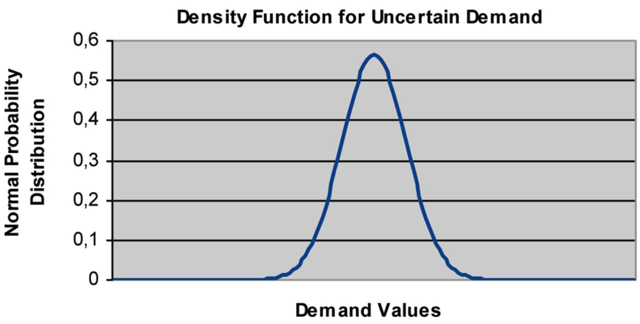

Figure 3. Demand uncertainty with a normal distribution.

cost function, whereas demand satisfaction is assumed as an independent expression. As a consequence, this second objective is considered as a ε constraint, to bring a set of Pareto-optimal solutions.

3. PROBLEM FORMULATION

3.1. Original Model. *3.1.1. Inventory Constraints.* The expected stock level at the beginning of each time period is determined in eq 1 and is calculated as the stock level in the previous period plus the expected quantity sent by the suppliers minus the expected demand to satisfy

$$s_{ft} = s_{f(t-1)} + eq_{f(t-1)} - d_{f(t-1)} \qquad \forall\, f \in F,\ \forall\, t \geq t_2 \in T \tag{1}$$

Variable S_{ft} defines the expected level in stock for each material family f at the beginning of period t. Variable $\mathrm{eq}_{f(t-1)}$ corresponds to the expected quantity sent by the suppliers in the previous term, which depends on the suppliers selected and the amount ordered from them. This restriction applies from period t_2 because the initial stock in period t_1 is given by the parameter IS_f.

Demand follows a normal probability distribution with known mean and standard deviation (Figure 3). Then, the variable d_{ft} indicates the amount of material f planned to satisfy the potential demand in period t. This variable is calculated in eq 2 according to the level of demand satisfaction (Demand_{pt}) desired

$$d_{ft} = \sum_{\forall p\, \in\, PF_{pf}} Demand_{pt}\, \alpha_{pf} \qquad \forall\, f \in F,\ \forall\, t \in T \tag{2}$$

$$\sum_{\forall f\, \in\, F} s_{ft} \leq SC \qquad \forall\, t \in T \tag{3}$$

Constraint 3 restricts the amount of materials in stock at the beginning of each period.

Equation 4 defines the initial stock of each family in the planning horizon. The definition of the average inventory level for each period and family is given in eq 5, where variable eoq_{jft} defines the delivery size from supplier j for family f in period t.

$$s_{ft_1} = IS_f \qquad \forall\, f \in F \tag{4}$$

$$savg_{ft} = \frac{s_{ft} + \sum_{\forall j\, \in\, J} eoq_{jft} + s_{f(t+1)}}{2} \qquad \forall\, f \in F,\ \forall\, t \in T \tag{5}$$

3.1.2. Purchase and Delivery Decisions. The purchase process presents several discrete decision variables that follow a hierarchy. A natural approach to model this situation is applied using nested disjunctions as shown in eq 6

$$\begin{bmatrix} Y_{1jft} \\ \sum_{k\, \in\, FK_{fk}} q_{jkt} \leq \sum_{k\, \in\, FK_{fk}} Q_{\max jkt} \\ \begin{bmatrix} Y_{2jkt} \\ q_{jkt} \leq Q_{\max jkt} \\ \bigvee_{\substack{c\, \in\, C \\ (c,t,t')\, \in\, TP_{ctt'}}} \begin{bmatrix} Y_{3jckt} \\ q_{jkt} \geq Q_{\min cj} \\ w_{jckt} = q_{jkt} PC_{jkt}(1 - \delta_{jc}) + FC_{jc} \\ w_{jckt} = m_{jckt'} \end{bmatrix} \end{bmatrix} \vee \begin{bmatrix} \neg Y_{2jkt} \\ q_{jkt} = 0 \end{bmatrix} \forall\, k \in FK_{fk} \end{bmatrix} \vee \begin{bmatrix} \neg Y_{1jft} \\ \sum_{k\, \in\, FK_{fk}} q_{jkt} \end{bmatrix}$$

$$\forall\, j \in J,\ \forall\, f \in F,\ \forall\, t \in T,\ \text{where } \delta_{jc_1} = 0; \delta_{jc_2} > 0; \delta_{jc_3} > \delta_{jc_2}; \delta_{jc_4} < 0 \tag{6}$$

where $\delta_{jc_1} = 0$, $\delta_{jc_2} > 0$, $\delta_{jc_3} > \delta_{jc_2}$, and $\delta_{jc_4} < 0$. Each disjunction represents one level in the decision process that restricts the decisions of the below levels. In the first step, Boolean variable y_{1jft} selects which families f are bought from each supplier j in period t. In the negative case, no material is ordered related to that family, supplier, and period. In the affirmative, the total quantity ordered must be lower than the maximum capacity of supplier j. In the following level, variable y_{2jkt} indicates the selection of material k from family f, according to FK_{fk}, in period t. Set FK_{fk} defines which materials correspond to each family f. In this term, the ordered of material k quantity in period t cannot be greater than the corresponding supplier capacity. In the third level, variable y_{3jckt} represents that a commercial alternative c must be chosen to order material k from supplier j in period t.

The first option, c_1, considers not signing any contract with the supplier. Then, a regular price is paid, and a minimum quantity is not required. The first contract, c_2, includes a discount over the total quantity ordered and restricts the minimum amount for the

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purchase order. Contract c_3 also offers a discount and requires a minimum quantity. It also promotes longer relationships with suppliers because it can be chosen only if some contract has been selected before for that material and supplier. Finally, contract c_4 also imposes a minimum order amount but offers a longer payment term. Because of this financial flexibility, an interest rate must be paid over the regular price. More details about contract types modeled can be found in Rodriguez and Vecchietti.[14] Many other articles support the idea of using contracts to formalize the purchase process[13,25−27] and propose different approaches to implement this strategy.

Despite the expression of the hierarchical decisions by means of nested disjunctions, the decisions cannot be implemented directly. To solve the problem, nested disjunctions must be transformed into GDP form as simple disjunctions.[28] For that purpose, the disjunctions in expression 6 must be rewritten as single disjunctions (eqs 7−9), and some additional constrains must be also included in the model.

$$\begin{bmatrix} y_{1jft} \\ \sum_{k \in FK_{fk}} q_{jkt} \leq \sum_{k \in FK_{fk}} Q_{\max jkt} \end{bmatrix} \vee \begin{bmatrix} \neg y_{1jft} \\ \sum_{k \in FK_{fk}} q_{jkt} = 0 \end{bmatrix} \quad \forall\, j \in J,\ \forall\, f \in F,\ \forall\, t \in T \tag{7}$$

$$\begin{bmatrix} y_{2jkt} \\ q_{jkt} \leq Q_{\max jkt} \end{bmatrix} \vee \begin{bmatrix} \neg y_{2jkt} \\ q_{jkt} = 0 \end{bmatrix} \quad \forall\, j \in J,\ \forall\, k \in FK_{fk},\ \forall\, t \in T \tag{8}$$

$$\bigvee_{\substack{c \in C \\ (c,t,t') \in TP_{vtt'}}} \begin{bmatrix} y_{3jckt} \\ q_{jkt} \geq Q_{\min cj} \\ q_{jkt} \leq Q_{\max cj} \\ w_{jckt} = q_{jkt}\, PC_{jkt}(1 - \delta_{jc}) + FC_{jc} \\ w_{jckt} = m_{jckt'} \end{bmatrix} \vee \begin{bmatrix} \neg y_{2jkt} \\ q_{jkt} = 0 \end{bmatrix} \quad \forall\, j \in J,\ \forall\, k \in K,\ \forall\, t \in T \tag{9}$$

As mentioned above, the expected amount of each material is determined by the selected suppliers and the quantities ordered from each of them. The formula to calculate eq_{ft} is derived as a function $g_{ift}(y_1)$ showing a binary relationship between the subscript i and the selection of suppliers given by the binary variable Y_{1jft}. This formula sets the value of the subscript i as

$$i_{ft} = \sum_{j=1}^{J} 2^{j-1} Y_{1jft} \quad \forall\, f \in F,\ \forall\, t \in T \tag{10}$$

where $i_{ft} \in I_{ft} = \{0, ..., 2^J - 1\}$ and $Y_{1jft} \in \{0,1\}$ is a binary variable indicating the choice of supplier for each family and period. Because of this procedure, the value of this subscript symbolizes what providers have been chosen. Considering all possible combinations in selecting suppliers leads to the whole representation of set I_{ft}.

Then, the expected quantity of each family and period can be determined from a new Boolean variable v_{ift}, which is equivalent to the set of selected suppliers according to the value of i_{ft}, as

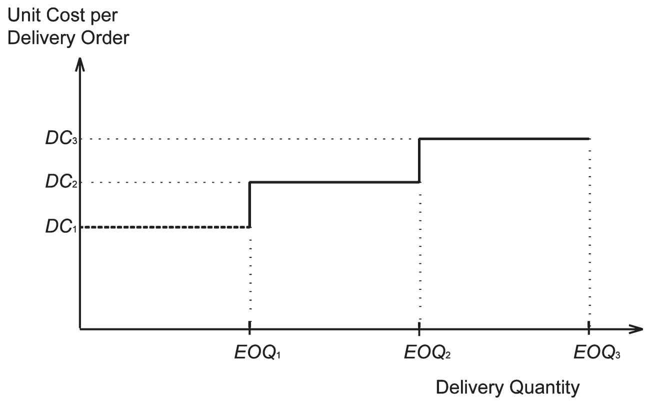

Figure 4. Fixed unit per delivery order.

shown in equation 11

$$\begin{bmatrix} v_{0ft} \\ eq_{ft} = g_{0ft} \end{bmatrix} \vee \begin{bmatrix} v_{1ft} \\ eq_{ft} = g_{1ft} \end{bmatrix} \vee \begin{bmatrix} v_{2ft} \\ eq_{ft} = g_{2ft} \end{bmatrix} \vee \cdots \vee \begin{bmatrix} v_{(2^J-1)ft} \\ eq_{ft} = g_{(2^J-1)ft} \end{bmatrix} \quad \forall\, f \in F,\ \forall\, t \in T \tag{11}$$

For instance, if the solution for family f in period t is given by $\neg y_{1j_4ft} \wedge \neg y_{1j_3ft} \wedge y_{1j_2ft} \wedge \neg y_{1j_1ft}$, meaning that only supplier j_3 is chosen for that family and period, then the procedure applies as follows

$$\neg\, y_{1j_4ft} \wedge \neg\, y_{1j_3ft} \wedge y_{1j_2ft} \wedge \neg\, y_{1j_1ft} \rightarrow \text{binary representation : 0010}$$

$$\begin{aligned} i_{ft} &= 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 \\ &= 0 + 0 + 2 + 0 = 2 \end{aligned}$$

Then, variable v_{2ft} is true and function $g_{2ft}(y_1)$ must be satisfied. For more details about this procedure, see Rodriguez and Vecchietti.[14]

As regards consistency of delivery and purchase decisions it is necessary to impose a restriction that the purchased quantities must be equal to the total shipped quantities of a given family, supplier, and period. This restriction is defined in the following equation

$$\sum_{k \in FK_{fk}} q_{jkt} = n_{jft}\, eoq_{jft} \quad \forall\, j \in J,\ \forall\, f \in F,\ \forall\, t \in T \tag{12}$$

where the integer variable n_{jft} represents the number of material shipments ordered for family f from provider j in period t and eoq_{jft} is the size of the delivery order.

The unit cost of each shipment is modeled as a fixed cost paid for all units delivered. As shown in Figure 4, this unit cost varies according to the total delivery size. For an order size less than or equal to EOQ_1, the fixed cost for each shipment is given by DC_1; if the amount exceeds EOQ_1 and is not greater than EOQ_2, then the associated fixed cost is DC_2. Finally, the upper bound for each delivery is given by EOQ_3. In this case, the cost of each shipment equals DC_3. Disjunction 13 models the unit transportation costs

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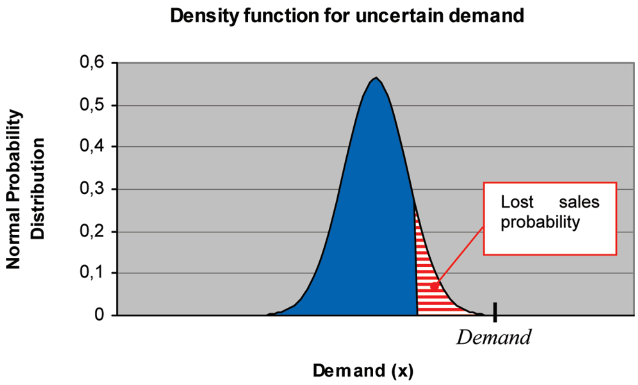

Figure 5. Lost sales probability.

in sections

$$\begin{bmatrix} u_{1jft} \\ eoq_{jft} \le EOQ_1 \\ dc_{jft} = DC_1 \end{bmatrix} \vee \begin{bmatrix} u_{2jft} \\ eoq_{jft} \le EOQ_2 \\ dc_{jft} = DC_2 \end{bmatrix} \vee \begin{bmatrix} u_{3jft} \\ eoq_{jft} \le EOQ_3 \\ dc_{jft} = DC_3 \end{bmatrix} \quad \forall\, j \in J,\ \forall\, f \in F,\ \forall\, t \in T \qquad (13)$$

Boolean variables u_{1jft}, u_{2jft}, and u_{3jft} select the means of transport to use, which conditions the maximum volume to carry per shipment and its unit cost. The total shipping cost for each material family, supplier, and time period, tdc_{jft}, is defined in constraint 14, as the bilinear product of the number of deliveries, n_{jft}, and the cost of each shipment, dc_{jft}

$$tdc_{jft} = n_{jft}\, dc_{jft} \quad \forall\, j \in J,\ \forall\, f \in F,\ \forall\, t \in T \qquad (14)$$

3.1.3. Logical Constraints. Some logical constraints are also considered to maintain the hierarchical meaning of the purchase decision process. The restriction

$$\sum_{\forall c \neq c_1} Y_{3jckt-1} \ge Y_{3jc_3kt} \quad \forall\, j \in J,\ \forall\, k \in K,\ \forall\, t \in T \qquad (15)$$

prevents the ordering of materials by signing contract c_3 if the company has not purchased the material from that supplier j previously. According to eq 16, the selection of contract c_3 is available from period t_2.

$$Y_{3jc_3kt_1} \le 0 \quad \forall\, j \in J,\ \forall\, k \in K \qquad (16)$$

Expression 17 forces the choice of at least one material from a family if that family has been selected from supplier j.

$$\sum_{\forall k \in FK_{fk}} Y_{2jkt} \ge Y_{1jft} \quad \forall\, j \in J,\ \forall\, f \in F,\ \forall\, t \in T \qquad (17)$$

The last logical constraint

$$\sum_{c} Y_{3jckt} = Y_{2jkt} \quad \forall\, j \in J,\ \forall\, c \in C,\ \forall\, k \in K,\ \forall\, t \in T \qquad (18)$$

requires that, when a material is chosen, one contract must be selected. On the contrary, if no material has been selected, a contract is also not selected.

3.1.4. Objective Functions. As discussed in the previous section, this optimization problem suggests the inclusion of several objectives to bring a comprehensive approach. In general, a multicriteria problem can be written as

$$\begin{aligned} &\min f_1(x) \\ &\min f_2(x) \\ &\vdots \\ &\min f_n(x) \\ &\text{s.t.} \\ &g(x) = 0 \\ &h(x) \ge 0 \\ &where\ x \in X \end{aligned} \qquad (19)$$

In this problem, we first want to minimize the costs of raw material purchasing, inventory holding, and transportation. Although the objectives from procurement, inventory management, and transportation can be very different and even conflicting with each other, it is possible to unify them in economic terms to handle a single objective function, such as the one given by

$$\min \sum_t \frac{\left(\sum_j \sum_c \sum_k m_{jckt} + \sum_f savg_{ft} COSTavg_{ft} MS + \sum_f \sum_j tdc_{jft} \right)}{(1 + RR)^t} \qquad (20)$$

However, the consideration of uncertain demand introduces new issues to analyze. On one hand, we must define which objective is pursued in this case. Because customer satisfaction is a crucial component that is difficult to weigh in economic terms, minimizing lost sales is considered an appropriate objective. This goal is defined as the expected customer orders that cannot be satisfied because of lack of materials to produce what is required.

Given the random demand shown in Figure 3, it is possible to estimate the probability of lost sales for a product as the area under the curve such that the actual demand exceeds Demand, where Demand is a variable that represents the sales target of the company. This concept is shown in Figure 5 as the red stripped area under the curve of the density function.

In this case, the objective is to minimize the expected unmet demand, that is, the expected lost sales. Assuming a normal density function to describe uncertain demand, the expected lost sales, $L(\text{Demand},\mu,\sigma)$, are given by

$$L(Demand, \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} \int_{Demand}^{\infty} (x - Demand) \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2 \right] dx \qquad (21)$$

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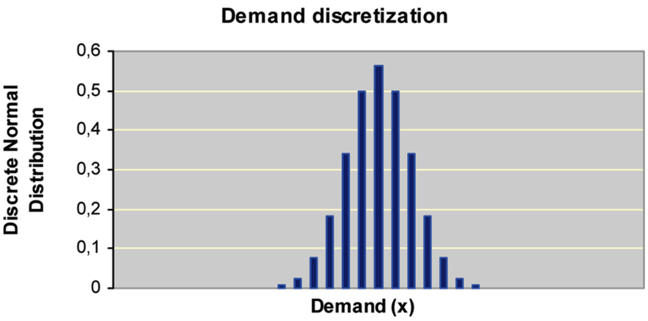

Figure 6. Uncertain demand discretization.

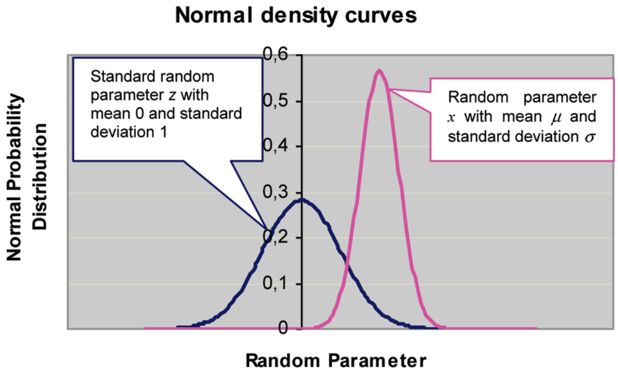

Figure 7. Normal density curves for variable z and demand.

For p products and t periods, this objective can be generalized as

$$\min L(Demand_{pt}, \mu_{pt}, \sigma_{pt}) \quad \forall\ p \in P,\ \forall\ t \in T \tag{22}$$

Because the presented problem corresponds to a multicriteria one, the strategy used to solve it is given by the ε-constraint method. This procedure, originally proposed by Haimes et al.,[10] selects one of the objectives of the problem as a priority. The other functions are used to form additional restrictions such as $f_o(x) \leq \varepsilon_o$, where ε_o determines an upper bound for the objective function to be minimized. The primary advantage of this method is that a systematic variation of the parameter ε_o leads to a Pareto-optimal solution set.[29] On the other hand, it should be noted that the inappropriate choice of parameters ε_o could result in infeasible formulations.[8] In this case, the second objective shown in eq 22 is considered as a constraint in the formulation assuming different values for parameter ε, as shown in eq 23

$$\frac{1}{\sigma\sqrt{2\pi}}\int_{Demand_{pt}}^{\infty}(x_{pt} - Demand_{pt}) \exp\left[-\frac{1}{2}\left(\frac{x_{pt}-\mu_{pt}}{\sigma_{pt}}\right)^2\right] dx_{pt} \leq \varepsilon_{pt} \quad \forall\ p \in P,\ \forall\ t \in T \tag{23}$$

Finally, the original problem is formulated as a multiobjective model using eqs 1−5, 7−18, 20, and 23, which is a nonlinear and nonconvex model because of bilinearities in constraints 12 and 14 and the integral calculation in eq 23. The model is thus transformed to give a global optimal solution.

3.2. Linear Transformation. The discrete nature of one of the variables involved in the bilinear terms enables the use of a disjunctive representation to linearize these constraints. Each Boolean variable represents one possible value of the discrete variable n_{jft}, which is the number of shipments for a given family, supplier, and period. Restrictions 12 and 14 are reformulated through the single disjunction

$$\begin{bmatrix} \beta_{jft}^{1} \\ n_{jft} = 1 \\ \sum_{k \in FK_{fk}} q_{jkt} = eoq_{jft} \\ tdc_{jft} = dc_{jft} \end{bmatrix} \vee \begin{bmatrix} \beta_{jft}^{2} \\ n_{jft} = 2 \\ \sum_{k \in FK_{fk}} q_{jkt} = 2eoq_{jft} \\ tdc_{jft} = 2dc_{jft} \end{bmatrix} \vee \cdots \vee \begin{bmatrix} \beta_{jft}^{N} \\ n_{jft} = N \\ \sum_{k \in FK_{fk}} q_{jkt} = N \times eoq_{jft} \\ tdc_{jft} = N \times dc_{jft} \end{bmatrix} \quad \forall\ j \in J,\ \forall\ f \in F,\ \forall\ t \in T \tag{24}$$

As regards the transformation of eq 23, it is necessary to consider some approximation of the integral to turn it into a linear constraint. One possible strategy is to discretize the density function of demand as shown in Figure 6. Although this technique does not present the same degree of accuracy as the original formulation, it allows a straightforward way to obtain the expected values of the demand distribution function. Other authors[7] have assumed a triangular distribution to approximate the random behavior of demand; however, this representation is further from the original density function (assuming that normal behavior best reflects the random nature of demand).

Because of the assumption that demand responds to a normal probability pattern, it is relatively easy to obtain discrete values from this distribution function. This information can be gathered from statistical sources presented in tabular format for standard normal random variables. Another alternative would be to apply Monte Carlo simulation. The first strategy is applied in this work given the connection between any random variable x with a normal distribution and the standard variable z, whose probabilistic behavior is known.

Figure 7 shows a comparison between the probability distribution of a random variable x and the distribution of a normal standard variable z. The relationship between these variables is given by

$$z = \frac{x - \mu}{\sigma} \tag{25}$$

In the problem under consideration, the generic variable x is replaced by the variable Demand$_{pt}$ described above. Then, eq 25 is rewritten as

$$z_{pt} = \frac{Demand_{pt} - \mu_{pt}}{\sigma_{pt}} \quad \forall\ p \in P,\ \forall\ t \in T \tag{26}$$

In terms of the standard variable, the expected lost sales, $L(\mathrm{Demand}_{pt}, \mu_{pt}, \sigma_{pt})$, can be determined as

$$L(Demand_{pt}, \mu_{pt}, \sigma_{pt}) = L(z_{pt})\sigma_{pt} \quad \forall\ p \in P,\ \forall\ t \in T \tag{27}$$

Table 1. Material Families

family	material	average costs ($COSTavg_{ft}$) t_1	t_2	t_3	initial stock (IS_f)
f_1	k_1, k_2, k_3	0.599	0.659	0.725	40
f_2	k_4, k_5, k_6, k_7	0.615	0.677	0.744	45
f_3	k_8, k_9, k_{10}	0.735	0.808	0.889	40
f_4	k_{11}, k_{12}, k_{13}	0.600	0.660	0.726	40

Table 2. Material Regular Costs (PC_{jkt})

	j_1			j_2			j_3			j_4		
material	t_1	t_2	t_3	t_1	t_2	t_3	t_1	t_2	t_3	t_1	t_2	t_3
k_1	0.50	0.55	0.61	0.52	0.57	0.63	0.51	0.56	0.62	0.55	0.61	0.67
k_2	0.47	0.52	0.57	0.49	0.54	0.59	0.48	0.53	0.58	0.45	0.50	0.55
k_3	0.80	0.88	0.97	0.81	0.89	0.98	0.79	0.87	0.96	0.82	0.90	0.99
k_4	0.71	0.78	0.86	0.78	0.86	0.94	0.69	0.76	0.84	0.70	0.77	0.85
k_5	0.75	0.83	0.91	0.72	0.79	0.87	0.73	0.80	0.88	0.77	0.85	0.93
k_6	0.51	0.56	0.62	0.52	0.57	0.63	0.53	0.58	0.64	0.54	0.59	0.65
k_7	0.49	0.54	0.59	0.45	0.50	0.55	0.48	0.53	0.58	0.47	0.52	0.57
k_8	0.78	0.86	0.94	0.68	0.75	0.82	0.72	0.79	0.87	0.70	0.77	0.85
k_9	0.72	0.79	0.87	0.81	0.89	0.98	0.71	0.78	0.86	0.70	0.77	0.85
k_{10}	0.75	0.83	0.91	0.74	0.81	0.90	0.72	0.79	0.87	0.79	0.87	0.96
k_{11}	0.45	0.50	0.55	0.55	0.61	0.67	0.52	0.57	0.63	0.51	0.56	0.62
k_{12}	0.46	0.51	0.56	0.48	0.53	0.58	0.47	0.52	0.57	0.48	0.53	0.58
k_{13}	0.78	0.86	0.94	0.81	0.89	0.98	0.88	0.97	1.07	0.81	0.89	0.98

Given the possible discrete values for standard variable z_{pt}, variable $L(z_{pt})$ is calculated as

$$L(z_{pt}) = \sum_{\forall h \in H} yz_{hpt} L_h \quad \forall\ p \in P,\ \forall\ t \in T \tag{28}$$

where

$$\sum_{\forall h \in H} yz_{hpt} = 1 \quad \forall\ p \in P,\ \forall\ t \in T \tag{29}$$

According to eq 28, $L(z_{pt})$ depends on the z_{pt} value, which is selected by the new binary variable yz_{hpt} and the corresponding parameter L_h, which represents the expected value of the standard variable for all $z^* > z_{pt}$. As mentioned above, this value is obtained from known statistics tables.[24] Additionally, eq 29 guarantees that only one value of variable z_{pt} is chosen for each product p and period t. Then, z_{pt} is rewritten in terms of the binary variable yz_{hpt}, and eq 26 is transformed to

$$\sum_{\forall h \in H} yz_{hpt} Z_h = \frac{Demand_{pt} - \mu_{pt}}{\sigma_{pt}} \quad \forall\ p \in P,\ \forall\ t \in T \tag{30}$$

where Z_h is a parameter that represents the value chosen for the standard random variable. These transformations allow reformulation of restriction 23 as

$$\sigma_{pt} \sum_{h} yz_{hpt} L_h \leq \varepsilon_{pt} \quad \forall\ p \in P,\ \forall\ t \in T \tag{31}$$

Finally, linear formulation is given by eqs 1−5, 7−11, 13, 15−18, 20, and 28−31.

Table 3. Minimum and Maximum Quantities Required[a]

	Q_{mincj}				Q_{maxcj}			
	c_1	c_2	c_3	c_4	c_1	c_2	c_3	c_4
j_1	0	30	55	70	–	60	155	160
j_2	0	40	60	75	–	63	150	155
j_3	0	32	52	77	–	55	100	122
j_4	0	38	55	80	–	60	140	150

[a] Payment terms ($\forall j$): c_1, $t = t'$; c_2, $t = t'$; c_3, $t = t'$; c_4, $t + 2 = t'$.

Table 4. Discount and Interest Rate (δ_{jc}) for Each Supplier and Contract

supplier	contract c_1	c_2	c_3	c_4
j_1	0	0.120	0.150	−0.130
j_2	0	0.060	0.120	−0.090
j_3	0	0.095	0.150	−0.125
j_4	0	0.055	0.121	−0.078

4. RESULTS

In this section, a case study is presented with the aim of showing model performance. To generate Pareto-optimal points, problem resolution was carried through several runs considering different values of the parameter ε_{pt}. Models were implemented in the GAMS 22.7 system and executed on an Intel Pentium D PC with a 2.8 GHz processor. Disjunctions were modeled with LogMIP, using convex hull relaxation.

4.1. Case Study. Although the approach proposed is general enough to be applied to a variety of industries, in this case, the company comes from a paper supply chain. This company produces corrugated board boxes handling several paper families as main raw materials. Each family is formed by several paper types with similar characteristics. For instance, white papers with different grammages are grouped into a family called "white". According to historical data, mean demand and its standard deviation are obtained for each product, regular and average costs are calculated, and contract characteristics are provided by the potential suppliers. To preserve confidential data, numbers presented in the following tables are modified from the original values.

A three-period planning horizon is assumed in this case study, where four potential suppliers offer raw materials to the company. The selection of suppliers allows the supply channel to be reconfigured in each period and for each material family. As mentioned in the previous section, the providers offer different contracts or purchase terms that the company can choose for each material and period.

Table 1 presents the materials corresponding to each family, their average costs in the planning horizon, and the initial stock considered. Note that there are four families comprising a total of 13 materials. Materials belonging to a given family can be used indistinctly to satisfy the families' demand presenting different costs, as shown in Table 2.

Tables 3 and 4 list the central characteristics of the contracts considered such as payment terms, minimum quantities, and discount or interest rate required. Table 5 lists the maximum supply capacity of each supplier for each material offered. In this case, it is assumed that this capacity is not modified in each time

Table 5. Provision Maximum Capacities (Q_{maxjkt})

material	supplier			
	j_1	j_2	j_3	j_4
k_1	150	160	150	160
k_2	50	70	55	20
k_3	50	65	45	64
k_4	145	180	165	175
k_5	60	80	65	60
k_6	50	80	40	77
k_7	60	50	53	52
k_8	158	160	159	142
k_9	60	45	50	40
k_{10}	65	60	66	70
k_{11}	45	80	42	70
k_{12}	148	150	140	160
k_{13}	50	50	60	22

Table 6. Suppliers' Failure Probabilities

supplier	failure (r)			
	0−10%	10−40%	40−70%	70−100%
j_1	57%	30%	10%	3%
j_2	50%	30%	13%	7%
j_3	50%	25%	15%	10%
j_4	53%	28%	15%	4%

Table 7. Mean Demand and Standard Deviation for Each Product and during Each Period

product	μ_{pt}			σ_{pt}		
	t_1	t_2	t_3	t_1	t_2	t_3
p_1	52.5	85.0	80.0	26.0	42.5	40.0
p_2	50.0	55.0	60.0	25.0	27.0	30.0
p_3	60.7	58.3	60.0	30.3	29.0	30.0
p_4	27.5	30	32.5	13.8	15	8.8

period. As regards provision uncertainty, Table 6 reports the discrete probability function for suppliers' failures.

Table 7 presents mean demand and its standard deviation for each product and period considered.

Table 8 establishes the possible values for the standard uncertain variable z, which estimates the value of the demand objective in eq 30.

These z_h values are associated with the loss values shown in Table 9, which are used to calculate the expected value of lost sales in eq 31.

To solve this problem, successive runs were executed with different values assigned to the parameter ε_{pt}, which limits the second objective of minimizing the unmet demand. With the purpose of giving the same weight to lost sales of all products, it was decided to quantify this value (ε_{pt}) as a percentage of the average demand. First, this ensures that equal importance is given to each produc,t and second, it guarantees that the areas under the curve of probability density are the same. However, it is clear that if one decided to give different weights to various products, it would also be possible to define a particular value to each of them. Then, parameter ε_{pt} is determined by

$$\varepsilon_{pt} = \delta\mu_{pt} \quad \forall\ p \in P,\ \forall\ t \in T \tag{32}$$

Parameter δ represents the rate being varied in each "Pareto" instance considered. The values used to build the curve points are presented in Table 10.

Characteristics and performance of the proposed model (linear transformation) are reported in Table 11. Every instance considered was executed in less than 1 min.

Table 12 presents the main results of the case study. For each value of δ allowed, the table lists the corresponding value of total

Table 8. Discrete Values for Variable z[a]

	Z_h
h_1	−4
h_2	−3.5
h_3	−3
h_4	−2.5
h_5	−2
h_6	−1.5
h_7	−1
h_8	−0.5
h_9	0
h_{10}	0.5
h_{11}	1
h_{12}	1.5
h_{13}	2
h_{14}	2.5
h_{15}	3
h_{16}	3.5
h_{17}	4

[a] Parameter in the model.

Table 9. Expected Values of Discrete Standard Losses

	L_h
h_1	4
h_2	3.5001
h_3	3.0004
h_4	2.5020
h_5	2.0085
h_6	1.5293
h_7	1.0833
h_8	0.6978
h_9	0.3989
h_{10}	0.1978
h_{11}	0.0833
h_{12}	0.0293
h_{13}	0.0085
h_{14}	0.002
h_{15}	0.0004
h_{16}	0.0001
h_{17}	0

Table 10. Parameter δ Used to Calculate ε_{pt}

δ (%)	δ (%)
0.00	7.50
1.25	10.00
2.50	12.50
5.00	

Table 11. Solution Performance

number of equations	3906
total number of variables	3777
number of discrete variables	1708
CPU s (t)	$30 < t < 55$

Table 12. Solution Comparison

expected lost sales, δ (% of mean demand)	OF (total cost)	OF difference (%)
0.00	2641.79	
1.25	1713.58	35
2.50	**1481.87**	**44**
5.00	1270.83	52
7.50	1261.26	52
10.00	1048.67	60
12.50	1040.38	61

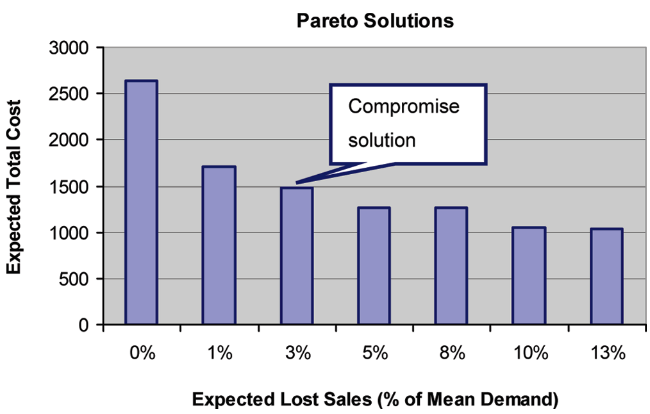

Figure 8. Pareto solutions.

costs (major objective function, OF). In the third column is presented the improvement in the OF with respect to the first solution (with δ = 0%). Even though the utopia point method can be used to define a compromise solution,[8] by simple inspection of Table 12, it can be concluded that, for a relatively low value of expected lost sales (2.5%), the OF reaches a 44% improvement over the first point. In the following solutions, demand satisfaction gets worse without a significant change in costs. By applying simple inspection, it is assumed that this solution achieves the best balance between the two objectives. These values are also reflected in Figure 8.

Results shown below correspond to the compromise solution adopted. Table 13 presents the amounts of each material ordered and the provider in the planning horizon. Supplier j_1 was chosen for most materials, whereas supplier j_3 was not selected. Note that the convenience of one or another provider is associated with both an economic/financial component and a risk component given the uncertainty in the material supply.

Table 13. Amounts Ordered from Each Supplier

	period		
material	t_1	t_2	t_3
	Supplier j_1		
k_1	148	150	150
k_2	50	50	50
k_4			30
k_6	50	50	50
k_7	60	60	60
k_9	58		
k_{11}	39		45
k_{12}	148	148	148
	Supplier j_2		
k_1		150	
k_6	80	80	80
k_7	50	50	50
	Supplier j_4		
k_1			148
k_6	73	77	76
k_7	52	52	52
k_8		121	133
k_{12}		123	

Table 14. Contracts Selected

	j_1			j_2			j_3		
material	t_1	t_2	t_3	t_1	t_2	t_3	t_1	t_2	t_3
k_1	c_4	c_3	c_3		c_4				c_4
k_2	c_2	c_2	c_2						
k_4			c_2						
k_6	c_2	c_2	c_2	c_4	c_4	c_4	c_2	c_2	c_2
k_7	c_2	c_2	c_2	c_2	c_2	c_2	c_2	c_2	c_2
k_8								c_4	c_4
k_9	c_2								
k_{11}	c_2		c_2						
k_{12}	c_4	c_3	c_3					c_4	

Table 14 reports the selection of purchase alternatives. Because the amounts ordered (Table 13) were not high, in general, contract c_2 was mostly chosen. Contract c_4, in turn, was selected as a second option. Although it requires the payment of interest, the possibility of paying in future periods reduces current costs through the application of the return rate (IR) in the objective function.

The expected amounts of each family and period are shown in Table 15. If we analyze Table 13, the total quantity of family f_1 ordered in the period t_1 is given by the amounts of materials k_1 and k_2 ordered from supplier j_1, so the total amount of this family is 198 units. In turn, the expected quantity for this family and

Table 15. Expected Material Quantities

	period		
family	t_1	t_2	t_3
f_1	162	279	280
f_2	288	291	315
f_3	48	96	105
f_4	153	218	157

Table 16. Delivery Sizes (eoq_{jft})

	period		
family	t_1	t_2	t_3
	Supplier j_1		
f_1	99	100	100
f_2	110	110	140
f_3	58		
f_4	93	148	97
	Supplier j_2		
f_1		150	
f_2	130	130	130
	Supplier j_4		
f_1			148
f_2	125	129	128
f_3		121	133
f_4		123	

Table 17. Number of Shipments (n_{jft})

	period		
family	t_1	t_2	t_3
	Supplier j_1		
f_1	2	2	2
f_2	1	1	1
f_3	1		
f_4	2	1	2
	Supplier j_2		
f_1		1	
f_2	1	1	1
	Supplier j_4		
f_1			1
f_2	1	1	1
f_3		1	1
f_4		1	

Table 18. Selection of Parameter Z_h

	period		
product	t_1	t_2	t_3
p_1	Z_{12}	Z_{12}	Z_{12}
p_2	Z_{12}	Z_{12}	Z_{12}
p_3	Z_{12}	Z_{12}	Z_{12}
p_4	Z_{12}	Z_{12}	Z_{11}

Table 19. Expected Unsatisfied Demand (L_{pt})

	period		
product	t_1	t_2	t_3
p_1	0.7618	1.2453	1.1720
p_2	0.7325	0.7911	0.8790
p_3	0.8888	0.8497	0.8790
p_4	0.4029	0.4395	0.7289

Table 20. Target Demand Level ($Demand_{pt}$)

	period		
product	t_1	t_2	t_3
p_1	91.5	148.8	140.0
p_2	87.5	95.5	105.0
p_3	106.2	101.8	105.0
p_4	48.1	52.5	41.3

period is 162 units. We can then conclude that, according to the failure distribution function of this provider (Table 6), it is expected that 36 units will not be delivered.

Regarding delivery decisions, Table 16 presents the delivery sizes selected, given by eoq_{jft}, and variable n_{jft} is presented in Table 17. Comparing the information presented in these two tables and again the quantities ordered (Table 13), one can see that the 198 units of materials k_1 and k_2 from supplier j_1 will be sent in two shipments of 99 units each.

According to eq 30, the demand level that the company aims to satisfy is a decision variable ($Demand_{pt}$) determined by binary variable yz_{hpt}, which selects the value of z (Z_h). Table 18 shows that the parameters Z_{11} and Z_{12} were chosen to replace z according to the product and the period. From Table 8, $Z_{11} = 1$ and $Z_{12} = 1.5$, and for those values, the corresponding expected loss of the standard variable z, L_h, are $L_{11} = 0.0833$ and $L_{12} = 0.0293$, respectively, obtained from Table 9.

Consistent with this result, the estimated unmet demand values are listed in Table 19. In all cases, the upper bound for this value was calculated as $\varepsilon_{pt} = 0.025\mu_{pt}$, and its calculation is given by the equation

$$L_{pt} = \sigma_{pt} \sum_{\forall h \in H} yz_{hpt} L_h \qquad \forall\ p \in P,\ \forall\ t \in T \tag{33}$$

In turn, Table 20 presents the demand to be satisfied, given by

$$Demand_{pt} = \sum_{\forall h \in H} zy_h Z_h \sigma_{pt} + \mu_{pt} \qquad \forall\ p \in P,\ \forall\ t \in T \tag{34}$$

As mentioned above, because we considered the same parameter value δ for all products p and periods t, the area under the curve from a certain demand level is the same for all products. Thus, if we take the demand of product p_1 in period t_1 from the tradeoff solution (δ = 2.5%), its density function and expected

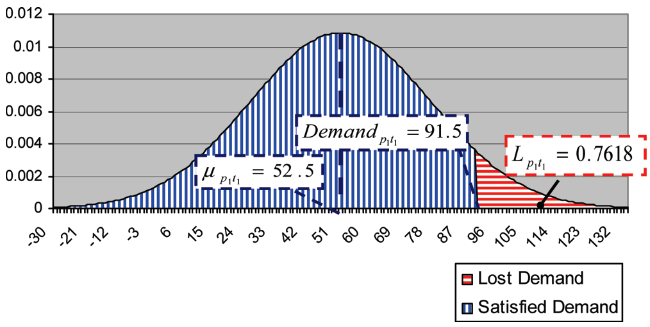

Figure 9. Uncertain demand for product p_1 in period t_1.

loss is representative of the other products' behavior. Assuming a continuous representation, the uncertain demand distribution for the product p_1 in t_1 is shown in Figure 9.

From Figure 9, it is clear that the objective demand ($\text{Demand}_{p_1t_1}$) is at 91.5 units to ensure that the expected unmet demand is lower than 2.5% of the average level (52.5 units). Actually, the expected loss, L_{pt}, is 0.7618 which is less than 0.025×52.5. Finally, if we had considered a deterministic context, the demand parameter would have probably been $\mu_{p_1t_1}$, in which case much of the potential demand would have been ignored and the risk of stock-out would have been increased. The same conclusions can be applied to the other products of this problem, and therefore, the impact on costs due to lost sales would have been significant.

5. DISCUSSION AND CONCLUSIONS

This article proposes a general model that successfully integrates various decision levels concerning the provision process. This approach allows a company to define an optimal provision plan considering raw material purchasing, supplier selection, contract alternatives, inventory levels, and delivery decisions under demand and provision uncertainty.

The wide decision ranges of the problem have led to various optimization objectives that can be generalized as minimizing purchasing, inventory, and transportation costs, as well as customer dissatisfaction. Given the difficulty of combining these goals in a single expression, the ε-constraint method was considered to solve the multicriteria problem. The main advantage of this technique is the possibility of maintaining the objectives as separate goals, thereby avoiding a subjective evaluation. Additionally, each objective can then be expressed in its natural units. Then, it is possible to obtain various Pareto-optimal solutions and select the one that best satisfies the objectives involved.

Both delivery decisions and demand uncertainty introduce nonlinearities into the formulation that lead to a nonconvex problem. To overcome this difficulty, two different linearization strategies were applied. Bilinear constraints given by delivery decisions were transformed using a disjunctive technique. Regarding demand uncertainty, the probability distribution was discretized by adding new binary variables to the formulation to choose the level of met demand, the second objective of the problem. This approach avoided the integral calculus of an exponential function and facilitated a linear formula to calculate expected lost sales.

Finally, the case study presented shows an efficient resolution. The main results that can be obtained by this approach are the following:

- a raw materials purchase plan in a medium-term is determined;
- the most advantageous contracts are selected given their requisites and benefits, as well as the specific requirements of target demand to meet;
- suppliers are selected in each period and for each material, so that it is possible to restructure purchase relationships in each period; and
- multiple shipments to meet the quantities purchased are also determined.

The proposed method also presents several positive notable characteristics:

- Failure probabilities in the provision process are taken into account in the selection of suppliers, and it is even possible to calculate the amounts expected to be received from them.
- The use of disjunctions, logical relations, and Boolean variables makes a more expressive formulation.
- Uncertainty in product demand is modeled using a normal distribution that is then discretized from the original formulation.
- The transformed model guarantees the global optimal solution.
- To minimize unmet demand and cost objectives, a multicriteria problem is generated so that it clearly shows the tradeoff between the objectives.

All of the results obtained from the model and its characteristics summarize the main contributions of the approach proposed in this article.

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NOTATION

Sets

C = contracts
F = material families
H = discrete points of variable z
J = suppliers
K = materials
L = set introduced for disjunctive transformation
PF_{pf} = set relating products p to material families f
R = failure range
T = periods
$\text{TP}_{ctt'}$ = set that determines that contract c signed in period t must be paid in period t'

Positive Variables

d_{ft} = quantinty of family f consumed to satisfy target demand in period t
dc_{jft} = fixed delivery cost to receive family f from supplier j in period t
Demand_{pt} = target demand of product p to be satisfied in period t
eoq_{jft} = delivery size of family f from supplier j in period t
eq_{ft} = expected quantity provided of family f in period t

$L(z_{pt})$ = expected value of lost sales of product p in period t according to z_{pt} value
m_{jckt} = amount paid to buy material k from supplier j with contract c in period t
q_{jkt} = quantity of material k ordered from supplier j in period t
qf_{jft} = quantity of family f ordered from supplier j in period t
qr_{jrft} = expected quantity of family f ordered from supplier j according to failure range r in period t
s_{ft} = stock of family f at the beginning of period t
$savg_{ft}$ = average stock of family f in period t
tdc_{jft} = total delivery cost for family f from supplier j in period t
w_{jckt} = purchase cost of material k from j with contract c in period t
z_{pt} = standard normal variable with an equivalent random behavior of product p in period t

Boolean Variables

v_{ift} = selection of supplier to provide family f in period t
y_{1jft} = selection of supplier j to buy family f in period t
y_{2jkt} = selection of material k from supplier j in period t
y_{3jckt} = selection of contract c with supplier j to order material k in period t
β^{l}_{jft} = Boolean variable from disjunctive transformation

Binary Variables

Y_{1jft} = selection of supplier j to buy family f in period t
Y_{2jkt} = selection of material k from supplier j in period t
Y_{3jckt} = selection of contract c with supplier j to order material k in period t
yz_{hpt} = selection of one possible value for standard variable z_{pt}

Integer Variable

n_{jft} = number of deliveries for family f from supplier j in period t

Parameters

$COSTavg_{ft}$ = average cost of family f in period t
DC_1 = fixed delivery cost when delivery size is not greater than EOQ_1
DC_2 = fixed delivery cost when delivery size is not greater than EOQ_2
DC_3 = fixed delivery cost when delivery size is not greater than EOQ_3
EOQ_1 = maximum delivery size under cost DC_1
EOQ_2 = maximum delivery size under cost DC_2
EOQ_3 = maximum delivery size under cost DC_3
FC_{jc} = fixed cost of contract c from supplier j
IS_f = initial stock for family f
L_h = expected loss values considered for a standard discretized variable
MS = percentage of raw material costs assigned to inventory holding costs
p_{jr} = probability that supplier j fails in range r
PC_{jkt} = regular price of material k ordered from supplier j in period t
$price_{pt}$ = price of product p in period t
RR = return rate
Q_{maxcj} = maximum quantity according to contract c of supplier j
Q_{maxjkt} = maximum quantity of material k available from supplier j in period t
Q_{mincj} = minimum quantity required established for c according to supplier j
SC = maximum quantity in stock at the beginning of each period
SS_f = security stock for family f
Z_h = discrete possible values for standard variable z_{pt}
α_{pf} = consumption of family f to produce one unit of product p
δ_{jc} = discount or interest range of contract c and supplier j
ε_{pt} = parameter applied for the ε-constraint method
μ_{pt} = mean value of demand for product p in period t
σ_{pt} = standard deviation of demand for product p in period t

Function

$g_{ift}(y^{1})$ = determines the selection of suppliers according to subindex i for family f in period t

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After this paper was published online August 23, 2011, a change was made to eq 9. The revised version was published August 25, 2011.