

## Supply Chain Design and Inventory Management Optimization in the Motors Industry

Maria Analia Rodriguez<sup>a</sup>, Aldo R. Vecchietti<sup>a</sup>, Ignacio E. Grossmann<sup>b\*</sup>, Iiro Harjunoski<sup>c</sup>

<sup>a</sup>INGAR (CONICET - UTN) Avellaneda 3657, Santa Fe, 3000, Argentina

<sup>b</sup>Department of Chemical Engineering, Carnegie Mellon University, 5000 Forbes Avenue, Pittsburgh, 15213, PA, United States.

<sup>c</sup>ABB AG, Corporate Research Germany, Wallstadter Straße 59, 68526 Ladenburg, Germany  
[grossmann@cmu.edu](mailto:grossmann@cmu.edu)

This article studies the supply chain redesign under demand uncertainty over a multi-period planning. We propose an optimization model to solve the problem taking into account strategic and tactical plans. This model is applied to the electric motors industry but it can be easily extended to other supply chains. Long term decisions involve new installations, expansions and elimination of warehouses. Tactical decisions include deciding inventory levels (safety stock and expected inventory) for each type of product in distribution centers and customer plants, as well as the connection links between the supply chain nodes. Capacity constraints are also considered when planning inventory levels. At the tactical level it is analyzed how demand of failing motors is satisfied, and whether to use new or used motors. The uncertain demand is addressed by defining the optimal amount of safety stock that guarantees certain service level at a customer plant. In addition, the risk-pooling effect is taken into account when defining inventory levels in distribution centers and customer zones. Due to the nonlinear nature of the original formulation, a piecewise linearization approach is applied to obtain a tight lower bound of the optimal solution.

### 1. Introduction

The integration of supply chain redesign and tactical decisions such as defining inventory levels and how supply chain nodes are connected is a challenging problem that can greatly impact the financial performance at a company. Rising transportation costs are key factors in decisions about both, where to place factories and distribution centers, and how much inventory to store. In addition, optimal inventory management has become a major goal in order to simultaneously reduce costs and improve customer service in today's increasingly competitive business environment (Daskin and Coullard, 2002). For that reason, over the last few years, there has been an increasing interest in developing enterprise-wide optimization (EWO) models to solve problems that are broad in scope and integrate several decision levels (Grossmann, 2005).

In the case of the electric motors industry, the relevance of this problem is given by some key issues. On the one hand, electric motors are expensive products, keeping them in inventory means tying a significant amount of capital. On the other hand, a motor malfunction may block the entire production of a customer's plant, and therefore obtaining a spare motor as soon as possible is critical.

Most contributions in the literature assume that the products are only moved forward in the supply chain and only the demand of new products is considered. In this case, the situation is more complex. Demand process is uncertain, and it can be originated by new customers or new investments at customer sites, but also by motors that are already in use at customer sites and fail. When that happens, clients require a spare motor in order to replace the one that failed. An important decision in this context is whether to replace failed motors with new units or with repaired products. An efficient inventory management of new and used motors in the supply chain warehouses is another challenge of this problem.

Customer plants typically have tens or more different types of motors in their production processes, and identical motors can be used for a variety of purposes. According to the type of motor and application, the criticality of a given unit can be very different so the time a customer allows a motor to be out of service until another one replaces it is case dependent. If the time requirement is very tight, it might be necessary to store some motors at the customer sites.

You and Grossmann (2010) propose an optimization model to design a multi-echelon supply chain and the associated inventory systems under demand uncertainty in the chemical industry. The supply chain involves one product, and design decisions consider the installation of new distribution centers, but no expansions or elimination of installed warehouses are considered. In addition, the model assumes one planning period so investment costs are annualized and capacity constraints are not analyzed.

We develop a multi-period optimization model to redesign the supply chain of the electric motors industry under demand uncertainty from strategic and tactical perspectives. The main objective is to redesign the supply chain minimizing costs and deciding where to place warehouses, which installed warehouses should be eliminated, what are the stock capacities and safety stocks required, as well as how to connect the different echelons of the supply chain in order to satisfy uncertain demand of motors.

The uncertain demand is addressed by defining the optimal amount of safety stock that guarantees certain service level at a customer plant. In addition, the risk-pooling effect described by Eppen (1979) is taken into account when defining inventory levels in distribution centers and customer zones. Due to the nonlinear and large size nature of the original formulation, a piece-wise linearization algorithm is applied to obtain the optimal solution.

## 2. Problem formulation

A three echelon supply chain is considered with a given set of fixed factories and warehouses, as well as potential factories and warehouses that can produce and deliver spare motors to end customers.

Long term decisions involve new installations, expansions, and elimination of factories and warehouses handling multiple products. It is also considered which warehouses should be used as repair work-shops in order to store, repair and deliver the used motors to customers. In addition, the connections between factories, warehouses and end customers must also be selected. We assume that several factories can serve one warehouse with the same standard motor, while each end customer is assumed to be served by only one warehouse. Figure 1 represents the main design decisions regarding the supply chain structure.

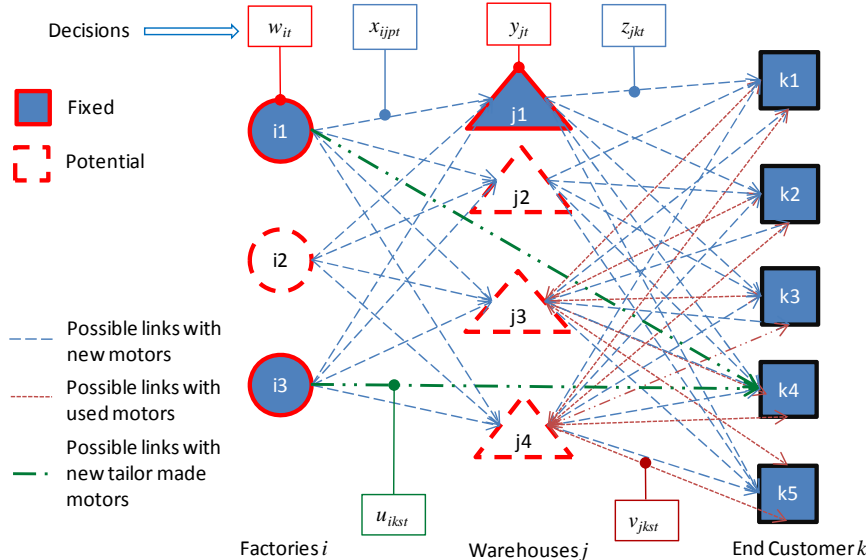


Figure 1: Design decisions in the supply chain

Some end customers allow that their demand for spare motors due to failure can be satisfied either by new or by used motors. It is also assumed that some warehouses can potentially repair workshops where motors are repaired. As an example, Figure 1 shows that warehouses  $j3$  and  $j4$  may repair motors and deliver them to end customers. In this figure, it is not distinguished what motors can be repaired from each end customer, but it is shown that all end customers accept used motors for at least one motor type. The

model decides what part of the spare motor demand is satisfied by new motors and what part by used ones. There is a certain probability that a given special motor from a given end customer can be repaired. Deciding whether a customer order is satisfied with new or repaired motors has also an impact on the required delivery route. Figure 2 shows that when new motors are needed, the traditional route is from factories to warehouses, and then from warehouses to customers. When used repaired motors are selected the route is from customers to repair workshops (which are also warehouses), and then the motor goes back to the customer. Transportation costs, times and stock management are different in both cases.

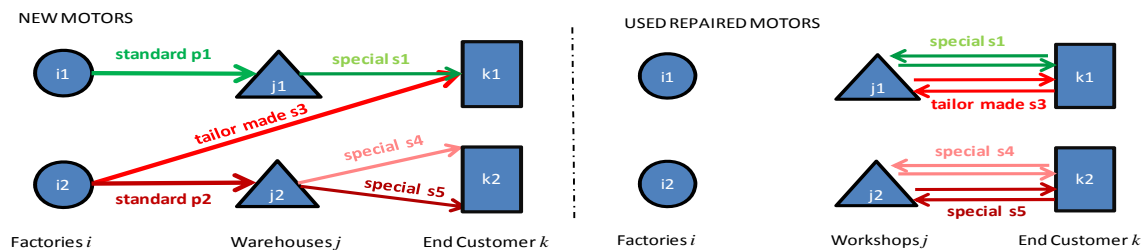


Figure 2: Motors route options

Current motors installed at end customers can fail during their operation. Spare motor demand due to failure rate is considered uncertain assuming it is a random continuous parameter with Poisson distribution, where the mean is given by the average failure rate  $\mu$  (motors/year) and standard deviation  $\sigma = \sqrt{\mu}$ . Safety stock is calculated in order to cope with demand uncertainty.

The end customers define a service level for each motor they have in use, which is directly correlated with the maximum time the end customer expects the provider company to have a spare motor delivered after a failure occurs. This time is called guaranteed service time. In order to achieve that target, it is necessary to have safety stocks both at the end customers' sites and warehouses, the levels of which are calculated considering the safety factor as well as the standard deviation of demand over the net lead time.

Since a multi-stage inventory system is assumed, the guaranteed service time approach is applied. The main idea of this approach is that each node guarantees a certain service time, in which the demand of products or materials will be satisfied. Therefore, considering the different times involved at each node of the supply chain, the net lead time is calculated, which means the time span for which safety stock is required to cover demand variation.

Tactical decisions include deciding inventory levels (safety stock and expected inventory) for each type of motor in distribution centers and customer plants. Capacity constraints are also considered when planning inventory levels. At the tactical level it is determined how to satisfy the demand of failed motors, and whether to use new or used motors.

### 3. Model description

A mathematical programming model is formulated in order to solve the problem. Due to the space limit, equations are not included in this article.

The model involves a set of logical constraints in order to guarantee coherence in the supply chain decisions. These constraints involve binary variables, which are used to determine design decisions such as new locations, expansions, elimination of assets and links between nodes. Demand constraints define the amount of motors provided by each warehouse and factory to each customer, and also what type of motor is selected (a proportion of demand can be satisfied with used repaired motors).

According to the transportation times, guaranteed service times and order processing time, the net lead times for the warehouses and the customer are calculated by a set of equations. Safety stocks in the different echelons of the supply chain are also determined. These constraints involve square root terms and bilinear terms. Two types of constraints are presented regarding warehouse capacity. First, according to investments (new installations and expansions) and elimination of warehouses, capacity available in each period for each warehouse is calculated. Second, the demand that each location satisfies must consider the capacity, which is given by another set of equations.

Finally, the objective function is to minimize the total present costs. The costs considered include installation costs, operational fixed costs, expansion costs (investment), elimination cost (fixed cost to uninstall), and operational variable costs of the warehouses, as well as operational variable costs at factories, repairing costs at workshops, transportation costs, inventory costs and safety stock costs.

The formulation gives rise to a mixed integer non-linear programming (MINLP) problem due to bilinear and square root terms. A mixed integer linear programming (MILP) formulation, which provides a lower bound to the original model, is obtained applying exact linearization of the bilinear equations and piece-wise relaxation of the square root.

**4. Case study**

The MILP version of the model is illustrated in this section. Two different case-studies are presented. Both examples were executed in GAMS 23.7 using CPLEX 12.3 over a CPU Intel Core i3, 2.20 GHz with a 6 GB of RAM.

**4.1 Electric motors supply chain**

The first case study considers the supply chain of electric motors over a five-year planning horizon. In this example case, all installed and potential warehouses are located in Sweden, while factories are located in Europe and Asia. The initial supply chain is given by seven factories, one warehouse which is not a repair workshop (J1) and 27 customers. No investments in new factories are allowed. Four additional warehouses can be installed in any of the five periods considered, and three of them can be repair workshops. A demand for 50 different motor types is considered.

In this case, the optimal solution indicates that two new warehouses (J3 and J4) are installed in period 1 as shown in Figure 3. The first one is only used for storage of new motors, while the second one is exclusively used as a repair workshop. No expansions nor eliminations are implemented.

The performance and size of this model is presented in Table 1. The termination is given by the solver that run out of memory after 7723 sec of execution (the last tree size reported by the solver is 3345.73 MB). Safety stock required at warehouses is shown in Table 2.



Figure 3: Supply chain design of case study 1

Table 1: Model performance and objective function in case study 1

Objective Function	CPU time (integer gap 1.99%)	Equations	Positive Variables	Binary Variables
\$16881715.92	7723 s	212407	115064	12639

Table 2: Safety stock at warehouses in case study 1

Warehouse	Motor ID	t1	t2	t3	t4	t5
	P1	1.812	2.066	2.066	2.066	2.167
	P2	2.030	2.316	2.316	2.316	2.429
	P3	1.737	1.980	1.980	1.980	2.077
Warehouse	Motor ID	t1	t2	t3	t4	t5
J1	P4	2.689	3.066	3.066	3.066	3.216
	P5	2.082	2.375	2.375	2.375	2.491

	P1	0.501	0.572	0.572	0.572	0.600
	P2	0.562	0.641	0.641	0.641	0.672
	P3	0.542	0.619	0.619	0.619	0.649
	P4	0.873	0.996	0.996	0.996	1.044
J3	P5	0.574	0.655	0.655	0.655	0.687

#### 4.2 Wind generators supply chain

This example considers a supply chain of wind turbines. End customers are located in different and distant regions of the world. In this case, 50 customers are assumed. The initial supply chain is given by three factories and three warehouses (J1, J2 and J3), all of them are repair workshops. No expansions in factories are allowed. However, warehouses can be expanded, and also nine (or less) new warehouses can be installed in any period. Nine different generator types (products) are considered. It is also assumed that the demand increases in each period. The main results and computational performance are presented in Table 3. In this case, the stopping criteria is given by 10000 s computational time limit.

Table 3: Model performance and objective function in case study 2

Objective Function	CPU time (integer gap 1.12%)	Equations	Positive Variables	Binary Variables
\$845840702.15	10000 s	115143	62171	9537

Regarding model decisions from the optimum solution, warehouse J1 is expanded in periods 3, 4 and 5. In period 5, this warehouse has more than the double of its original capacity. Four new warehouses are installed in period 1 (J7, J9, J11 and J12), which are not expanded. No elimination of warehouses is done. The warehouse capacity profiles are shown in Figure 4.

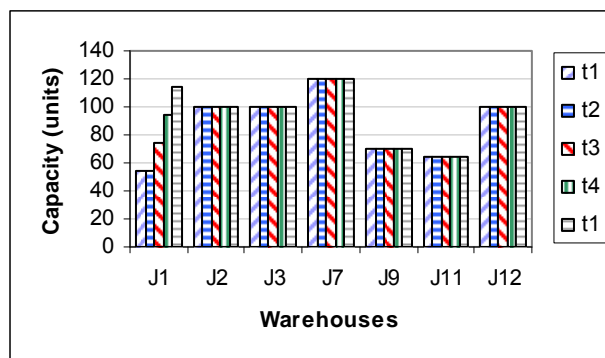


Figure 4: Capacity profiles of warehouses (J1-J12) in case study 2

Figure 5 shows the connections between the different points in the supply chain in period one for new motors to the left, and for used motors (green line) to the right. These decisions can be changed in any period. In the left, links from factories to warehouses (red line) are shown as well as from warehouses to end customers (blue line). In these figures it is also possible to analyze that some "small" warehouses have no links to any customer or factory. This means that these warehouses were potential investments that were not selected by the model. Since all new warehouses are decided in period 1, the same supply chain structure would be shown for any other period. In the right side of Figure 5, the links (green lines) correspond to used repaired motors in period 1. Therefore, they connect repair workshops to customers.

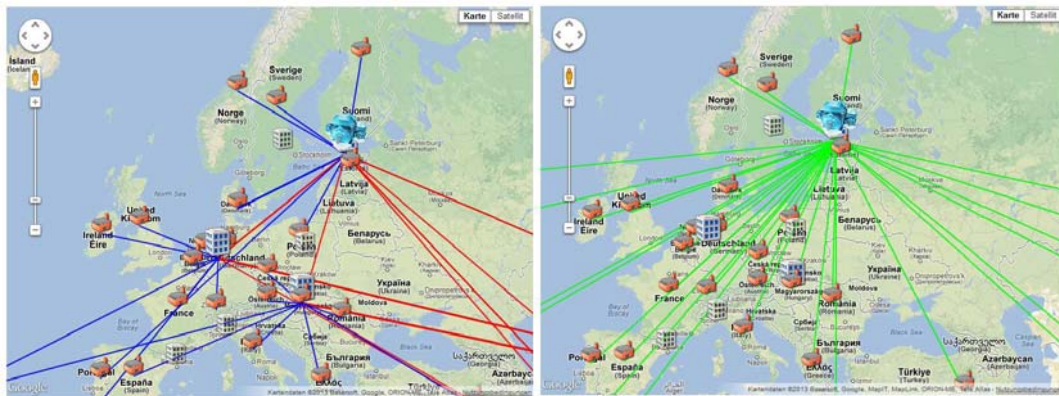


Figure 5: Supply chain design of case study 2

## 5. Conclusions

We have developed an MINLP model that defines the optimal supply chain structure over a multi-period horizon planning considering demand uncertainty. Special characteristics from the electric motors industry are considered. However, this model is generic and can also be applied to other type of industries. New investment, expansion and elimination decisions considered allow that the model be used not only for design but also for evaluation and re-design of a supply chain that is already in operation. From the inventory management perspective, safety stock, mean stock levels and capacity constraints are also taken into account to satisfy customer orders according to the company commitments. Future work includes to define a decomposition approach in order to solve larger instances as well as to obtain a lower gap in reasonable computational time.

## References

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