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Modeling transition diffusive–nondiffusive transport in a turbid media and application to time-resolved reflectance

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ABSTRACT

In this work a generalized solution for the photon density, $\Phi_{gen}(\mathbf{r},t)$, is applied to two types of experiments in turbid media carried out in the last years. Both involve small typical distances, where it is known that the diffusion approximation ceases to be valid. In one case, the use of time-resolved reflectance at small or null source-detector separation using fast single photon gating to localize small inhomogeneities embedded in diffusive media has been proposed. In other type of experiments, it is addressed the transition between the ballistic and the diffusive regimes, measuring the transmitted light within a relatively narrow solid angle. The model proposed here corroborates the importance of the solid angle of the measurement device in order to see the ballistic photons, and the given generalized solution provides valid answers to problems posed by the mentioned experiments. Furthermore, it permits the description of diffusive photons when the absorption coefficient is relatively high, where the diffusion approximation is not valid.

1. Introduction

As a general definition, turbid media are those presenting both, absorption and multiple scattering properties. Even though the fundamental magnitudes in collision processes are the cross sections, turbid media can be characterized by the following parameters: the scattering coefficient μ_s, the absorption coefficient, μ_a, the anisotropy factor g, which is the mean value of the cosine of the polar scattering angle, and the refractive index n. Related to these parameters are the mean free path between collisions, $l_s = 1/\mu_s$, the reduced scattering coefficient $\mu_s' = \mu_s(1-g)$ and the diffusion coefficient, D. In principle, the diffusion coefficient is defined as $D = 1/(3(\mu_s' + \alpha\mu_a)$, being the parameter $\alpha = 1$ if the Diffusion Equation (*DE*) is deduced from the Radiative Transfer Equation (*RTE*) and $\alpha = 0$ if it is deduced from the kinetic theory. Experiments show that for absorption coefficients up to values as high as $\mu_s'/\mu_a \simeq 3$ the diffusion coefficient defined with $\alpha = 0$ provides better results [1–3]. Therefore, because our expressions are based on the kinetic theory approach, we will adopt $D = 1/(3\mu_s')$.

In the few last years, appeared two types of experiments involving the photon transport in diffusive media when the typical experimental distances are very short as compared with those required for the validity of the diffusion approximation ($l \geq 8/\mu_s'$). In one case, some researchers have proposed the use of time-resolved reflectance at small or null source-detector separation, using fast single photon gating, to localize small inhomogeneities embedded in diffusive media [4–6]. In other type of experiments, it was studied the transition between the ballistic and the diffusive regimes, measuring the transmitted

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light within a relatively narrow solid angle [7,8]. Our model corroborates the importance of the solid angle of the measurement device in order to see the ballistic photons.

As it is known, light diffusion in turbid media can be described accordingly to the following hierarchy of equations: (i) the *RTE*, which is an equation concerning the specific intensity, $I(\mathbf{r},\mathbf{s},t)$, (ii) the Equation of the Telegraphist (*TE*), which is about the fluence $\Phi(\mathbf{r},t)=\int_\Omega I(\mathbf{r},\mathbf{s},t)\, d\Omega$ and finally (iii) disregarding some terms and using Fick's law to approximate the flux **J**, the *DE* is obtained. It is well established that the first two approaches are numerically cumbersome, and most of work is done using the *DE*. It is valid for media with $\mu_s' \gg \mu_a$ and when the distance d between the source and the observation point satisfies the condition $d \gtrsim 8/\mu_s'$, and thus the independence between μ_s and g is lost. Therefore, if short propagation distances need to be considered, a better alternative to the *DE* is mandatory. Because of this, for cases concerning with short propagation distances, several authors have proposed to return to the equations such as the RTE [9,10] or, more specifically, to the Telegraphist Equation [11], which, as stated above, are difficult to handle. In a previous work [12], some of the authors have proposed a generalized solution, $\Phi_{gen}(\mathbf{r},t)$, to the problem of light diffusion in homogeneous media, based on the statistical moments of multiple scattering. It follows the previous works of Zaccanti et al. [13], Lutomirski et al. [14] and Kolinko et al. [15] and describes the light propagation in terms of two different variances, $\sigma_x^2(\equiv\sigma_y^2)$ and σ_z^2. This solution has the particular characteristic of describing not only the diffusive regime, but it also takes into account both, ballistic and snake photons (or regimes). Additionally, it can be readily adapted to semi-infinite or slab media by addition of the corresponding image sources. The obtained results are in very good agreement with those of Monte Carlo (MC) simulations. This is so because the theoretical approach to obtain the σ_i's is based on the repetitive use of the matrix of Euler angles to calculate the position of the photon after eachscattering event, as it is precisely the case in MC simulations [16].

In this work we will show that with our function $\Phi_{gen}(r,t)$ it is possible to retrieve the particular cases (ballistic and snake), not taken into account by the Diffusion Approximation (*DA*), but avoiding the complex treatment required by the RTE and/or the Telegraphist equations. Two of the most interesting applications, which geometries are shown in Fig. 1, are: (i) transmission experiments through relatively "thin" slabs of thickness $s<8/\mu_s'$ and (ii) time-resolved diffuse reflectance experiments for which the optodes are at small or null source-detector separation [6]. In particular for this last situation, if the detector is far enough from the source, the *DA* is verified, but in general, a lot of light is lost in the process; because of this, measuring at shorter source—detector distances could be convenient. Moreover, if inside the bulk there is an inclusion next to the surface, the diffusive point of view may be inadequate. As it will be shown, the approach of this work also includes these situations.

This article is organized as follows: in Section 2, a brief summary of the principal equations is made; in Section 3, the limits for both, ballistic and snake photons are presented as well as the behavior of the diffusive photons when the absorption is high: $\mu_a \lessapprox \mu_s'$. Then, we present short sections applying our approach to diffuse reflectance at short distances, the transitions from the ballistic to the diffusive regime and the proposal of a new differential equation to first order in time. Finally, we compare analytical results with Monte Carlo simulations and we present our conclusions.

Note: Along this work and when it shows to be convenient, we will use a generalized dimensionless "time", $N=\mu_s vt$, representing the number of collisions, with $v=c/n$ ($c=3\times10^{11}$ cm/s), instead of the time t[s].

2. Theory

The solution to the *DE* in an infinite medium, in response to a point source $S(\mathbf{r},t)=\delta(\mathbf{r})\delta(t)$, has the general form of a Gaussian with variance $V\equiv\sigma^2$, where $2\sigma^2=4Dvt$:

$$\frac{\Phi_{diff}(x,y,z,t)}{m^{-2}s^{-1}} = \frac{v}{(4\pi Dvt)^{3/2}} \times \exp\left(-\frac{x^2+y^2+(z-\langle z_N\rangle)^2}{4Dvt}\right)\exp(-\mu_a vt). \tag{1}$$

On the other side, in a similar way as in the works of Refs. [13,14], Kolinko deduces some very convenient expressions for the first and second order moments [15]. From these it is possible to calculate both, the center of the diffusive photon "cloud", $m_1=\langle z_N\rangle$ and the variances, $V=m_2-m_1^2$, giving $\sigma_x^2=\langle x_N^2\rangle$ and $\sigma_z^2=\langle z_N^2\rangle-\langle z\rangle^2$, that we repeat for later convenience, when calculating ballistic and snake limit cases:

$$\langle z_N\rangle = \frac{1}{\mu_s}\frac{1-g^N}{1-g}, \quad \langle x_N\rangle = \langle y_N\rangle = 0, \tag{2}$$

$$\langle z_N^2\rangle = \frac{2}{3\mu_s^2(1-g)}\left[N-\frac{1-g^N}{1-g^2}(-2+g+g^2+2g^{N+1})\right], \tag{3}$$

and

$$\langle x_N^2\rangle = \langle y_N^2\rangle = \frac{2}{3\mu_s^2(1-g)}\left[N-\frac{1-g^N}{1-g^2}(1+g+g^2-g^{N+1})\right]. \tag{4}$$

For a slightly simplified description, that we will use below, we can introduce a measure of the variance given by the weighted average:

$$\langle r_N^2\rangle = \frac{\langle z_N^2\rangle+2\langle x_N^2\rangle}{3} = \frac{2}{3\mu_s^2(1-g)}\left[N-\frac{g(1-g^N)}{(1-g)}\right] \equiv \frac{2}{3\mu_s^2(1-g)}[N-\delta_g(N)], \tag{5}$$

where we have defined $\delta_g(N)=g(1-g^N)/(1-g)$. If the diffusive regime has been reached, it holds $N\gg1$, and in all cases $\sigma_x^2=\sigma_z^2=2Dvt$.

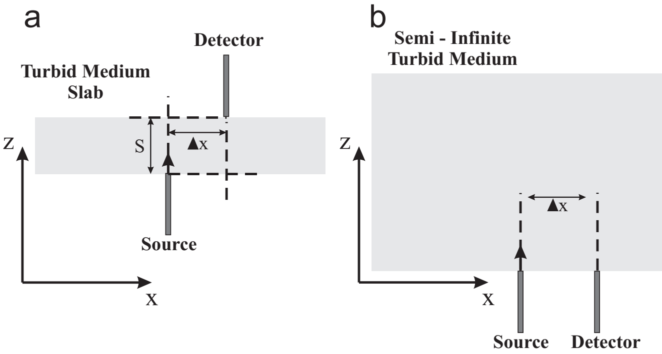

Fig. 1. The two geometrical arrangements discussed in this work, (a) slab and (b) semi-infinite reflectance.

In Ref. [12] we have presented the arguments to propose a generalized expression for the photon density:

$$\frac{\Phi_{gen}(x,y,z;t)}{m^{-2}s^{-1}} = \frac{v\,\exp\left[-\left(\frac{(x^2+y^2)}{2\sigma_x^2}+\frac{(z-\langle z_N\rangle)^2}{2\sigma_z^2}\right)\right]}{(2\pi\sigma_x^2)(2\pi\sigma_z^2)^{1/2}} \times H(t-r/v)\,\exp(-\mu_a v t), \quad (6)$$

which, reproduces a series of particular results obtained by other authors and gives, at the same time, an adequate generalized expression for Eq. (1). In Eq. (6) σ_x^2 and σ_z^2 are calculated considering Eqs. (2)–(4).

In particular, the Heaviside function $H(t-r/v)$ takes into account the behavior of Φ at short distances, while the independence between the values of g and μ_s (valid for small values of N) is explicit in the expressions for the σ_i's. Additionally, it is simple to verify one of the empirical laws about the distance for which the *DE* is valid. In fact, by comparison of the graphs corresponding to Eqs. (6) and (1) it can be concluded that for delta-like sources, $\delta(\mathbf{r})\delta(t)$, the condition $d \gtrsim 8/\mu_s'$ is required, as it will be shown later. It must be mentioned here, that our function is independent of the concrete expressions for the variances; though Kolinko et al. provide simpler expressions [15], numerical evaluations give indistinguishable results if compared with those obtained using the expressions given by Zaccanti et al. [13] and/or by Lutomirski et al. [14].

To better visualize the relation between Φ_{gen} and Φ_{diff} we assume $\sigma_x^2 \approx \sigma_z^2 = \langle r_N^2\rangle$, $r^2 = x^2+y^2+z^2$, and then it results, in terms of the explicit time variable t

$$\Phi_{gen}(\mathbf{r},t) = \left(\frac{\mu_s v t}{\mu_s v t-\delta_g(t)}\right)^{3/2} \exp\left[-\frac{0.75 g r^2}{v^2 t^2}\right]\Phi_{diff}(\mathbf{r},t)H(t-r/v). \quad (7)$$

3. Ballistic, snake and diffusive regimes

3.1. Ballistic

For the case of photons colliding with $\lim g = 1$ the following expressions result:

$$\lim_{g\to 1}\left[N-\frac{1-g^N}{1-g^2}(-2+g+g^2+2g^{N+1})\right] = \frac{3}{2}N(N+1)(1-g), \quad (8)$$

and

$$\lim_{g\to 1}\left[N-\frac{1-g^N}{1-g^2}(1+g+g^2-g^{N+1})\right] = \frac{1}{3}(N^3-N)(1-g)^2, \quad (9)$$

then, from Eqs. (3) and (4) it follows

$$\lim_{g\to 1}\langle z_N^2\rangle = \frac{N^2+N}{\mu_s^2},$$

$$\lim_{g\to 1}\langle x_N^2\rangle = \lim_{g\to 1}\langle y_N^2\rangle = \frac{2}{9}\frac{(N^3-N)(1-g)}{\mu_s^2}, \quad (10)$$

and, as a consequence

$$\sigma_z^2 = \langle z_N^2\rangle - \langle z_N\rangle^2 = \frac{N}{\mu_s^2}; \quad \sigma_x^2 = \sigma_y^2 = 0. \quad (11)$$

In Ref. [12] we have already argued that, from a theoretical point of view, the solution for Φ_{ball} should include, to describe null dispersion in the coordinates x, y, a factor of the general form $\delta(x)\delta(y)m^{-2}$, in such a way that

$$\Phi_{ball} = \frac{v\,\exp[-\mu_s^2 z^2/(2N)]\,\exp(-N\mu_a/\mu_s)}{(2\pi N/\mu_s^2)^{1/2}}H(N-\mu_s z)\delta(x)\delta(y). \quad (12)$$

From an experimental point of view, it should be necessary to build a collimating system with an acceptance angle $\delta\Omega$ as small as possible. However, it must be mentioned that it is nonsense to make this angle smaller than the Gaussian spread of the beam. We take into account that, in practice, it is impossible to separate truly ballistic photons from quasi-ballistic (snake) photons and a portion of the diffusive photons traversing an angle $\delta\Omega/4\pi$. Anticipating the result given by Eq. (16) (see below), we choose $\sigma_x^2 = \sigma_y^2 = N^3\langle\theta^2\rangle/(9\mu_s^2)$, and we replace in the above expression for Φ_{ball}, $\delta(x)\delta(y) = (2\pi\sigma_x^2)^{-1}$, giving:

$$\Phi_{ball}(N) = \frac{9}{4}\frac{v\mu_s^3}{\pi N^3\langle\theta^2\rangle} \times \frac{\exp[-\mu_s^2 z^2/(2N)]\,\exp(-N\mu_a/\mu_s)}{\sqrt{\pi N}}H(N-\mu_s z). \quad (13)$$

We see clearly, in the above equation, the remarks made by Yaroshevsky et al. [8]: the dependence of the solid angle $\langle\theta^2\rangle$ of the measurement device to see the ballistic photons.

Using now the dimensionless time scale previously defined, with $N=\mu_s vt$ and $z=vt$,[1] we obtain

$$\Phi_{ball}(\mathbf{r},t)=\frac{9\sqrt{2}}{4\pi\mu_s^{1/2}v^2t^3\langle\theta^2\rangle}\times\frac{\exp(-\frac{1}{2}\mu_s vt)\exp(-\mu_a vt)}{\sqrt{\pi vt}}H(t-z/v), \quad (14)$$

where it is readily seen that explicitly appears μ_s instead of μ_s', as it would be the case when the medium is diffusive.

In total agreement with the works of Refs. [7,8], when the parameters are of relevance in Biomedical Optics ($\mu_s\simeq 5\ \text{mm}^{-1}$, $g\simeq 0.8$, and therefore $\mu_s'\simeq 1\ \text{mm}^{-1}$), the ballistic photons represent an important fraction of the total only for distances $\lesssim 1$ mm (See Fig. 2).

3.2. Snake photons (small angle approximation)

The light intensity due to snake photons is measured in a transmittance setup using a collimator system as the one mentioned above. However, in Ref. [8] it is mentioned that in the range of interest for Biomedical Optics, the influence of these *quasi-ballistic* photons is of no significance; this is confirmed in our analysis. We are going to consider values of g in the range $g\to 1$, which leads to

$$\sigma_z^2=\frac{N}{\mu_s^2}, \quad (15)$$

as it was for the former case, while, remembering that $g=\langle\cos\theta\rangle$

$$\sigma_x^2=\sigma_y^2=\frac{2}{9}\frac{(N^3-N)(1-g)}{\mu_s^2}\approx\frac{2}{9}\frac{N^3(1-g)}{\mu_s^2}=\frac{2}{9}\frac{N^3(1-\langle\cos\theta\rangle)}{\mu_s^2}\approx\frac{2}{9}\frac{N^3(\langle\theta^2\rangle/2)}{\mu_s^2}=\frac{N^3\langle\theta^2\rangle}{9\mu_s^2}. \quad (16)$$

Let us write Φ_{snake} in terms of N taking into account Eqs. (6) and (13):

$$\Phi_{snake}(N)=\exp\left(-\frac{9}{2}\frac{\mu_s^2(x^2+y^2)}{N^3\langle\theta^2\rangle}\right)\times\Phi_{ball}(N),$$

whereas, if we now write it as a function of t, it is

$$\Phi_{snake}(\mathbf{r},t)=\exp\left[-\frac{9}{2}\frac{(x^2+y^2)}{\mu_s v^3t^3\langle\theta^2\rangle}\right]\Phi_{ball}(\mathbf{r},t). \quad (17)$$

We also see the behavior t^3 for the variance, as it is found in the works from [17,18] in their small angle approximation for light propagation under water and in the atmosphere.

As it is the case for ballistic photons, the relative amount of snake photons is important for propagation distances up to $z\sim 1$ mm if biological media are considered. This is shown in Fig. 2. It must, however, been taken into account, that in a real experiment it is always the sum $\Phi_{ball}+\Phi_{snake}$ what is measured (plus a portion of diffusive photons), since the existence of the angle $\delta\Omega$ introduced above precludes their separation.

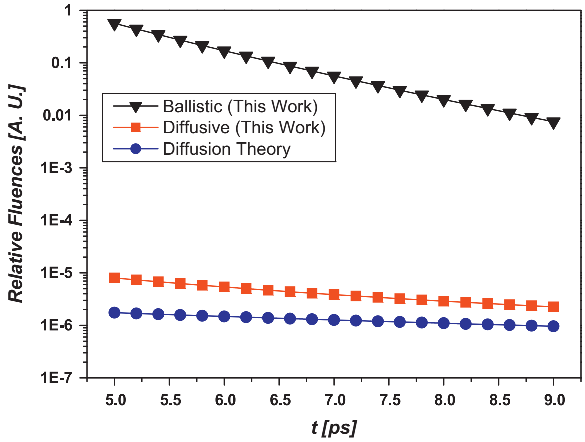

Fig. 2. Relative fluences for ballistic and diffusive regimes for on-axis measurements ($\Delta x=0$) when the distance between source and detector is set to $\Delta z=1$ mm. For this figure we took $\mu_s=5\ \text{mm}^{-1}$ and g=0.8, $\mu_a=0.01\ \text{mm}^{-1}$ and n=1.33. In the horizontal scale, 5 ps represents an average value of $N\approx 5.6$ collisions.

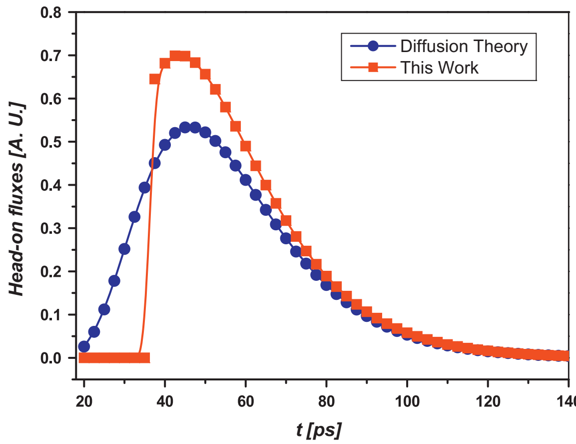

Fig. 3. Temporal distribution of the relative fluxes for on axis source and detector and for the case of high absorption. For this figure the axial distance between source and detector is z=8 mm and the parameters are: $\mu_s=5\ \text{mm}^{-1}$ and g=0.8, $\mu_a=0.3\ \text{mm}^{-1}$ and n=1.33.

3.3. Diffusive photons in the high absorption limit

If both conditions, $N\gg 1$ and $\mu_a\ll\mu_s'$ are satisfied, it was shown that $\sigma_x^2=\sigma_z^2=2vt/(3\mu_s')$ and then it follows that only diffusive photons must be considered; in symbols $\Phi(\mathbf{r},t)\to\Phi_{\text{diff}}(\mathbf{r},t)$. However, an additional situation remains for the case of experiments carried out in media for which the inequality $\mu_a\ll\mu_s'$ does not hold. In real cases, related for example to biomedical optics, this situation could be achieved by using highly absorbing dyes to stain some particular regions of the tissue containing inhomogeneities. As seen, the result obtained in Eq. (6) is independent of the value of μ_a relative to μ_s', and it is thus capable of considering this case. In Fig. 3 it is

[1] Note that for ballistic photons this expression is exact. See Ref. [3], page 98. For snake photons $z\simeq vt$, Ref. [14], page 7129.

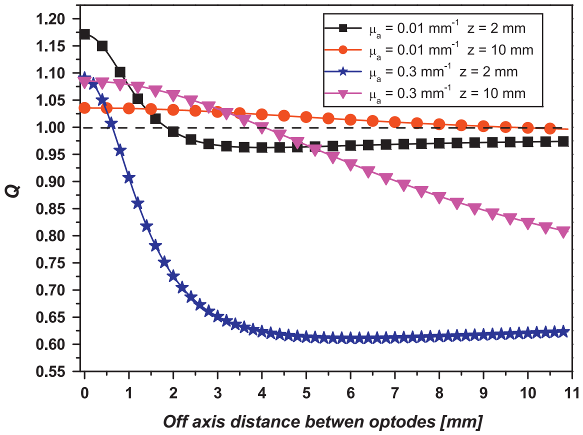

Fig. 4. Ratio, Q, of the integrated fluences predicted by this work and by the diffusion theory as a function of the off-axis distances, Δx, between optodes and for two cases of absorption combined with two on-axis distances, z.

shown the temporal distribution of the relative fluxes for on-axis, i.e. $\Delta x=0$, source and detector (see Fig. 1a) for the case of high absorption. The distance between source and detector is set to z=5 mm and the parameters are: $\mu_s=5\ \text{mm}^{-1}$ and g=0.8, $\mu_a=0.3\ \text{mm}^{-1}$ and n=1.33. In Fig. 4 we plot the ratio, Q, of the integrated fluences predicted by this work and the diffusion theory as a function of the off-axis distances, Δx, between optodes and for two cases of absorption combined with two on-axis distances, z.

4. Diffuse reflectance for homogeneous media; temporal and integrated behaviors

We briefly consider now the case of Diffuse Reflectance, schematized in Fig. 1b. Diffusive theory is again expected to fail for very short propagation times (or distances Δx). In order to verify this assumption we compare our results with diffusive values and Monte Carlo simulations in both cases: for time distributions (Fig. 5a and b) and integrated reflectances (Fig. 6). In Fig. 5a, the lateral offset of the optodes is $\Delta x=1$ mm, whereas in Fig. 5b, $\Delta x=5$ mm. In particular, in Fig. 6 our model shows how, for a medium with optical properties typical for biological tissues ($\mu_s'=1\ \text{mm}^{-1}$, $\mu_a=0.01\ \text{mm}^{-1}$) both, this proposal and the Diffusion Theory tend to coincide as the distance Δx from the light injection point increases. On the contrary, for very short distances, the discrepancy between the fluence predicted by the *DA* and the one predicted by our approach is noticeable.

5. The transition from the ballistic to the diffusive regime

The detection of the ballistic and diffusive light was experimentally presented by Kempe et al. [7] using confocal and heterodyne imaging systems in transillumination. Using different numerical apertures, they found that ballistic photons were important up to lengths of the order of $z\sim 1-1.2$ mm. In another experiment, carried out by Yaroshevsky et al. [8], the distance between the sample

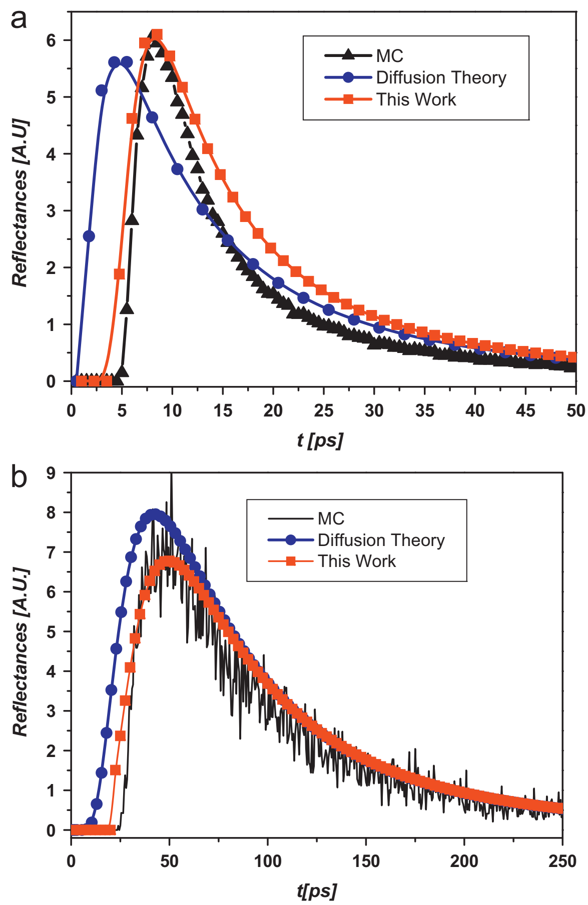

Fig. 5. (a) Time dependent reflectances predicted by this work, Monte Carlo simulation for 10^9 photons and the diffusion theory for $\Delta x=1$ mm, and for $\mu_s=5\ \text{mm}^{-1}$, g=0.8, $\mu_a=0.01\ \text{mm}^{-1}$ and n=1.33. Results for diffusion theory and this work are raw data, whereas MC values were normalized to long times, where all curves tend to coincide and (b) time dependent reflectances predicted by this work, Monte Carlo simulation for 10^9 photons and the diffusion theory for $\Delta x=5$ mm, and for $\mu_s=5\ \text{mm}^{-1}$, g=0.8, $\mu_a=0.01\ \text{mm}^{-1}$ and n=1.33. Results for diffusion theory and this work are raw data, whereas MC values were normalized to long times, where all curves tend to coincide.

and the detector was set to L=300 mm and the diameter of the aperture in front of the detector was d=1.5 mm; therefore, the solid angle results $\delta\Omega/4\pi \cong (d/4L)^2 \cong 1.5\times 10^{-6}$. Their measurements of the transmission through diffusive medium vs. sample width also indicate that ballistic photons were important up to $z\approx 1$ mm, as in the case of Kempe et al. In Fig. 7 we show the results of calculating that transmission using our approach for the same parameters used by Kempe et al., that is: $\mu_s=14\ \text{mm}^{-1}$, g=0.8 and, therefore, $\mu_s'=2.8\ \text{mm}^{-1}$. Explicitly, we calculate the sum of the ballistic photons plus the portion of the diffusive photons collected within the solid angle $\delta\Omega$, using the expression [8]

$$\rho=\rho_B+(\delta\Omega/4\pi)\rho_D$$

and considering our Eqs. (6), (14) and (17). The quantitative agreement between our Fig. 7 and the Fig. 2 of Yaroshevsky et al. is highly remarkable.

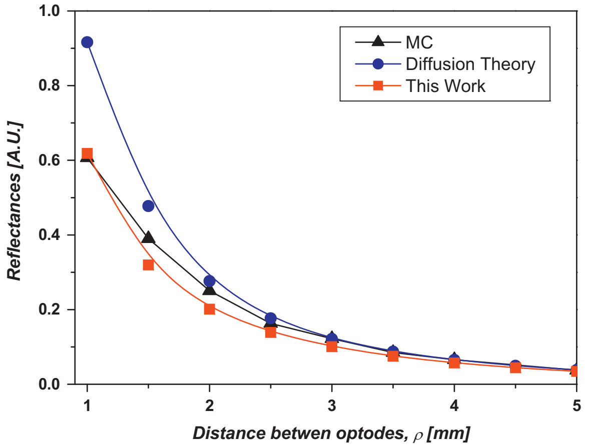

Fig. 6. Integrated reflectances in terms of the off axis distance, Δx, comparing the results of this work with the diffusion theory and MC simulations.

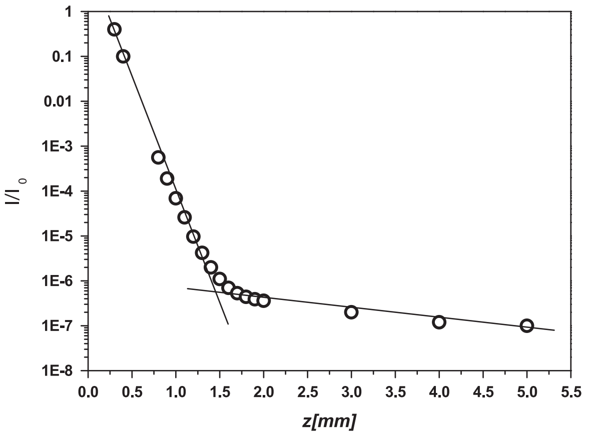

Fig. 7. Calculated transmission through diffusive media as a function of the turbid medium thickness, z, for $\mu_s = 14\ \text{mm}^{-1}$, $\mu_a = 0.004\ \text{mm}^{-1}$ and $g=0.8$. Compare this figure to Fig. 2 in Ref. [8].

6. Results and conclusions

In this work we have used our expression of Ref. [12] in several cases where experiments involving short propagation distances and or short times were reported.

We have tested our proposed expression for $\Phi_{gen}(\mathbf{r},t)$ by comparison to the *DA*, to experiments carried out by other authors and to MC simulations, this last normally taken as the gold standard in this type of phenomena. Diverse criteria could be used to contrast our results with other approaches. Concerning the ballistic and snake photons, the general conclusion is that, for optical parameters found in Biomedical Optics, they are of a certain importance for very small distances ($\lesssim 1$ mm) and negligible otherwise. Ballistic photons are, in general, more abundant that snake ones, in accordance with the works of Refs. [7,8] (see Fig. 2). On the other hand, as it is known, ballistic and snake photons are important for foggy media [19].

Additionally, when treating with media with high absorption, as can be the case of using highly absorbing dyes to stain some particular regions of tissues, the diffusive distribution of photons calculated with our method is very different of that obtained using the *DA* (see Fig. 3). The integrated fluxes, as compared with the *DA* show that, for off-axis measurements, the discrepancy increases with increasing z as can be seen from Fig. 4.

Concerning temporally distributed reflectances, the peaks of the distributions calculated with our approach coincide with those simulated by Monte Carlo methods, whereas *DA* starts from the nonsense value $t=0$ (see Fig. 5a and b). On the other hand, integrated values show that, for our parameters of interest, all values tend to coincide for $\Delta x \gtrsim 4$ mm, whereas for short distances, MC simulations are closer to our approach ($\Delta x \lesssim 1.5$ mm) and intermediate values occur between $1.5\ \text{mm} \le \Delta x \le 4\ \text{mm}$ (see Fig. 6).

Related with the experiments of Ref. [6] and considering the results presented in Fig. 5a and b, it is clear that if the *DA* is used as a model to retrieve optical properties, large errors or even nonsense values may occur.

In Fig. 7 we present the results of calculating the transmission of photons through scattering media, showing the transition from the ballistic to the diffusive regime for the optical parameters as those used by Yaroshevsky et al.; our figure is striking similar to Fig. 2 of Ref. [8].

It is worthwhile to notice, that in a recent paper by Liemert and Kienle [10], it is derivated an infinite space Green function of the time-dependent RTE. The approach of these authors is more rigorous but the numerical implementation is heavier than ours. Despite the differences between approaches, the time resolved fluences for different anisotropy factors and different optical properties show similar behavior.

Accordingly to above paragraphs, the main conclusion is that our expression (6) provides a convenient model to study the transition between ballistic and diffusive regime, by independently varying μ_s and g and also to model reflectance experiments at very short optode distances.

Acknowledgments

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