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Fuzzy Programming Model for Procurement Management under Delivery Shortage

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S Supporting Information

ABSTRACT: This paper proposes a FMIP (fuzzy mixed integer program) to model the procurement process of a manufacturing company which contemplates uncertainty in the delivery of raw materials. The focus of this article lies on the use of fuzzy sets to represent the percentages of failure in the delivery of the amount of materials requested and include it in a mathematical model as an evaluation measure in the performance of each supplier. The main objective of the proposed model is to select the most promising suppliers in order to optimize the quantitative and qualitative performance of the company, by maximizing the net present value (NPV) and providing a better customer service, respectively, in relation with the commitment of delivery of the company's suppliers. To solve the problem raised, the FMIP model proposed is transformed into an equivalent MILP (mixed integer linear program), and then, several scenarios are solved. An illustrative example is presented to show the utility of the model.

1. INTRODUCTION

Material procurement is a crucial activity for a manufacturing company where raw material availability and cost is vital to optimize production process cost, product quality, and customer satisfaction. For many companies, an important percentage of the manufacturing performance depends on the procurement process and materials supply. Zeydan et al.[1] remark that adequate and efficient supplier selection is a key aspect to achieve an effective procurement process, becoming an essential tool for reducing costs, waiting times, and lost sales by shortage of materials. The authors also highlight the importance of expanding the traditional criteria of supplier selection based only on the evaluation of prices offered, by introducing more complex measurements of supplier performance. A common situation in some manufacturing sectors is the lack of material supply that affects both production process and customer's demand satisfaction. The deficient supply may occur because of several factors, such as accidents at the provider's plants, strikes, materials whose demand exceeds supply, etc. These circumstances make that production amount, product delivery, and customer satisfaction uncertain on the basis of the amount of raw materials received for manufacturing. The procurement of goods is also related to the company supply chain activities, where information sharing and coordination with company's suppliers is a critical issue. Coordination between the different units can be achieved by appropriate commitments; one option is to sign contracts between the company and its suppliers or between the company and its customers.[2]

There are several articles in the open literature analyzing the procurement of goods and contract signature with suppliers to reduce uncertainty in the production and delivery process. Li and Zabinsky[3] developed two multiobjective supplier selection problems with business volume discounts to find a minimal set of suppliers to achieve quality and delivery goals. The objective function is the minimization of cost and the risk of having insufficient supply to meet demand. They consider uncertainties coming from supplier capacity and unknown demand. The uncertainties in demand and supply are captured by scenarios or with a probability distribution. Several scenarios are proposed to handle different levels of uncertainty. The ϵ-constraint method is used to generate Pareto-optimal solutions. The authors claim that the model provides a tool to explore the balance between the risk of not meeting the demand, the benefits of having a reduced number of suppliers, and the cost. A similar work can be found in Zhang and Chen.[4] In this paper the authors present a model dealing with the procurement problem under uncertain demand, for a single period and a single item. The decision variables of the model are the supplier selection and quantities to order to satisfy the uncertain demand in the next year with a minimal total cost. If the volume ordered is larger than the demand, an extra amount of stocking cost must be paid. On the other hand, if the demand exceeds the ordering quantity, the shortage must be satisfied by an emergent purchase at a higher price. In the model, the supplier offers a discount based on the quantity ordered. Once the buyer selects a particular supplier, a fixed selection cost is incurred. The objective is the minimization of the following costs: supplier selection, purchase, holding, and shortage. The proposed model is a mixed integer nonlinear problem (MINLP) and the authors propose an algorithm based on generalized Bender's decomposition to solve it.

Park et al.[5] studied the purchasing process by a disjunctive programming model, focusing on supplier selection and purchasing contracts. They showed that signing contracts is a business practice that helps to reduce uncertainty in the supply of raw materials. Naraharisetti et al.[6] and Laínez et al.[7] state that

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the signature of contracts with suppliers is a strategic decision that may be essential to reduce the risks in the supply process. Rodriguez and Vecchietti[8] present a formulation for selecting purchase contracts having uncertainties in material provision. Uncertainty is handled assuming a known fault distribution for each supplier. The decision variables of this problem are the supplier selection, the contract type to sign, the purchase quantities involved and the stock of materials to maintain along the time horizon.

Driven by the obvious influence of the unmet demand of materials onto the quality of service offered, this paper proposes a linear mathematical programming model for supplier selection and procurement of raw materials where uncertainty in delivery of goods is represented using fuzzy sets. The use of fuzzy sets can provide beneficial information to the company management, which cannot be efficiently represented with deterministic or probabilistic approaches. Bellman and Zadeh[9] pointed out the need of distinguishing between vagueness and randomness in order to develop programming models with more details in the formulation. Because of the nature of the problem studied in this article, the use of fuzzy sets is more appropriate than probabilistic distributions, since in most cases it is not possible to establish a distribution function of the failure in supply processes but rather to use the experience to describe the effectiveness or shortcoming of them. Zimmermann[10] introduces the use of fuzzy sets in modeling linear problems as an alternative to represent the vagueness in the knowledge of the real world. Fuzziness is employed by Petrovic et al.[11] to develop a supply chain model having uncertainty in the demand and in the suppliers reliability. Amid et al.[12] pointed out that a correct provider selection is becoming a new strategy to have a more competitive organization. They also assert that the use of fuzzy sets is a useful tool to handle uncertainty for cases where information is not precise. Lin[13] deals with the optimal selection of suppliers as a crucial activity for the optimal operation of the supply chain. This author develops several fuzzy linear program models having uncertainty in the formulation.

The main interest of this work lies in the incorporation of variable data in the supply of materials using fuzzy sets. As was shown in the previous paragraphs, several works can be found in the literature addressing uncertainty in the supply of raw materials, in most of them, the selection of suppliers is based on traditional criteria such as prices offered, delivery time, geographic location of the supplier, and transportation costs, among others. However, little attention is given to the impact caused by deficiencies in the supply of goods to a company. This situation is included in the approach of this article using fuzzy sets, allowing the supplier evaluation and selection and its consequences in the company operation. This work presents an alternative of modeling to the one developed by Rodriguez and Vecchietti[8] since the imprecise data are formulated using fuzzy sets instead of having a normal failure distribution. The use of vagueness in the supplier delivery gives a different problem perspective in which the information about provision failures is uncertain. With this approach, it is possible to make a scenario analysis assuming that each supplier fails on a delivery percentage of the total expected. In this study, the group of suppliers that optimizes the performance and effectiveness of the company can be determined. In addition, the model gives the type of contracts to sign with them in order to establish a more reliable and beneficial relationship.

This article is outlined as follows: section 2 describe the problem to solve, section 3 introduces the problem formulation in terms of objective function and constraints, section 4 shows a case study and its results, and finally conclusions are presented in section 5.

2. PROBLEM DESCRIPTION

The problem addressed in this paper focuses on the supplier and more promising purchase contracts selection for the commercial activity of a company, in a time horizon divided into several periods. The objective function of the model presented is maximization of the net present value (NPV), which enables the determination of a financial analysis since it is based on the use of the actual cash flows, that is, the amounts of money that really enter and disburse due the operations in the present period or due the transactions in the previous ones.

Materials with similar characteristics are grouped into families, each family can be provided by a single supplier in each time period. Other problem data are the maximum delivery capacity for each material. In addition, we consider the possibility of delivery failures, meaning there could exist a difference between the amount of materials ordered (delivery obligation assumed by the supplier) and the amount received (delivery actually carried out); this difference depends directly on the delivery commitment of each supplier. The amount received of each family of material is modeled by a fuzzy set. This set is defined as an interval called range of confidence or variability, in which the most optimistic value or upper limit is the total provision of the requested materials, while the most pessimistic case or lower limit is the delivery of a percentage of the total amount requested, which is determined from historical data or by experience and expectation of the decision makers.

The decision variables of the model are the material and supplier selected to provide it, the amount delivered, and type of contract to sign with the supplier. Regarding this last item, three types of contracts are included in the model, $C = \{c | 1 \le c \le 3\}$; each one influences the calculation of the final cost. In all contracts, a minimum purchase amount must be ordered that depends on the selected provider and the type of contract characterizing the business relationship. Moreover, in the first case ($c = 1$) the discount offered is the lowest because the number of units required for the purchase is minimal. In the second type ($c = 2$), the purchase obligation as well as the discount rate on the price of goods increases; this contract can be selected only in the case that the same material has been provided by the same supplier in a previous period. Finally, the third type of contract ($c = 3$) includes a greater requirement of the amount to purchase because it allows flexibility in the payments; the cancellation of the debt can be made θ periods after the delivery time, where $\theta \ge 0$.

3. PROBLEM FORMULATION

The formulated model has several discrete decisions. One of them is the supplier assignation to provide a specific material or family, and another is the type of contract chosen to be signed with the provider. To represent these decisions, Big-M constraints and binary variables are employed for both the suppliers and contract selection. Thus, the variables $y_1(j, f, t)$ and $y_2(j, f, k, t)$ take the value 1 when the supplier j is chosen to supply the family f or the product k of the family f in period t, respectively; while $y_3(j, c, k, t)$ is 1 if the contract c has been

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selected for the purchase of the material k to supplier j in period t.

On the other hand, the deferred payment of contract $c = 3$ shows the need of defining a larger number of time intervals, say $T = \{t | 1 \leq t \leq l, l = n + \theta, \theta \geq 0\}$ where n is the total number of periods of the planning horizon and θ the number of later periods in which the payment is made.

Assigning Suppliers. Equation 1 establishes that if the supplier j is selected to provide the family f ($y_1(j, f, t) = 1$) to which k belongs ($k \in FK(f, k)$), the number of units of material k ordered to the provider j ($q(j, k, t)$) is positive and limited by the maximum capability of each j ($Q\max(j, k)$ parameter); otherwise $q(j, k, t)$ is zero. Furthermore, in eq 2 the number of suppliers that can satisfy each family is limited.

$$q(j, k, t) \leq Q\max(j, k) \cdot y_1(j, f, t), \quad \forall\, j \epsilon J, \forall\, k \epsilon FK(f, k), \forall\, f \epsilon F, \forall\, t \leq n \tag{1}$$

$$\sum_j y_1(j, f, t) = 1, \quad \forall\, f \epsilon F, \forall\, t \leq n \tag{2}$$

Equation 3 is similar to eq 1 but for the case where supplier j is selected to provide the material k ($y_2(j, f, k, t) = 1$), instead of the family f.

$$q(j, k, t) \leq Q\max(j, k) \cdot y_2(j, f, k, t), \quad \forall\, j \epsilon J, \forall\, k \epsilon FK(f, k), \forall\, f \epsilon F, \forall\, t \leq n \tag{3}$$

Equation 4 determines that the summation of the amount provided of material k ($k \in FK(f, k)$), by supplier j at time t must be less than or equal to the summation of the maximum capability of that supplier for all material belonging to that family.

$$\sum_{k \epsilon FK(f,k)} q(j, k, t) \leq \sum_{k \epsilon FK(f,k)} Q\max(j, k) \cdot y_1(j, f, t), \quad \forall\, j \epsilon J, \forall\, f \epsilon F, \forall\, t \leq n \tag{4}$$

Equation 5 shows that the supplier j can be selected to provide all materials belonging to a family, when j has been selected to supply this family. Note that $|f|$ represents the cardinality of the set f.

$$\sum_{k \epsilon FK(f,k)} y_2(j, f, k, t) \leq \max_{f \epsilon F} |f| \cdot y_1(j, f, t), \quad \forall\, j \epsilon J, \forall\, f \epsilon F, \forall\, t \leq n \tag{5}$$

Equation 6 provides that if the material k does not belong to the family f, which will be provided by j, or the period is higher than the time horizon, then binary variable $y_2(j, f, k, t)$ is zero.

$$y_2(j, f, k, t) = 0, \quad \forall\, j \epsilon J, \forall\, f \epsilon F, \forall\, k \notin FK(f, k) \vee t \geq n + 1 \tag{6}$$

Stock and Sales. The stock of the family at the beginning of each period t ($s(f, t)$) is calculated as the sum of the number of units received $\widetilde{qr}(f, t - 1)$ and the existing stock in the previous period, minus sales of $t - 1$, $d(f, t-1)$, as shown in eq 7. Note that $\widetilde{qr}(f, t - 1)$ is a fuzzy set that contains the uncertainty in the supply of raw materials.

$$s(f, t) = s(f, t - 1) - d(f, t - 1) + \widetilde{qr}(f, t - 1), \quad \forall\, f \epsilon F, \forall\, 2 \leq t \leq n \tag{7}$$

The initial stock IS (f) and the maximum storage capacity SC, employed in eq 8 and eq 9 are model parameters.

$$s(f, t) = \mathrm{IS}(f), \quad \forall\, f \epsilon F, t = 1 \tag{8}$$

$$\sum_{f \epsilon F} s(f, t) \leq \mathrm{SC}, \quad \forall\, t \leq n \tag{9}$$

Equation 10 states that the quantities ordered for each family cannot exceed the period demand ($\mathrm{FD}(f, t)$), which is a model parameter. Similarly, eq 11 limits the sales of the family ($d(f, t)$) in each period.

$$\sum_{k \epsilon FK(f,k)} \sum_j q(j, k, t) + s(f, t) \leq \mathrm{FD}(f, t), \quad \forall\, f \epsilon F, \forall\, t \leq n \tag{10}$$

$$d(f, t) \leq \mathrm{FD}(f, t), \quad \forall\, f \epsilon F, \forall\, t \leq n \tag{11}$$

Assignment of Contracts. Equations 12 to 15 are related to assignation of a contract type to sign with supplier j when it is chosen to supply material k in period t. Equation 12 states that only one contractual form must be selected to provide material k of family f in period t.

$$\sum_{c \epsilon C} y_3(j, c, k, t) = y_2(j, f, k, t), \quad \forall\, j \epsilon J, \forall\, f \epsilon F, \forall\, k \epsilon \mathrm{FK}(f, k), \forall\, t \leq n \tag{12}$$

Equations 13 and 14 assert that the contract $c = 2$ can only be chosen in the case that the material k has been provided by the supplier j in a previous period.

$$y_3(j, c, k, t) = 0, \quad \forall\, j \epsilon J, \forall\, k \epsilon K, t = 1, c = 2 \tag{13}$$

$$y_3(j, c, k, t) \leq \sum_{c \epsilon C} y_3(j, c, k, t - 1), \quad \forall\, j \epsilon J, \forall\, k \epsilon K, \forall\, t \leq n, c = 2 \tag{14}$$

Equation 15 models the fact that quantities ordered of each item k must exceed a minimum number of units $Q\min(c, j)$, which is a model parameter, specified by each supplier in the chosen contract c.

$$q(j, k, t) \geq Q\min(c, j) \cdot y_3(j, c, k, t), \quad \forall\, j \epsilon J, \forall\, k \epsilon K, \forall\, c \epsilon C, \forall\, t \leq n \tag{15}$$

Cost Constraints. Equations 16 and 17 determine that the purchase price of the item k, ordered to the supplier j in period t under contract c, $w(j, c, k, t)$, is equal to the regular price offered by supplier j ($\mathrm{PC}(j, k)$) minus the discount $\delta(j, c)$ specified in contract c for that amount. Big-M expressions (second term of the right-hand side of the inequalities) have been introduced in order to make the constraint active if the supplier j and contract c is selected to deliver material k for period t ($y_3(j, c, k, t) = 1$). BM is a scalar large enough such that when $y_3(j, c, k, t) = 0$, the constraint becomes redundant.

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$$w(j, c, k, t) \leq \widetilde{qr}(f, t) \cdot \mathrm{PC}(j, k) \cdot (1 - \delta(j, c)) + \mathrm{BM} \cdot (1 - y_3(j, c, k, t)), \quad \forall\, j \epsilon J, \forall\, f \in F, \forall\, k \epsilon \mathrm{FK}(f, k), \forall\, c \epsilon C, \forall\, t \leq n \tag{16}$$

$$w(j, c, k, t) \geq \widetilde{qr}(f, t) \cdot \mathrm{PC}(j, k) \cdot (1 - \delta(j, c)) - \mathrm{BM} \cdot (1 - y_3(j, c, k, t)), \quad \forall\, j \epsilon J, \forall\, f \in F, \forall\, k \epsilon \mathrm{FK}(f, k), \forall\, c \epsilon C, \forall\, t \leq n \tag{17}$$

Equation 18 is a constraint that forces the purchase price of the material k to be null in the case that the supplier j has not been selected to provide it.

$$w(j, c, k, t) \leq \mathrm{BM} \cdot y_3(j, c, k, t), \quad \forall\, j \epsilon J, \forall\, k \epsilon K, \forall\, c \epsilon C, \forall\, t \leq n, \forall\, t \epsilon T \tag{18}$$

Equation 19 is related to contract $c = 3$ which has a deferred payment; it specifies that the money to be paid at time t_θ when $c = 3$, $m(j, c, k, t_\theta)$, is due to the purchase of material k in the period t; that is, $m(j, c, k, t_\theta)$ is equal to the purchase price $w(j, c, k, t)$ where $t_\theta = t + \theta$, $0 \leq \theta \leq l - n$ and $c = 3$. Otherwise, if the type of contract selected is $c = 1$ or $c = 2$ (eq 20) the payment amount matches the purchase price of the item k in the same period; that is, $t_\theta = t$.

$$m(j, c, k, t_\theta) = w(j, c, k, t), \quad \forall\, j \epsilon J, \forall\, k \epsilon K, t \leq n, t < t_\theta \leq l, c = 3 \tag{19}$$

$$m(j, c, k, t) = w(j, c, k, t), \quad \forall\, j \epsilon J, \forall\, k \epsilon K, t \leq n, \forall\, c \epsilon C, c \neq 3 \tag{20}$$

Objective Function. To know the company profitability the maximization of the NPV is proposed as the objective function. It takes into account the inflows and outflows of money and the value of future cash flows by using a discount rate (eq 21). In this equation, the numerator is the difference between the gross income ($\mathrm{IB}(t)$) and the total outflows ($\mathrm{TC}(t)$). The income is calculated in eq 22 as the quantity sold ($d(f, t)$) multiplied by its price ($\mathrm{AP}(f, t)$); while the outflows represented in eq 23 involve costs of purchasing materials (first term), loss of sales (second term), inventory cost (third term), and processing costs (fourth term).

$$\mathrm{NPV} = \sum_{t \epsilon T} \frac{IB(t) - \mathrm{TC}(t)}{(1 + \mathrm{RR})^t} \tag{21}$$

$$\mathrm{IB}(t) = \sum_{f \epsilon F} \mathrm{AP}(f, t) \cdot d(f, t), \quad \forall\, t \epsilon T \tag{22}$$

$$\mathrm{TC}(t) = \sum_{j \epsilon J} \sum_{k \epsilon K} \sum_{c \epsilon C} m(j, c, k, t) + \sum_{f \epsilon F} \mathrm{PLS}(f, t) \cdot (\mathrm{FD}(f, t) - d(f, t)) + \sum_{f \epsilon F} \mathrm{MS}(f, t) \cdot s(f, t) + \sum_{f \epsilon F} \mathrm{PrC}(f, t) \cdot \widetilde{qr}(f, t), \quad \forall\, t \epsilon T \tag{23}$$

In eq 21 the parameter RR is a rate of return corresponding to the capital cost. Moreover, in eq 23 the coefficients $\mathrm{PLS}(f, t)$, $\mathrm{MS}(f, t)$ and $\mathrm{PrC}(f, t)$ represent the penalty for lost sales (\$/unit), the cost of storage (\$/unit), and the processing cost (\$/unit), respectively.

Finally, the proposed FMIP model consists of the eqs 1 to 23.

Modeling Uncertainty. The uncertainty in the amount of materials received from each family $f \in F$ and in each period $t \leq n$ is represented by fuzzy sets whose membership function $\mu_{\widetilde{qr}}: \mathbb{R} \to [0,1]$ is shown in Figure 1 and defined in eq 24:

$$\mu_{\widetilde{qr}}(x) = \begin{cases} \dfrac{x - qr^{\max}(f, t)}{qr^{\max}(f, t) - qr^{\min}(f, t)} + 1, & \text{if } qr^{\min}(f, t) \leq x \leq qr^{\max}(f, t) \\ 0, & \text{if } x < qr^{\min}(f, t) \vee x > qr^{\max}(f, t) \end{cases} \tag{24}$$

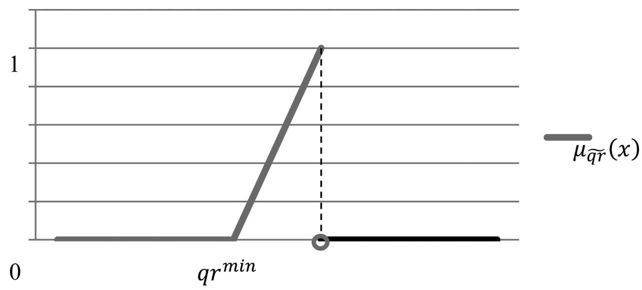

Figure 1. Membership function for some family f in period t.

The membership function determines the degree of membership for all $x \in \mathbb{R}$. For the set $[qr^{\min}(f, t), qr^{\max}(f, t)]$, the top end is the more optimistic value expected $qr^{\max}(f, t)$, which consists in the total receipt of the order placed to each supplier (eq 25), the lower end is the most pessimistic value $qr^{\min}(f, t)$, which is calculated as the sum of the amounts ordered minus a percentage of failure $r(j, k)$ in the delivery (eq 26).

$$qr^{\max}(f, t) = \sum_{j \epsilon J} \sum_{k \epsilon FK(f,k)} q(j, k, t), \quad \forall\, f \epsilon F, \forall\, t \leq n \tag{25}$$

$$qr^{\min}(f, t) = \sum_{j \epsilon J} \sum_{k \epsilon FK(f,k)} (1 - r(j, k)) \cdot q(j, k, t), \quad \forall\, f \epsilon F, \forall\, t \leq n \tag{26}$$

Equation 27 specifies the different values within the range of uncertainty that the delivery amount can take with a not-null membership value. For this purpose a convex combination between the extremes of the interval $[qr^{\min}(f, t), qr^{\max}(f, t)]$ is used.

$$\widetilde{qr}(f, t) = \alpha \cdot qr^{\max}(f, t) + (1 - \alpha) \cdot qr^{\min}(f, t), \quad \forall\, f \epsilon F, \forall\, t \leq n \tag{27}$$

where $0 \leq \alpha \leq 1$. Then replacing eq 27 into eq 7, eq 16, eq 17 and eq 23 leads to the equations 28, 29, 30 and 31.

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$$s(f, t) = s(f, t-1) - d(f, t-1) + \alpha \cdot qr^{\max}(f, t-1) + (1-\alpha) \cdot qr^{\min}(f, t-1), \quad \forall f \epsilon F, \forall t \leq n, 0 \leq \alpha \leq 1 \quad (28)$$

$$w(j, c, k, t) \leq [\alpha \cdot qr^{\max}(f, t) + (1-\alpha) \cdot qr^{\min}(f, t)] \cdot [\mathrm{PC}(j, k) \cdot (1 - \delta(j, c))] + \mathrm{BM} \cdot (1 - y_3(j, c, k, t)), \quad \forall j \epsilon J, \forall k \epsilon K, \forall c \epsilon C, \forall t \leq n \quad (29)$$

$$w(j, c, k, t) \geq [\alpha \cdot qr^{\max}(f, t) + (1-\alpha) \cdot qr^{\min}(f, t)] \cdot [\mathrm{PC}(j, k) \cdot (1 - \delta(j, c))] - \mathrm{BM} \cdot (1 - y_3(j, c, k, t)), \quad \forall j \epsilon J, \forall k \epsilon K, \forall c \epsilon C, \forall t \leq n \quad (30)$$

$$\mathrm{TC}(t) = \sum_{j \epsilon J} \sum_{k \epsilon K} \sum_{c \epsilon C} m(j, c, k, t) + \sum_{f \epsilon F} \mathrm{PLS}(f, t) \cdot (\mathrm{FD}(f, t) - d(f, t)) + \sum_{f \epsilon F} \mathrm{MS}(f, t) \cdot s(f, t) + \sum_{f \epsilon F} \mathrm{PrC}(f, t) \cdot [\alpha \cdot qr^{\max}(f, t) + (1-\alpha) \cdot qr^{\min}(f, t)], \quad \forall t \epsilon T \quad (31)$$

In this way, the FMIP is transformed to an equivalent linear programming model (MILP). The MILP consist of eqs 1−6, 8−15, 18−22, 25, 26, and 28−31.

To solve the problem, some scenarios are created by fixing the value of α, which represents a variation in the uncertainty interval, as shown in Figure 2.

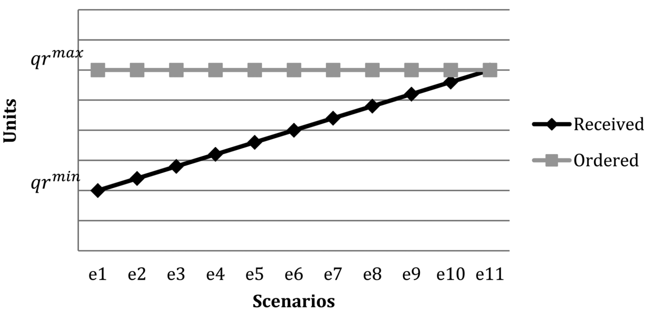

Figure 2. Order and delivery of materials.

4. CASE STUDY

The case study presented in this section is based on a company that manufactures boxes of corrugated cardboard, where the cost of raw material in the final product has the highest percentage of the total cost, followed by the processing cost. For confidentiality reasons, the data used are not the real ones but follow the characteristics of that manufacturing company.

The case study consists of 3 time periods ($n = 3$) and 32 materials ($K = \{k | 1 \leq k \leq 32\}$), grouped into 10 families ($F = \{f | 1 \leq f \leq 10\}$), as shown in Table 1.

We assumed the existence of 8 potential suppliers $J = \{j | 1 \leq j \leq 8\}$ with different percentages of failures to deliver the orders requested. Also we assumed, for each supplier, that the failure rate is constant with respect to the materials provided; for this reason $r(j, k) = r(j)$, $\forall k$. Data are detailed in Table 2.

Table 1. Material k Belonging to the Family f (FK(f,k))

families F	materials FK (f,k)
$f = 1$	$\{k = i | i = 1,2,3\}$
$f = 2$	$\{k = i | i = 4,5,6,7\}$
$f = 3$	$\{k = i | i = 8,9,10\}$
$f = 4$	$\{k = i | i = 11,12,13\}$
$f = 5$	$\{k = i | i = 14,15,16,17\}$
$f = 6$	$\{k = i | i = 18,19,20\}$
$f = 7$	$\{k = i | i = 21,22\}$
$f = 8$	$\{k = i | i = 23,24,25\}$
$f = 9$	$\{k = i | i = 26,27,28,29\}$
$f = 10$	$\{k = i | i = 30,31,32\}$

Table 2. Percentages of Failures of Each Supplier ($r(j)$)

suppliers							
$j = 1$	$j = 2$	$j = 3$	$j = 4$	$j = 5$	$j = 6$	$j = 7$	$j = 8$
20%	10%	30%	20%	20%	30%	20%	30%

As mentioned in the previous sections, we consider three types of contracts $C = \{c | 1 \leq c \leq 3\}$, where payment flexibility admitted by the contract $c = 3$ is $\theta = 2$. Therefore, the set of time periods, T, defined to illustrate the implementation of the model is $T = \{t | 0 \leq t \leq l, l = n + \theta = 5\}$.

The conditions for each contract and supplier, like the minimum number of units to be ordered (Qmin(c,j)) and the discount rates ($\delta(j, c)$) are presented in the Supporting Information. In this file are also added the tables that contain the values of the initial stock (IS(f)), the average sale prices (AP(f, t)), the unit processing cost (PrC(f, t)) and the penalty cost for lost sales (PLS(f, t)). Note that, we have PrC(f, t) = PrC(f) and PLS(f, t) = PLS(f) because it is assumed that these are constant over the time period. Furthermore, the parameter for lost sales is calculated as a percentage of the profit expected per family; that is, PLS(f, t) = $\beta(f)$·AP(f, t). Such percentages, together with the maximum delivery capacity (Qmax(j, k)) and the unit purchase price for each item ($PC(j, k)$), can be found in the Supporting Information.

Finally, the stock capacity (SC) is set to a value of 5000 units and the return rate (RR) is 0.15. Furthermore, the unit storage cost is assumed constant throughout the time periods considered and equal for all product families. Therefore, MS(f, t) = MS and takes the value \$0.25.

Solution of the Case Study. The model presented in the previous sections was posed in the General Algebraic Modeling System (GAMS) and executed on a PC with Intel Core i7 processor. With the aim of performing a scenario analysis, 11 scenarios (e1−e11) were defined varying the value of α between 0 and 1, with a step of 0.1. The solution time of the 11 scenarios was about 1 min, using the Gurobi solver.

The amount of material received is proportional to the increase of the α parameter given the different scenarios shown in Figure 3, the amount increases from scenario e1 to e11 when $\alpha \to 1$ (see eq 27), this affects directly the result of the activity. The NPV rises while receiving a higher number of materials to be processed, as seen in Figure 4.

The individual costs involved in business activities are detailed in Table 3. In this table and in Figure 5, it can be seen that the most influential costs on the performance of the company are the acquisition of materials, followed by the processing cost. Moreover, it should be noted that the greatest impact in the costs resulting by incorporating uncertainty in the

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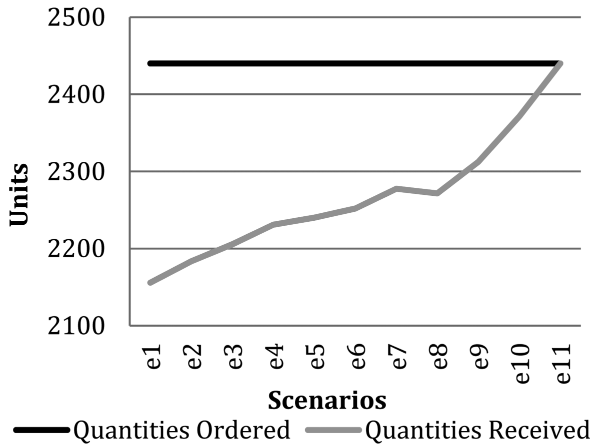

Figure 3. Total units ordered and received in each case.

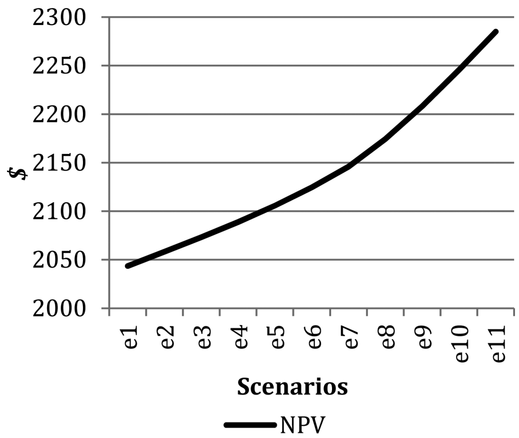

Figure 4. NPV obtained for the different scenarios.

Table 3. Costs Due to the Purchase, Inventory, and Processing of Material and the Cost for the Lost Sales

	purchase cost (PC)	storage cost (SC)	lost sales cost (LSC)	processing cost (PrC)
scenarios	$	$	$	$
e1	844.09	8.30	60.08	117.96
e2	854.62	8.30	54.43	119.53
e3	859.67	8.30	50.10	120.81
e4	867.11	8.30	45.04	122.18
e5	857.85	8.30	43.14	123.06
e6	848.69	8.30	40.39	124.08
e7	850.29	8.30	35.23	125.29
e8	818.17	8.30	36.12	124.86
e9	821.16	8.30	27.88	127.31
e10	838.97	8.30	15.91	130.52
e11	863.09	8.30	1.91	134.25

supply of materials is reflected in the growth of costs due to unsatisfied demand (greater than 5%). Since there is a close relationship between the supply of raw materials and the quality of service offered, the decision to find an optimal selection of suppliers is an effective means to ensure a sufficient supply of materials, which allows us to strengthen the performance and therefore the profits.

A selection at first glance would indicate that the supplier with the best performance is $j = 2$ for its low failure rate and moderate prices. Moreover, $j = 1$, $j = 4$, $j = 5$, and $j = 7$ are

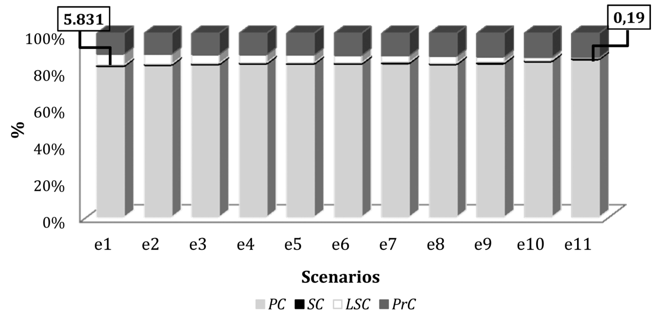

Figure 5. Percentage of the total cost due to the purchase of raw materials (PC), storage (SC), loss of sales (LSC), and processing (PrC) in each scenario defined.

postulated as promising suppliers. Even more, considering the prices and benefit rates offered for providers $j = 1$ and $j = 5$, they would be positioned as attractive creditors to consolidate the purchase of raw materials. This analysis omits a lot of information and requires much time to make a more complete and deeper study in order to cover different combinations, resulting from evaluating different aspects defining the supplier service quality. The model presented in this article allows the determination of the most promising suppliers selection through a simultaneous performance evaluation in response to different scenarios.

Figure 6 shows the quantity of material ordered and received from the selected supplier for each scenario. The performance

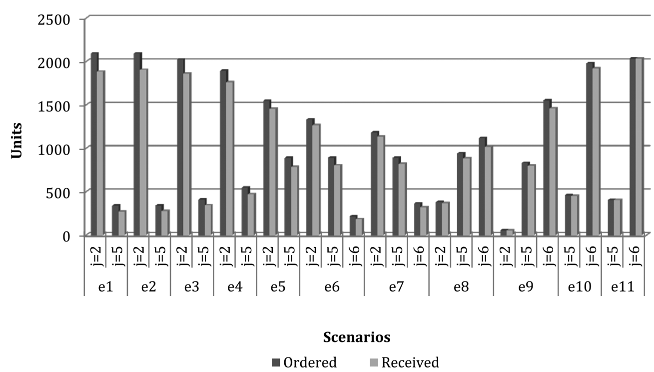

Figure 6. Amount of units ordered and received of each supplier in the different scenarios.

of each supplier can be seen in the delivery of orders and the selection trend in the different situations considered.

Note that, in the case for which no failures of the suppliers exist (e11), the selection is inclined to $j = 5$ and $j = 6$. However, when taking into account the erratic behavior in delivery of the providers, we see that the involvement of $j = 6$ decreases rapidly, requiring the addition of $j = 2$ into the list of potential suppliers. This behavior is explained because of the benefit of low prices offered by $j = 6$ which are not convenient because of the high percentage of failure in the delivery. Moreover, $j = 2$ acquires a strong role starting from scenario e1 until scenario e7, being responsible for delivery of up to 87% of the order in those with higher failures rates. Meanwhile, $j = 5$ has a participation in all scenarios, but never exceeds 40% of the total order even when it is postulated as one of the suppliers with the lowest percentage of failure. Therefore, we conclude that in order to guarantee supply into the largest possible number of

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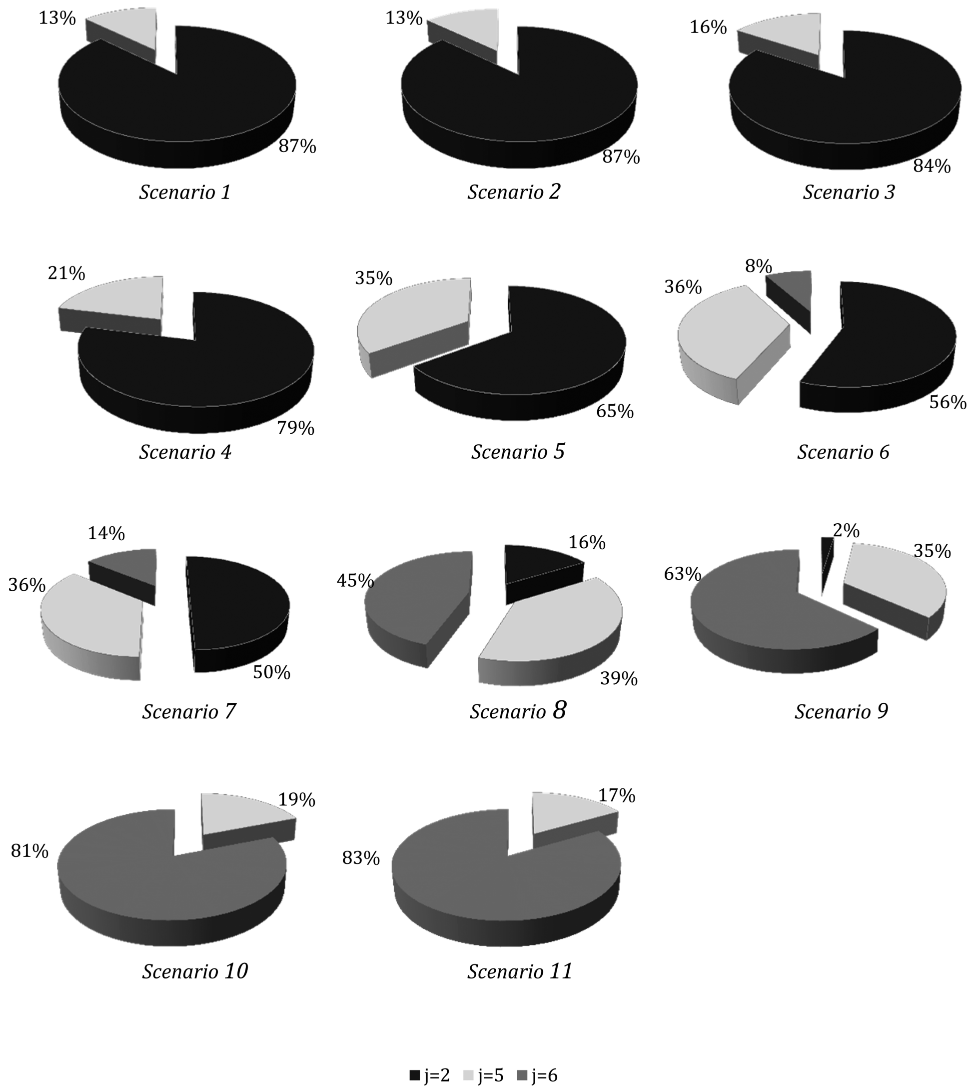

Figure 7. Participation rate of each supplier on the total quantities of materials delivered in each scenario.

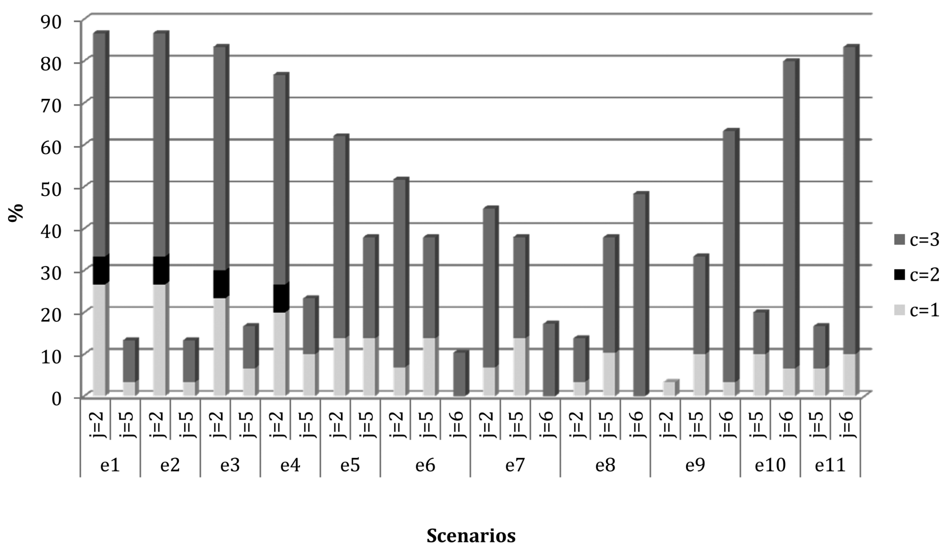

Figure 8. Percentage of purchasing contracts selected to formalize the transaction with each selected suppliers.

failure scenarios, the priority is to strengthen trade ties with j = 2, j = 5, and j = 6.

The percentages of the total amount received from each supplier, in the different scenarios, are presented in Figure 7.

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Finally, we have highlighted in the previous sections the benefits of establishing not only a promising selection of suppliers but also purchasing contracts with them to maximize the benefit obtained in each case. In Figure 8 the choice of purchasing contracts is summarized, appearing in percentage for each case. Here we note that the most used purchase contract is $c = 3$, followed by $c = 1$. It is important to note that the contracts correspond to those having lower discount compared with $c = 2$. However, only in scenarios with greater uncertainty, and exclusively with $j = 2$, the model selects the contractual business relationship that exceeds a period ($c = 2$). This responds to the fact that this supplier is the one with less uncertainty in delivery.

5. CONCLUSIONS

The shortage in the supply of goods is a highly relevant factor for the good functioning of production companies, strongly affecting the quality of service and the business profit. This paper presents an alternative modeling of the uncertainty in the acquisition of raw materials for manufacturing companies, with the aim of improving competitiveness through an efficient supplier selection. This is intended to optimize the benefits derived from the efficient operation and coordination between a company and its suppliers, in particular those related to the provision of materials. The target is to establish a reliable business relationship with suppliers avoiding discontinuities in the production, noneconomic transactions, and overstock. This impacts significantly the quantitative performance of the company, namely the optimization of purchasing, production, and storage costs, and also the qualitative functioning by improving the customer service provided in terms of commitment in the amount delivered.

The proposed model intends to go beyond the traditional criteria of supplier selection based on prices to evaluate qualitative aspects and incorporate in the assessment the efficiency percentage in goods delivery. This extension of evaluation criteria is a challenge as it introduces imprecise parameters in the model. In this case, one resorts to the use of fuzzy sets as a technique to represent the data variability. Therefore, the model formulated is a FMIP, in which the amounts received of raw materials are inaccurate data that varies within a range defined by the failure percentage to each provider on delivering the amount requested. The failure rate is generally built with information gathered from historical records of each supplier or by experience of a decision maker. To know the most promising business decisions to optimize the results and effectiveness of the company, this uncertainty range is evaluated at different points (scenarios). Thus, in each scenario defined, a MILP problem is solved, and the solution reached is indeed optimal considering a delivery fixed value. Therefore, the adopted modeling technique allows a description of the uncertainty in the operation of the delivery service, while providing a robust tool for generating scenarios, which subsequently is reused to carry out the decision making of the procurement process. The model also contemplates the signature of trade agreements that regulate the interaction between the company and selected suppliers to ensure the obligations compliance undertaken for them. These legal forms also introduce various guidelines for the choice of providers, limiting quantities ordered, prices, and benefits offered.

■ ASSOCIATED CONTENT

Supporting Information

Tables that contain the values of the initial stock (IS(f)), the average sale prices (AP(f, t)), the unit processing cost (PrC(f, t)) and the penalty cost for lost sales (PLS(f, t)). This material is available free of charge via the Internet at http://pubs.acs.org.

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Notes

The authors declare no competing financial interest.

■ NOMENCLATURE

Set of Indices

$T = \{t\}$ = time periods
$K = \{k\}$ = products
$F = \{f\}$ = product families
$J = \{j\}$ = suppliers

Scalars

α = constant that varies between 0 and 1
SC = maximum storage capacity
BM = constant large value for Big-M constraint
RR = rate of return

Parameters

$r(j, k)$ = percentage of failure in the delivery of material k by the supplier j
$\delta(j, c)$ = setting rate (discount or increase) of the purchase price offered by j under the contract c
IS(f) = initial stock of the family f
AP(f, t) = sale price of the family f at time t
MS(f, t) = cost of storage of the family f in the period t
FD(f, t) = demand of the family f at time t
PC(j, k) = price offered by the supplier j to purchase the product k
PrC(f, t) = processing cost of the family f at time t
Qmax(j, k) = maximum capacity of the supplier j to provide k
Qmin(c, j) = minimum number of units to be ordered from the supplier j under the contract c
$qr^{\min}(f, t)$ = minor number of units of the family f received at time t
$qr^{\max}(f, t)$ = maximum number of units of the family f received at time t
PLS(f, t) = penalty for lost sales of the family f at time t

Binary Variables

$y_1(j, f, t)$ = if j is the supplier selected to provide the items of the family f at time t
$y_2(j, f, k, t)$ = if the product k of the family f is provided by the supplier j in the period t
$y_3(j, c, k, t)$ = if the contract c is chosen for the buy of the product k to the supplier j at time t

Positive Variables ($\tilde{}$ Represents the Fuzzy Data)

$s(f, t)$ = units in stock of the family f at the beginning of the period t
$d(f, t)$ = number of units of the family f sold in the time period t
$q(j, k, t)$ = units of the material k ordered to the supplier j at time t
$\widetilde{qr}(f, t)$ = units of the family f received at period t

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$w(j, c, k, t)$ = purchase price of the product k bought from j under the contract c at time t
$m(j, c, k, t)$ = money paid to j by the purchase of the item k under contract c at time t
IB(t) = gross income at time t
TC(t) = total outflows at time t
NPV = net present value

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