Two recent works attempt to extend results for the conductance $G$ through a quantum dot described by the particle-hole symmetric (PHS) impurity Anderson model out of the PHS case using renormalized perturbation theory in $U$ (RPTU) up to $U^2$ [1,2]. Contrary to what is stated in the “Note added” of Ref. [2], previous results for the PHS case (where the occupation of the dot is $n = 1$ by symmetry) and general coupling to the left and right leads and chemical potentials $\mu_{L(R)} = \pm \alpha_{L(R)} \text{eV}$ [3] are recovered by the first approach [Eq. (30) of Ref. [1]]. The dependence on temperature $T$ is also correct. Ward identities are trivially satisfied because they were used [see the paragraph above Eq. (23)]. A limitation of this approach is that out of the PHS case, the coefficients of the expansion of $G$ in terms of $T$ and voltage $V$ contain derivatives of $n$ or the real part of the retarded self-energy $\Sigma'$, which for an interacting system seem to depend on high energy properties hard to capture in a Fermi liquid approach. Exceptions are the linear term in $T$ for $V = 0$ (which vanishes) and the linear term in $V$ for $T = 0$ (addressed below).

Instead, Ref. [2] provides explicit expressions for all coefficients up to second order in $T$ and $V$ for $\alpha_L \Gamma_R = \alpha_R \Gamma_L$ and $n \to 1$. Unfortunately, the authors have made mistakes in the evaluation of the lesser quantities $\Sigma^{-}$ and $G^{-}$ ($-\Sigma^{<}$ and $G^{<}$ in our notation) already in the PHS case. This implies that also the greater quantities can be eliminated from the expression of $G$, they play a crucial role precisely in this conservation [Eqs. (80)–(89) of Ref. [4]], and therefore the approach seems unreliable. One incorrect result is that $\Sigma^{<}_{\text{MBK}}(\omega) = 2i f_{\text{eff}}(\omega) \text{Im}[\Sigma'(\omega)]$, where $f_{\text{eff}}(\omega)$ is the average of the Fermi function at the two leads, weighted by the corresponding $\Gamma_n$. In addition, the authors claim to demonstrate that the term proportional to the noninteracting lesser Green function $g^{<}$ in the expression for $G^{<}$ [first term in Eq. (73) of Ref. [4]] vanishes [although it can be written as $2i f_{\text{eff}}(\omega) \Delta |G'|^2$ [Eqs. (7) and (8) of Ref. [5]] and uses this result to claim that $G^{<}_{\text{MBK}} = -|G'|^2 \Sigma^{<}_{\text{MBK}}$, CC would follow from the form of $G^{<}_{\text{MBK}}$, $\Sigma^{<}_{\text{MBK}}$, and known relations between the different Green functions.

Unfortunately, the demonstration is flawed because Eq. (76) of Ref. [4] is used, which misses the term $2i \Delta$. The correct form of this equation is $(G')^{-1} - (G')^{-1} = \Sigma^{<} - \Sigma' + 2i \Delta$. This comes trivially from the definition of $G'$ [third line below Eq. (13) of Ref. [2]] and its complex conjugate $G'^*$. In Fig. 1, we compare $\Sigma^{<}_{\text{MBK}}(\omega)$ at $T = 0$ with the correct one, obtained integrating numerically the RPTU expressions [5,6]. We also display the analytical result [1,6] up to total second order in $\omega$ and $V$ [Eq. (20) or Ref. [1]]. As it is known [6], the correct result is continuous. Instead, $\Sigma^{<}_{\text{MBK}}$ has jumps at $\mu_L$ and $\mu_R$, and strongly disagrees with the correct result except at energies far away from both $\mu_n$. It is difficult to say how these mistakes affect the reported expansion coefficients. The linear term in $V$ can be written in the form $c_{\text{Vel}} = 2(\alpha_L - \alpha_R) \cos(\pi n/2)$, which coincides with the result of Ref. [1], taking $\alpha_L \Gamma_R = \alpha_R \Gamma_L$ and $n \to 1$. In any case, CC is an essential requisite for any nonequilibrium theory. RPTU conserves the current in the PHS case and up to order $V^3$ in the general case for $T = 0$ [1].

This work was sponsored by PIP 1821 of CONICET and PICT R1776 of the ANPCyT, Argentina. A. A. A. is partially supported by CONICET.