Radiation Characteristics of Electromagnetic Eigenmodes at the Corrugated Interface of a Left-Handed Material

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(Received 29 May 2009; revised manuscript received 2 July 2009; published 25 August 2009)

We study the radiation characteristics of electromagnetic surface waves at a periodically corrugated interface between a conventional and a negatively refracting (or left-handed) material. In this case, and contrary to the surface plasmon polariton in a metallic grating, surface plasmon polaritons may radiate on both sides of the rough interface along which they propagate. We find novel radiation regimes which provide an indirect demonstration of other unusual phenomena characteristic of electromagnetic wave propagation in left-handed materials, such as negative refraction or backward wave propagation.

DOI: 10.1103/PhysRevLett.103.097401 PACS numbers: 78.68.+m, 42.25.Ja, 42.79.Dj, 73.20.Mf

A well-known [1–4] property of electromagnetically *impenetrable* conventional materials which exhibit a real negative electric permittivity—such as metals or plasmas below the plasma frequency—is their capacity to guide surface plasmon polaritons (SPPs) along their boundary. This guidance property is also shared by *transparent* materials with negative refractive index (NRI) [5], the so-called left-handed materials, which have both negative electric permittivity and magnetic permeability. NRI materials are characterized by an unusual negative refraction (the incident and refracted beams at the interface between the NRI and ordinary media lie on the same side of the normal to the interface) and possess other unusual properties, in particular, the possibility of creating a perfect lens with subwavelength resolution [6].

It has been shown [7–9] that the unusual properties of NRI media also manifest themselves in the propagation characteristics of SPPs supported by a flat boundary: while conventional media with negative permittivity only support p-polarized SPPs with a total energy flux in the same direction as that of wave propagation, transparent NRI media can support both p- and s-polarized SPPs with a total energy flux either parallel (forward SPPs) or opposed (backward SPPs) to the direction of wave propagation. The purpose of this Letter is to report unusual SPP radiation characteristics, not present in the metallic case, that appear when the flat boundary of a transparent NRI medium is perturbed with a periodic corrugation.

SPPs are electromagnetic waves that are trapped at the interface and whose electromagnetic fields decay exponentially into both media. Hence, the interface forms a peculiar resonator [10] with eigenmodes that are localized along the normal to the interface and can propagate freely along the interface. Since SPPs propagate with a wave vector greater than that corresponding to a free space photon of the same frequency, resonances cannot be excited by a propagating incident electromagnetic wave on a flat surface. To overcome this fact, special phase matching techniques, such as prism or grating coupling, have been extensively used [1–4,10].

Recent investigations [11] on the diffraction of a plane wave by a corrugated grating have shown that the excitation of SPPs on NRI media can give optical responses very different from those obtained for the excitation of SPPs on metallic gratings. Since the SPP resonant response of a corrugated surface can be thought of as a superposition of the fields generated by the surface without corrugation and the fields radiated—via corrugation—by the SPP, the results in [11] indicate that the SPP-photon coupling mechanisms at the corrugated boundary of transparent NRI media can be very different from those occurring at the corrugated boundary of a metal. Taking into account that while the metal is a region of forbidden electromagnetic propagation and the NRI media is not, it would be reasonable to think that the origin of these differences is rather self-evident: in the metallic grating the SPP radiates on one side only, while in the NRI grating the SPP radiates on both sides. Here we reveal that, in contrast to this expectation, this is not always the case. Contrary to what has been known so far for the SPP-photon coupling mechanism in metallic gratings, with SPPs always radiating into the "air side," we find that the periodically corrugated boundary between a conventional dielectric and a transparent NRI medium can exhibit novel SPP-photon coupling regimes, with SPPs radiating either into the conventional dielectric but not into the NRI medium, or into the NRI medium but not into the conventional dielectric, or into both the conventional dielectric and the NRI medium.

In order to find the SPPs propagating along a periodically corrugated boundary at the frequency ω, we solve the homogeneous boundary value problem for a diffraction grating [12]. We use a perturbative approach—valid in the limit of small groove depth to wavelength ratio h/λ ($\lambda = 2\pi c/\omega$)—which closely follows the formalism presented in [13] for magnetic materials with positive refractive index. We consider that the periodically corrugated boundary $y = f(x) = f(x + d)$ separates a conventional material [$y > f(x)$, electrical permittivity ε_1, magnetic permeability μ_1] and a NRI medium [$y < f(x)$, electrical permittivity ε_2, magnetic permeability μ_2], with d being the corrugation period. Both materials are homogeneous, isotropic, and linear. To focus on the key aspects of the propagation, we assume ideal lossless materials, although the dispersive nature of the media necessarily implies the existence of dissipation. Note that under this condition the propagation constant for a flat surface is real, but the propagation constant for a corrugated surface can have a nonzero imaginary part, in order to take radiation losses into account. The SPPs propagating along the boundary in the (x, y) plane are represented by Rayleigh expansions in terms of spatial harmonics

$$\phi(x, y) = \sum_{m=-\infty}^{+\infty} R_m e^{i(\alpha_m x + \beta_m^{(1)} y)}, \qquad y > f(x), \quad (1)$$

$$\phi(x, y) = \sum_{m=-\infty}^{+\infty} T_m e^{i(\alpha_m x - \beta_m^{(2)} y)}, \qquad y < f(x), \quad (2)$$

where $\phi(x, y)$ represents the z-directed component of the total electric field for the s-polarization case or the z-directed component of the total magnetic field for the p-polarization case, R_m and T_m are complex amplitudes, $\alpha_m = \frac{\omega}{c}\kappa(h/\lambda) + \frac{2\pi}{d}m$ is the x component of the wave vector for the mth spatial harmonic, $\beta_m^{(j)} = \sqrt{\frac{\omega^2}{c^2}\epsilon_j\mu_j - \alpha_m^2}$, $\kappa(h/\lambda)$ is the propagation constant of the eigenmodes in the corrugated structure, and h is the corrugation height. After imposing the boundary conditions, we arrive at a dispersion equation which can be numerically solved by using an iterative method initialized with the value $\kappa(0)$ corresponding to the flat interface. The details of the calculation will be given elsewhere; here we present results that illustrate novel radiation characteristics of SPPs which cannot be found in the metallic case. The conventional material is vacuum ($\varepsilon_1 = 1$, $\mu_1 = 1$) and the corrugation is sinusoidal [$y = \frac{h}{2}\sin(\frac{2\pi}{d}x)$] in all the examples.

First we consider constitutive parameters $\epsilon_2 = -0.2$ and $\mu_2 = -1.5$, for which the flat interface supports an s-polarized forward SPP [8,9] with dimensionless propagation constant $\kappa(0) = 1.2490$. When the flat surface is perturbed by a weak corrugation with $h = 0.008\lambda$ and $d = 2/3\lambda$, the propagation constant is $\kappa = 1.2492 + i5.359 \times 10^{-4}$. As in the metallic case, the periodic corrugation induces an increase in radiation losses, leading to a nonzero value of Imκ. Using the spatial harmonics of the perturbed eigenmodes, in Fig. 1(a) we have plotted the map and lines of the power flux (Poynting vector). Near the surface, the picture is very similar to that corresponding to a flat surface: flux lines are almost parallel to the grating, pointing in the $+x$ direction on the vacuum side and in the $-x$ direction on the NRI side. Moreover, on the vacuum side, the picture is also similar to that obtained for metallic gratings, with upper lines forming an angle $\theta_1 = 14.52°$ with the y axis and corresponding to radiation losses associated to the spatial harmonic with $m = -1$ in Eq. (1). However, contrary to the case of metallic gratings, and in virtue of the NRI medium transparency, flux lines are also radiated into the lower medium at an angle $\theta_2 = 27.25°$ with the $-y$ axis, corresponding to radiation losses associated to the spatial harmonic with $m = -1$ in Eq. (2). Note that the angles θ_1 and θ_2 satisfy $\sin\theta_1 = \sqrt{\epsilon_2\mu_2}\sin\theta_2$ and that flux lines radiated into the upper and lower media do not emerge on the same side of the normal to the mean corrugation plane, in agreement with the fact that the surface is negatively refracting. In Fig. 1(b) we show the normalized energy density map near the grating surface. We observe that the energy density in vacuum is greater than the energy density in the NRI medium and that the corrugation maintains the forward character of the SPP corresponding to the flat surface, as indicated by the fact that the energy density decreases in the propagation direction ($+x$).

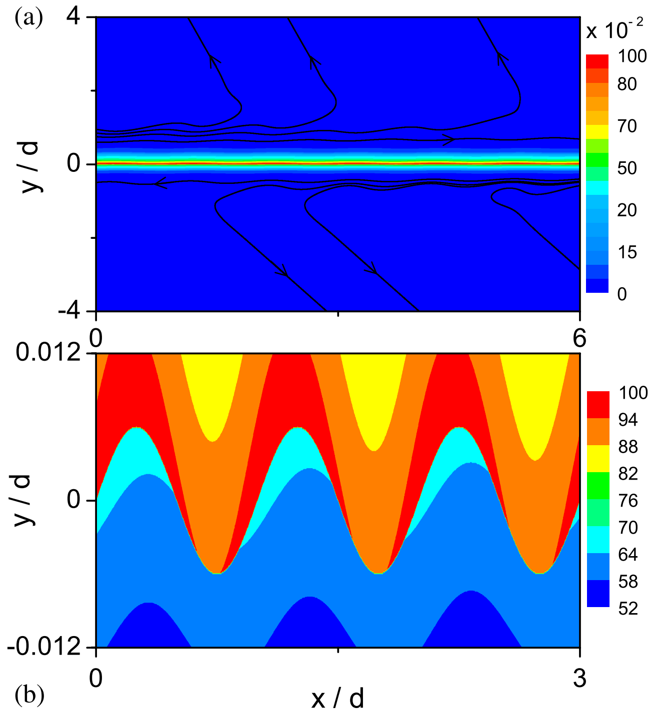

FIG. 1 (color online). (a) Map and lines of the power flux (Poynting vector) and (b) map of normalized Poynting vector modulus near the grating surface for a boundary given by $y = \frac{h}{2}\sin(\frac{2\pi}{d}x)$, with $h = 0.008\lambda$ and $d = 2/3\lambda$, separating a vacuum from a NRI medium with $\epsilon_2 = -0.2$ and $\mu_2 = -1.5$.

In order to emphasize the unusual features of the SPP-photon grating coupling mechanism at transparent interfaces, we next show that a very different SPP coupling regime can be induced in the same boundary considered in our previous example by changing only the period of the corrugation. For example, if we choose $d = 10/19\lambda$, the propagation constant is $\kappa = 1.2493 + i18.45 \times 10^{-4}$ except now both the far and near fields of the SPP are similar to those obtained for metallic gratings, with the SPP radiating into the upper conventional dielectric but not into the lower medium, as can be seen in Fig. 2. In spite of the NRI medium transparency, the SPP is not allowed to radiate into this medium for this value of d/λ. This is true because in this case all the spatial harmonics in Eq. (2) have a phase velocity less than that corresponding to a photon of the same frequency propagating in the NRI medium. A somewhat different, but equivalent, view of the situation may be noted by considering that the flux lines radiated into the vacuum side form an angle $\theta_1 = 40.59°$ with the y axis and that this angle is greater than the critical angle of total reflection corresponding to the flat surface. Note that the energy density carried by the SPP in both this and our previous example decreases in the propagation direction $(+x)$, a fact observed in Figs. 1(a) and 2 through a decrease in the density of radiation lines along this direction.

Another SPP-photon coupling regime which radically differs from what has been known so far for metallic gratings occurs in the regions of constitutive parameters supporting backward SPPs. In our final example we consider a corrugated boundary between vacuum and a NRI medium with $\epsilon_2 = -0.44$ and $\mu_2 = -4.54$. Without corrugation, this boundary supports p-polarized backward SPPs [8,9] with dimensionless propagation constant $\kappa(0) = 1.495\,70$. For a sinusoidal corrugation with $h = 0.005\,77\lambda$ and $d = 10/26\lambda$, the solution of the homogeneous problem gives $\kappa(h/\lambda) = 1.495\,76 - i9.5081 \times 10^{-4}$. As in the previous cases, the periodic corrugation induces an increase in radiation losses, leading to an increase in the value of $|\mathrm{Im}\kappa|$, except now $\mathrm{Im}\kappa < 0$. The situation is identical to that of backward SPPs at a flat surface when the NRI has intrinsic losses [14] and so $\mathrm{Im}\kappa < 0$ is a consequence of the fact that the energy carried by the SPP must attenuate in the direction of power flow $(-x)$, that is, opposite to the propagation direction $(+x)$. The map and lines of the power flux are shown in Fig. 3(a) and the normalized energy density map near the grating surface in Fig. 3(b). We observe that the near field with the corrugation is very similar to that corresponding to a flat surface, with flux lines almost parallel to the grating, pointing in the $+x$ direction on the vacuum side and in the $-x$ direction on the NRI side. We observe that in this case flux lines are only radiated into the lower media, at an angle $\theta_2 = 38.62°$ with the $-y$ axis. Contrary to the case of metallic gratings, the SPP-photon coupling can now occur when all the spatial harmonics in Eq. (1) propagate with a wave vector greater than that corresponding to a photon of the same frequency in vacuum. Note that the energy density in the NRI medium is greater than the energy density in vacuum, in strict contrast with the behavior of SPPs in metallic gratings, for which the energy density is always concentrated on the "air side." Also note that the energy density and the density of radiation lines decrease in the $-x$ direction, evidencing the fact that the energy carried by the SPP attenuates in the direction of lateral power flow $(-x)$.

In conclusion, by using a perturbative approach we have revealed the novel radiation characteristics of SPPs at the

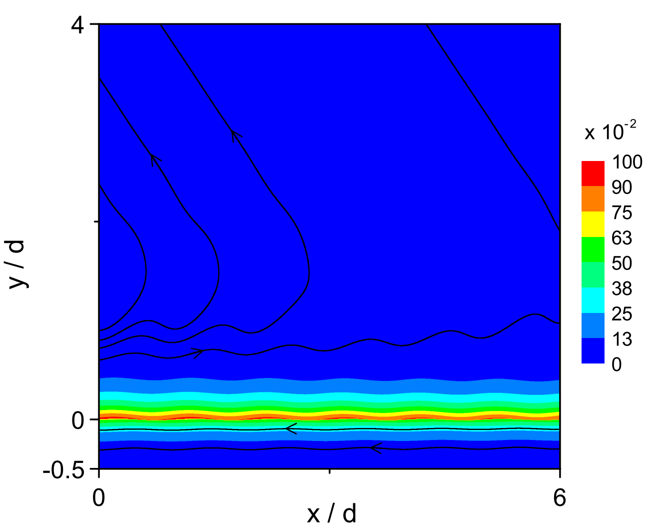

FIG. 2 (color online). Map and lines of the power flux (Poynting vector) for a boundary given by $y = \frac{h}{2}\sin(\frac{2\pi}{d}x)$, with $h = 0.008\lambda$ and $d = 10/19\lambda$, separating a vacuum from a NRI medium with $\epsilon_2 = -0.2$ and $\mu_2 = -1.5$.

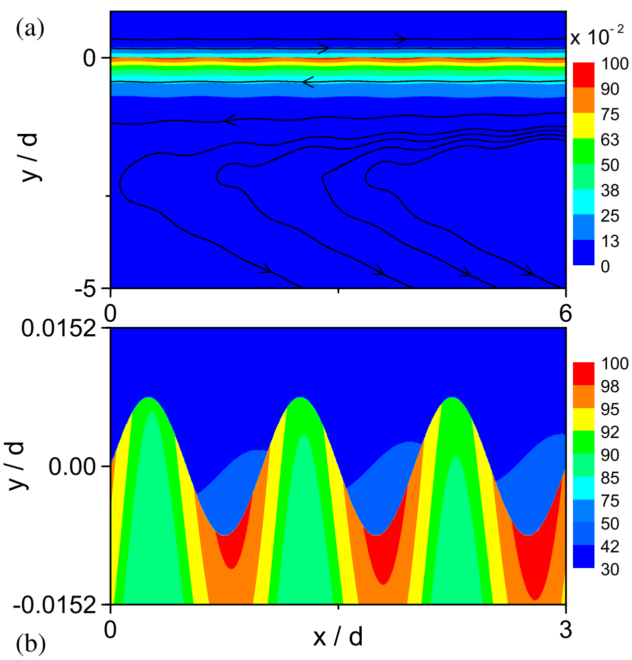

FIG. 3 (color online). (a) Map and lines of the power flux (Poynting vector) and (b) map of normalized Poynting vector modulus near the grating surface for a boundary given by $y = \frac{h}{2}\sin(\frac{2\pi}{d}x)$, with $h = 0.005\,77\lambda$ and $d = 10/26\lambda$, separating a vacuum from a NRI medium with $\epsilon_2 = -0.44$ and $\mu_2 = -4.54$.

corrugated boundary of transparent NRI media and we have shown that these characteristics are very different from those occurring at the corrugated boundary of metals. Taking into account that the SPP resonant response of a corrugated surface can be thought of as a superposition of the fields generated by the surface without corrugation and the fields radiated—via corrugation—by the SPP, the radiation characteristics reported here are essential to understand the physical origin of the novel optical responses shown in [11]. We have seen that the differences between the radiation characteristics of SPPs in the metallic and the NRI cases are not self-evident and that they cannot be completely ascribed to the opacity-transparency characteristics of the boundary. Whereas in metallic gratings SPPs always radiate into the "air side," we have shown that in NRI gratings SPPs can radiate into both the conventional dielectric and the transparent NRI medium, but they can also radiate either into the conventional dielectric but not into the NRI medium, or into the NRI medium but not into the conventional dielectric. Furthermore, we have demonstrated that the radiation behavior of SPPs in corrugated NRI media also provide an indirect but vivid demonstration of other unusual phenomena, such as negative refraction or backward wave propagation, that are a signature of electromagnetic wave propagation in left-handed materials.

We acknowledge financial support from Consejo Nacional de Investigaciones Científicas y Técnicas, Agencia Nacional de Promoción Científica y Tecnológica (BID 1728/OC-AR PICT-11-1785) and Universidad de Buenos Aires.

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